Debt, Innovation, and Growth[*]

Thomas Geelen[†]   Jakub Hajda[‡]   Erwan Morellec[§]

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Abstract

Recent empirical studies show that innovative firms heavily rely on debt financing. Debt overhang implies that debt hampers investment by incumbents. We show that a second effect of debt is that it stimulates entry of new firms and, therefore, innovation. Using a Schumpeterian growth model in which firms' dynamic R&D and financing choices are endogenously determined, we demonstrate that this second effect always dominates, so that debt fosters innovation and growth at the aggregate level. Our paper shows that debt financing has large effects on firm turnover and industry structure. It also predicts substantial intra-industry variation in leverage and innovation.

Keywords: debt, innovation, industry dynamics, growth.

JEL Classification: G32, O30.

[†]Copenhagen Business School and Danish Finance Institute, Solbjerg Plads 3, 2000 Frederiksberg, Denmark. E-mail: tag.fi@cbs.dk

[‡]HEC Lausanne and Swiss Finance Institute, Quartier UNIL Chamberonne, Extranef, 1015 Lausanne, Switzerland. E-mail: jakub.hajda@unil.ch

[§]Corresponding author. EPF Lausanne, Swiss Finance Institute, and CEPR, Quartier UNIL Chamberonne, Extranef, 1015 Lausanne, Switzerland. E-mail: erwan.morellec@epfl.ch

Over the last few decades, the US economy has become innovation driven. Public firms now spend twice as much on research and development than on capital expenditures, and fixed assets have fallen from 34% to less than 20% of total assets between 1975 and 2016 (see for example Corrado and Hulten (2010) or Doidge, Kahle, Karolyi, and Stulz (2018)). Creative destruction has been a driving force of this transition to a knowledge-based economy. A good example of this phenomenon is the swift rise to power of Apple and Samsung in the mobile phone industry, replacing Nokia as the market leader. This example of creative destruction was driven by the innovative success of Apple and Samsung, even though all three firms devoted large amounts of resources to R&D.[1]
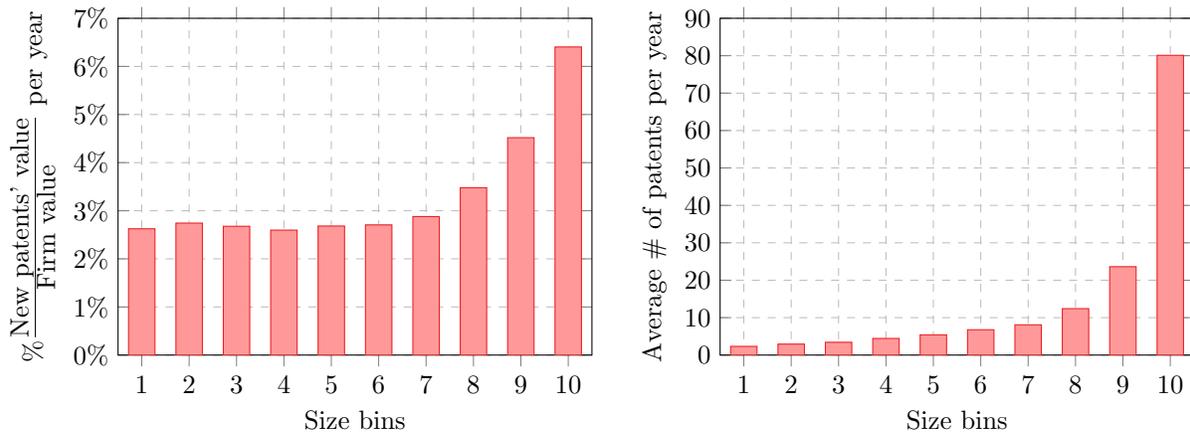

Figure 1: **Innovation quality and intensity.** The innovation data is based on Kogan, Papanikolaou, Seru, and Stoffman (2017) and the firm size data is from Compustat. The averages are conditional on issuing a new patent.

As shown in Figure 1, large firms play an important role for aggregate levels of innovation. Decades of empirical research have shown that debt is a key source of financing for these firms; see e.g. Graham, Leary, and Roberts (2015). In addition, even though debt is widely cast as an unlikely way to fund young and risky ventures, recent empirical studies show that small and young firms also heavily rely on debt financing. For example, Robb and Robinson

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[1]See https://thenextweb.com/plugged/2019/03/29/24-years-global-phone-sales-graph-visualization/ for an impressive visualization of this change in market share.

(2014) find that formal debt financing (business bank loans, credit lines, and owner-backed bank loans) provides about 40% of firms' initial startup capital. The reliance on formal credit channels holds true even for the smallest firms at the earliest stages of founding. Looking only at those firms that access equity sources, such as venture capital or angel financing, the average firm still has around 25% of its capital structure in the form of debt. A recent study by Hochberg, Serrano, and Ziedonis (2018) further documents a widespread use of loans to finance technology startups, even in early stages of development. Relatedly, Davis, Morse, and Wang (2018) find that venture debt is often a complement to equity financing, with over 40% of all financing rounds including some amount of debt.[2]

Given the change to an innovation-based economy and the heavy reliance of innovative firms on debt financing, a number of questions naturally arise. First, how does debt financing influence innovation at the firm level? Second, how do innovation and creative destruction in turn feed back into firms' financing policies? Third, what are the implications of debt financing in innovative firms for aggregate levels of innovation and growth?

This paper attempts to answer these questions by developing a Schumpeterian growth model in which firms' innovation and financing policies are endogenously determined. In this model, each incumbent has a portfolio of products and invests in R&D. Innovations by firms improve the quality of the goods produced. Firms therefore expand into new product lines when R&D is successful, which allows them to profit from their own innovations. But these profits are compromised when competitors develop better products. The force of creative destruction therefore affects firms R&D policies, as each product remains profitable until it is overtaken by another firm's innovation.

In line with Figure 1, firms decide on both R&D intensity, that is the rate at which they generate new innovations, and R&D quality, that is the expected number of products

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[2]While start-ups cannot typically obtain debt financing from traditional banks, major U.S. banking institutions, public firms, and private firms specialize in providing loans to the very start-ups that banks turn away. In related research, Mann (2018) shows that patents are pledged as collateral to raise significant debt financing, and that the pledgeability of patents contributes to the financing of innovation. Suh (2019) finds that firm ownership of patents increases firms' total debt-to-assets ratio by 18%. Xu (2019) shows that firms use trademarks as collateral for debt financing.

that each innovation creates. Shareholders' choice of R&D therefore determines firms' cash flow dynamics, which feeds back into their financing decisions. In the model, R&D and financing policies maximize shareholder wealth. As a result, financing choices reflect conflicts of interest between shareholders and debtholders, on top of the standard trade-off between the tax advantage of debt, the costs of issuing securities, and default costs.[3]

After solving for individual R&D, investment, and financing choices, we embed the single-firm model into an industry equilibrium in which the rate of creative destruction is endogenously determined. We derive a steady state equilibrium in which new product lines replace existing ones and entrants replace incumbents that default and exit the industry. Firms in this equilibrium exhibit a wide variation in leverage, size, and innovation rates. Furthermore, all industry-wide equilibrium variables are constant over time, although individual firms continue innovating, investing, and adjusting their capital structure.

In this equilibrium, capital structure and R&D influence each other through three main channels. First, R&D policy influences firms' risk profile and the aggregate level of creative destruction, which in turn affects their capital structure decisions. Second, levered firms are subject to debt overhang, which alters their incentives to innovate and the level of competition. Third, debt financing changes the surplus from entering the industry, which again influences the aggregate level of creative destruction and competition.

Starting with firm-level policies, we find that there is significant interaction between leverage and innovation. Notably, high levels of debt lead to less innovation by *incumbents* due to debt overhang, in that shareholders endogenously cut R&D and investment when their benefits mostly accrue to debtholders by rendering debt less risky. We find that the effect of debt on innovation by incumbents is sizeable, larger for firms with fewer products, and present both when firms follow a static debt policy or can dynamically adjust their debt levels. We also show that R&D policies and the industry rate of creative destruction play a key role in determining financing choices by affecting cash flow risk and the probability of

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[3]A simplified version of this model has been shown to capture the main stylized facts about corporate leverage (Strebulaev (2007), Morellec, Nikolov, and Schürhoff (2012), and Danis, Rettl, and Whited (2014)).

default. Our model predicts substantial intra-industry variation in leverage and innovation, in line with the evidence in MacKay and Phillips (2005) and Kogan et al. (2017). It also shows that debt financing has large effects on firm turnover and industry structure.

Underinvestment by incumbents suggests that debt may hamper innovation and growth. A novel and striking result of the paper is to demonstrate that debt financing does in fact lead to more investment and growth at the aggregate level. This is the outcome of two opposing forces. First, as discussed above, innovation by incumbents is negatively associated with debt. Second, debt increases the surplus from entering the industry (by providing a tax shield), leading to a higher entry rate and to greater innovation by entrants. We demonstrate that the latter effect always dominates in our endogenous growth model, so that debt financing fosters creative destruction and growth.

Importantly, the economic mechanism underlying this result suggests that measuring the effects of debt on innovation and growth using shock-based causal inference can potentially be problematic. Indeed, according to our model, any exogenous policy shock that would make debt more valuable—e.g., a decrease in debt issuance costs due to a change in regulation— would lead to higher leverage ratios and to lower innovation rates for incumbents. This could lead to a negative relation between innovation rates of firms and debt levels in the data. Yet, at the aggregate level, cheaper debt would foster entry and spur innovation and growth. Shock responses would therefore not recover theory-implied causal effects as they would not capture the influence of debt financing on entry.

Remarkably, our result that debt fosters creative destruction and growth does not hinge upon the specific trade-off we use to determine firms' financing decisions but on the fact that debt increases the surplus from entering the industry by reducing frictions. This result would also hold for example if debt reduced the cost of informational asymmetries between insiders and outsiders (Myers (1984)) or the cost of free cash flow and managerial flexibility (Jensen (1986)), as in both cases debt financing would increase the surplus of entrants (thereby stimulating entry), and reduce innovation by and facilitate exit of incumbents.[4]

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[4]Because informational asymmetries or managerial agency conflicts are more more difficult to calibrate

Importantly, while debt subsidizes entry, it also leads to underinvestment for incumbents and, therefore, does not work like government policies targeted at inducing innovation (e.g. patent policy) which would stimulate innovation by both entrants and incumbents.

Lastly, this result is consistent with the evidence in Kerr and Nanda (2009), who examine entrepreneurship and creative destruction following US banking deregulation. Their empirical analysis shows that US banking reforms, that made bank debt widely available and cheaper by increasing competition, brought growth in both entrepreneurship and business closures (see also Amore, Schneider, and Žaldokas (2013) and Chava et al. (2013)).

We also illustrate how the conclusions reached in the single-firm model, when ignoring equilibrium feedback effects, can be fundamentally altered, or even reversed, when the rate of creative destruction is endogenized in industry equilibrium.

Consider for example the effects of innovation costs on equilibrium quantities. Increasing innovation costs leads to a drop in the level of innovation and in the value of future innovations. This reduces the cost of debt (overhang) and leads firms to increase financial leverage. These effects are stronger in a single-firm model that does not incorporate the industry-wide response. Indeed, an effect that is absent when ignoring industry dynamics is that the drop in innovation quantity and the increase in leverage feed back into the equilibrium rate of creative destruction. As shown in the paper, the effect on innovation is generally first order, leading to a negative relation between innovation costs and the rate of creative destruction. This decrease in the rate of creative destruction—and the corresponding increase in the expected life of product lines—spurs innovation, partly offsetting the higher innovation costs. Lastly, in industry equilibrium these mechanisms translate to a lower turnover rate

than the corporate tax rate, in this paper we look at the whether tax advantage of debt provided by debt financing is sufficient to overturn the negative effects of debt financing on investment. We also do not use a mechanism based on exogenous financing constraints because these models generally require debt to be fully collateralized and lead to the counterfactual prediction that all debt is risk-free. In addition, the empirical studies of Robb and Robinson (2014), Hochberg et al. (2018), or Davis et al. (2018) show that debt financing is used by innovative firms that have access to and use both debt and equity financing. Lastly, consistent with the mechanism in the paper, Kerr and Nanda (2009) and Chava, Oettl, Subramanian, and Subramanian (2013) find that debt affects the entry incentives of young firms.

as innovation costs increase. By contrast, when industry feedback effects are ignored, the sharp increase in leverage due to increasing innovation costs increases the turnover rate.

Our article contributes to several strands of the literature. First, we contribute to the large literature studying the effects of debt on investment, following the seminal contribution of Myers (1977). This literature generally shows that debt financing hampers investment at the firm level, due to debt overhang.[5] A number of empirical studies have recently provided empirical support for this prediction; see for instance Chava and Roberts (2008), Giroud, Mueller, Stomper, and Westerkamp (2012), or Favara, Morellec, Schroth, and Valta (2017). This literature generally emphasizes the negative effects of debt on investment. By contrast, we show that in aggregate debt fosters investment and growth by stimulating entry. This result goes against standard economic intuition and highlights an important and overlooked effect of debt on investment.

Second, our paper relates to the literature on dynamic capital structure choice initiated by Fischer, Heinkel, and Zechner (1989) and Leland (1994). Models in this literature generally maintain the Modigliani and Miller (1958) assumption that investment and financing decisions are independent by assuming that the assets of the firm are exogenously given. This allows them to focus solely on the liability side of the balance sheet (see for example Fan and Sundaresan (2000), Duffie and Lando (2001), Hackbarth, Miao, and Morellec (2006), Gorbenko and Strebulaev (2010), Glover (2016), or DeMarzo and He (2018)). Our paper advances this literature by endogenizing not only firms' capital structure choices but also their investment policy. We find that debt financing has a negative effect on innovation and investment at the firm level, due to debt overhang. The distortions in investment due to debt financing are large and imply important feedback effects of (endogenous) investment on capital structure choice. A key contribution with respect to this literature is that we embed the individual firm choices into a Schumpeterian industry equilibrium in which the rate of

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[5]Mello and Parsons (1992) and Parrino and Weisbach (1999) examine the effects of debt financing on corporate investment in dynamic models of the firm. Our study departs from prior work by endogenizing capital structure choices and by embedding the single-firm model in an industry equilibrium.

creative destruction is endogenous. We show that while debt leads to underinvestment by incumbents, it increases creative destruction and growth by stimulating entry.

Third, our paper relates to the literature on debt in industry equilibrium. In a closely related paper, Miao (2005) builds a competitive equilibrium model in which firms face idiosyncratic technology shocks and can issue debt at the time of entry before observing their profitability. In this model, all firms have the same debt level. However, the model has heterogeneity in firm size because firms are allowed to invest after entry. An important assumption in Miao (2005) is that there are no costs of adjusting capital. As a result, there is no debt overhang in the sense of Myers (1977) because the absence of adjustment costs or frictions make investment independent of financing (Manso (2008)).[6] By contrast, firms have different (endogenous) debt levels in our model and can adjust capital structure after entry as profitability evolves. In addition, investment and financing decisions interact, leading to debt overhang and underinvestment by incumbents.[7] Other important contributions to this literature include Fries, Miller, and Perraudin (1997) and Zhdanov (2007), which respectively study static and dynamic capital structure choices in the Leahy (1993) model. In these models, incumbent firms are exposed to a single industry shock. They all have the same assets and the same debt level and there is no investment.

Lastly, our paper contributes to the literature on innovation in Schumpeterian growth models. Schumpeterian growth theory has been widely used in the literature on innovation and industry structure and evolution; see for example Klette and Kortum (2004), Lentz and Mortensen (2008), Aghion, Akcigit, and Howitt (2014), Akcigit and Kerr (2018), and Acemoglu, Akcigit, Alp, Bloom, and Kerr (2018). However, to the best of our knowledge, this literature has not studied the effects of debt financing on innovation, Schumpeterian competition, and industry dynamics. This is relatively surprising given that innovative firms

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[6]In Miao (2005), firms underinvest in that levered firms exit the industry at a higher rate than unlevered firms would. This feature is also present in our model.

[7]In related research, Malamud and Zucchi (2019) develop a model of cash holdings, innovation, and growth in the presence of Schumpeterian competition. Firms are all equity financed in their model. Maksimovic and Zechner (1991) develop a three-period model in which investment decisions reflect debt choices in industry equilibrium. They do not study entry and exit decisions, which are central to our analysis.

heavily rely on debt financing. Our paper fills this gap by extending the model proposed by Klette and Kortum (2004) to incorporate debt financing. In our model, firms hold debt and default, which influences their R&D policies and the industry level of creative destruction.[8]

This article is organized as follows. Section 1 describes individual firm choices and then embeds the single-firm model into an industry equilibrium. Section 2 analyzes the model implications. Section 3 closes the model in general equilibrium. Section 4 concludes.

1   Model

We present the model in steps, starting with the investment and financing decisions of an individual firm. We then embed the single-firm model into an industry equilibrium.

1.1   Assumptions

Throughout the paper, time is continuous and shareholders and creditors are risk neutral and discount cash flows at a constant rate $r > 0$. The economy consists of a unit mass of differentiated goods that are produced by incumbent firms.

A firm is defined by the portfolio of goods it produces. The discrete number of different products supplied by any given firm at time $t \geq 0$, denoted by P_t, is defined on the integers and is bounded from above by $\bar{p}$. As a result of competition between firms, each good is produced by a single firm and yields a profit flow of one. The profit flow of the firm evolves through time as a birth-death process that reflects product creation and destruction.

To increase the number of goods it produces, a firm invests in innovative effort, i.e. spends resources on R&D. A firm's R&D choice is two-dimensional. Each instant, it chooses both the frequency of arrival of new innovations $\lambda_t \in [0, \bar{\lambda}]$ and the quality of new innovations $\theta_t \in [0, 1]$. The arrival intensity λ_t determines the Poisson rate at which innovations arrive.

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[8]Another departure from Klette and Kortum (2004) is that we introduce heterogeneity in the quality of innovations, which is key to match the patterns in Figure 1.

Conditional on an innovation, the number of new product lines generated is given by

$$X_t = \min\left(Y_t, \bar{p} - P_{t-}\right) \ \text{ with } Y_t \sim Bin(n, \theta),$$

where $n < \bar{p}$ is an exogenous upper bound on the number of new product lines that can be developed following an innovation and $Bin(n, \theta)$ is the binomial distribution.[9] This specification implies that the expected number of new product lines is approximately $n\theta$. Therefore, a higher quality θ leads to a higher expected number of new product lines. Bounding the number of new product lines X_t from above by $\bar{p} - P_{t-}$ ensures that P_t never exceeds $\bar{p}$. These assumptions imply that the total number of product lines the firm has developed up to time t, denoted by I_t, evolves as

$$dI_t = X_t dN_t^I,$$

where dN_t^I is a Poisson process with intensity λ_t.

A firm's existing product lines can become obsolete because some other firm innovates on a good it is currently producing. In this case, the incumbent producer loses the good from its portfolio due to creative destruction. Since any firm is infinitesimal, we can ignore the possibility that it innovates on a good it is currently producing. Because of creative destruction, each product becomes obsolete at an exponentially distributed time with intensity f. We call f the rate of creative destruction, that each firm takes as given. Subsection 1.3 embeds the single-firm model into an industry equilibrium and endogenizes the rate f of creative destruction. The total number O_t of product lines lost by the firm up to time $t \geq 0$ because of creative destruction evolves as

$$dO_t = dN_t^O,$$

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[9]In the static debt model, this bound implies that the firm does not grow out of its debt. In the dynamic model, this bound is irrelevant as the firm optimally issues additional debt as the firm's prospects improve.

where dN_t^O is a Poisson process with intensity fP_{t-}. The total number product lines in a firm's portfolio P_t is therefore given by

$$P_t = I_t - O_t.$$

A firm with zero product lines exits the economy at time $\tau_0 \equiv \inf\{t > 0 : P_t = 0\}$.

A firm performing R&D with intensity and quality (λ_t, θ_t) incurs flow costs $q(P_t, \lambda_t, \theta_t)$. To make sure that shareholders are better off with more product lines, we impose that the R&D cost function does not increase too fast in the number of product lines in that

$$q(p + 1, \lambda, \theta) - q(p, \lambda, \theta) < 1. \tag{1}$$

An incumbent firm's operating profit is the profit that comes from the operation of the product lines minus the endogenous costs of performing R&D:

$$P_t - q(P_t, \lambda_t, \theta_t).$$

Profits are taxed at the constant rate $\pi > 0$. As a result, firms have an incentive to issue debt to reduce corporate taxes.[10] To stay in a simple time-homogeneous setting, we follow the literature (e.g. Leland (1994), Duffie and Lando (2001), and Manso (2008)) and consider debt contracts that are characterized by a perpetual flow of coupon payments c. The firm incurs a proportional cost ξ when issuing debt. Because of creative destruction, firms may default on their debt obligations. Default risk leads to endogenous distortions in the firm's R&D decisions when close to distress, reflecting debt overhang. An additional cost of debt is that default leads to exit and liquidation with a fraction $\alpha > 0$ of assets in place being lost

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[10]The main benefit of debt in our model is that it provides tax savings thereby raising the value of incumbents and, therefore, the surplus from entering the industry. We could similarly assume that firms obtain better financing terms with debt for example due to lower sensitivity to informational asymmetries. Because the later is more difficult to calibrate, in this paper we look at the whether tax advantage of debt provided by debt financing is sufficient to overturn the negative effects of debt financing on investment.

as a frictional cost. When choosing the amount of debt, shareholders balance the benefits of debt against its costs. Internet Appendix A allows firms to dynamically optimize their capital structure.

As in Klette and Kortum (2004), a mass of entrants invests in R&D to become producers upon a successful innovation. An entrant that generates an innovation becomes an incumbent. Similarly to an incumbent, the entrant chooses its R&D intensity λ_t and quality θ_t. The entrant has R&D cost function $q_e(\lambda, \theta)$. Because an entrant has no product lines before becoming an incumbent, it has (optimally) no debt and its optimal innovation strategy is time-homogenous: $\lambda_t = \lambda_e$ and $\theta_t = \theta_e$. As soon as an entrant has an innovative breakthrough and knows how many product lines this breakthrough generates, it has the possibility to issue debt. The cost of becoming an entrant is denoted by $H > 0$. We consider that entry costs are tax deductible, e.g. through depreciation. This assumption ensures that taxes have no bearing on innovation when firms are unlevered so that our results are directly comparable to those in prior work.

Figure 2 illustrates the life cycle of a firm. An entrepreneur first pays the entry cost and becomes an all-equity financed entrant, which incurs R&D expenses until it innovates for the first time. At τ_e, the entrant experiences a breakthrough resulting in new product lines, decides how much debt to issue, and becomes an incumbent. Once the firm becomes an incumbent, it generates profits from its portfolio of products and continues to make R&D decisions, which influences the intensity at which new innovations arrive as well as their quality. This process continues until the firm exits at time τ_D in case of default or at time τ_0 in case it loses all of its product lines to competitors.

1.2 Optimal Financing and Investment

We start by analyzing the case in which debt policy is static (as in e.g. Leland (1994), Duffie and Lando (2001), Manso (2008), or Antill and Grenadier (2019)) and solve the model recursively, starting with the value of levered equity for a given financing policy. Since each
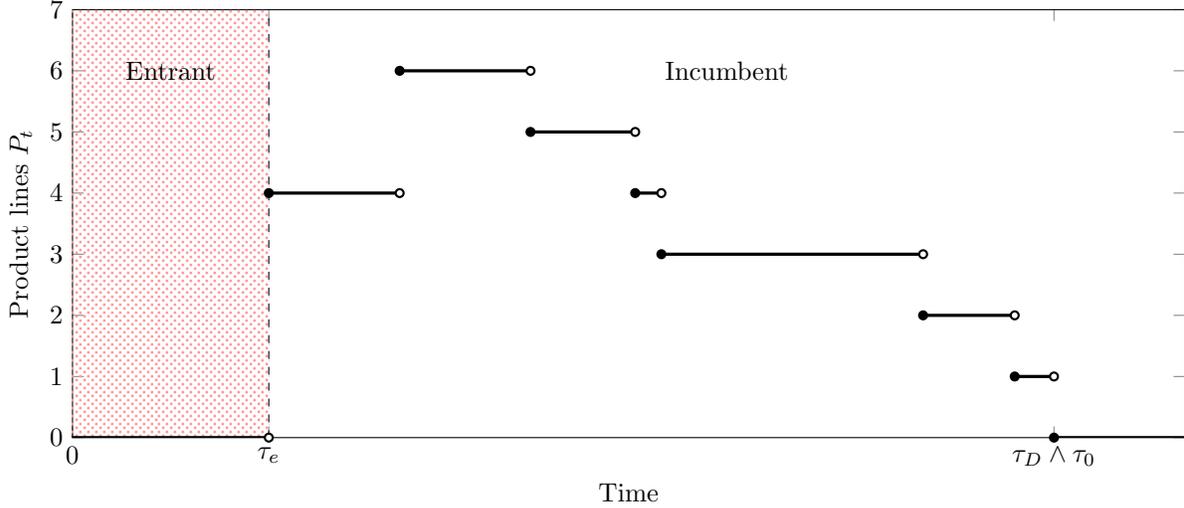
Figure 2: **Life-cycle of a firm.** The firm starts as an entrant and becomes an incumbent (with 4 product lines) at τ_e. The number of product lines then evolve over time and the firm defaults at $\tau_D \wedge \tau_0$ where $x \wedge y = \inf\{x, y\}$.

good generates the same flow of profits, we only need to keep track of the number of goods it produces and the coupon when describing the state of the firm.

After debt has been issued, shareholders maximize equity value by choosing the firm's default and R&D policy. As a result, equity value for a given coupon c satisfies

$$E(p, c) = \sup_{\{\lambda_t, \theta_t\}_{t \geq 0}, \tau_D} \mathbb{E}_p \left[\int_0^{\tau_D \wedge \tau_0} e^{-rt}(1 - \pi) \left(P_t - c - q(P_t, \lambda_t, \theta_t) \right) dt \right], \tag{2}$$

where $\mathbb{E}_p[\cdot] = \mathbb{E}_0[\cdot | P_0 = p]$, τ_0 is the first time the firm has zero product lines, and $x \wedge y = \inf\{x, y\}$. As shown by (2), shareholders receive the after-tax profits from P_t product lines minus the coupon payments c and R&D expenses $q(P_t, \lambda_t, \theta_t)$ until they decide to default or the firm exists with zero products. They select the R&D strategy $\{\lambda_t, \theta_t\}_{t \geq 0}$ and default time τ_D to maximize the equity value. Debt and creative destruction alter shareholders' incentives to invest in R&D or to continue operations.

From equation (2), it follows that equity value solves the Hamilton-Jacobi-Bellman equa-

tion

$$rE(p,c) = \sup \left\{ 0, (1-\pi)(p-c) + \underbrace{fp\left(E(p-1,c) - E(p,c)\right)}_{Creative\ destruction} \right. \tag{3}$$

$$\left. + \underbrace{\sup_{\lambda,\theta} \left\{ \lambda \left(\mathbb{E}^{\theta} \left[E(\min\{p+x,\bar{p}\},c) \right] - E(p,c) \right) - (1-\pi)q(p,\lambda,\theta) \right\}}_{R\&D} \right\},$$

where $\mathbb{E}^{\theta}$ takes the expectation over $x \sim Bin(n,\theta)$ and $E(0,c) = 0$. We then have the following result.

Theorem 1 (Equity Value). *A unique solution to the equity value* (2) *exists. Equity value is non-decreasing in p and therefore the optimal default strategy is a barrier strategy $\tau_D = \inf\{t > 0 | P_t \leq p_D\}$. If the optimal level of R&D is interior $((\lambda,\theta) \in (0,\bar{\lambda}) \times (0,1))$, it solves*

$$\mathbb{E}^{\theta} \left[E(\min\{p+x,\bar{p}\},c) \right] - E(p,c) = (1-\pi)\frac{\partial q(p,\lambda,\theta)}{\partial \lambda},$$

$$\lambda \frac{\partial \mathbb{E}^{\theta} \left[E(\min\{p+x,\bar{p}\},c) \right]}{\partial \theta} = (1-\pi)\frac{\partial q(p,\lambda,\theta)}{\partial \theta}.$$

The optimal R&D strategy, if interior, equates the marginal benefits and the marginal costs of R&D.[11] The marginal cost depends on the R&D cost function $q(p,\lambda,\theta)$. If an innovation arrives, the expected increase in equity value is

$$\underbrace{\mathbb{E}^{\theta} \left[E(\min\{p+x,\bar{p}\},c) \right]}_{Post\ innovation} - \underbrace{E(p,c)}_{Pre\ innovation},$$

which is the marginal gain from increasing the arrival rate of innovations λ. Similarly, higher R&D quality θ increases the expected number of new product lines when an innovation

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[11]If there exists a λ^* such that for any $\lambda > \lambda^*$ and $\theta \in [0,1]$

$$\frac{\partial q(p,\lambda,\theta)}{\partial \lambda} \geq \frac{\theta n}{r}$$

then in equilibrium $\lambda < \lambda^*$ and imposing the bound on λ becomes void.

arrives. The marginal increase in equity value from higher R&D quality θ is

$$\lambda \frac{\partial \mathbb{E}^\theta \left[E(\min\{p + x, \bar{p}\}, c) \right]}{\partial \theta}.$$

The presence of debt in the firm's capital structure implies that shareholders do not fully capture the benefits of investment in that cash flows to shareholders are truncated at $\tau_D \wedge \tau_0$. This in turn implies that the level of R&D that maximizes shareholder value is lower in a levered firm, notably when close to distress (see Section 2).

We also perform a comparative statics analysis with respect to the model's parameters:

Proposition 1 (Equity Value: Comparative statics). *If $E(p, c) > 0$, equity value is decreasing in the tax rate π, the coupon c, the rate of creative destruction f, and the cost $q(p, \lambda, \theta)$ of performing R&D.*

An increase in these parameters decreases the cash flows to shareholders or the expected lifetime of the firm and, therefore, reduces equity value.

Given the rate of creative destruction f and shareholders' optimal R&D $\{\lambda_t, \theta_t\}_{t \geq 0}$ and default τ_D policies, the debt value $D(p, c)$ is the discounted value of the coupon payments until the time of default plus the liquidation value of assets in default. That is, we have

$$D(p, c) = \mathbb{E}_p \left[\int_0^{\tau_D \wedge \tau_0} e^{-rt} c \, dt + e^{-r(\tau_D \wedge \tau_0)} (1 - \alpha) \frac{(1 - \pi) P_{\tau_D \wedge \tau_0}}{r + f} \right]. \tag{4}$$

Finally, we can also determine the value of an entrant given the rate of creative destruction f. Let τ_e be the time at which the entrant has a breakthrough and can develop its first product lines, which happens with intensity λ_e. The entrant's shareholders choose the R&D intensity and quality that maximize their equity value, which consists of the proceeds once

14

there is a breakthrough minus the tax-deductible R&D costs. That is, we have

$$E^e(f) = \sup_{\lambda_e,\theta_e} \mathbb{E}_0 \left[e^{-r\tau_e} V(f,\theta_e) - \int_0^{\tau_e} e^{-rt}(1-\pi)q_e(\lambda_e,\theta_e)dt \right]$$
$$= \sup_{\lambda_e,\theta_e} \left(\frac{\lambda_e V(f,\theta_e) - (1-\pi)q_e(\lambda_e,\theta_e)}{r+\lambda_e} \right), \tag{5}$$

where

$$V(f,\theta_e) = \mathbb{E}^{\theta_e} \left[\sup_{c \geq 0} \{E(p_0,c) + (1-\xi)D(p_0,c)\} \right], \tag{6}$$

with $p_0 = \min(x,\bar{p})$ and $x \sim Bin(n,\theta_e)$. As shown by equation (6), shareholders select the coupon that maximizes the value of their claim once they know how many product lines their innovative breakthrough generates. Because the debt choice is affected by the number of product lines, the heterogeneity in entrants' R&D outcomes naturally leads to cross-sectional variation in the amount of debt issued, even in the static debt model.

Lastly, an entrepreneur pays the cost $H(1-\pi) > 0$ upon becoming an entrant. The free entry condition then implies that

$$E^e(f) \leq H(1-\pi), \tag{7}$$

which becomes an equality when there is a positive mass of entrants. In equilibrium, competition implies that the value of becoming an entrant can never exceed the cost of entry.

1.3 Industry Equilibrium

This section incorporates the individual-firm decisions into a Schumpeterian industry equilibrium. We look for a Markovian steady state industry equilibrium in which the number of firms and product lines is constant over time. In this industry equilibrium, both incumbents and entrants maximize their equity value. That is, incumbents optimally choose their R&D and default decisions and entrants optimally choose their R&D and capital structure deci-

sions. Given that we look for a Markovian steady state equilibrium, incumbents' optimal policies are a function of the number of product lines they own and the coupon payment on their debt, which is a function of the number of product lines at entry p_0. Entrants' optimal policies are time-homogenous. Finally, the free entry condition ensures that new entrants continue to enter as long as entry is profitable.

Definition 1 (Industry Equilibrium). *The parameters and policies*

$$\Psi^* = \{f^*, c^*(p_0), \lambda^*(p|p_0), \theta^*(p|p_0), p_D^*(p_0), \lambda_e^*, \theta_e^*\}$$

are an industry equilibrium if:

1. **Incumbents:** *Given the rate of creative destruction f^* and coupon payment $c^*(p_0)$, incumbents level of R&D $(\lambda^*(p|p_0), \theta^*(p|p_0))$ and default decision $p_D^*(p_0)$ maximize shareholder value.*

2. **Entrants:** *Given the rate of creative destruction f^*, entrants level of R&D $(\lambda_e^*, \theta_e^*)$ and capital structure upon becoming an incumbent $c^*(p_0)$ maximize shareholder value.*

3. **Entry:** *The free entry condition holds:*

$$E^e(f^*) \leq H(1 - \pi),$$

and the inequality binds when there is creative destruction $f^* > 0$.

Figure 3 shows an industry equilibrium in which new product lines replace existing ones and entrants replace incumbents that default and exit the industry. The size of the circles indicates the mass of firms of each type. In a steady state equilibrium, the size of these circles is constant over time. Incumbents can move up due to innovations, which generate new product lines, and move down due to creative destruction. Because an innovation can generate more than one product line and the number of product lines generated is random,
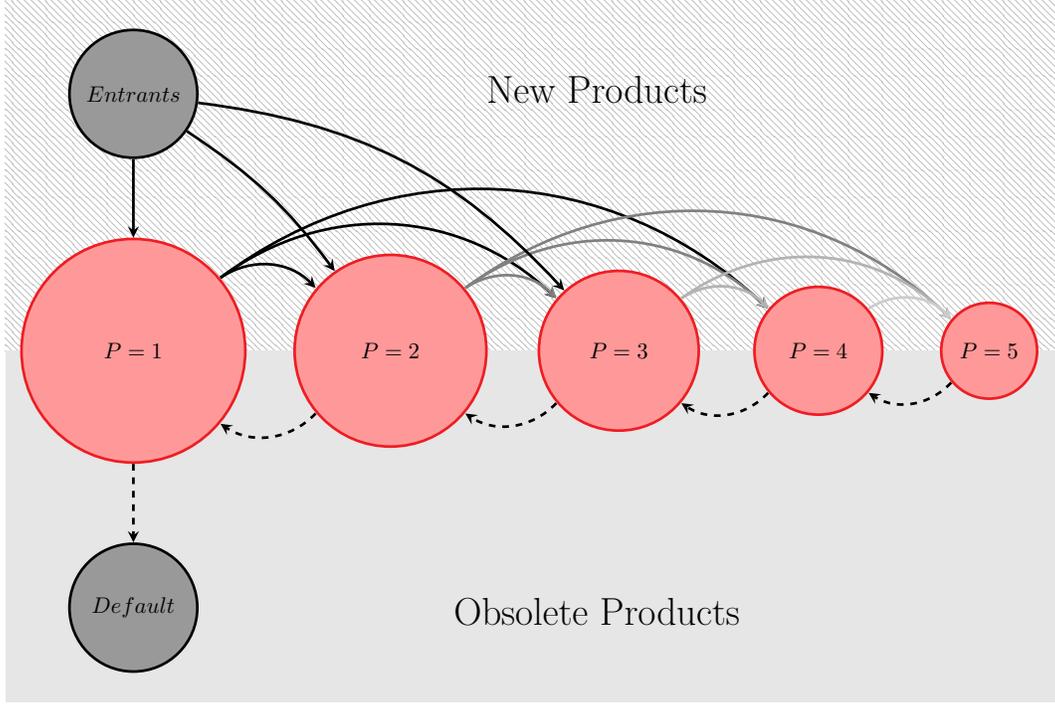
16

Figure 3: **Steady state equilibrium.** This figure gives an example of a steady state distribution in which there is entry. Appendix 3 derives the steady state firm size distribution.

there are multiple upward flows. In this equilibrium, firms exit when they have zero product lines and therefore there is a positive mass of entrants. All industry-wide variables are constant over time, even though individual firms can create new product lines, can lose product lines to competitors, and can even exit. Debt financing affects industry structure and dynamics by changing firms' R&D policies and the rate of creative destruction.

In industry equilibrium, creative destruction arises because of innovations by incumbents and entrants. The level of innovation by incumbents depends on the mass of incumbents in the economy M_i and the flow of innovations they generate per firm $\mathbb{E}\left[\lambda X\right] \approx \mathbb{E}\left[\lambda n \theta\right]$,[12] where an innovation generates X new product lines and the expectation is taken over the steady state distribution of incumbent firms. The level of innovation by entrants depends on the mass of entrants M_e and the average number of innovations per entrant $\lambda_e n \theta_e$. Altogether,

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[12]The equality is approximate since a firm can never have more than $\bar{p}$ product lines.

17

this implies that

$$f^* = f^i + f^e = \underbrace{M_i \mathbb{E}\left[\lambda X\right]}_{Incumbents} + \underbrace{M_e \lambda_e n \theta_e}_{Entrants},$$

where we denote by f^i and f^e the respective contributions of incumbents and entrants to the rate of creative destruction. As this equation makes clear, innovation and investment are driven by both incumbents and entrants in industry equilibrium.

The following theorem establishes equilibrium existence:

Theorem 2 (Equilibrium Existence). *If Assumption 1 in Appendix 2 holds then there exists an industry equilibrium Ψ^*.*

Under additional conditions, we can establish that all equilibria have the same rate of creative destruction f^*:

Proposition 2 (Uniqueness of the Rate of Creative Destruction). *If the debt value is strictly decreasing in the rate of creative destruction f, then all equilibria have the same rate of creative destruction f^*.*

The condition that debt value is strictly decreasing in f ensures, when combined with Proposition 1, that firm value is strictly decreasing in f so that a higher rate of creative destruction makes the firm worse off. Therefore, there can only exist one level of creative destruction for which the free entry condition binds.

In the model, firms that innovate choose to issue debt because debt financing increases firm value (i.e. issuing some debt is positive NPV). By increasing the value of incumbents, debt increases the surplus from entering the industry. This leads to an increase in the entry rate and in innovation, thereby increasing the rate of creative destruction. Notably, we have the following result:

Proposition 3 (Debt Financing and Creative Destruction). *Let $f^*_{No\ Debt}$ be the equilibrium rate of creative destruction in case firms are restricted to have no debt. Then there exists an*

industry equilibrium with a rate of creative destruction

$$f^* \geq f^*_{No\ Debt}.$$

While prior research has emphasized the negative effects of debt on investment at the firm level following the seminal contribution by Myers (1977), Proposition 3 demonstrates that debt financing fosters innovation and creative destruction at the aggregate level. As we show below, this is the outcome of two opposing forces. First, debt hampers innovation by incumbents due to debt overhang. Second, debt increases the surplus from entering the industry, thereby stimulating entry. As shown by Proposition 3, the latter effect dominates in equilibrium so that debt spurs innovation and growth. Importantly, our results are consistent with the evidence in Kerr and Nanda (2009), who examine entrepreneurship and creative destruction following US banking deregulation. Their empirical analysis shows that US banking reforms—that made bank debt widely available and cheaper by increasing competition—brought growth in both entrepreneurship and business closures.

1.4 Refinancing

Internet Appendix A extends the model by allowing firms to dynamically optimize their capital structure as their portfolio of products evolves. Notably, firms that perform well may releverage to exploit the tax benefits of debt. We show in this appendix that all the results derived in this section go through when we allow firms to restructure and demonstrate that there exists an industry equilibrium.

2 Model Analysis

This section examines the implications of the model for innovation, financing policy, and industry dynamics. To do so, we calibrate the model to match the observed characteristics of innovation and capital structure policies of an average US public firm, using firms' financial

data from Compustat and the data on firms' innovation activity from Kogan et al. (2017).

2.1 Parameter Values

We first set the interest rate r at 4.2% as in Morellec et al. (2012). We choose a tax rate π of 15%, consistent with the estimates of Graham (1996). The bankruptcy cost α is set to 45%, in line with the estimates of Glover (2016). The proportional cost of debt issuance ξ is set to 1.09%, consistent with the evidence on debt underwriting fees in Altinkilic and Hansen (2000). We choose a cost function separable in R&D intensity and quality, as in Akcigit and Kerr (2018). Notably, we assume that:

$$q(p, \lambda, \theta) = p \left(\beta_i \left(\frac{\lambda}{p} \right)^{\frac{1}{\gamma}} + \beta_q \theta^{\frac{1}{\gamma}} \right),$$

$$q_E(\lambda, \theta) = \beta_i \lambda^{\frac{1}{\gamma}} + \beta_q \theta^{\frac{1}{\gamma}},$$

where $\beta_q = 2\beta_i$. This specification captures the notion that investment in innovation quality is more expensive than investment in innovation intensity. To obtain the remaining parameter values, we focus on matching several key moments of interest in the data: the mean and variance of the leverage ratio, the mean of the innovation value per patent, and the turnover rate. Firms' choice of leverage is tightly linked to the parameters governing the R&D cost function β and γ. Furthermore, innovation quantity is directly linked to the maximum number of new products per innovation n. These parameters also determine the cost of performing R&D and are thus informative about the innovation value per patent. Lastly, the entry cost H pins down the turnover rate. Panel A of Table 1 summarizes the baseline values of the parameters.

To compute the data counterparts of the model-implied variables, we use the Kogan, Papanikolaou, Seru, and Stoffman (2017) data on patent quantity and value merged with accounting variables from Compustat. We use the sample period 1980 - 2010. Furthermore, we apply standard Compustat filters and remove firms with negative book equity and market-

Panel A: Baseline parameter values		
Parameter	**Symbol**	**Value**
Max # products per firm	$\bar{p}$	25
Interest rate	r	4.2%
Tax rate	π	15%
Bankruptcy cost	α	45%
Debt issuance cost	ξ	1.09%
Max # new products per innovation	n	3
After-tax entry cost	$H(1-\pi)$	5
Innovation curvature	γ	0.345
Innovation intensity: scale	β_i	26
Innovation quality: scale	β_q	52

Panel B: Variable definitions		
Moment	**Model**	**Data**
Leverage	$\frac{D(P_t,c_t)}{D(P_t,c_t)+E(P_t,c_t)}$	$\frac{dltt_t+dlc_t}{dltt_t+dlc_t+prcc\_f_t*csho_t}$
Innovation value per patent	$\frac{E(P_t+n,c_t)-E(P_t,c_t)}{nE(P_t,c_t)}$	$\frac{tsm_t}{prcc\_f_t*csho_t*fnpats_t}$
Tax benefit	$\frac{\mathbb{E}[\pi D(P_t,c_t)]}{\mathbb{E}[V(P_t,c_t)]}$	

Table 1: **Baseline parameter values and definitions of moments.**

to-book larger than 15. All variables are then winsorized at 1% and 99% in each fiscal year. Panel B of Table 1 presents the definitions of the moments of interest in the data as well as their model counterparts. We compute the model-implied moments by simulating a balanced panel of $N = 15000$ firms over $T = 15$ years, similar to the ones observed in the data. Firms that exit are replaced with entrants to keep the panel balanced.

2.2   Baseline Calibration and Model-Implied Moments

We calibrate the model parameters using the static debt version of the model and report the model-implied variables in Table 2. The numbers in the table suggest that the model succeeds in replicating the magnitude of observed financing and innovation policies. In particular, the average (market) leverage ratio is equal to 21.47% in the static debt specification and to 28.21% in the dynamic debt specification, both of which are close to the empirical value of 22%. As we will show later on, the relatively low value of leverage in the model is the result of the endogenous rate of creative destruction that disciplines firms' financing policy and the endogenous R&D policy that feeds back in financing decisions. The model also closely matches the variance of leverage, which equals 1.8% in the data and 2.2% in the model, thus generating sizeable variation in financing policy. The average innovation quality per patent is close to the observed value of 0.5%. The model generates a turnover rate of 1.21%, which is close to the observed turnover rate of 1.1% reported by Corbae and D'Erasmo (2017).

Baseline calibration. All values are in %.					
	Leverage Mean	Leverage Variance	Value p.p. Mean	Tax benefit	Turnover rate
No debt	0.00	0.00	0.43	0.00	0.54
Static debt	21.47	2.24	0.41	3.21	1.21
Dynamic debt	28.21	2.51	0.40	4.23	0.94

Table 2: **Baseline calibration of the model.**

A comparison between the static debt and no debt case indicates that debt lowers R&D

investment and facilitates firm exit by increasing the turnover rate. The results also show that despite the debt induced distortions in R&D, firms benefit substantially from debt financing. The implied tax benefit of debt is around 3.21% of firm value, which is close to the estimates of Korteweg (2010) and van Binsbergen, Graham, and Yang (2010).

Table 2 reveals that there are quantitative differences between the static and dynamic specifications. In particular, when the maximum number of new product lines that can be developed following an innovation is low, firms initially select a low debt level in the static debt model to avoid default and cannot subsequently readjust. This implies that the average firm has a lower leverage ratio in the static debt model than in the dynamic debt model, in which firms that perform well can issue additional debt. Finally, because the model with dynamic debt generates the same qualitative results as the model with static debt, we focus hereafter on analyzing the predictions of the model with static debt.

Table .4 shows how changes in the firm's environment affect outcome variables in the static debt case. The table illustrates that frictions (i.e. the corporate tax rate or the cost of issuing debt) and the quality of the firm's investment opportunity set have important effects on financing decisions and the industry turnover rate (Internet Appendix C reports a similar table for the dynamic debt case). The next subsection provides an in-depth analysis of the relation between debt financing, innovation, and competition.

2.3   Debt and Innovation

As shown by Proposition 3, a surprising and novel prediction of our model is that debt financing fosters innovation and creative destruction at the aggregate level by increasing the surplus from entering the industry and, therefore, the entry rate. To better understand this mechanism, we turn to analyzing the effects of debt financing on R&D investment by incumbents and entrants.

	Leverage Mean	Leverage Variance	Value p.p. Mean	Tax benefit	Turnover rate
Comparative statics. All values are in %.					
Max # new products per innovation					
$n = 2$	34.42	4.82	0.30	5.16	1.01
$n = 3$	21.47	2.24	0.41	3.22	1.21
$n = 4$	20.35	1.79	0.47	3.05	1.52
After-tax entry cost					
$H = 4$	27.38	2.24	0.37	4.11	2.87
$H = 5$	21.47	2.24	0.41	3.22	1.21
$H = 6$	20.99	2.22	0.39	3.15	0.71
Innovation cost scale					
$\beta = 23$	17.43	1.36	0.51	2.61	1.33
$\beta = 26$	21.47	2.24	0.41	3.22	1.21
$\beta = 29$	27.64	1.84	0.45	4.15	1.09
Innovation cost curvature					
$\gamma = 0.333$	20.91	1.94	0.44	3.14	1.41
$\gamma = 0.345$	21.47	2.24	0.41	3.22	1.21
$\gamma = 0.357$	27.97	3.68	0.37	4.20	1.24
Tax rate					
$\pi = 0.10$	16.26	1.47	0.42	1.63	0.97
$\pi = 0.15$	21.47	2.24	0.41	3.22	1.21
$\pi = 0.20$	30.45	2.37	0.40	6.09	1.06
Debt issuance cost					
$\xi = 0\%$	21.81	2.40	0.41	3.27	1.23
$\xi = 1.09\%$	21.47	2.24	0.41	3.22	1.21
$\xi = 4.36\%$	21.27	2.13	0.41	3.19	1.16

Table 3: **Comparative statics of selected moments.**

2.3.1 R&D Investment and Debt Overhang

We first examine the effects of debt financing on investment by incumbents. To do so, we first show how debt affects investment in innovation intensity λ and quality θ depending on incumbent's size, as captured by the number of product lines p. Notably, Figure 4 shows the effects of debt overhang by plotting the difference between R&D investment in the static first-best case, in which R&D policy maximizes firm value, and the static debt case, in which R&D policy maximizes shareholder value and is therefore subject to debt overhang. The first-best case uses the optimal coupon from the static debt case.
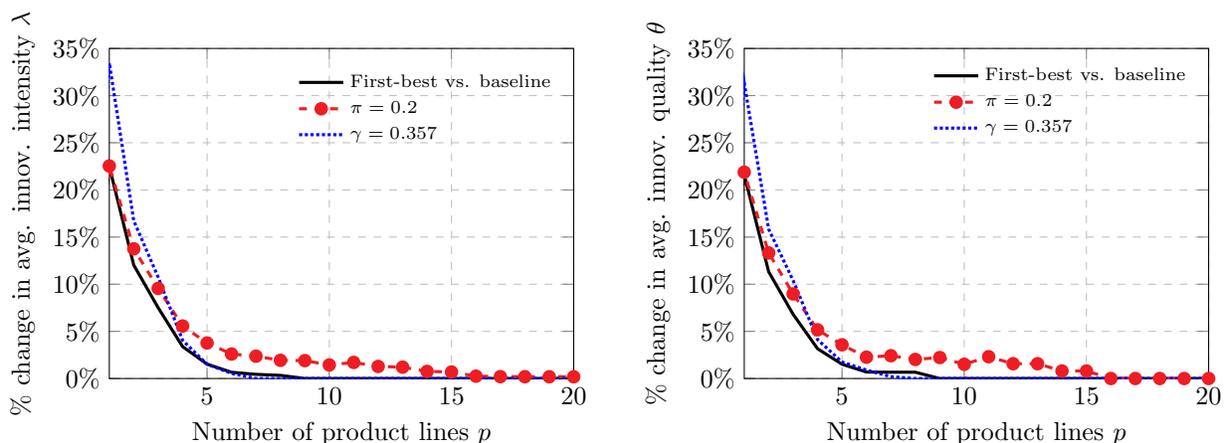

Figure 4: **Debt overhang.** The figure plots the change in innovation intensity and quality by incumbents due to debt overhang as a function of firm size p.

Figure 4 shows that when investment decisions maximize shareholder value and firms have debt outstanding, firms not only spend less on R&D overall, but also innovate less on each margin. The effects of debt overhang are substantial in the model. Depending on firm size p and leverage, firms invest up to 23% more in innovation intensity and quality in the first-best case compared to the baseline case. This distortion, that is solely due to debt, is especially strong for small firms. These effects tend to become smaller when firm size p increases as debt becomes less risky. As a result, wealth transfers to debtholders due to new investment are limited and so are the distortions in investment policy due to debt overhang.

Figure 4 also demonstrates that the magnitude of these effects varies with input parameter values. Distortions in investment are greater when the tax rate is larger or when the quality of the investment opportunity set worsens, as firms adopt higher leverage ratios (see Table .4). Overall, the analysis indicates that debt has first-order effects on incumbents' R&D policy, notably for smaller firms close to distress.
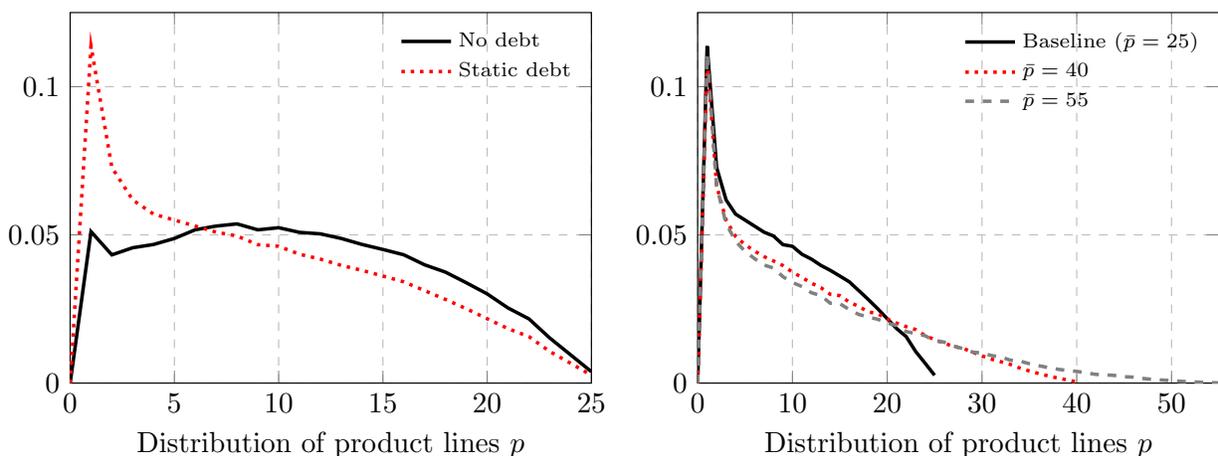

Figure 5: **Distribution of the number of products** p**.** The figure in the left panel shows the distribution of the number of products p in the no debt (solid) and static debt (dotted) cases. The figure in the right panel shows the distribution of the number of products p depending on the number of maximum product lines $\bar{p}$.

Because of its effects on investment, debt financing also has important implications for the size distribution of firms. To illustrate these implications, the left panel of Figure 5 presents this distribution for the no debt and static debt cases. The figure shows that the distribution is positively skewed when firms are allowed to issue debt. This change can be attributed to the higher entry and turnover rates and to debt overhang, which reduces incumbents' incentives to innovate and grow (see Subsection 2.3). The right panel of Figure 5 documents that the skew is largely unaffected by the maximum number of product lines $\bar{p}$. When increasing $\bar{p}$, the mass of firms with few products remains relatively unchanged as it depends on the incentives of entrants, which are insensitive to $\bar{p}$, and the forces of creative destruction, which are stronger for firms with more product lines.

26

2.3.2 Debt and Entry

To further characterize the effects of debt financing on innovation, we now examine the effects of debt financing on entry and the change in the contribution of entrants to the rate of creative destruction.[13] The left panel of Figure 6 plots the increase in f^e (i.e. $\frac{f^e - f^e_{No\ Debt}}{f^e_{No\ Debt}}$) due to debt financing. In our base case environment, allowing firms to issue debt increases f^e by 161%, which indicates that debt does indeed significantly foster turnover and entry. This increase in the entry rate is greater when the benefits of debt increases and reaches 270% when the tax rate increases to 20%. It is also greater when the cost of debt decreases (i.e. n decreases or γ increases). This increase in f^e due to debt financing is also illustrated by Figure 5, in which debt financing increases the positive skew in the size distribution.

As discussed above, a second effect of debt is that it leads to underinvestment by incumbents. To assess the relative magnitudes of these effects, we compare the change in f^e due to debt to the change in the aggregate rate of creative destruction f due to debt:

$$\frac{f^e - f^e_{No\ Debt}}{f - f_{No\ Debt}} \approx 1.43.$$

In our base case environment, about 143% of the increase in the rate of creative destruction f due to debt can be attributed to the increase in the entry rate. Underinvestment by incumbents acts as a balancing force and has a negative effect on the aggregate rate of creative destruction. That is, the net effect of debt financing on the rate of creative destruction results from two large and opposing forces, that partially offset each other in equilibrium.

Lastly, we can examine the effects of debt on firm value by computing the increase in the value of incumbents due to debt financing. The left panel of Figure 7 corresponds to the exercise undertaken in the empirical studies of Korteweg (2010) and van Binsbergen et al. (2010). It shows that debt financing leads to a significant increase in incumbent firm

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[13]The number of product lines that entrants generate every period is given by the mass of incumbent firms times the turnover rate of incumbent firms times the expected number of product lines an innovation by an entrant generates, conditional on the entrant generating at least one product line.
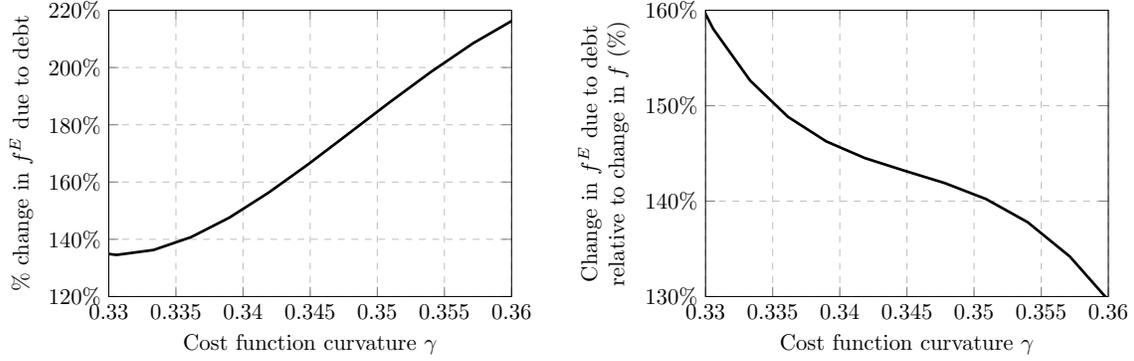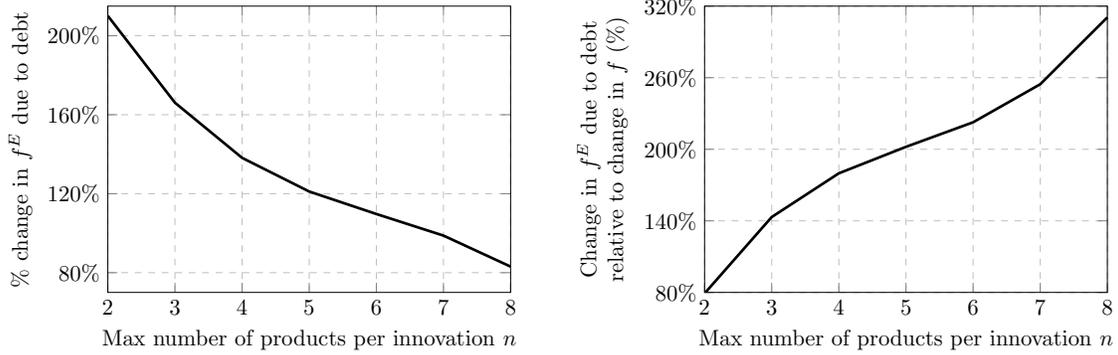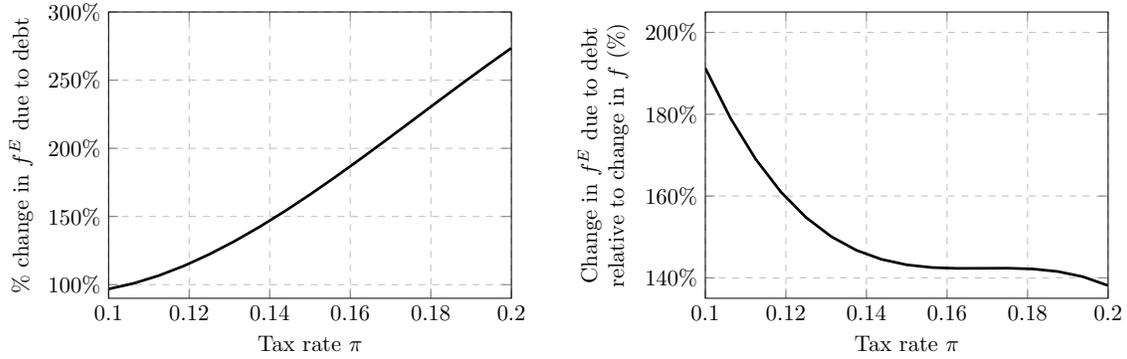
Figure 6: **The effects of debt financing on innovation by entrants.** The left figures show the effects of using debt financing on innovation by entrants. The right figures show entrants contribution to the aggregate increase in creative destruction due to debt financing. The comparative statics are smoothed using a third-order polynomial.
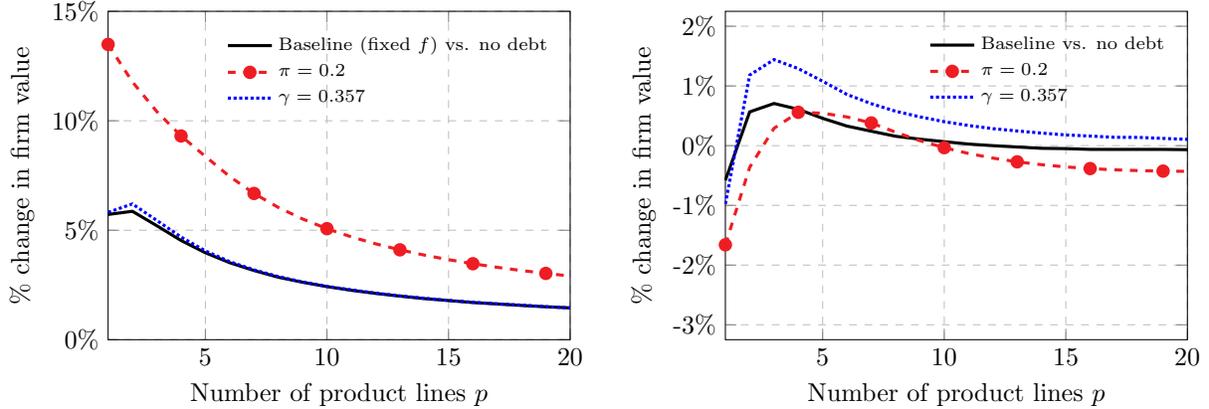
Figure 7: **Net benefits of debt.** The figure plots the relative change in firm value due to debt financing with exogenous rate of creative destruction (left panel) and endogenous rate of creative destruction (right panel).

value *when holding fixed the rate of creative destruction* (i.e applying the policy change to a single firm by assuming that $f = f^*_{No\ Debt}$). This increase is larger when firms have greater incentives to issue debt, due e.g. to a higher tax rate or to a higher cost of innovation. This increase in the value of incumbents leads to an increase in the benefits of entry and, therefore, to an increase in the entry rate and in the rate of creative destruction.

The right panel of Figure 7 shows the change in the value of incumbents due to debt financing when the rate of creative destruction is endogenized. In equilibrium, debt financing leads to an increase in the rate of creative destruction, which is such that the entry condition (given by equation (7)) binds. This leads to a dampening of the effects of debt financing on firm value. In our base case calibration, the entry condition binds for low p since the number of new products that can be developed following an innovation is low ($n = 3$).

2.3.3 Financing Policy and Investment Opportunities

Shareholders choose a leverage ratio that balances the marginal benefits and marginal costs of debt. Interest expenses on debt are tax deductible, which gives shareholders an incentive to issue debt. The presence of debt gives shareholders an option to default, which is costly.

29

Debt also reduces the benefits of innovation to shareholders because part of the benefits of investment accrue to creditors (due to the fact that debt becomes less risky). Therefore, debt distorts innovation incentives and leads to underinvestment by incumbents. These distortions in innovation policy feed back into firms' cash flow dynamics which influences the optimal leverage choice. Investment and financing policy are therefore jointly determined.

To illustrate these mechanisms, Figure 8 shows how leverage is affected by several key parameters describing the quality of the firms' investment opportunities: The cost function curvature γ, the cost function level β_i, the maximum number of new products per innovation n, and the maximum number of product lines $\bar{p}$.
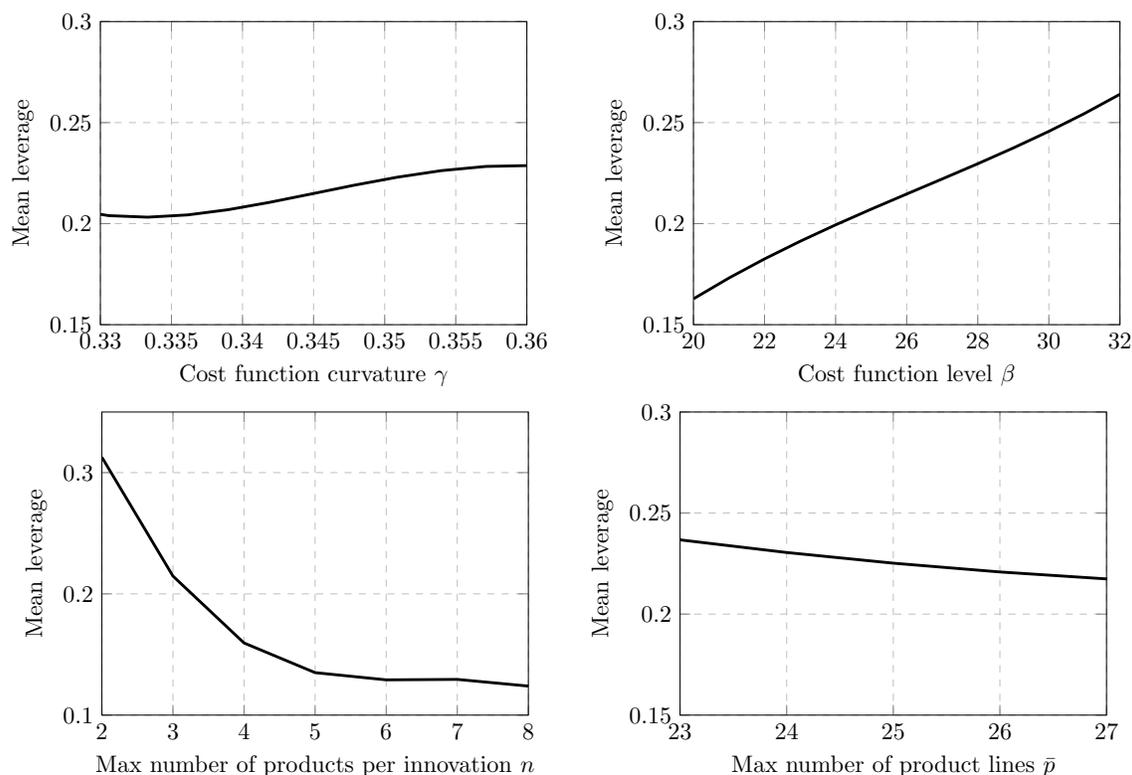

Figure 8: **Investment opportunities and financing policy.** The figure shows the effects of the quality of investment opportunities on financing decisions. The comparative statics are smoothed using a third-order polynomial.

Figure 8 shows that higher costs of innovation lower individual firms' incentives to inno-

vate, so that a smaller amount of their value comes from growth opportunities. In response, firms increase financial leverage. Figure 8 also shows that when each innovation has the potential of creating more product lines (as n gets larger), the potential costs of debt overhang are larger and firms issue less debt. The effect of changing $\bar{p}$ on leverage is more muted. This is due to the fact that $\bar{p}$ has been chosen large enough so that its effects on firm policies are limited. Overall, these results show that investment decisions feed back into financing choices. Our results are consistent with evidence in Smith and Watts (1992) and Barclay and Smith (1995) that firms with better growth opportunities adopt lower leverage ratios.

2.4 Industry Equilibrium

In equilibrium, the industry rate of creative destruction and firms' capital structure decisions are jointly and endogenously determined. To better understand the underlying economic mechanism, Panel A of Figure 9 shows how changing the cost of innovation γ affects equilibrium quantities. The top left graph of Panel A shows that increasing the cost of innovation γ lowers firms' investment in R&D. Interestingly, when f is fixed, the drop in R&D is much stronger as it does not incorporate the feedback from the industry. Because firms face worse growth opportunities when the cost of R&D investment is high, much of their value is attributable to assets in place. As a result, they increase leverage, as shown by the top right panel of the figure. The effect is again weaker in industry equilibrium as the effects of γ on R&D get muted. The drop in innovation quantity and the increase in leverage in turn feedback into the equilibrium rate of creative destruction, as illustrated in the bottom left panel of the figure. In equilibrium, the effect on innovation quantity is first order, leading to a negative relation between γ and f. This decrease in the rate of creative destruction—and therefore the longer expected productive life of each product line—spurs innovation, partly offsetting the higher innovation costs (top left graph). Lastly, as illustrated by the bottom right graph, these mechanisms translate to a lower turnover rate as γ increases. By contrast, in the single-firm model in which f is fixed, the sharp increase in leverage leads to a sharp

31

Panel A: Cost of innovation
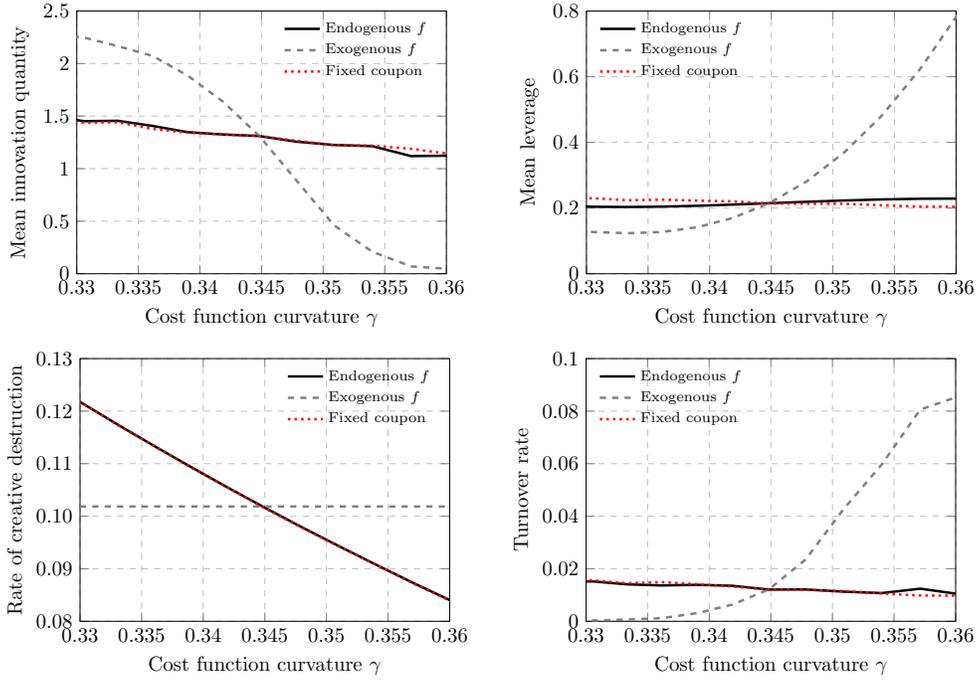
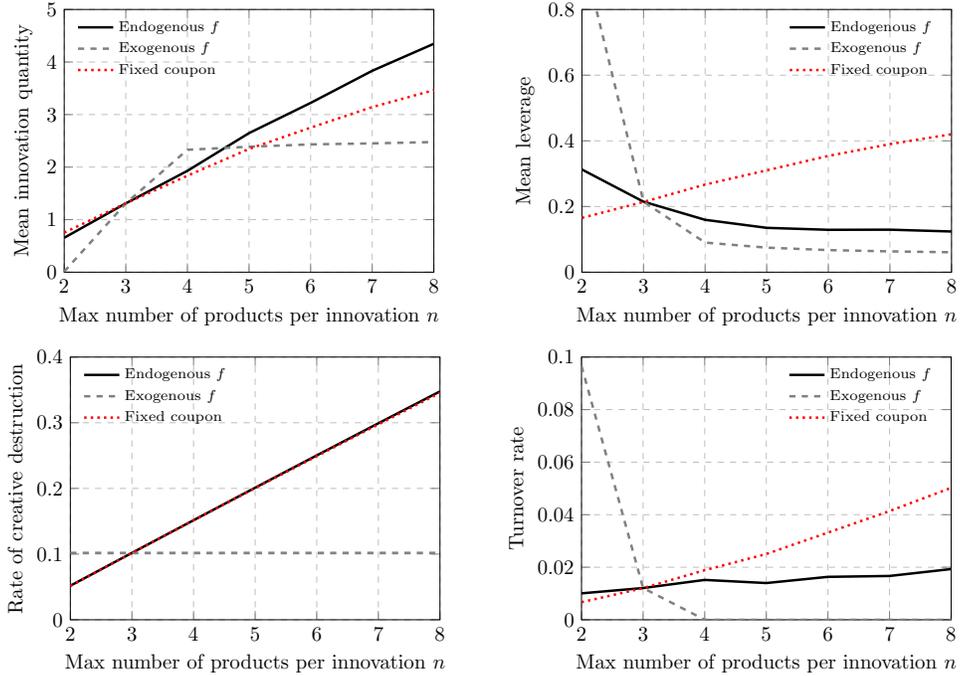## Panel B: Quality of investment opportunities

Figure 9: **The effects of the endogenous rate of creative destruction.** The figure shows the effects of changing the cost of innovation and the quality of investment opportunities on outcome variables in the single-firm model and in industry equilibrium. The comparative statics for leverage are smoothed using a third-order polynomial

increase in the turnover rate.

Panel B of Figure 9 shows the effects of varying the maximum number of new products per innovation on outcome variables. There again, endogenizing the rate of creative destruction has first order effects on model predictions. For instance the bottom right figure of Panel B shows that when f is endogenous, increasing n increases the turnover rate due to the large increase in the rate of creative destruction (and even more so when the coupon is fixed). By contrast, when f is exogenous, increasing n decreases the turnover rate. Again this is due to the fact that in the latter case, an increase in n only leads to a decrease in leverage. Another result illustrated by the top right figure of Panel B is that an increase in n leads to a decrease in leverage when financing decisions are endogenous. By contrast, with a fixed coupon, leverage increases with n because the higher rate of creative destruction impairs firm value.

3   General Equilibrium

This section closes the model in general equilibrium to endogenize the growth rate, labor demand, and the interest rate in the economy. The general equilibrium setup builds on Klette and Kortum (2004). We study a stationary equilibrium with a balanced growth path. This subsection describes the key features of the general equilibrium framework. Appendix 4 provides a detailed and formal description.

There is a representative household with logarithmic preferences who perfectly elastically supplies labor at a fixed wage. Entrants and incumbents use labor to perform R&D and produce goods. All costs in the model come in the form of labor costs, and therefore aggregate production equals aggregate consumption.

An innovation improves a product's production technology and therefore increase aggregate production and consumption. Each firm uses one unit of labor for each product line. Given the representative agent's preferences, this setup implies that a firm's profits

per product line only depend on the wage rate, which allow us to use the industry equilibrium framework we developed before. As a consequence, all the results derived in industry equilibrium still hold in general equilibrium.

This also implies that Proposition 3, which shows that creative destruction is higher in an industry equilibrium with debt, still holds true in general equilibrium. As we show in Appendix 4, this higher rate of creative destruction implies that the growth rate is also higher in the presence of debt. The following proposition formalizes this result.

Proposition 4 (Debt Financing and Growth). *Let $g^*_{No\ Debt}$ be the equilibrium growth rate in case firms are restricted to have no debt. There exists an equilibrium with growth rate*

$$g^* \geq g^*_{No\ Debt}.$$

This result follows directly from Propostion 3 and the fact that, as we show in Appendix 4, the growth is proportional to the rate of creative destruction f^*. When firms are allowed to issue debt, levered incumbents face debt overhang which lowers investment. But the possibility to issue debt also increases firm value, which spurs entry and therefore innovation and growth.

4   Conclusion

This paper investigates the relation between debt financing, innovation, and growth in a Schumpeterian growth model in which firms' dynamic R&D and financing choices are jointly and endogenously determined. In the model, each firm's R&D policy influences its risk profile, which feeds back in its capital structure decisions. In addition, a levered firm's R&D policy can be altered by its financing decisions, due to conflicts of interest between shareholders and debtholders. As a result, financing and investment are intertwined at the firm level.

We embed the single-firm model into a Schumpeterian industry equilibrium that endogenizes the rate of creative destruction and derive a steady state equilibrium in which innovating firms introduce new products that replace existing ones, and new entrants replace exiting incumbents. In this equilibrium, firms' R&D and capital structure decisions affect the aggregate level of creative destruction, which in turn feeds back in their policy choices.

Based on the resulting equilibrium, the paper delivers several novel results. First, we show that while debt hampers innovation by incumbents due to debt overhang, it also stimulates entry, thereby fostering innovation and growth at the aggregate level. Second, we show that debt financing has large effects on firm entry, firm turnover, and industry structure and evolution. Third, we show that our model predicts substantial intra-industry variation in leverage and innovation, in line with the empirical evidence.

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Appendix

The appendix consists of four parts. We solve the static debt case (Theorem 1 and Proposition 1) in Section 1. Section 2 embeds this static debt model into an industry equilibrium (Theorem 2 and Proposition 2). Section 3 derives the steady state firm size distribution. Section 4 closes the model in general equilibirum.

1   Debt Financing

First, we establish the individual firm results (Theorem 1) and intermediate results that show that the equity value is continuous and decreasing in f and c (Lemma 1). Finally, we prove the comparative statics results (Proposition 1).

In the static debt model an incumbent's coupon is constant. Therefore, we write the equity value as

$$E(p) = E(p, c)$$

and use this notation when it does not lead to confusion. Furthermore, the equity value indirectly depends on the parameters f and c. When necessary, we make this dependence explicit by writing $E(p|f, c)$.

Theorem 1 (Equity Value). *A unique solution to the equity value (2) exists. Equity value is non-decreasing in p and therefore the optimal default strategy is a barrier default strategy $\tau_D = \inf\{t > 0 | P_t \leq p_D\}$. If the optimal level of R&D is interior $((\lambda, \theta) \in (0, \bar{\lambda}) \times (0, 1))$, it solves*

$$\mathbb{E}^\theta \left[E(\min\{p + x, \bar{p}\}, c) \right] - E(p, c) = (1 - \pi) \frac{\partial q(p, \lambda, \theta)}{\partial \lambda},$$

$$\lambda \frac{\partial \mathbb{E}^\theta \left[E(\min\{p + x, \bar{p}\}, c) \right]}{\partial \theta} = (1 - \pi) \frac{\partial q(p, \lambda, \theta)}{\partial \theta}.$$

Proof. The proof has several steps. First, we establish existence of the equity value. Then we show that it is increasing in the number of product lines p. Finally, we derive the first-order conditions for the internal optimal level of R&D.

1. Equation (3) shows that the equity value for $p \in \{1, ..., \bar{p}\}$ can be rewritten as

$$E(p) = \sup_{\theta, \lambda, \tau_D} \left\{ \mathbb{E}_p \left[\int_0^{\tau_D} e^{-(r + \lambda + pf)t} (1 - \pi)(p - c - q(p, \lambda, \theta)) dt \right] \right.$$
$$\left. + \mathbb{E}_p \left[\int_0^{\tau_D} e^{-(r + \lambda + pf)t} \left(\lambda \mathbb{E}^\theta \left[E(\min\{p + x, \bar{p}\}) \right] + pf E(p - 1) \right) dt \right] \right\}.$$

with $E(0) = 0$. Define $\mathcal{M}(E)$ as the mapping

$$\mathcal{M}(E) = \sup_{\theta,\lambda,\tau_D} \left\{ \mathbb{E}_p \left[\int_0^{\tau_D} e^{-(r+\lambda+pf)t}(1-\pi)(p-c-q(p,\lambda,\theta))dt \right] \right.$$
$$\left. + \mathbb{E}_p \left[\int_0^{\tau_D} e^{-(r+\lambda+pf)t} \left(\lambda \mathbb{E}^\theta \left[E(\min\{p+x,\bar{p}\}) \right] + pfE(p-1) \right) dt \right] \right\}.$$

Any fixed point of this mapping is bounded from above by $\bar{p}/r$ and from below by zero. Furthermore, the mapping is monotone in E and finally,

$$\mathcal{M}(E+L) = \sup_{\theta,\lambda,\tau_D} \left\{ \mathbb{E}_p \left[\int_0^{\tau_D} e^{-(r+\lambda+pf)t}(1-\pi)(p-c-q(p,\lambda,\theta))dt \right] \right.$$
$$+ \mathbb{E}_p \left[\int_0^{\tau_D} e^{-(r+\lambda+pf)t} \lambda \mathbb{E}^\theta \left([E(\min\{p+x,\bar{p}\})] + L \right) dt \right]$$
$$\left. + \mathbb{E}_p \left[\int_0^{\tau_D} e^{-(r+\lambda+pf)t} pf \left(E(p-1) + L \right) dt \right] \right\},$$
$$\mathcal{M}(E+L) \leq \mathcal{M}(E) + \frac{\bar{\lambda} + \bar{p}f}{r + \bar{\lambda} + \bar{p}f} L,$$

because $\lambda \leq \bar{\lambda}$ by assumption. Therefore, the mapping $\mathcal{M}(E)$ satisfies Blackwell's sufficient conditions for a contraction (see Theorem 3.3 on page 54 in Stokey, Lucas, and Prescott (1989)) and it is a contraction mapping, which implies that a fixed point exists and is unique. The equity value is the fixed point of this mapping.

2. The next step is to show that equity value is non-decreasing in p. We do this by showing that having one extra product line improves a firm's cash flows even if shareholders run the firm as if it does not have this extra product line. Assume today the firm has $p+1$ product lines and that it separates one product line and runs the firm as if it had only p product lines. The firm receives cash flows from this extra product line until the product line becomes obsolete, the firm's non-separated number of product lines reaches $\bar{p}$ or zero, or the firm defaults. The firm receives the extra (gross) profits from this separated product line but it also incurs higher R&D costs (since they depend on P_t). The equity value of this $p+1$ firm with a separated product line is given by

$$E(p) + \mathbb{E}_p \left[\int_0^{\tau_D(p) \wedge \tau_0(p) \wedge \tau_{\bar{p}}(p)} e^{-(r+f)t}(1-\pi)\left(1 - q(P_t+1, \lambda_t, \theta_t) + q(P_t, \lambda_t, \theta_t)\right) dt \right],$$

where $\tau_D(p)$ is the optimal default time of a firm that starts with p product lines, $\tau_0(p)$

is the first time the firm has zero product lines if it starts with p product lines, and $\tau_{\bar{p}}(p)$ is the first time a firm with p product lines has $\bar{p}$ product lines. The first term is the cash flows from the p product line firm, and the second term is the cash flow from the separated product line minus the changes in R&D costs. The conditions on the R&D cost function ensure that the second term is non-negative. Furthermore, the optimal R&D and default strategy followed by a $p+1$ product line firm (weakly) dominates the one chosen by a firm that separates one product line and uses the strategy from a p product line firm. Therefore,

$$
\begin{aligned}
E(p) \leq &E(p) \\
&+ \mathbb{E}_p \left[\int_0^{\tau_D(p) \wedge \tau_0(p) \wedge \tau_{\bar{p}}(p)} e^{-(r+f)t}(1-\pi)\left(1 - q(P_t+1, \lambda_t, \theta_t) + q(P_t, \lambda_t, \theta_t)\right) dt \right]
\end{aligned}
$$

$$
\leq E(p+1),
$$

which shows that the equity value $E(p)$ is non-decreasing in p. This also implies that a barrier default strategy is the optimal default strategy.

3. Finally, the (internal) optimal levels of R&D should satisfy the first-order conditions that follow from equation (3).

$\square$

Lemma 1. *The equity value $E(p|f, c)$ is continuous and non-increasing in f and c. If $E(p|f, c) > 0$ then the equity value is decreasing in f and c.*

Proof. We first show that equity value decreases with the rate of creative destruction f.

1. Fix $f_2 < f_1$. Let P_t^1 be the number of product lines of a firm facing a rate of creative destruction f_1. We know that

$$
E(p|f_1) = \mathbb{E}_p \left[\int_0^{\tau_D^1 \wedge \tau_0^1} e^{-rt}(1-\pi)\left(P_t^1 - c - q(P_t^1, \lambda_t^1, \theta_t^1)\right) dt \right],
$$

where $\{\lambda_t^1, \theta_t^1\}, \tau_D^1$ are shareholders optimal strategy given f_1. The dynamics of P_t^1 are

$$
dP_t^1 = dI_t^1 - dO_t^1 = \max\left(Y_t^1, \bar{p} - P_{t-}^1\right) dN_t^1 - dO_t^1
$$

with

$$
\mathbb{E}\left[dP_t^1\right] = \lambda_t^1 \mathbb{E}^{\theta_t^1}\left[\max\left(Y_t^1, \bar{p} - P_{t-}^1\right)\right] dt - f_1 P_{t-}^1 dt.
$$

43

2. Define $\tilde{P}_t^2$ as,

$$d\tilde{P}_t^2 = d\tilde{I}_t^1 - X_t dO_t^1 - dH_t,$$

where

$$\tilde{I}_t^1 = \max\left(Y_t^1, \bar{p} - \tilde{P}_{t-}^2\right) dN_t^1,$$

$$X_t \sim Bin\left(1, \frac{f_2}{f_1}\right),$$

$$H_t \sim Poisson\left(f_2\left(\tilde{P}_t^2 - P_t^1\right)\right).$$

The construction of X_t and H_t implies that,

$$\mathbb{E}_t\left[X_t d\tilde{O}_t^1 - dH_t\right] = \frac{f_2}{f_1} f_1 P_{t-}^1 dt - f_2\left(\tilde{P}_t^2 - P_{t-}^1\right) dt = f_2 \tilde{P}_{t-}^2 dt.$$

These dynamics imply that $\tilde{P}_t^2$ evolves according to the R&D strategy $\{\lambda_t^1, \theta_t^1\}$ given a failure intensity of f_2. The construction $\tilde{P}_t^2$ ensures that

$$P_t^1 \le \tilde{P}_t^2.$$

If $\tilde{P}_{t-}^2 = P_{t-}^1$ then innovation dynamics are the same $dI_t^1 = d\tilde{I}_t^2$. Furthermore, product line failure is higher for P_{t-}^1 since $f_2/f_1 < 1$ and if a product line fails for $\tilde{P}_{t-}^2$ then it fails for P_{t-}^1. Therefore, if $\tilde{P}_{t-}^2 = P_{t-}^1$ then $\tilde{P}_t^2 \ge P_t^1$. If $\tilde{P}_{t-}^2 > P_{t-}^1$ then product line failure can never imply $\tilde{P}_t^2 < P_t^1$ since product lines drop by only one. Furthermore, by construction innovation happens at the same time and the number of product lines created for both is either Y_t or $\bar{p}$ is reached. This implies that if at time t product lines are created and $\tilde{P}_{t-}^2 > P_{t-}^1$ then $\tilde{P}_t^2 = \min\left(\tilde{P}_{t-}^2 + Y_t, \bar{p}\right) \ge \min\left(P_{t-}^1 + Y_t, \bar{p}\right) = P_t^1$. Therefore, if $\tilde{P}_{t-}^2 > P_{t-}^1$ then $\tilde{P}_t^2 \ge P_t^1$.

3. Given the assumptions on the cost function the equity value satisfies

$$E(p|f_1) = \mathbb{E}_p\left[\int_0^{\tau_D^1 \wedge \tau_0^1} e^{-rt}(1-\pi)\left(P_t^1 - c - q(P_t^1, \lambda_t^1, \theta_t^1)\right) dt\right]$$

$$\le \mathbb{E}_p\left[\int_0^{\tau_D^1 \wedge \tau_0^1} e^{-rt}(1-\pi)\left(\tilde{P}_t^2 - c - q(\tilde{P}_t^2, \lambda_t^1, \theta_t^1)\right) dt\right]$$

$$\le E(p|f_2).$$

If the equity value is positive then $\tau_D^1 \wedge \tau_0^1 > 0$, and the second inequality becomes a strict inequality. This shows that $E(p|f)$ is non-increasing in f and strictly decreasing in f when $E(p|f) > 0$.

4. The next step is showing that the equity value is continuous in f. The mapping $\mathcal{M}(E|f)$ is continuous in f. Therefore, for every $\epsilon > 0$ there exists a $\delta > 0$ such that for $f' \in (f - \delta, f + \delta)$,

$$\|\mathcal{M}(E(p|f)|f') - E(p|f)\| = \|\mathcal{M}(E(p|f)|f') - \mathcal{M}(E(p|f)|f)\| < \epsilon.$$

Fix one such ϵ. Define $\mathcal{M}^m(E|f)$ as applying the mapping $\mathcal{M}(\cdot|f)$ m times to E. Applying the mapping $\mathcal{M}$ again leads to,

$$\|\mathcal{M}^2(E(p|f)|f') - \mathcal{M}(E(p|f)|f')\| < U\|\mathcal{M}(E(p|f)|f') - E(p|f)\| < U\epsilon.$$

where

$$U = \frac{\bar{\lambda} + \bar{p}f'}{r + \bar{\lambda} + \bar{p}f'}.$$

This process can be repeated and leads to

$$\|\mathcal{M}^{m+1}(E(p|f)|f') - \mathcal{M}^m(E(p|f)|f')\| < U^m\epsilon.$$

Therefore, the distance between $E(p|f)$ and $E(p|f')$ is bounded by

$$
\begin{aligned}
\|E(p|f) - E(p|f')\| &= \|E(p|f) - \mathcal{M}^\infty(E(p|f)|f')\| \\
&\leq \sum_{i=0}^{\infty} \|\mathcal{M}^{i+1}(E(p|f)|f') - \mathcal{M}^i(E(p|f)|f')\| \\
&< \epsilon \sum_{i=0}^{\infty} U^i \\
&= \epsilon \frac{1}{1 - U} \\
&= \epsilon \frac{r + \bar{\lambda} + \bar{p}(f + (f' - f))}{r} \\
&= \epsilon \frac{r + \bar{\lambda} + \bar{p}(f + \delta)}{r}.
\end{aligned}
$$

45

Take an $\tilde{\epsilon} > 0$ and set

$$\epsilon = \tilde{\epsilon}\frac{r}{r + \bar{\lambda} + \bar{p}(f+1)}.$$

Then define $\tilde{\delta} = \min\{\delta, 1\}$. We get that for $f' \in (f - \tilde{\delta}, f + \tilde{\delta})$

$$\frac{r + \bar{\lambda} + \bar{p}\left(f + (f' - f)\right)}{r} \leq \frac{r + \bar{\lambda} + \bar{p}\left(f + 1\right)}{r} = \tilde{\epsilon}.$$

This implies that for every $\tilde{\epsilon} > 0$ there exists a $\tilde{\delta} > 0$ such that for $f' \in (f - \tilde{\delta}, f + \tilde{\delta})$,

$$\|E(p|f) - E(p|f')\| < \tilde{\epsilon}$$

Therefore, $E(p|f)$ is continuous in f. The same argument shows that $E(p|c)$ is continuous in c.

5. The final step is showing that the equity value is non-increasing in c and decreasing if $E(p|c) > 0$. The mapping $\mathcal{M}(E|c)$ is non-increasing in c and non-decreasing in E. Therefore, for a $c < c'$ we have that

$$\begin{aligned}
E(p|c) &= \mathcal{M}(E(p|c)|c) \\
&\geq \mathcal{M}(E(p|c)|c') \\
&\geq \mathcal{M}^2(E(p|c)|c') \\
&\geq \mathcal{M}^{n>2}(E(p|c)|c') \\
&\geq \mathcal{M}^{\infty}(E(p|c)|c') \\
&= E(p|c'),
\end{aligned}$$

which proves the result. The first inequality becomes a strict inequality when $E(p|c) > 0$, which shows the decreasing result.

$\square$

Proposition 1 (Comparative Statics: Equity Value)**.** *If $E(p, c) > 0$, equity value is decreasing in the tax rate π, the coupon c, the rate of creative destruction f, and the cost $q(p, \lambda, \theta)$ of performing R&D.*

Proof. The result for c and f follows from Lemma 1. Take any other parameter (or the function $q(p, \lambda, \theta)$) and call it Ξ. If $E(p|\Xi) > 0$ then the mapping $\mathcal{M}(E|\Xi)$ is decreasing in

Ξ and increasing E. Therefore, we have

$$
\begin{aligned}
E(p|\Xi) &= \mathcal{M}(E(p|\Xi)|\Xi) \\
&> \mathcal{M}(E(p|\Xi)|\Xi') \\
&\geq \mathcal{M}^2(E(p|\Xi)|\Xi') \\
&\geq \mathcal{M}^{n>2}(E(p|\Xi)|\Xi') \\
&\geq \mathcal{M}^{\infty}(E(p|\Xi)|\Xi') \\
&= E(p|\Xi'),
\end{aligned}
$$

which proves the result. $\qquad\square$

2 Industry Equilibrium

We first establish the existence of an industry equilibrium (Theorem 2). We then derive conditions under which there is a unique rate of creative destruction (Proposition 2).

    To establish the existence of an equilibrium, we make the following assumption:

Assumption 1. *For the firm value, the order of the limit with respect to f and the supremum over c can be interchanged:*

$$
\lim_{f' \to f} \sup_c \{E(p,c|f') + (1-\xi)D(p,c|f')\} = \sup_c \lim_{f' \to f} \{E(p,c|f') + (1-\xi)D(p,c|f')\}.
$$

Theorem 2 (Equilibrium Existence). *If Assumption 1 holds then there exists an industry equilibrium Ψ^*.*

Proof. The proof has several steps:

1. The first step is showing that the equity value converges to zero when $f \to \infty$. Assume this is not the case then for some p we have that $E(p|f) > 0$ when $f \to \infty$. From equation (3) it follows that for any $p > 0$ with $E(p|f) > 0$

$$
\begin{aligned}
0 = &\frac{-rE(p|f) + (1-\pi)(p-c)}{f} \\
&+ \frac{\max_{(\lambda,\theta)} \left\{\lambda \left(\mathbb{E}^\theta \left[E(\min\{p+x,\bar{p}\})\right] - E(p|f)\right) - (1-\pi)q(p,\lambda,\theta)\right\}}{f} \\
&+ p\{E(p-1|f) - E(p|f)\}.
\end{aligned}
$$

Given that $E(p|f) \leq \bar{p}/r$ and $\lambda \leq \bar{\lambda}$, taking $f \to \infty$ implies that

$$
0 = p\{E(p-1|f=\infty) - E(p|f=\infty)\}
$$

47

and therefore that

$$E(p|f = \infty) = E(p - 1|f = \infty)$$

for any p for which $E(p|f = \infty) > 0$. Given that $E(0|f = \infty) = 0$ this implies that

$$E(p|f = \infty) = 0,$$

which is a contradiction. Therefore, the equity value does converge to zero.

2. The debt value also goes to zero when $f \to \infty$ since the default time and the recovery value in default go to zero. Therefore, firm value $V(f, \theta)$ and also entrant value $E^e(f)$ goes to zero as $f \to \infty$.

3. Define firm value as

$$F(p_0|f, c) = E(p_0|f, c) + (1 - \xi)D(p_0|f, c).$$

4. By Lemma 1, equity value is continuous in f and therefore

$$\lim_{f' \to f} \|E(p|f, c) - E(p|f', c)\| = 0.$$

As a result, the dynamics of P_t will also be the same under f and $f' \to f$. If in addition the default threshold is the same then

$$\lim_{f' \to f} \|D(p|f, c) - D(p|f', c)\| = 0$$

since the default times will converge. Since the equity value is continuous in f, if the default threshold is not the same then at f shareholders must be exactly indifferent between default and no default. Take an arbitrary small ϵ, because the equity value is decreasing in c, for either $c - \epsilon$ or $c + \epsilon$ the default threshold under $f' \to f$ will be the same as the default threshold under f (and c). Furthermore, because the equity value is continuous in f and c the dynamics of P_t will be continuous in both as well. This implies that

$$\lim_{\epsilon \to 0} \lim_{f' \to f} \|D(p|f, c) - D(p|f', c \pm \epsilon)\| = 0$$

since the default time will converge. This implies that

$$\lim_{\epsilon \to 0} \lim_{f' \to f} |F(p_0|f, c) - F(p_0|f', c \pm \epsilon)| = 0,$$

5. The previous step shows that for a given f, c, and $f' \to f$ there exists an $c' = \lim_{\epsilon \to 0} c \pm \epsilon$ such that the firm value is continuous in f. This implies that

$$\sup_c F(p_0|f',c) = \sup_c \lim_{f' \to f} F(p_0|f',c) = \lim_{f' \to f} \sup_c F(p_0|f',c).$$

The last step follows from Assumption 1. This shows that $\sup_c F(p_0|f,c)$ is continuous in f.

6. The above also implies that

$$V(f,\theta) = \mathbb{E}^\theta \left[\sup_c \left\{ F(p_0|f,c) \right\} \right]$$

with $p_0 = \min(y,\bar{p})$ and $y \sim Bin(n,\theta)$ is continuous in f and θ.

7. If there exists an $E^e(f) \geq H(1-\pi)$ then the intermediate value theorem ensures existence of an f such that $E^e(f) = H(1-\pi)$, which is an industry equilibrium.

8. If for all f $E^e(f) < H(1-\pi)$ then entry is never optimal. Given the fact that P_t is non-decreasing for $f = 0$, it follows that for $p > 0$ and $c = 0$ the equity value is positive $E(p|c = 0, f = 0) > 0$. Therefore, a steady state equilibrium exists in which all firms have $\bar{p}$ product lines and no one innovates.

$\square$

Proposition 2 (Uniqueness of the Rate of Creative Destruction). *If the debt value is strictly decreasing in f then all equilibria have the same rate of creative destruction f^*.*

Proof. The proof has several steps:

1. First, we show the entrant value is strictly decreasing in f. Since the equity value (for any positive value) and debt value are strictly decreasing in f, the optimal firm value $V(f,\theta)$ must be strictly decreasing in f as well. Take an $f_1 < f_2$ then

$$\begin{aligned} V(f_2,\theta) &= \mathbb{E}^\theta \left[E(\min\{x,\bar{p}\}|f_2,c_2) + D(\min\{x,\bar{p}\}|f_2,c_2) \right] \\ &< \mathbb{E}^\theta \left[E(\min\{x,\bar{p}\}|f_1,c_2) + D(\min\{x,\bar{p}\}|f_1,c_2) \right] \\ &\leq V(f_1,\theta), \end{aligned}$$

where c_2 is the firm value maximizing coupon given θ and f_2. Because the entrants value is

$$E^e(f) = \sup_{\{\lambda,\theta\}} \left(\frac{\lambda V(f,\theta) - (1-\pi)q_E(\lambda,\theta)}{r+\lambda} \right),$$

49

it is also strictly decreasing in f.

2. There are now two cases. If $E^e(0) \leq H(1-\pi)$ then $E^e(f) < H(1-\pi)$ for all $f > 0$ and the only equilibrium rate of creative destruction is $f^* = 0$. If $E^e(0) > H(1-\pi)$ then there exists a unique f^* such that

$$E^e(f^*) = H(1-\pi),$$

which is a condition that needs to be satisfied in equilibrium if $f^* > 0$. This proves that any equilibrium must have a rate of creative destruction f^*.

$\square$

Proposition 3 (Debt versus No Debt). *Let $f^*_{No\ Debt}$ be the equilibrium rate of creative destruction in case firms are restricted to have no debt. Then there exists an industry equilibrium with a rate of creative destruction*

$$f^* \geq f^*_{No\ Debt}.$$

Proof. The proof has several steps:

1. By assumption the option to issue debt increases shareholder value. This implies that,

$$E^e(f^*_{No\ Debt}) \geq E^e_{No\ Debt}(f^*_{No\ Debt}).$$

2. If $f^*_{No\ Debt} = 0$ then from Theorem 2 it directly follows that there exists an

$$f^* \geq f^*_{No\ Debt}.$$

3. If $f^*_{No\ Debt} > 0$ then

$$E^e(f^*_{No\ Debt}) \geq E^e_{No\ Debt}(f^*_{No\ Debt}) = H(1-\pi).$$

The proof of Theorem 2 shows that the entrant value is continuous in f and that $\lim_{f \to \infty} E^e(f) = 0$. Therefore, there exists an $f^* \geq f^*_{No\ Debt}$ such that

$$E^e(f^*) = H(1-\pi).$$

This f^* is an industry equilibrium.

$\square$

3 Steady State Distribution

In this appendix, we derive the steady state firm size distribution. Let $S(p|p_0)$ be the steady state distribution of firms that started initially with p_0 product lines and coupon $c^*(p_0)$. If firms with p product lines decided to default, then $S(p|p_0) = 0$. Assuming the firm does not default, the steady state distribution for p product lines $S(p|p_0)$ solves

$$0 = - \underbrace{\lambda(p|p_0) * (1 - \psi(p, 0, n, \theta(p|p_0))) * S(p|p_0)}_{\text{Exit: Product line creation}}$$

$$- \underbrace{f * p * S(p|p_0)}_{\text{Exit: Product line becomes obsolete}}$$

$$+ \underbrace{\sum_{i=1}^{\min(n,p)} \lambda(p-i|p_0) * \psi(p-i, i, n, \theta(p-i|p_0)) * S(p-i|p_0)}_{\text{Entry: Product line creation}}$$

$$+ \underbrace{f * (p+1) * S(p+1|p_0)}_{\text{Entry: Product line becomes obsolete}}$$

$$+ \underbrace{s * \mathbb{I}_{\{p=p_0\}}}_{\text{Entry: Entrants}},$$

where $\psi(p, X, n, \theta)$ is the pdf of $\min(X, \bar{p} - p)$ with $X \sim Bin(n, \theta)$.

Firms can exit for two reasons. First, they can create new product lines (first term). Second, one of their product lines can become obsolete (second term). Firms can enter for three reasons. First, a firm with less than p product lines can create new product lines and become a p-product line firm (third term). Second, a product line of a firm with $p+1$ product lines can become obsolete (fourth term). Third, there is endogenous entry (fifth term).

The term s determines the flow of entrants that become incumbents with p_0 product lines. In steady state, the constant s ensures that the outflow of firms is equal to the inflow of firms. The mass of firms that flow out is given by

$$f * (\min(p_D(p_0), 0) + 1) * S(\min(p_D(p_0), 0) + 1|p_0),$$

where $p_D(p_0)$ is the optimal default threshold. Given that the optimal default strategy is of a barrier type, firms can exit by either flowing into the default state or into the state with zero product lines, which one of the two happens first. Setting

$$s = f * (\min(p_D(p_0), 0) + 1) * S(\min(p_D(p_0), 0) + 1|p_0)$$

ensures that the inflow of entering firms is equal to the outflow of defaulting firms.

Given that $p_0 \sim \min\{x, \bar{p}\}$ with $x \sim Bin(n, \theta_e)$, the steady state firm size distribution is

$$S(p) = \sum_{p_0=1}^{n} \frac{\psi(0, p_0, n, \theta_e) * S(p|p_0)}{1 - \psi(0, p_0, n, \theta_e)}.$$

4 General Equilibrium Setup

In this appendix, we embed our model into a general equilibrium setup. This endogenizes the growth rate of the economy, the labor supply, and the interest rate. The general equilibrium setup is similar to Klette and Kortum (2004) and leads to a stationary equilibrium with a balanced growth path.

Production

There is a unit mass of differentiated goods in the economy, which are indexed by $i \in [0, 1]$. A measure L^P of labor is used for production, a measure $L^{R\&D}$ of labor performs R&D, and a meausre L^E of labor is used to generate entrants. Labor supply L^S is perfectly elastic, and it receives a wage w per unit supplied in each of these activities.

Incumbent firms use labor and installed product lines to produce goods. An improvement in the production technology increases the amount of the consumption good that one unit of labor produces.

For each type of product there is a leading producer, as in the industry equilibrium model. The production technology of good i's leading producer is q_t^i and determines the number of products that one unit of labor produces.

A firm that innovates on product i improves the production technology and becomes the leading producer. Each innovation is a quality improvement applying to a good drawn at random. The innovation increases the production technology proportionally. That is, when an innovation arrives at time t, the production technology increases from q_{t-}^i to $q_t^i = (1+\delta)q_{t-}^i$ with $\delta > 0$.

A firm that is the leading producer for product i is a monopolist for that good and can choose to supply or not supply that good. If the firm supplies the good then it uses one unit of labor to generate q_t^i units of the product. If the firm does not supply the good, its output and profits are zero.[14]

Let y_t^i be the amount of good i produced at time t. As in Klette and Kortum (2004) or Aghion, Bloom, Blundell, Griffith, and Howitt (2005), the aggregate consumption good is

---

[14]We can obtain equivalent results when each production line has as production function $q_t^i(l - \mathbb{I}_{\{l \geq 1\}} k(l-1))$ where l is the amount of labor used, $k(0) = 0$, $k'(\cdot) > 0$, and the firm produces the maximum amount of the good among production quantities that maximize its profits.

produced using a logarithmic aggregator

$$\ln(Y_t) = \int_0^1 \ln\left(y_t^i\right) di,$$

with Y_t the aggregate production of the consumption good.[15]

Innovation

Firms can invest in R&D. Investment in R&D leads to product innovations, which improve the amount of a product that one unit of labor produces. R&D investment costs come in the form of labor costs. Innovation costs are a function of the wage rate multiplied by the number of hours spend on R&D:

$$q(p, \lambda, \theta) = w * \tilde{q}(p, \lambda, \theta). \tag{4.1}$$

Therefore, a firm with p products that has an R&D policy (λ, θ) requires $\tilde{q}(p, \lambda, \theta)$ units of labor.[16] We define the innovation cost function for an entrant in a similar way:

$$q_E(\lambda, \theta) = w * \tilde{q}_E(\lambda, \theta).$$

Default and Entry

Debt distorts investment in R&D and can lead to default. If a firm with profitable product lines defaults, creditors continue producing these goods until the products become obsolete after which they exit. Furthermore, creditors do not perform R&D and run the firm as an all-equity financed firm. Their expected payoff in default is therefore

$$\frac{(1 - \pi)P_{\tau_D}(1 - w)}{r + f}.$$

---

[15]This is a limiting case of the Dixit and Stiglitz (1977) aggregator when the elasticity of substitution ϵ goes to 1

$$\lim_{\epsilon \to 1} \ln\left(\left(\int_0^1 \left(y_t^i\right)^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}\right) = \lim_{\epsilon \to 1} \frac{\ln\left(\int_0^1 \left(y_t^i\right)^{\frac{\epsilon-1}{\epsilon}} di\right)}{\frac{\epsilon-1}{\epsilon}} = \lim_{\epsilon \to 1} \frac{\int_0^1 \ln\left(y_t^i\right) \left(y_t^i\right)^{\frac{\epsilon-1}{\epsilon}} di}{\int_0^1 \left(y_t^i\right)^{\frac{\epsilon-1}{\epsilon}} di} = \int_0^1 \ln\left(y_t^i\right) dt.$$

[16]The condition on the R&D cost that ensures that the equity value is non-decreasing in p, see (1), in the general equilibrium framework boils down to

$$q(p + 1, \lambda, \theta) - q(p, \lambda, \theta) \leq 1 - w.$$

This setup implies that the debt value is the same as in the industry equilibrium model with $\alpha = 0$; see equation (4). In this model, default costs are therefore uniquely related to the distortions in investment policy triggered by default (and debt overhang).

Because firms exit, in a stationary equilibrium there must be entry. As in the industry equilibrium model, entrants have no product lines but perform R&D in the hope of developing innovations, so they can become the leading producer for at least one product. In the industry equilibrium model, entrants pay a fixed entry cost $H(1 - \pi)$ to become an entrant. In our general equilibrium model, these fixed costs are replaced by labor costs (as in e.g. Klette and Kortum (2004) or Lentz and Mortensen (2008)). An entrepreneur can hire one unit of labor, which costs him after tax $w(1 - \pi)$, and that generates an idea with Poisson intensity h. Once the entrepeneur has generated this idea he can become an entrant. Since in equilibrium the cost and benefits should equate for an entrepreneur, the free entry condition becomes

$$E^e(f)h = w(1 - \pi).$$

Representative Household

There is a representative household with logarithmic preferences:

$$U_0 = \int_0^\infty e^{-rt} \left(\ln\left(\mathcal{C}_t\right) - wL_t^S \right) dt$$

where $\mathcal{C}_t$ is aggregate consumption and r is the discount rate. The representative household's labor supply L_t^S is perfectly elastic at a wage rate w.

Equilibrium Properties

Since our model is a closed economy and all costs come in the form of labor costs and collected taxes are paid out to the representative household, consumption equals production for each good i, and therefore aggregate consumption and production are also equal

$$\mathcal{C}_t = Y_t.$$

The logarithm of aggregate consumption $\ln(\mathcal{C}_t)$ is the numeraire in this economy. The representative household owns all (financial) assets in the economy and receives all labor income.

Using the logarithm of consumption $\ln(\mathcal{C}_t)$ as the numeraire, the representative household's optimal consumption across goods implies that the price of good i should be

$$\frac{1}{y_t^i} = p_t^i,$$

where the marginal benefit of good i is equal to its marginal cost. The average cost of production are

$$\frac{w}{q_t^i}.$$

Therefore, the profits on product i are given by

$$q_t^i \left(\frac{1}{q_t^i} - \frac{w}{q_t^i} \right) = 1 - w.$$

This result implies that the equity value is as in the industry equilibrium framework (see equation (2)), except that the profit flow from a product line is $1 - w$ instead of 1 and the R&D cost depend on the wage rate w (see equation (4.1)).

In equilibrium, the growth rate g, the interest rate $\tilde{r}$, and the labor supply L^S are determined by market clearing. Since we use the logarithm of consumption as the numeraire, the agent is effectively risk-neutral in the numeraire and therefore,[17]

$$\tilde{r} = r.$$

Consumption grows at a rate of

$$d\ln(\mathcal{C}_t) = d\int_0^1 \ln(y_t^i)di = \ln(1+\delta)fdt = gdt$$

where f is the rate of creative destruction in the economy, which results from innovations by incumbents and entrants.

Finally, there is a labor supply L^S which is used for production L^P, for research $L^{R\&D}$, and to generate entrants L^E:

$$L^P = 1,$$
$$L^{R\&D} = \int_{\mathcal{F}_t^I} \tilde{q}(P_t^j, \lambda_t^j, \theta_t^j)dj + \tilde{q}_E(\lambda_E, \theta_E)\int_{\mathcal{F}_t^E} dj,$$

where subscript j indicates firm j, $\mathcal{F}_t^I$ is the set of active incumbents, and $\mathcal{F}_t^E$ is the set of active entrants. The labor supply is set such that the labor market clears at a wage w:

$$L^S = L^P + L^{R\&D} + L^E.$$

The utility of the representative household is

$$
\begin{aligned}
U_0 &= \int_0^\infty e^{-rt}(\ln(\mathcal{C}_0) + gt - wL^S)dt \\
&= \frac{\ln(\mathcal{C}_0) - wL^S}{r} + \left[\frac{-1}{r}e^{-rt}gt\right]_0^\infty + \int_0^\infty \frac{1}{r}e^{-rt}gdt \\
&= \frac{r\ln(\mathcal{C}_0) + g - rwL^S}{r^2}.
\end{aligned}
$$

The higher the growth rate in the economy the higher the representative household's utility.

The formal equilibrium definition is

Definition 2 (General Equilibrium)**.** *The parameters and policies*

$$\Psi^* = \{g^*, L^{S*}, r^*, f^*, c^*(p_0), \lambda^*(p|p_0), \theta^*(p|p_0), p_D^*(p_0), \lambda_e^*, \theta_e^*\}$$

are a general equilibrium if:

1. **Incumbents:** *Given the rate of creative destruction f^*, the interest rate r^*, and coupon $c^*(p_0)$, incumbents production decision, level of R&D ($\lambda^*(p|p_0), \theta^*(p|p_0)$), and default decision $p_D^*(p_0)$ maximize their equity value.*

2. **Entrants:** *Given the rate of creative destruction f^* and the interest rate r^*, entrants*

level of R&D $(\lambda_e^*, \theta_e^*)$ and capital structure upon becoming an incumbent $c^*(p_0)$ maximize their equity value.

3. **Entry:** The free entry condition holds:

$$E^e(f^*) \le \frac{w(1-\pi)}{h},$$

and the inequality binds when there is creative destruction $f^* > 0$.

4. **Labor:** The labor supply L^{S*} ensures that the labor market clears:

$$L^{S*} = L^P + L^{R\&D} + L^E$$

for a wage rate w.

5. **Growth and interest rate:** The growth and interest rate follow from the Euler equation and the rate of creative destruction:

$$d\ln(\mathcal{C}_t) = g^*dt = \ln(1+\delta)f^*dt,$$
$$r^* = r.$$

Internet Appendix (not for publication)

The internet appendix consists of two parts. Section 1 solves the model with refinancing (Theorem 3 and Theorem 4). Section 2 provides additional numerical results from the model with refinancing.

1   Debt Refinancing

This appendix extends the model by allowing firms to dynamically optimize their capital structure. Notably, firms that perform well may releverage to exploit the tax benefits of debt. For simplicity, we assume that firms can only reduce their indebtedness in default.[18] We consider that firms can call their debt at price $\rho\left(p^I\right)c$ with $\rho(p^I) > 0$, where p^I is the number of product lines the firm had when it previously issued debt. The ability to buyback the debt for $\rho(p^I)$ implies that we have to keep track of the number of product lines the firm had the last time it issued debt p^I. We restrict the firm to refinance at most K times and assume that $c \leq \bar{c}$. In this section, we present the solution for the stationary case when $K \to \infty$. Our results also hold for any finite K.

Define firm value as the equity value plus the debt value minus the issuance cost:

$$F\left(p, c, p^I\right) = E\left(p, c, p^I\right) + (1 - \xi) D\left(p, c, p^I\right).$$

The exact definition of the equity and debt value in case the firm can refinance its debt is given below. The payoff to shareholders of restructuring the firm's debt is given by the value of the firm after refinancing minus the cost of buying back the debt:

$$\sup_{c' > c} F(p, c', p) - \rho\left(p^I\right) c.$$

This implies that the equity value, with the possibility to dynamically optimize the firm's capital structure, is given by

$$E\left(p, c, p^I\right) = \sup_{\{\lambda_t, \theta_t\}_{t \geq 0}, \tau_D, \tau_R} \left\{ \mathbb{E}_p\left[\int_0^{\tau_D \wedge \tau_0 \wedge \tau_R} e^{-rt}(1 - \pi)\left(P_t - c - q(P_t, \lambda_t, \theta_t)\right) dt\right] \right.$$
$$\left. + \mathbb{E}_p\left[\mathbb{I}_{\{\tau_R < \tau_D \wedge \tau_0\}} e^{-r\tau_R}\left(\sup_{c' > c} F(P_{\tau_R}, c', P_{\tau_R}) - \rho\left(p^I\right) c\right)\right] \right\},$$

where τ_R is the restructuring time chosen by shareholders. Shareholders receive the revenues

---

[18]While in principle management can both increase and decrease debt levels, Gilson (1997) finds that transaction costs discourage debt reductions outside of renegotiation. Hugonnier, Malamud, and Morellec (2015) show in a Leland-type model that reducing debt is never optimal for shareholders if debt holders are dispersed and have rational expectations. That is, there is no deleveraging along the optimal path.

generated by the portfolio of products minus the coupon payments, the R&D cost, and corporate taxes until either the firm defaults or changes its capital structure. In default, equity value drops to zero. When refinancing, shareholders repurchase existing debt at price $\rho\left(p^I\right)c$ and obtain the (after issuance cost) optimal firm value with a larger coupon $F(P_{\tau_R}, c', P_{\tau_R})$.

Debt value also takes into account the possibility that the firm refinances and is given by:

$$
\begin{aligned}
D\left(p, c, p^I\right) = & \mathbb{E}_p\left[\int_0^{\tau_D \wedge \tau_0 \wedge \tau_R} e^{-rt}cdt + \mathbb{I}_{\{\tau_D \wedge \tau_0 \leq \tau_R\}}e^{-r(\tau_D \wedge \tau_0)}(1-\alpha)\frac{(1-\pi)P_{\tau_D \wedge \tau_0}}{r+f}\right] \\
& + \mathbb{E}_p\left[\mathbb{I}_{\{\tau_R < \tau_D \wedge \tau_0\}}e^{-r\tau_R}\rho\left(p^I\right)c\right].
\end{aligned}
$$

This equation shows that creditors receive coupon payments until either the firm defaults or refinances its debt. When the firm defaults ($\tau_D \wedge \tau_0 \leq \tau_R$), creditors get the present value of the firm cash flows net of the proportional default costs α. When the firm refinances its debt ($\tau_R < \tau_D \wedge \tau_0$), creditors get $\rho\left(p^I\right)c$.

In the numerical analysis, we set $\rho\left(p^I\right)$ such that debt is called at a fraction κ of its risk-free value. The buyback price $\rho(p^I)$ therefore solves

$$
\rho(p^I)c = \frac{\kappa c}{r}.
$$

The entrant value is the same as in equation (5) with $V(f, \theta_e)$ defined as

$$
V(f, \theta_e) = \mathbb{E}^{\theta_e}\left[\sup_{c \geq 0}\left\{E\left(p_0, c, p_0\right) + (1-\xi)D\left(p_0, c, p_0\right)\right\}\right],
$$

An industry equilibrium is defined as before, except that firms' optimal policies additionally depend on the number of product lines the firm had the last time it issued debt p^I.

In the next part of this appendix, we establish existence of the equity value, which is the equivalent of Theorem 1 in the model with static debt, and existence of an equilibrium.

Proof of Equilibrium Existence

First, we establish existence of the equity and debt values (Theorem 3). Next, we establish the existence of an industry equilibrium under Assumption 2 (Theorem 4).

In this appendix we denote by

$$
E_K(p, c, p^I)
$$

the equity value for a firm that can still restructure its debt K times. The debt value

$D_K(p, c, p^I)$ and firm value $F_K(p, c, p^I)$ are similarly defined. Furthermore, define

$$E(p, c, p^I) = \lim_{K \to \infty} E_K(p, c, p^I),$$
$$D(p, c, p^I) = \lim_{K \to \infty} D_K(p, c, p^I),$$
$$F(p, c, p^I) = \lim_{K \to \infty} F_K(p, c, p^I).$$

Theorem 3. *The equity and debt values exist. If the optimal level of R&D is internal* $((\lambda, \theta) \in (0, \bar{\lambda}) \times (0, 1))$ *then it solves*

$$\mathbb{E}^\theta \left[E(\min\{p + x, \bar{p}\}, c, p^I) \right] - E(p, c, p^I) = (1 - \pi) \frac{\partial q(p, \lambda, \theta)}{\partial \lambda},$$
$$\lambda \frac{\partial \mathbb{E}^\theta \left[E(\min\{p + x, \bar{p}\}, c, p^I) \right]}{\partial \theta} = (1 - \pi) \frac{\partial q(p, \lambda, \theta)}{\partial \theta}.$$

Proof. We establish existence of the equity and debt value recursively.

1. From Theorem 1 it follows that the equity value for a firm that does not have the option to refinance exists. Therefore, also the debt value exists. Let this equity and debt values define the firm value:

$$F_0(p, c, p^I) = E_0(p, c, p^I) + (1 - \xi)D_0(p, c, p^I).$$

The state variable p^I plays no role if the firm cannot restructure.

2. Assume that $F_{K-1}(p, c, p^I)$ exists. First, observe the equity value $E_K(p, c, p^I)$ does not depend on $D_K(p, c, p^I)$ since the price at which the existing debt is bought back is $\rho(p^I)c$. The equity value for a firm that has K restructuring options is

$$E_K(p, c, p^I) = \sup_{\{\lambda_t, \theta_t\}_{t \geq 0}, \tau_D, \tau_R} \left\{ \mathbb{E}_p \left[\int_0^{\tau_D \wedge \tau_0 \wedge \tau_R} e^{-rt}(1 - \pi) \left(P_t - c - q \left(P_t, \lambda_t, \theta_t \right) \right) dt \right] \right.$$

$$\left. + \mathbb{E}_p \left[\mathbb{I}_{\{\tau_R < \tau_D \wedge \tau_0\}} e^{-r\tau_R} \left(\sup_{c' > c} F_{K-1}(P_{\tau_R}, c', P_{\tau_R}) - \rho(p^I)c \right) \right] \right\}.$$

Given $F_{K-1}(p, c, p^I)$, this implies that the equity value $E_i(p, c, p^I)$ is a fixed point of

3

the mapping

$$\mathcal{M}_K(E) = \sup_{\lambda,\theta,\tau_D,\tau_R} \left\{ \mathbb{E}_p \left[\int_0^{\tau_D \wedge \tau_R} e^{-(r+\lambda+pf)t}(1-\pi)\left(p-c-q\left(p,\lambda,\theta\right)\right) dt \right] \right.$$

$$\mathbb{E}_p \left[\int_0^{\tau_D \wedge \tau_R} e^{-(r+\lambda+pf)t} \lambda \mathbb{E}^\theta \left[E(\min(p+x,\bar{p}),c,p^I) \right] dt \right]$$

$$\mathbb{E}_p \left[\int_0^{\tau_D \wedge \tau_R} e^{-(r+\lambda+pf)t} fpE(p-1,c,p^I) dt \right]$$

$$\left. + \mathbb{E}_p \left[\mathbb{I}_{\{\tau_R < \tau_D\}} e^{-(r+\lambda+pf)\tau_R} \left(F_{K-1}(p,c',p) - \rho(p^I)c \right) \right] \right\}$$

with $E_K(0,c,p^I) = 0$. The equity value is bounded from above by

$$\frac{(1-\pi)\bar{p} + \pi\bar{c}}{r}$$

and from below by zero, it is increasing in E, and

$$\mathcal{M}_K(E+L) \leq \mathcal{M}_K(E) + \frac{\bar{\lambda} + f\bar{p}}{r + \bar{\lambda} + f\bar{p}} L,$$

which holds even if the firm restructures its debt. Therefore, the mapping $\mathcal{M}_K(E)$ satisfies Blackwell's sufficient conditions for a contraction, see Theorem 3.3 on page 54 in Stokey et al. (1989), and it is a contraction mapping, which implies that a fixed point exists and is unique. Let $E_K(p,c,p^I)$ be the fixed point of this mapping.

3. The debt value $D_K(p,c,p^I)$ follows from the optimal policies of the firm and therefore firm value $F_K(p,c,p^I)$ also exists. These steps recursively establish existence of the value functions.

4. Optimality of an internal R&D policy implies that they solve the first-order conditions, which shows the last result.

$\square$

We need the following assumption for the equilibrium existence proof, which generalizes Assumption 1 from the static debt case:

Assumption 2. *For the firm value, the order of the limit with respect to f and the supremum over c can be interchanged:*

$$\lim_{f' \to f} \sup_c \left\{ E_K(p,c,p|f') + (1-\xi)D_K(p,c,p|f') \right\} = \sup_c \lim_{f' \to f} \left\{ E_K(p,c,p|f') + (1-\xi)D_K(p,c,p|f') \right\}.$$

Lemma 2. *The entrant value $E_e(f)$ is continuous in f.*

Proof. Continuity is shown recursively.

1. From the proof of Theorem 2 it follows that $\sup_{c'>c} F_0(p, c, p^I|f)$ is continuous in f and c.

2. Assume that $\sup_{c'>c} F_{K-1}(p, c, p^I|f)$ is continuous in f and c. The mapping $\mathcal{M}_K(E|f)$ is continuous in f. Therefore, for every $\epsilon > 0$ there exists a $\delta > 0$ such that for $f' \in (f - \delta, f + \delta)$ we have

$$\|\mathcal{M}_K(E_K(p, c, p^I|f)|f') - E_K(p, c, p^I|f)\|$$
$$= \|\mathcal{M}_K(E_K(p, c, p^I|f)|f') - \mathcal{M}_K(E_K(p, c, p^I|f)|f)\|$$
$$< \epsilon.$$

   Fix one such ϵ. Applying the mapping $\mathcal{M}_K$ again leads to,

$$\|\mathcal{M}_K^2(E_K(p, c, p^I|f)|f') - \mathcal{M}_K(E_K(p, c, p^I|f)|f')\|$$
$$\leq U\|\mathcal{M}_K(E_K(p, c, p^I|f)|f') - E_K(p, c|f)\|$$
$$< U\epsilon.$$

   where,

$$U = \frac{\bar{\lambda} + \bar{p}f'}{r + \bar{\lambda} + \bar{p}f'}.$$

   This process can be repeated and leads to

$$\|\mathcal{M}_K^{m+1}(E_K(p, c, p^I|f)|f') - \mathcal{M}_K^m(E_K(p, c, p^I|f)|f')\| < U^m\epsilon.$$

Therefore, the distance between $E_K(p, c, p^I|f)$ and $E_K(p, c, p^I|f')$ is bounded by

$$\|E_K(p, c, p^I|f) - E_K(p, c, p^I|f')\|$$
$$= \|E_K(p, c, p^I|f) - \mathcal{M}_K^\infty(E_K(p, c, p^I|f)|f')\|$$
$$\leq \sum_{i=0}^\infty \|\mathcal{M}_K^{i+1}(E_K(p, c, p^I|f)|f') - \mathcal{M}_K^i(E_K(p, c, p^I|f)|f')\|$$
$$< \epsilon \sum_{i=0}^\infty U^i$$
$$= \epsilon \frac{1}{1-U}$$
$$\leq \epsilon \frac{r + \bar\lambda + \bar p\,(f + (f' - f))}{r}.$$

Take an $\tilde\epsilon > 0$ and set

$$\epsilon = \tilde\epsilon \frac{r}{r + \bar\lambda + \bar p(f + 1)}$$

then define $\tilde\delta = \min\{\delta, 1\}$. We get that for $f' \in (f - \tilde\delta, f + \tilde\delta)$

$$\frac{r + \bar\lambda + \bar p\,(f + (f' - f))}{r} \leq \frac{r + \bar\lambda + \bar p\,(f + 1)}{r} = \tilde\epsilon.$$

This implies that for every $\tilde\epsilon > 0$ there exists a $\tilde\delta > 0$ such that for $f' \in (f - \tilde\delta, f + \tilde\delta)$,

$$\|E_K(p, c, p^I|f) - E_K(p, c, p^I|f')\| < \tilde\epsilon.$$

Therefore, $E_K(p, c, p^I|f)$ is continuous in f. The same argument shows that $E_K(p, c, p^I)$ is continuous in c.

3. Since the equity value $E_K(p, c, p|f)$ is continuous in f, similar steps as in the proof of Theorem 2 show that for $F_K(p_0, c, p_0|f)$ there exists an ϵ such that

$$\lim_{\epsilon \to 0} \lim_{f' \to f} |F_K(p_0, c, p_0|f) - F_K(p_0, c \pm \epsilon, p_0|f')| = 0.$$

4. The previous step shows that for a given f, c, and $f' \to f$ there exists a coupon $c' = \lim_{\epsilon \to 0} c \pm \epsilon$ such that the firm value is continuous in f. This implies that

$$\sup_c F_K(p_0, c, p_0|f') = \sup_c \lim_{f' \to f} F_K(p_0, c, p_0|f') = \lim_{f' \to f} \sup_c F_K(p_0, c, p_0|f').$$

6

The last step follows from Assumption 2. This shows that $\sup_c F(p_0|f, c)$ is continuous in f.

5. Applying the previous steps recursively ensures that

$$\sup_{c'>c} F_K(p, c', p|f)$$

is continuous in f. This result ensures that

$$V(f, \theta_E) = \mathbb{E}^{\theta_e} \left[\sup_{c \geq 0} \{ F(p_0, c, p_0) \} \right]$$

is continuous in f and therefore that the entrant value $E^e(f)$ is continuous in f.

□

Theorem 4 (Equilibrium Existence with Debt Refinancing). *If Assumption 2 holds, then there exists an industry equilibrium ψ^* in the model with debt refinancing.*

Proof. The proof has several steps

1. It follows from Theorem 2 that $F_0(p, c, p^I)$ converges to zero as $f \to \infty$. Assume $F_{K-1}(p, c, p^I|f)$ converges to zero as $f \to \infty$. If $E_K(p, c, p^I|f)$ does not converge to zero as $f \to \infty$ then for some p we have that $E_K(p, c, p^I|f) > 0$ when $f \to \infty$. This directly implies that the firm does not restructure for this p. Furthermore, from equation (3) it follows that for any $p > 0$ with $E_K(p, c, p^I|f) > 0$

$$0 = \frac{-r E_K(p, c, p^I|f) + (1-\pi)(p-c)}{f}$$

$$+ \frac{\max_{(\lambda, \theta)} \left\{ \lambda \left(\mathbb{E}^\theta \left[E_K(\min\{p+x, \bar{p}\}, c, p^I) \right] - E_K(p, c, p^I|f) \right) - (1-\pi)q(p, \lambda, \theta) \right\}}{f}$$

$$+ p \left\{ E_K(p-1, c, p^I|f) - E_K(p, c, p^I|f) \right\}.$$

Given that $E_K(p, c, p^I|f) \leq ((1-\pi)\bar{p} + \pi\bar{c})/r$ and $\lambda \leq \bar{\lambda}$, taking $f \to \infty$ implies that

$$0 = p \left\{ E_K(p-1, c, p^I|f = \infty) - E_K(p, c, p^I|f = \infty) \right\}$$

and therefore that

$$E_K(p, c, p^I|f = \infty) = E_K(p-1, c, p^I|f = \infty)$$

7

for any p for which $E_K(p, c, p^I | f = \infty) > 0$. Given that $E_K(0, c, p^I | f = \infty) = 0$ this implies that

$$E_K(p, c, p^I | f = \infty) = 0$$

which is a contradiction. Therefore, the equity value goes to zero as $f \to \infty$. The debt value also goes to zero when $f \to \infty$ since the default time and the recovery value in default go to zero. This result implies that $F_K(p, c, p^i)$ goes to zero as $f \to \infty$. Recursively applying this argument ensures that the entrant value $E^e(f)$ goes to zero as $f \to \infty$.

2. If $\exists f$ such that $E^e(f) > H(1 - \pi)$ then Lemma 2, the previous step, and the intermediate value theorem imply there exists an f^* such that

$$E^e(f^*) = H(1 - \pi),$$

which is an industry equilibrium.

3. If $\nexists f$ such that $E^e(f) > H(1 - \pi)$ then $f^* = 0$ is an industry equilibrium.

□

2 Dynamic Debt Model: Additional Results

We calibrate the dynamic debt model using an additional parameter κ, that captures the repurchase price of debt $\rho(p^I) = \frac{\kappa}{r}$. We set $\kappa = 87\%$.

	Leverage Mean	Leverage Variance	Value p.p. Mean	Tax benefit	Turnover rate
Comparative statics. All values are in %.					
Max # new products per innovation					
$n = 2$	29.20	2.67	0.31	4.38	0.75
$n = 3$	28.21	2.51	0.40	4.23	0.94
$n = 4$	27.93	1.62	0.47	4.19	1.13
After-tax entry cost					
$H = 4$	28.82	1.72	0.39	4.32	2.47
$H = 5$	28.21	2.51	0.40	4.23	0.94
$H = 6$	23.17	1.49	0.39	3.48	0.48
Innovation cost scale					
$\beta = 23$	28.11	1.66	0.50	4.22	1.03
$\beta = 26$	28.21	2.51	0.40	4.23	0.94
$\beta = 29$	27.64	1.84	0.45	4.15	1.09
Innovation cost curvature					
$\gamma = 0.333$	27.47	1.94	0.43	4.12	1.16
$\gamma = 0.345$	28.21	2.51	0.40	4.23	0.94
$\gamma = 0.357$	26.69	2.10	0.38	4.00	0.78
Tax rate					
$\pi = 0.10$	23.12	1.58	0.41	2.31	0.76
$\pi = 0.15$	28.21	2.51	0.40	4.23	0.94
$\pi = 0.20$	30.45	2.37	0.40	6.09	1.06
Debt issuance cost					
$\xi = 0\%$	28.62	2.17	0.41	4.29	0.99
$\xi = 1.09\%$	28.21	2.51	0.40	4.23	0.94
$\xi = 4.36\%$	25.24	2.55	0.41	3.79	0.77

Table .4: **Comparative statics of selected moments (dynamic debt model).**