# Short-Term Debt and Incentives for Risk-Taking*

Marco Della Seta[†] Erwan Morellec[‡] Francesca Zucchi[§]

June 21, 2019

**Abstract**

We challenge the view that short-term debt curbs moral hazard and analytically demonstrate that, in a world with financing frictions and fair debt pricing, short-term debt increases incentives for risk-taking. To do so, we develop a model in which firms are financed with equity and short-term debt and cannot freely optimize their default decision because of financing frictions. Using this model, we show that short-term debt can give rise to a "rollover trap," a scenario in which firms burn revenues and cash reserves to absorb severe rollover losses. In the rollover trap, shareholders find it optimal to increase asset risk in an attempt to improve interim debt repricing and prevent inefficient liquidation. These risk-taking incentives do not arise when debt maturity is sufficiently long.

Keywords: Short-term debt financing; rollover risk; risk-taking; financing frictions.

JEL Classification Numbers: G32, G35.

*We thank the editor (Toni Whited), two anonymous referees, Thomas Dangl, Engelbert Dockner, Hyunsoo Doh (CICF discussant), Sebastian Gryglewicz, Julien Hugonnier, Hayne Leland, Ye Li, Semyon Malamud, Konstantin Milbradt (AFA Discussant), Kristian Miltersen, Martin Oehmke, Steven Ongena, Dino Palazzo, Jay Ritter, Alejandro Rivera, Michael Sockin, Yuri Tserlukevich (ITAM discussant), Tak-Yuen Wong, Vijay Yerramilli (UT Dallas discussant), Josef Zechner, and participants at Université Paris Dauphine, the University of St Gallen, the Vienna University of Economics and Business, the 2017 China International Conference in Finance, the 2018 ITAM Finance Conference, the 2018 UT Dallas Fall Finance Conference, the 2019 AFA Annual Meeting, and the CAFIN workshop at UC Santa Cruz for comments. Erwan Morellec acknowledges financial support from the Swiss Finance Institute. The views expressed in the paper are those of the authors and should not be interpreted as reflecting the views of the Federal Reserve System or its staff.

[†]STOXX Ltd. E-mail: marco.della.seta@stoxx.com.

[‡]EPF Lausanne, Swiss Finance Institute, and CEPR. E-mail: erwan.morellec@epfl.ch.

[§]Federal Reserve Board of Governors. E-mail: francesca.zucchi@frb.gov.

# 1 Introduction

A central result in corporate finance is that equity holders in levered firms have incentives to increase asset risk, as they benefit from successful outcomes of high-risk activities while the losses from unsuccessful outcomes are borne by debtholders (see Jensen and Meckling (1976)).[1] As argued in the corporate finance literature, this "potential agency cost can be substantially reduced or eliminated by using shorter-term debt" (Leland and Toft (1996)).[2] Similarly, following Calomiris and Kahn (1991), much of the banking literature argues that short-term debt disciplines management, because the fragility induced by short-term debt prevents managerial moral hazard.

The view that short-term debt disciplines management and curbs moral hazard does not accord well, however, with the available empirical evidence. In their survey of corporate managers, Graham and Harvey (2001) find little evidence that short-term debt reduces the chance that shareholders take on risky projects. In a recent study, Chen and Duchin (2019) find that firms with high levels of *short-term* debt outstanding increase the riskiness of their assets when close to distress by investing in financial securities. Admati and Hellwig (2013), Admati, DeMarzo, Hellwig, and Pfleiderer (2013), and Eisenbach (2017) also question this theory by observing that the increasing reliance on short-term debt in the years before the financial crisis of 2007-2009 went hand in hand with exceedingly risky activities, a point also made in the final report of the Financial Crisis Inquiry Commission (2011).[3] Admati, DeMarzo, Hellwig, and Pfleiderer (2013) further note that "in addition to recent history, there are conceptual reasons to doubt the effectiveness of "debt renewal" as an optimal disciplining mechanism. Absent insolvency

[1] See Eisdorfer (2008), Gormley and Matsa (2011), Becker and Stromberg (2012), and Favara, Morellec, Schroth, and Valta (2017) for evidence on risk-shifting in non-financial firms. See Keeley (1990), Gan (2004), Laeven and Levine (2009), Ellul and Yerramilli (2013), and Jiménez, Ongena, Peydro, and Saurina (2014, 2017) for evidence on risk-shifting in financial institutions.

[2] This view was first expressed in Barnea, Haugen and Senbet (1980). Important contributions to this literature also include Leland (1998), Cheng and Milbradt (2012), or Huberman and Repullo (2015).

[3] In its final report, the commission writes: "Too many [...] institutions acted recklessly, taking on too much risk, with too little capital, and with too much dependence on short-term funding."

or market failure, debt can always be renewed at a sufficient yield."

In this paper, we develop a model that rationalizes this evidence using two important features of real world environments: Financing frictions and fair pricing of risky debt. Notably, we show that, in a world with financing frictions and fair debt pricing, short-term debt does not decrease but, instead, increases incentives for risk-taking. To demonstrate this result and examine its implications for corporate policies, we formulate a dynamic model in which firms are financed with equity and short-term debt and cannot freely optimize their default decisions because of financing frictions. In this model, debt is repriced continuously to reflect changes in firm performance. Firms operate risky assets and have the option to invest in risk-free, liquid assets such as cash reserves. They also have access to zero-NPV investments with random return—for instance derivatives contracts—that they can use to change asset risk. Firms maximize shareholder value by choosing their precautionary buffers of liquid assets as well as their payout, financing, risk taking, and (constrained) default policies.

As in Leland and Toft (1996), Leland (1998), He and Xiong (2012a), and much of the literature on short-term debt and rollover risk, we consider that when a short-term bond matures, the firm rolls it over at market price. When the proceeds from debt rollover net of debt issuance costs are lower than the principal of the maturing bond, the firm bears rollover losses. To avoid default, shareholders need to absorb these losses. A fundamental difference between our work and prior contributions is that we do *not* assume that outside equity can be issued instantly and at no cost to absorb rollover losses. Rather, firms face financing frictions, which may lead to forced, inefficient liquidations. This in turn provides shareholders with incentives to build up liquidity buffers that can be used to absorb operating or rollover losses and reduce expected refinancing costs and the risk of inefficient liquidation.

A first result of the paper is to show that combining fairly-priced short-term debt with financing frictions generates risk-taking incentives for shareholders, thereby rationalizing the evidence discussed above. Consider first the effects of financing frictions on shareholders' risk-taking incentives. As shown by previous corporate finance models (e.g., Décamps, Mariotti, Rochet, and Villeneuve (2011) or Bolton, Chen, and Wang (2011)), shareholders in a solvent firm facing financing frictions behave in a risk-averse fashion to avoid inefficient liquidation. In a different setup, Leland (1994a) and Toft and Prucyk (1997) similarly show that shareholders become effectively risk-averse when default can be exogenously triggered by debt covenants or by capital requirements. In these models, shareholders cannot freely optimize the timing of default. If the firm is liquidity constrained but fundamentally profitable, default is suboptimal to shareholders. In such instances, the equity value function becomes concave, and shareholders effectively behave as if they were risk-averse.[4]

In all these models, debt is either absent or has infinite maturity. Our main contribution is to show that introducing fairly-priced short-term debt yields radically different implications. Notably, when a firm experiences negative operating shocks, default risk increases. This leads to a drop in the price of newly-issued debt and to an increase in rollover losses. Rollover losses therefore compound operating losses, increasing further default risk. Because firms issuing debt with shorter maturity need to roll over a larger fraction of their debt, this amplification mechanism is stronger for firms financed with shorter-term debt. When firms are close to distress and debt maturity is short enough, rollover losses can become larger than net income. We call this scenario, in which expected net cash flows to shareholders are negative because of severe rollover losses and the firm "burns" cash reserves, the "rollover trap." In the rollover trap, the dynamic interaction of operating and rollover losses (and the resulting amplification of shocks) fuels default risk. As a result, shareholders have incentives to increase asset volatility in an attempt to improve firm performance and interim debt repricing and thereby reduce

[4] This is also the case in the Black and Scholes (1973) model, in which maximum leverage ratio or minimum interest coverage ratio requirements imply that equity is akin to a down-and-out call option on the firm's assets (see e.g. Black and Cox (1976)). In this case, shareholders do not have incentives to shift risk when firms fundamental worsen and asset value approaches the "knock-out" barrier corresponding to the protective covenant or regulatory requirement (see Derman and Kani (1996)).

the risk of inefficient liquidation.

Our result that short-term debt generates risk-taking incentives when debt is fairly priced is fundamentally driven by the presence of financing frictions and the ensuing inability of shareholders to freely optimize their default decision. Indeed, this result also obtains in Leland-type models if default decisions are constrained, for instance by debt covenants or capital requirements.[5] These risk-taking incentives in the presence of financing frictions do not arise when debt maturity is sufficiently long (or when firms are all-equity financed). In such cases, debt needs to be rolled over less often (or never), rollover losses are small (or absent), and expected net cash flows are always positive. As a result, the main effect of financing frictions is to expose shareholders to the risk of an inefficient liquidation, so that shareholders do not want to increase asset risk.

After demonstrating the effects of short-term debt on risk-taking incentives, we investigate whether risk-taking strategies give rise to agency conflicts between debtholders and shareholders. To this end, we allow asset risk to be changed through time via positions in zero-NPV investments with random return (as in e.g. Bolton, Chen, and Wang (2011) or Hugonnier, Malamud, and Morellec (2015a)) and examine the effects of risk-taking on the values of equity and risky debt. We show that agency conflicts arise *if* debt maturity is sufficiently short and the firm bears rollover losses. When rollover losses are moderate, only shareholders have risk-taking incentives close to distress. In this case, debtholders want to preserve their coupon and principal payments and have no incentives to increase asset risk. By contrast, when rollover losses are large, debtholders also have risk-taking incentives at the brink of distress, when their promised payments are at stake. Our results therefore imply that a conflict of interest between shareholders and debtholders arises. Indeed, because shareholders capture all the returns above those

[5]Our paper follows prior models on financing frictions (e.g. Décamps et al. (2011) or Bolton et al. (2011)) by assuming that the firm cash flows are governed by an arithmetic Brownian motion. This differs from Leland-type models in which cash flows are governed by a geometric Brownian motion. We show in the Supplementary Appendix that our result that short-term debt increases risk-taking incentives does not rest on specific assumptions about the stochastic process governing firm cash flows.

required to service debt and are protected by limited liability, they may have incentives to increase asset risk far from distress, when suboptimal for debtholders. We show that firms financed with short-term debt are more likely to face such agency problems when they have lower profitability, more volatile cash flows, less tangible assets, or when they face higher debt issuance costs. We also find that these effects are stronger when these zero-NPV investments are positively correlated with the firm's asset in place, as a positive correlation leads to larger negative shocks that magnify rollover losses.

Lastly, we demonstrate that our results are not driven by the specific way in which financing frictions are modeled. In fact, our results hold in the extreme case in which the firm does not have access to the equity market (and financing frictions are the largest) as well as when assuming that the cost of raising outside equity is time-varying. In the Supplementary Appendix to the paper, we also show the robustness of our results to a number of alternative setups. First, we consider the possibility for the firm to acquire additional debt via a credit line. We show that when credit lines are senior to market debt (as is typically the case), rollover losses are larger when the firm approaches distress, which magnifies shareholders' incentives for risk-taking. Second, we show that our results are not driven by the specific assumption about the stochastic process governing firm cash flows, but rather by the shareholders' inability to freely optimize their default timing. To do so, we relax the assumption that shareholders have deep pockets in a setup à la Leland (1994b, 1998) and confirm in this setup our result that short-term debt generates risk-taking incentives. We additionally consider a setup in which both equity issuance costs and firm profitability are time-varying and show that all of our results hold in this more general model as well. Finally, while our goal is to understand how debt maturity affects risk-taking incentives for firms facing financing frictions, we also investigate in the Supplementary Appendix how the mechanisms at play in the paper can affect the firm's ex-ante capital structure choice.

Our work is related to the recent papers that incorporate financing frictions into dynamic models of corporate financial decisions. These include Bolton, Chen, and Wang (2011, 2013), Décamps, Mariotti, Rochet, and Villeneuve (2011), Asvanunt, Broadie, and

Sundaresan (2011), Hugonnier, Malamud and Morellec (2015a), Décamps, Gryglewicz, Morellec, and Villeneuve (2017), or Malamud and Zucchi (2019). In this literature, it is generally assumed that firms are all-equity financed. Notable exceptions are Gryglewicz (2011), Bolton, Chen, and Wang (2015), and Hugonnier and Morellec (2017), in which firms and/or financial institutions are financed with equity and long-term (infinite maturity) debt. In these models, firms are fundamentally solvent and because financing frictions introduce the risk of forced liquidations, shareholders behave as if they were risk-averse. That is, convexity in equity value and risk-taking incentives do not arise in these models.[6] Our paper advances this literature by characterizing the interaction between debt maturity and corporate policies and by showing that short-term debt and rollover losses can foster risk taking when firms are close to financial distress.

Our paper also relates to the literature that examines the relation between short-term debt financing and credit risk by using dynamic models with rollover debt structure. Starting with Leland (1994b, 1998) and Leland and Toft (1996), these models show that short-term debt generally leads to an increase in default risk via rollover losses. Important contributions in this literature include Hilberink and Rogers (2002), Ericsson and Renault (2006), Hackbarth, Miao, and Morellec (2006), He and Xiong (2012a), He and Milbradt (2014), Dangl and Zechner (2016), DeMarzo and He (2018), or Chen, Cui, He, and Milbradt (2018). All of these models assume that shareholders have deep pockets and can inject funds in the firm at no cost (i.e. there are no financing frictions) or just do not allow firms to hoard precautionary cash reserves. In our model, firms face financing frictions and optimally retain part of their earnings in cash reserves to absorb potential rollover losses. Consistent with this modeling, Harford, Klasa, and Maxwell (2014) document that refinancing risk due to short-term debt financing represents a key motivation for cash hoarding in non-financial firms.

[6] Notable exceptions are Hugonnier, Malamud and Morellec (2015a) and Babenko and Tserlukevich (2017), in which equity value can be locally convex *away* from distress due to lumpy investment. In Bolton, Chen, and Wang (2013), convexity arises if shareholders want to time the equity market and issue equity before their cash reserves are depleted. In these models, firms are all-equity financed.

The relation between risk-taking incentives and debt maturity is also at the core of the paper by Cheng and Milbradt (2012). In their model, assets mature at a random time and with a random liquidation value (which may fall below the face value of debt), implying that creditors may have incentives to run. In Cheng and Milbradt (2012), debt that is too short term is inefficient as it leads to high run risk. Debt that is too long term increases risk-shifting incentives when the firm is far from default. In our model, assets-in-place are subject to Brownian shocks and, because they face financing frictions, firms optimally retain earnings to hedge these shocks. Debt is fairly priced to reflect time-varying operating performance and, as a result, debtholders have no incentives to run (in line with the intuition in Admati et al. (2013) discussed above).[7] Instead, default occurs when firms run out of cash. The dynamic interaction of operating and rollover losses fuels default risk and provides shareholders in firms financed with short-term debt with incentives to increase asset volatility when close to distress. Shareholders' risk-taking incentives decrease with debt maturity (as the amplification of operating losses due to debt rollover gets weaker) and do not arise if debt maturity is sufficiently long.

Our paper is also related to the early studies of Diamond (1991) and Flannery (1986, 1994), in which short-term debt can be repriced given interim news. Debt repricing implies that the yield on corporate debt changes over time to reflect the firm's operating performance. A central difference with these papers is that, in our dynamic model, there are always creditors who are willing to buy debt at a sufficient yield and debt repricing does not lead short-term debt to discipline shareholders.

Lastly, our paper also relates to the banking literature on the disciplining role of short-term debt; see e.g. Calomiris and Kahn (1991), Diamond and Rajan (2001), Diamond (2004), or Eisenbach (2017).[8] In this literature, the fragility induced by short-term debt financing prevents moral hazard problems. The experience leading up to the 2007-

[7]On page 1 of the paper, we report the following quote: "in addition to recent history, there are conceptual reasons to doubt the effectiveness of "debt renewal" as an optimal disciplining mechanism. Absent insolvency or market failure, debt can always be renewed at a sufficient yield."

[8]In Eisenbach (2017), short-term debt is effective as a disciplining device only if firms face purely idiosyncratic shocks. Otherwise, good aggregate states lead to excessive risk-taking while bad aggregate

2009 crisis calls into question the effectiveness of short-term debt as a disciplining device. Admati and Hellwig (2013) note, for example, that "in light of this experience, the claim that reliance on short-term debt keeps bank managers "disciplined" sounds hollow," as the heavy reliance on short-term debt was accompanied by overly risky activities. Our paper shows that short-term debt financing exacerbates incentives for risk-taking when debt is fairly priced and shareholders' cannot freely optimize their default decision because of financing frictions, regulatory constraints, or debt covenants.

The paper is organized as follows. Section 2 presents the model. Section 3 demonstrates the effects of short-term debt on risk-taking and discusses the key implications of the model. Section 4 analyzes optimal dynamic risk-taking strategies and their effects on the values of corporate securities. Section 5 examines the robustness of our predictions to alternative model setups. Section 6 concludes. Proofs are gathered in the Appendix.

## 2 Model and assumptions

Throughout the paper, time is continuous and all agents are risk neutral and discount cash flows at a constant rate $r > 0$. The subject of study is a firm held by shareholders that have limited liability. As in He and Xiong (2012a), one may interpret this firm as any firm, either financial or non-financial. However, our model is perhaps more appealing for financial firms because of their heavy reliance on short-term debt financing.[9]

Specifically, we consider a firm that owns a portfolio (or operates a set) of risky, illiquid assets whose size is normalized to one. These assets generate after-tax cash flows given by $dY_t$ and governed by:

$$dY_t = (1 - \theta)(\mu dt + \sigma dZ_t), \tag{1}$$

where $\mu$ and $\sigma$ are positive constants, $(Z_t)_{t \geq 0}$ is a standard Brownian motion representing

states suffer costly fire sales.

[9] A number of intermediaries, such as insurance companies, hedge funds, brokers/dealers, special purpose vehicles, and government-sponsored enterprises, do not take deposits directly from households, but in many ways behave like banks in debt markets (see Krishnamurthy (2010)).

random shocks to cash flows, and $\theta \in (0,1)$ is the corporate tax rate. Equation (1) implies that over any time interval $(t, t+dt)$, the after-tax cash flows from risky assets are normally distributed with mean $(1-\theta)\mu dt$ and volatility $(1-\theta)\sigma\sqrt{dt}$. This in turn implies that the firm can make profits as well as losses. This cash flow specification is similar to that used in the models examining the effects of financing frictions on firm decisions by Bolton, Chen, and Wang (2011), Décamps, Mariotti, Rochet, and Villeneuve (2011), or Hugonnier, Malamud, and Morellec (2015a) or in the contracting models of DeMarzo and Sannikov (2006) or DeMarzo, Fishman, He, and Wang (2012).

In addition to these assets in place, shareholders have access to zero-NPV investments with random returns that they can use to increase the riskiness of the firm's assets. Notably, we follow Bolton, Chen, and Wang (2011), Hugonnier, Malamud, and Morellec (2015a), and Décamps, Gryglewicz, Morellec, and Villeneuve (2017) and assume that the firm has access to futures contracts whose price is a Brownian motion $B_t$. To focus on risk-taking rather than risk management, we assume that the Brownian motion $B_t$ is positively correlated with the Brownian motion $Z_t$ driving the firm cash flows and denote the correlation coefficient by $\rho$, i.e.

$$\mathbb{E}\left[dZ_t dB_t\right] = \rho dt, \quad \rho \in [0,1].$$

This assumption implies that risk-taking strategies can lead to large losses by creating correlation across the firm's assets. Futures positions are generally constrained by margin requirements. We thus consider that the futures position $\gamma_t$ cannot exceed some fixed size $\Gamma > 0$. That is, in contrast with Leland (1998), we do not impose the increase in asset volatility to be fixed and exogenous but, rather, allow shareholders to endogenously choose their optimal increase in asset volatility between zero and a maximum value.

The firm is financed with equity and risky debt, which provides tax benefits. To make our results comparable with prior contributions, we consider finite-maturity debt structures in a stationary environment as in Leland (1998), Leland (1994b), or Hackbarth, Miao, and Morellec (2006). Notably, we assume that the firm has issued debt with constant principal $P$ and paying a constant total coupon $C < \mu$. At each moment in time, the firm rolls over a fraction $m$ of its total debt. That is, the firm continuously retires outstanding debt principal at a rate $mP$ and replaces it with new debt vintages of identical coupon, principal, and seniority. In the absence of default, average debt maturity equals $M \equiv 1/m$. As in He and Milbradt (2014), the firm pays a constant proportional debt issuance cost $\kappa \in [0, 1]$, which only plays a minor (quantitative) role for the results emphasized in the paper.

Equity financing also entails issuance costs. Following Bolton, Chen, and Wang (2013), we consider that the firm operates in an environment characterized by time-varying equity financing opportunities. Specifically, the firm can be in one of two observable states of the world. In the good state $G$, the firm can raise equity by incurring a fixed cost $\phi_G > 0$ and a proportional cost $p_G$, so that the firm gets $\xi$ when raising the amount $(1 + p_G)\xi + \phi_G$ from equity investors. In the bad state $B$, the firm has no access to outside equity or, equivalently, the funding costs $(\phi_B, p_B)$ are too high compared to the benefits of outside funds. The state switches from $G$ to $B$ (resp. from $B$ to $G$) with probability $\pi_G dt$ (resp. $\pi_B dt$) on any time interval $(t, t + dt)$.

Financing frictions provide incentives for the firm to retain earnings and build up cash reserves. We denote by $W_t$ the firm's cash/liquid reserves at time $t \geq 0$. Cash reserves earn a rate of interest $r - \lambda$ and can be used to cover operating and rollover losses if other sources of funds are costly or unavailable. The wedge $\lambda > 0$ represents a carry cost of liquidity.[10] When choosing its target level of cash reserves, the firm balances this carry cost with the benefits of liquidity.

We denote by $D_i(w)$ the market value of short-term debt in state $i = G, B$ for a level of cash reserves $w$. Debt rollover implies that short-term debt of a new vintage is issued at market price and has principal value and coupon payment given by $mP$

[10]The cost of holding cash includes the lower rate of return on these assets because of a liquidity premium and tax disadvantages (Graham (2000) finds that cash retentions are tax-disadvantaged because corporate tax rates generally exceed tax rates on interest income). This cost of carrying cash may also be related to a free cash flow problem within the firm, as in Décamps, Mariotti, Rochet, and Villeneuve (2011), Bolton, Chen, and Wang (2011), or Hugonnier, Malamud, and Morellec (2015a).

and $mC$, respectively. When the market value of newly issued debt net of the issuance cost is lower than the principal, the firm bears rollover losses. Otherwise, it enjoys rollover gains. Over any time interval $(t, t + dt)$, the rollover imbalance is given by $m[(1-\kappa)D_i(w) - P]dt$, and the dynamics of cash reserves satisfy

$$\begin{aligned} dW_t = (1-\theta)[(r-\lambda)W_t dt + (\mu - C)dt + \sigma dZ_t + \gamma_t dB_t] \\ + \underbrace{m\left[(1-\kappa)D_i(W_t) - P\right]dt}_{\text{Rollover gains/losses}} - dU_t + dH_t - dX_t. \end{aligned} \tag{2}$$

where $U_t$, $H_t$, and $X_t$ are non-decreasing, adapted processes representing respectively the cumulative payouts to shareholders, the firm's cumulative equity financing, and cumulative equity issuance costs until time $t$. Equation (2) shows that cash reserves grow with earnings net of taxes, outside financing, rollover gains, and the interest earned on cash holdings. Cash reserves decrease with payouts to shareholders, the coupon paid on outstanding debt, the cost of outside funds, and rollover losses.

The firm can be forced into default if its cash reserves reach zero following a series of negative shocks and it is not possible/optimal to raise outside funds. As in Bolton, Cheng, and Wang (2011), we assume that the liquidation value of risky assets, denoted by $\ell$, is a fraction of their book value in that $\ell \equiv 1 - \varphi$, where $\varphi \in [0,1]$ represents a haircut related to default costs. We denote by $\tau$ the stochastic default time of the firm.

Management chooses the firm's payout ($U$), financing ($H$), risk-taking ($\gamma$), and default ($\tau$) policies to maximize shareholder value. That is, management solves:

$$E_i(w) \equiv \sup_{(U,H,\gamma,\tau)} \mathbb{E}_{w,i}\left[\int_0^\tau e^{-rt}(dU_t - dH_t) + e^{-r\tau}\max\{0; \ell + W_\tau - P\}\right], \tag{3}$$

subject to (2). The first term on the right-hand side of equation (3) represents the flow of dividends accruing to incumbent shareholders, net of the claim of new shareholders on future cash flows. The second term represents the present value of the cash flow to shareholders in default. In the following, we focus on the case in which the liquidation value of assets is lower that the face value of outstanding short-term debt, i.e. $\ell < P$. Since $W_\tau = 0$ in default, short-term debt is risky.

*Discussion of assumptions*

Firms in our model have the same debt structure as firms in Leland (1994b, 1998), Hackbarth, Miao, and Morellec (2006), or Chen, Cui, He, and Milbradt (2018). As in these models, firm cash flows are stochastic and debt is repriced continuously to reflect changes in firm fundamentals. Thus, debt is always fairly priced and debtholders have no incentives to run. A key difference with our setup is that firms in these models do not face financing frictions and/or regulatory constraints. As a result, there is no role for cash holdings, the timing of default maximizes shareholder value, and shortening debt maturity decreases shareholders' incentives to increase asset risk.

Introducing financial or regulatory constraints in a setup à la Leland (1994b, 1998) implies that the firm can be forced into liquidation at a time that does *not* maximize equity value. In such instances, shortening debt maturity does not decrease but, instead, increases shareholders' incentives for risk-taking (see the Supplementary Appendix). That is, our main result is robust to different assumptions regarding the stochastic process governing the firm cash flows. In the baseline version of our model, we focus on a setup featuring precautionary cash reserves and cash flows following an arithmetic Brownian motion as in Décamps, Mariotti, Rochet, and Villeneuve (2011) or Bolton, Chen, and Wang (2011, 2013), because financing frictions are a key ingredient of our model. Consistently, Harford, Klasa, and Maxwell (2014) document that firms facing refinancing risk due to short-term debt financing have larger cash holdings.

The models of He and Xiong (2012b) and Cheng and Milbradt (2012) also share the debt structure described above. However, these models assume that firms deliver a constant cash flow through time, which is all paid out to debtholders. Because the firm's assets may be terminated at a random time and their liquidation value is assumed to fluctuate over time (and may fall below the face value of debt), debtholders have incentives to run if the liquidation value of assets falls below some endogenous threshold. Under these assumptions, Cheng and Milbradt (2012) show that optimal debt maturity balances the risk of runs of shorter term debt with the risk-shifting incentives associated with longer term debt. In contrast with these studies, our model allows periodic cash flows to vary randomly and liquidation does not occur at an exogenous Poisson time but when cash reserves are depleted. Because debt is fairly priced, debtholders have no incentives to run (in line with the intuition in Admati, DeMarzo, Hellwig, and Pfleiderer (2013) discussed in the introduction). Under these assumptions, we show that short-term debt financing can generate risk-taking incentives when close to financial distress.

Lastly, our model assumes that firms face proportional and fixed equity issuance costs, which are time-varying. To understand the effects of equity issuance costs on financing decisions and default risk, assume for now that equity issuance costs are constant over time. As shown by Décamps, Mariotti, Rochet, and Villeneuve (2011), two cases must be distinguished based on the level of issuance costs (see their Proposition 2 in Section III.A). In the first case, issuance costs are so high that it is never optimal to issue new equity and the following condition holds:

$$\max_{W_T} \left[E(W_T) - (1+p)W_T - \phi\right] \leq 0. \tag{4}$$

In this case, the firm is liquidated as soon as it runs out of cash and, as a result, debt is risky (recall that we focus on the case $\ell < P$). In the second case, issuance costs are sufficiently low that it is optimal for shareholders to raise funds and the following condition holds:

$$\max_{W_T} \left[E(W_T) - (1+p)W_T - \phi\right] > 0.$$

In this case, the firm raises new funds as soon as the cash buffer is depleted. The optimal issue size $W_T$ satisfies the optimality condition $E'(W_T) = 1 + p$, i.e. is such that the marginal benefit and cost of raising outside equity are equalized. The firm is never liquidated and, therefore, debt is risk-free. Because default and rollover risks are central to our analysis, we do not consider this case until Section 5.1, in which financing conditions are time-varying and default can happen at least in one state of the world.

## 3 The rollover trap: Short-term debt and risk-taking

In the model, management chooses the firm's payout, financing, savings, risk-taking, and default policies to maximize shareholder value. Because creditors have rational expectations, the price at which maturing short-term debt is rolled over reflects these policy choices and feeds back into the value of equity by determining the magnitude of rollover imbalances.

To aid in the intuition of the model and demonstrate the effects of short-term debt on risk taking in a world with financing frictions and fair debt pricing, we focus in this section on an environment in which the firm only raises new funds by rolling over short-term debt (facing the proportional issuance cost $\kappa$) and does not have access to outside equity. This is the case when the cost of equity financing is too high (due to, e.g., a liquidity crisis) and equity issuance costs are such that condition (4) is satisfied. In Section 5.1, we analyze a model in which the firm can raise outside equity and faces time-varying financing conditions (as described above) and show that all of our results hold in this more general model.

In addition, to better understand shareholders' risk-taking *incentives*, we start by assuming that shareholders cannot alter the volatility of cash flows. We then move on to analyzing the relation between short-term debt and dynamic risk-taking in Section 4, where we allow shareholders to dynamically alter the riskiness of assets using the zero-NPV investments with random return described in Section 2.

### *3.1 Valuing corporate securities*

We start our analysis by deriving the value of equity. In our model, financing frictions lead the firm to value inside equity and, therefore, to retain earnings. Keeping cash inside the firm, however, entails an opportunity cost $\lambda$ on any dollar saved. For sufficiently large cash reserves, the benefit of an additional dollar retained in the firm is decreasing. Since the marginal cost of holding cash is constant, we conjecture that there exists some target level $W^*$ for cash reserves where the marginal cost and benefit of cash reserves are equal and it is optimal to start paying dividends.

To solve for equity value, we first consider the region in $(0, \infty)$ over which it is optimal for shareholders to retain earnings. In this region, the firm does not deliver any cash flow to shareholders and equity value satisfies:

$$rE(w) = \Big[(1-\theta)((r-\lambda)w+\mu-C)+\underbrace{m\,[(1-\kappa)D(w)-P]}_{\text{Rollover gains/losses}}\Big]E'(w)+\frac{((1-\theta)\sigma)^2}{2}E''(w), \quad (5)$$

where we omit the subscript $i$ because there is only one financing state. The left-hand side of this equation represents the required rate of return for investing in the firm's equity. The right-hand side is the expected change in equity value in the earnings retention region. The first term on this right-hand side captures the effects of cash savings and reflects debt rollover. That is, one important aspect of this equation is that the value of short-term debt feeds back into the value of equity via rollover imbalances. The second term captures the effects of cash flow volatility.

Equation (5) is solved subject to the following boundary conditions. First, when cash reserves exceed the target level $W^*$, the firm places no premium on internal funds and it is optimal to make a lump sum payment $w - W^*$ to shareholders. We thus have

$$E(w) = E(W^*) + w - W^*$$

for all $w \geq W^*$. Subtracting $E(W^*)$ from both sides of this equation, dividing by $w-W^*$, and taking the limit as $w$ tends to $W^*$ yields the condition:

$$E'(W^*) = 1.$$

The equity-value-maximizing payout threshold $W^*$ is then the solution to the high-contact condition (see Dumas (1991)):

$$E''(W^*) = 0.$$

When the firm makes losses, its cash buffer decreases. If its cash buffer decreases sufficiently, the firm may be forced to raise new equity or to default. When the firm has no access to outside equity, it defaults as soon as its cash reserves are depleted. As a result, the condition

$$E(0) = \max\{\ell - P; 0\} = 0$$

holds at zero, and the liquidation proceeds are used to partially repay debtholders.

Consider next the value of short-term debt. Denote by $D^0(w,t)$ the date-$t$ value of short-term debt issued at time 0. Since a fraction $m$ of this original debt is retired continuously, these original debtholders receive a payment rate $e^{-mt}(C + mP)$ at any time $t \geq 0$ as long as the firm is solvent. The value of total outstanding short-term debt is defined by $D(w) \equiv e^{mt}D^0(w,t)$. Because $D(w)$ receives a constant payment rate $C + mP$, it is independent of $t$. In the following, we only need to derive the function $D(w)$, i.e. the value of total short-term debt. From this value, we can also derive the value of newly issued short-term debt, denoted by $d(w,0)$. The Appendix shows that it satisfies: $d(w,0) = mD(w)$.

To solve for the value of total short-term debt $D(w)$, we first consider the region in $(0,\infty)$ over which the firm retains earnings. In this region, $D(w)$ satisfies:

$$\begin{aligned}(r+m)D(w) = {} & [(1-\theta)((r-\lambda)w + \mu - C) + m((1-\kappa)D(w) - P)]D'(w) \\ & + \frac{1}{2}((1-\theta)\sigma)^2 D''(w) + C + mP.\end{aligned} \tag{6}$$

The left-hand side of equation (6) is the return required by short-term debtholders. The right-hand side represents the expected change in the value of total short-term debt on any time interval. The first and second terms capture the effects of a change in cash reserves and in cash flow volatility on debt value. The third and fourth terms are the coupon and principal payments to short-term debtholders.

Equation (6) is solved subject to the following boundary conditions. First, the firm defaults the first time that its cash buffer is depleted. The value of short-term debt at this point is equal to the liquidation value of assets:

$$D(0) = \min\{\ell, P\} = \ell.$$

Second, the value of short-term debt does not change when dividends are paid out, because dividend payments accrue exclusively to shareholders. We thus have:

$$D'(W^*) = 0.$$

### *3.2 The economic mechanism*

Before proceeding with the model analysis and demonstrating our main results, we provide some intuition on the economic mechanism underlying these results—in particular, how short-term debt can generate risk-taking incentives.

Our model incorporates two important features of real world environments: Financing frictions and fair pricing of risky debt. Consider first the effects of financing frictions on shareholders' risk taking incentives. As shown by previous dynamic models, shareholders in a profitable firm facing financing frictions behave in a risk-averse fashion to preserve equity value and prevent inefficient liquidations (see, e.g., Décamps et al. (2011) or Bolton et al. (2011)). Similarly, Leland (1994a) and Toft and Prucyk (1997) show that equity value can become a concave function of asset value in Leland-type models when the possibility of inefficient liquidation is introduced, e.g., via protective debt covenants or liquidity constraints. In these environments, shareholders cannot freely optimize the timing of default and, if the firm is fundamentally solvent (implying that the default option has a negative payoff), the equity value function is concave and shareholders are effectively risk-averse.

In all of these models, debt is either absent or has infinite maturity. The main contribution of our paper is to show that allowing for fairly-priced short-term debt financing in the presence of financing frictions yields radically different implications. Notably, when debt has finite maturity, it needs to be rolled over. If the firm cash flows deteriorate, the market value of newly-issued debt drops, leading to rollover losses. If rollover losses become sufficiently large, expected net cash flows may turn negative. When this is the case, shareholders have incentives to increase asset risk and "gamble for resurrection" to improve firm performance and avoid inefficient closure.

To single out this economic mechanism, consider a counterfactual firm financed with equity and infinite maturity debt (as in Leland (1994a), Bolton, Chen, and Wang (2015), or Hugonnier and Morellec (2017)). Since this firm does not need to roll over debt, its equity value $E_\infty(w)$ satisfies

$$rE_\infty(w) = (1-\theta)\left[(r-\lambda)w + \mu - C\right] E'_\infty(w) + \frac{1}{2}\left((1-\theta)\sigma\right)^2 E''_\infty(w)$$

in the earnings retention region. This equation is solved subject to the following boundary conditions: $E_\infty(0) = E'_\infty(W^*_\infty) - 1 = E''_\infty(W^*_\infty) = 0$, where $W^*_\infty$ is the optimal payout trigger for shareholders when debt maturity is infinite. The value of risky, infinite-maturity debt in turn satisfies

$$rD_\infty(w) = (1-\theta)\left[(r-\lambda)w + \mu - C\right] D'_\infty(w) + \frac{1}{2}\left((1-\theta)\sigma\right)^2 D''_\infty(w) + C$$

in the earnings retention region, which is solved subject to $D_\infty(0) - \ell = D'_\infty(W^*_\infty) = 0$.

Three important features differentiate a firm financed with infinite-maturity debt from a firm financed with finite-maturity debt. First, while the value of debt reflects the equity value-maximizing payout/savings policy ($W^*_\infty$ enters the debt's boundary conditions), the market value of infinite-maturity debt *does not* directly affect the market value of equity, because debt does not need to be rolled over. By contrast, when maturity is finite, the repricing of debt affects the market value of equity via debt rollover.

Second, expected net cash flows from assets in place net of coupon payments and corporate taxes are given by $(1-\theta)(\mu - C)dt > 0$ when the firm has issued infinite maturity debt, i.e. they are time-invariant and positive. As a result, the expected change in cash reserves on each interval of length $dt$ is given by

$$(1-\theta)[(r-\lambda)w + \mu - C]dt > 0,$$

and is always positive because $\mu > C$ and $w \geq 0$. By contrast, expected net cash flows are given by $[(1-\theta)(\mu - C) + m((1-\kappa)D(w) - P)]dt$ in the finite-maturity case and can become negative if rollover losses are sufficiently large. As a result, the expected change in cash reserves on each interval of length $dt$ is given by

$$[(1-\theta)((r-\lambda)w + \mu - C) + m((1-\kappa)D(w) - P)]dt, \tag{7}$$

and can become negative if rollover losses are sufficiently large.

Third, because the firm is solvent and its expected net cash flows are positive in the infinite maturity debt case, shareholders behave in a risk-averse fashion. The reason is that shareholders want to avoid inefficient liquidation (or save on refinancing costs in the model with time-varying costs analyzed in Section 5.1) and have no incentives to increase asset risk, even when the firm is levered. By contrast, expected net cash flows as well as the expected change in cash reserves can become negative in the finite debt maturity case because of rollover losses. In these instances, the firm is temporarily unprofitable and, as a result, shareholders' incentives to keep the firm away from default weaken. The value of equity becomes convex (because of shareholders' limited liability), and shareholders have incentives to increase the riskiness of assets in order to improve firm fundamentals and debt repricing close to distress, as we show next.

### *3.3 Risk-taking generated by short-term debt financing*

When a firm is financed with finite maturity debt (i.e. $m \neq 0$), it needs to roll over maturing debt. Fair debt pricing implies that the value of newly-issued debt may differ from the principal repayment on maturing debt, leading to rollover imbalances. Over each time interval of length $dt$, rollover imbalances are given by the difference between the market value of newly-issued debt (net of issuance costs) and the repayment on maturing debt:

$$R(w)dt \equiv m[(1-\kappa)D(w) - P]dt.$$

Because liquidation becomes less likely as cash reserves $w$ grow, the value of debt is monotonically increasing in $w$ in the earnings retention region. Thus, there exists at most one threshold $\overline{W}$ at which the rollover imbalance is zero, such that: $(1-\kappa)D(\overline{W}) = P$ (see Appendix A.2.1 for a proof). The firm bears rollover losses for any $w < \overline{W}$, as the market value of debt net of issuance costs is smaller than the principal repayment $P$. That is, lower cash reserves are associated with higher default risk, which reduces the value of newly-issued debt. Conversely, for $w \in (\overline{W}, W^*]$, the firm is financially strong and default risk is low. The proceeds from newly-issued debt exceed the principal repayment of maturing debt.

As we show next, short-term debt financing (and the associated rollover losses) can generate convexity in equity value and, thus, risk-taking incentives when firms face financing frictions. The reason is that as the firm approaches financial distress, the market value of debt decreases, and rollover losses increase. As a result, when the firm is sufficiently close to default, the expected change in cash reserves (i.e., expression (7)) can be negative and the firm can become temporarily unprofitable. This leads to the following proposition (see Appendix A.2).

**Proposition 1 (Short-term debt and convexity in equity value)** *When a firm is financed with finite maturity debt, equity value is locally convex when rollover losses are sufficiently large that the inequality*

$$(1-\theta)[(r-\lambda)w + \mu - C] + m[(1-\kappa)D(w) - P] \leq 0 \tag{8}$$

*holds. In such instances, short-term debt financing provides shareholders with risk-taking incentives in a right interval of* $w = 0$.

A direct implication of Proposition 1 is that, in the presence of financing frictions and fair pricing of short-term debt, shareholders have risk-taking incentives if expected net cash flows are negative so that condition (8) is satisfied. To understand this result, note that as long as (8) is satisfied, the sum of the expected net cash flows from assets-in-place (net of coupon payments), the interest earned on cash holdings, and the proceeds from newly issued debt (net of issuance costs) is lower than the repayment of maturing debt. In other words, rollover losses are larger than net income. As a result, when condition (8) holds, the value of an additional unit of cash to shareholders is low because it plays a minor role in helping the firm escape financial distress.[11] Indeed, that unit of cash

[11] This result is consistent with the evidence in Faulkender and Wang (2006), who show that shareholders place a relatively low value on cash when they are burdened by sizable debt obligations.

will be used to repay maturing debt and not to rebuild cash reserves. In expectation, the firm makes rollover losses, further reducing its cash reserves and increasing the risk of inefficient liquidation. In such instances, shareholders want to improve firm fundamentals and interim debt repricing to turn cash flows from negative to positive, which provides them with incentives to increase risk.

As we show next, condition (8) is more likely to hold when the rollover frequency is greater or, equivalently, when the average maturity of outstanding debt $M \equiv \frac{1}{m}$ is shorter. Risk-taking incentives decrease as debt maturity $M$ increases because the fraction of debt that needs to be rolled over on each time interval is smaller (and so are rollover losses) and do not arise with infinite maturity debt ($M \to \infty$). In fact, if the firm is financed with infinite-maturity debt (so that debt is never rolled over as $m \to 0$), there are no rollover losses (the second term on the left hand side of (8) is equal to zero) and condition (8) never holds as the first term on the left hand side of (8) is strictly positive. The next Proposition formalizes this intuition (see Appendix A.3):

**Proposition 2 (Short-term debt and incentives for risk-taking)** *Shareholders' risk-taking incentives arise if the firm is financed with debt with maturity shorter than the critical maturity* $\overline{M} = \frac{P-\ell(1-\kappa)}{(1-\theta)(\mu-C)}$ *that makes condition* (8) *bind at* $w = 0$*. Risk-taking incentives do not arise if debt maturity exceeds* $\overline{M}$*.*

Proposition 2 shows that short-term debt (as opposed to long-term debt) can generate incentives for risk-taking—in fact, if debt maturity is sufficiently long, risk-taking incentives do not arise. Whenever $M < \overline{M}$, convexity in the value of equity arises whenever rollover losses are large enough that condition (8) holds. In this case, the firm "burns" cash reserves in expectation, and expected net cash flows are negative because of severe rollover losses. We call this scenario "the rollover trap." When a firm is in the rollover trap, the marginal value of cash progressively increases as the firm approaches the break-even point at which (8) becomes positive. The marginal value of cash to shareholders only starts decreasing with cash reserves—and equity value becomes concave—when expected cash flows become sufficiently large to guarantee that an additional unit of cash helps increase cash reserves rather than cover rollover losses.

Proposition 2 shows that shareholders' risk-taking incentives arise if debt maturity is sufficiently short. That is, in contrast with the long-standing idea that short-term debt curbs risk-taking incentives, we show that when firms face financing frictions and debt is fairly priced, short-term debt is more likely to generate risk-taking incentives, in line with the evidence discussed in the introduction. In addition, the degree of shareholders' effective risk-aversion is negative and larger in absolute magnitude as maturity decreases below the critical level $\overline{M}$, as we show next.

**Proposition 3 (Short-term debt and incentives for risk-taking)** *For any debt maturity below the critical level $\overline{M}$, the shorter is debt maturity, the more shareholders are effectively risk-loving and have risk-taking incentives.*

Shareholders are effectively risk averse if $M > \overline{M}$, as in dynamic models with financing frictions such as Décamps et al. (2011) or Bolton, Chen, and Wang (2011). In turn, Proposition 3 shows that shareholders are effectively risk-loving if $M < \overline{M}$, and more so as debt maturity decreases below this critical level. As a result, shareholders have stronger incentives to engage in risk-taking activities (see Appendix A.4).

Our result that short-term debt is associated with larger risk-taking incentives contrasts with previous models of rollover risk in which shareholders have deep pockets and can optimally choose the timing of default; e.g. Leland and Toft (1996) or Leland (1998). In these models, equity value is convex, and short-term debt reduces incentives for risk-taking. As shown by Leland (1994a) and Toft and Prucyk (1997), equity value can become a concave function of asset value in Leland-type models when the possibility of inefficient liquidation is introduced, e.g., via protective debt covenants or capital requirements. In these environments, shareholders are effectively risk-averse and have no risk-taking incentives. We show in the Supplementary Appendix that short-term debt can restore the convexity of equity value in such a setup as well. The reason is that short-term debt can make net cash flow negative because of severe rollover losses. Whenever the firm experiences severe rollover losses, shareholders value the option to default, are no longer risk-averse and, thus, have incentives to increase asset risk (they hold an out-of-the-money option). This extension shows that our results are not driven by the specific assumption about the stochastic process governing the firm cash flow.

It is also important to note that the principal and the coupon payment on outstanding aggregate debt are fixed in our model, as in Leland and Toft (1996), Leland (1998), He and Xiong (2012a), or He and Milbradt (2014) among many others. This assumption does not trim the generality of our results. Suppose indeed that shareholders are allowed to take on more debt when close to distress, to cover operating losses. As the face value of debt increases, rollover losses get larger when the firm approaches distress, which magnifies shareholders' incentives for risk-taking. (The Supplementary Appendix illustrates this point by allowing the firm to take on more debt via credit line drawdowns.) Suppose instead that the firm is allowed to decrease leverage by buying back some of its debt at par value. Debt reductions would reduce the firm's ability to cover operating losses, because the firm would need to use its cash balances to repurchase debt. Additionally, repurchasing debt in the region where the firm makes rollover losses would transfer wealth from shareholders to debtholders (since $D(w) < P$), making it subptimal for shareholders to buy back debt. Thus, we deem this scenario as unrealistic—because the wealth transfers associated with debt repurchases would lead to a leverage ratchet effect—as well as unfeasible—because the firm would have to use cash to finance the leverage reduction, thereby getting even closer to a forced default.[12]

Lastly, an alternative strategy for shareholders could be to decrease debt maturity after a series of negative shocks, in order to pay a smaller yield on corporate debt. However, as discussed by He and Milbradt (2014), the presence of a positive cost of issuing debt ($\kappa > 0$) rules out a strategy in which shareholders keep shortening maturity ($M \to 0$) and always manage to avoid default by reducing maturity in response to a

[12]The firm's reluctance to reduce debt in this context is consistent with the "leverage ratchet effect" discussed in Admati, DeMarzo, Hellwig, and Pfleiderer (2018) and Hugonnier, Malamud, and Morellec (2015b), according to which "shareholders pervasively resist leverage reduction no matter how much such reductions may enhance firm value."

sequence of negative operating shocks. In addition, after such a sequence of negative shocks, reducing debt maturity would increase the fraction of debt that needs to be rolled over on any $dt$, thereby magnifying the amplification of operating shocks via rollover losses, widening the rollover trap, and strengthening risk-taking incentives.

## *3.4 Incentive compatibility problems*

An important question is whether risk-taking incentives generated by short-term debt financing are a source of agency conflicts. Agency conflicts arise if shareholders have risk-taking incentives (i.e., the value of equity is convex) whereas debtholders do not (i.e., the value of debt is concave). This section seeks to answer this question.

The dynamics of the value of short-term debt in the earnings retention region are given by equation (6). Now, consider a firm that expects cash reserves to decrease because of large rollover losses (i.e. condition (8) is satisfied). A key difference between debt and equity is that debtholders receive the periodic payments $C + mP > 0$ (coupon plus principal payments) in the earnings retention region. Because debtholders want to preserve these periodic payments, they only have incentives to increase asset risk at the very brink of distress, when rollover losses are substantial and these payments are at stake. To see this, consider the valuation equation for equity when condition (8) in Proposition 1 holds:

$$\underbrace{rE(w)}_{\geq 0} = \underbrace{\left[(1-\theta)((r-\lambda)w + \mu - C) + m((1-\kappa)D(w) - P)\right] E'(w)}_{\leq 0} + \frac{1}{2}\left((1-\theta)\sigma\right)^2 E''(w). \tag{9}$$

Because shareholders have limited liability and equity value is increasing in cash reserves, equity value is convex whenever condition (8) holds. Now, consider the valuation equation for debt when condition (8) holds:

$$\underbrace{(r+m)D(w)}_{\geq 0} = \underbrace{[(1-\theta)((r-\lambda)w+\mu-C)+m((1-\kappa)D(w)-P)]D'(w)}_{\leq 0} \qquad (10)$$
$$+\frac{1}{2}\left((1-\theta)\sigma\right)^2 D''(w)+\underbrace{C+mP}_{\geq 0}.$$

Because of the periodic payment to debtholders $C + mP \geq 0$ (the last two terms on the right-hand side), debt value can become convex *if* the left-hand side of condition (8) is sufficiently negative. That is, condition (8) is necessary but not sufficient for convexity in debt value to arise and the region of convexity for the value of risky debt is always smaller than the region of convexity for equity value (or may not exist). An incentive compatibility problem therefore exists for the range of cash reserves for which the value of equity is convex and the value of debt is concave. This leads to the following proposition (see Appendix A.5).

**Proposition 4 (Agency conflicts and risk-taking)** *Rollover losses can give rise to convexity in debt value. The region of convexity in debt value is smaller than the region of convexity in equity value, giving rise to agency conflicts between shareholders and debtholders.*

Figure 1 illustrates the results in Proposition 4. When debt maturity is sufficiently long, both shareholders and debtholders are effectively risk averse and there are no agency conflicts (top panel). When debt maturity is sufficiently short so as to generate convexity in equity value, two scenarios are possible. First, only shareholders have incentives to increase asset risk, and an agency conflict arises when cash reserves are close to zero (middle panel). Second, both shareholders and debtholders have incentives to increase asset risk at the very brink of distress. In this case, an agency problem still arises for intermediate levels of cash reserves (bottom panel).

Insert Figure 1 Here

It is worth noting that our predictions are different from the Jensen and Meckling (1976) result that risk-shifting incentives are larger when firms are close to default. In our model, maturity plays a key role in determining risk-taking incentives—i.e., shareholders have no incentives to increase asset risk if the firm is financed with debt with sufficiently long maturity. In addition, risk-taking incentives lead to agency conflicts for intermediate levels of cash reserves, but risk taking may be optimal at the very brink of distress for both shareholders and debtholders.

### *3.5 Numerical implementation*

While our result on the relation between short-term debt and risk-taking incentives is analytical, it is useful to provide a numerical implementation of our model.

Insert Table 1 Here

The baseline values of the model parameters are reported in Table 1. We set the cash flow drift $\mu = 0.09$, which is consistent with van Binsbergen, Graham, and Yang (2010), Chen and Duchin (2019) as well as is in the range of values reported by Whited and Wu (2006) and Hugonnier, Malamud, and Morellec (2015a). We set $\sigma = 0.08$, which is consistent with the estimates in Graham and Leary (2018). We set the corporate tax rate to $\theta = 0.3$, which is slightly below the average value reported by Graham, Leary, and Roberts (2015) for the period 1920-2010. We take a smaller value to reflect recent regulation aimed at decreasing the tax burden of U.S. firms—our value is closer to the tax rate parameter value considered for example in Bolton, Chen, and Wang (2015), He and Xiong (2012a), or He and Milbradt (2014). The value of the opportunity cost of cash $(r - \lambda)$ is set to 0.01, as in Bolton, Chen, and Wang (2011) and Décamps, Mariotti, Rochet, and Villeneuve (2011). The risk-free rate is set to $r = 3.5\%$. We base the value of liquidation costs on the estimates of Glover (2016) and set $\varphi = 0.45$, which gives an effective bankruptcy recovery rate that is consistent with the value in He and Milbradt (2014). Following Altinkilic and Hansen (2000), we set the proportional cost of debt issuance to $\kappa = 0.01$. Finally, we set the aggregate principal and coupon payment on debt to $P = 0.75$ and $C = 0.036$, respectively. Since assets-in-place are normalized to one in our baseline model, the ratio interests/assets is in line with the value reported by van Binsbergen, Graham, and Yang (2010). Moreover, our principal value implies that the ultimate recovery rate is consistent with the value reported by He and Milbradt (2014). In the following, we provide a number of comparative statics that illustrate the robustness of our results to different parameter values.

Figure 2 plots rollover imbalances as a function of cash reserves and shows that they are markedly asymmetric in that rollover losses are larger in absolute value than rollover gains. The reason is that at the target cash level, positive operating shocks are paid out to shareholders, and debt value is insensitive to these shocks (i.e., $D'(W^*) = 0$). As a result, debt value is almost insensitive to cash inflows or outflows when cash reserves are sufficiently large. The top left panel of the figure also shows that rollover losses are more severe when debt maturity is shorter, because the fraction of debt that needs to be rolled over on each time interval is larger. The top right panel shows that rollover losses are larger if the firm is less profitable (i.e., $\mu$ decreases). If profitability deteriorates, the market value of debt decreases and rollover imbalances become more negative, all else equal. Figure 2 also shows that rollover losses are increasingly larger as liquidation costs increase (because the market value of debt is lower) and as debt issuance costs get larger (because rolling over outstanding debt is more expensive).

Insert Figure 2 Here

Figure 3 plots the value of equity $E(w)$ and the marginal value of cash to shareholders $E'(w)$ as functions of cash reserves, $w \in [0, W^*]$. Figure 3 shows that the value of equity is increasing in cash reserves. However, Figure 3 also shows that the relation between value of equity, debt maturity, and cash reserves is non-trivial and reflects the potential losses generated by debt rollover. A shorter debt maturity decreases (respectively, increases) the value of equity when cash reserves are small (large) due to rollover losses (gains). Equity value is concave and shareholders are quasi risk-averse for any $w$ for long debt maturities. Equity value can be locally convex close to liquidity distress (i.e., when $w$ is close to zero) if debt maturity $M$ is sufficiently short (in our base case environment, $\overline{M}$ is about 5.4).

Insert Figure 3 Here

To understand under which conditions short-term debt is more likely to generate incentives for risk-taking, Figure 3 also plots the value of equity $E(w)$ and the marginal value of equity $E'(w)$ as functions of cash reserves for varying levels of asset profitability $\mu$, cash flow volatility $\sigma$, liquidation costs $\varphi$, and debt issuance costs $\kappa$. A decrease in asset profitability or an increase in cash flow volatility both lead to an expansion in the region of convexity in equity value. That is, less profitable firms as well as firms with more volatile cash flows face larger cost of debt, implying that both rollover losses and shareholders' risk-taking incentives are larger. Figure 3 additionally shows that larger liquidation costs are associated with a larger region of convexity for equity value. A lower recovery rate makes debt more risky and rollover losses more severe, which in turn fuels risk-taking incentives. Figure 3 also shows that an increase in debt issuance cost $\kappa$ decreases equity value and leads to larger rollover losses and to a larger region over which shareholders have risk-taking incentives. Consistent with these results, a lower asset profitability $\mu$ or firm liquidation value $\ell$, or a greater debt issuance costs $\kappa$, lead to a greater $\overline{M}$, so that the range of debt maturities for which shareholders have risk-taking incentives is wider.

We next turn to incentive compatibility problems. To understand when agency conflicts are more likely to arise, Table 2 reports the inflection points for debt ($W_D$) and equity ($W_E$), the size of the region over which equity value is convex and debt value is concave (the agency region $AR$), as well as the target cash level ($W^*$) for different debt maturities ($M$), cash flow drift ($\mu$), cash flow volatility ($\sigma$), liquidation costs ($\varphi$), and debt issuance cost ($\kappa$).

Insert Table 2 Here

Table 2 confirms our analytical results and shows that incentive compatibility problems are more likely to arise if debt maturity is short. When debt maturity is sufficiently long, equity and debt values are concave for any level of cash reserves, and both classes of claimholders behave as if they were risk-averse (this case is depicted in the top panel of Figure 1). In this case, $W_D \notin (0, W^*]$ and $W_E \notin (0, W^*]$. In Table 2, we indicate these cases using "n.a." for the values of $W_D$ and $W_E$. The last column of this panel also shows that shorter debt maturity is associated with larger cash holdings, which is consistent with the evidence reported by Harford, Klasa, and Maxwell (2014).[13]

We further investigate how risk-taking incentives vary as a function of other firm characteristics, when fixing debt maturity at $M = 1$. The second panel of Table 2 shows that both risk-taking incentives and agency problems (i.e., the agency region $AR$) decrease with profitability $\mu$. Indeed, a decrease in profitability makes debt (and debt rollover) more costly and makes it more likely that the inequality (8) is satisfied. The third panel shows that increasing volatility results in a decrease (respectively, increase) in debtholders' (shareholders') risk-taking incentives. As a result, agency problems are more likely to arise if $\sigma$ is large, all else equal. The fourth panel shows that liquidation costs increase the risk-taking incentives of both shareholders and debtholders. Liquidation costs decrease the market value of debt when close to distress, magnifying rollover losses and fueling risk-taking incentives for both shareholders and debtholders. The last panel shows that, all else equal, the region of agency conflicts is wider if debt issuance costs are greater. Overall, our results show that firms financed with short-term debt are more likely to face such agency problems when they have larger liquidation costs, lower profitability, more volatile cash flows, and greater costs of debt rollover. When $M = \infty$, agency conflicts do not arise when varying other model parameters (for instance, $\mu$, $\sigma$,

[13]In unreported results, we find that the target cash level *can* be locally increasing in debt maturity at the higher end of the maturity spectrum (especially if leverage is relatively low). The reason is that when debt maturity is sufficiently long, rollover losses are minimal. Thus, the main effect of shortening debt maturity is a decrease in the cost of debt, which leads to a decrease in the precautionary need of cash and, thus, in the target cash level.

$\varphi$, or $\kappa$), and all claimholders are effectively risk-averse.

Insert Figure 4 Here

Lastly, Figure 4 plots the value of debt $D(w)$ and the marginal value of cash to debtholders $D'(w)$ as functions of cash reserves, for different debt maturities. $D(w)$ increases with maturity, as a shortening of maturity implies an increase in rollover losses and, thus, in liquidation risk. In addition, while debtholders suffer from the risk implied by a shorter debt maturity due to larger rollover losses, they do not capture the upside potential due to any rollover gains. Figure 4 also shows that the convexity is less pronounced for debtholders than for equityholders.

## 4 Dynamic risk-taking strategies

We have just shown that, in a world with financing frictions and fair debt pricing, short-term debt financing generates a local convexity in the value of equity and, to a lower extent, in the value of risky debt when the firm is close to financial distress (i.e. in a right neighborhood of $w = 0$, as established in Appendix A.2.1). This naturally raises the question of whether this convexity may lead to a gambling behavior, in which management engages in zero NPV investments with random returns in an attempt to improve firm value. Financial markets provide a natural way for the firm to take such gambles. In this section, we analyze the effects of risk-taking strategies on the value of corporate securities by allowing the firm to take positions in the future contracts described in Section 2. This setup is consistent with the evidence in Chen and Duchin (2019), who show that distressed firms are more likely to engage in risk-shifting using financial (as opposed to real) assets.

A position $\gamma_t$ in future contracts changes after-tax cash flows from $dY_t$ to

$$dY_t + (1-\theta)\gamma_t dB_t = (1-\theta)\left[\mu dt + (\sigma + \gamma\rho)\, dZ_t + \gamma\sqrt{1-\rho^2} dB_t^Z\right]$$

where $B^Z$ is a Brownian motion that is independent of $Z$. That is, a position in these contracts only changes the volatility component of cash flows, i.e. the riskiness of the firm's assets, but not the cash flow drift. Futures positions are generally constrained by margin requirements. To capture these requirements, we consider that the futures position $\gamma_t$ cannot exceed some fixed size $\Gamma > 0$. Our setup does not require the increase in asset volatility to be fixed and exogenous (as, for instance, in Leland, 1998) but, rather, allows shareholders to endogenously choose their optimal increase in asset volatility between zero and a realistically finite, maximum value.

Assuming frictionless trading in derivatives contracts, standard arguments show that in the region over which the firm retains earnings, equity value satisfies:

$$\begin{aligned} rE(w) = {} & [(1-\theta)((r-\lambda)w + \mu - C) + m((1-\kappa)D(w) - P)]E'(w) \\ & + \frac{(1-\theta)^2}{2} \max_{0 \leq \gamma \leq \Gamma} \left\{ \left[ (\sigma + \gamma\rho)^2 + \gamma^2(1-\rho^2) \right] E''(w) \right\}. \end{aligned} \tag{11}$$

where the last term on the right-hand side of equation (11) captures the effects of risk-taking on equity value.

By differentiating with respect to $\gamma$, we can analytically determine the optimal risk-taking strategy for shareholders. Under the assumption that $\rho \geq 0$, it is optimal to take on the maximal risk $\gamma = \Gamma$ if $E''(w) > 0$. That is, using the results in Appendix A.2.1 that there is at most one region of convexity for the function $E(w)$, the value of equity is defined over three intervals: $[0, W_E(\Gamma))$, $[W_E(\Gamma), W^*(\Gamma))$, and $[W^*(\Gamma), \infty)$, where $W_E(\Gamma)$ represents the threshold below which shareholders engage in risk-taking and $W^*(\Gamma)$ is the optimal payout threshold. Equity value solves equation (11) subject to boundary conditions at zero and at the target cash level $W^*(\Gamma)$, as well as continuity and smoothness conditions at $W_E(\Gamma)$. This leads to the following Proposition (see Appendix A.6 for analytical details).

**Proposition 5 (Optimal dynamic risk-taking strategy for shareholders)** *For all $w$ such that $E''(w) > 0$, shareholders find it optimal to increase the volatility of assets by taking the maximum position in future contracts. For all $w$ such that $E''(w) < 0$, shareholders behave as if they were risk-averse and do not take positions in future contracts.*

*As a result, the optimal risk-taking policy is given by:*

$$\gamma = \begin{cases} \Gamma & \text{if } 0 \leq w < W_E(\Gamma), \\ 0 & \text{if } W_E(\Gamma) \leq w < W^*(\Gamma). \end{cases}$$

Proposition 5 reveals that the optimal risk-taking policy is of bang-bang type: If risk-taking is optimal for shareholders because equity value is convex, shareholders choose the riskiest strategy (see Appendix A.6). Our result can therefore rationalize the evidence in Gan (2004) that if (financial) firms are hit by a shock that wipes out their profits, they tend to choose either the minimal or the maximal feasible risk. Our model illustrates that firms take the minimal or the maximal risk depending on debt maturity: If maturity is sufficiently short (respectively, long), firms will take on the maximal (minimal) risk. Moreover, our result that risk-taking arises for firms approaching distress is consistent with the evidence in Eisdorfer (2008), Becker and Strömberg (2012), Favara, Morellec, Schroth, and Valta (2017), or Chen and Duchin (2019), among others.

Insert Figure 5 Here

The top panel of Figure 5 shows the effect of different risk-taking strategies on the value of equity, when debt maturity $M$ is one year (left panel) and two years (right panel). The figure shows that risk-taking increases shareholder value when the value of equity is convex—that is, when debt maturity is short and cash reserves are low. The figure also shows that the increase in equity value due to risk-taking is greater when debt maturity is shorter, in which case the region of convexity is larger. Moreover, when equity value is convex, the strategy associated with the largest increase in cash flow volatility (i.e., the largest $\Gamma$) is the one that increases equity value the most.

Proposition 4 and Proposition 5 together lead to the following result.

**Corollary 6 (Risk-taking and debt value)** *Risk-taking leads to: (1) an increase in the value of debt in the region over which debt value is convex, (2) a decrease in the value of debt in the region over which equity value is convex but debt value is concave.*

To illustrate the results in Corollary 6, recall the scenarios represented in the middle and bottom panels of Figure 1. In the middle panel, debt value is concave for any level of cash reserves. In this case, risk-taking strategies lead to a decrease in the value of debt. This result is in line with previous models following Jensen and Meckling (1976). Shareholders have incentives to increase asset risk in distress, and this is detrimental to debtholders. As a result, risk taking increases credit risk.

The bottom panel of Figure 1 illustrates a different scenario, in which debt value can be locally convex. As shown in Section 3.4, the value of debt is locally convex when rollover imbalances are large and the firm is sufficiently close to distress. In such instances, risk-taking strategies increase the values of equity *and* debt. This result is illustrated in the bottom panel of Figure 5, which shows that increasing asset volatility leads to a modest decrease in yield spreads (i.e., a modest increase in debt value) at the very brink of distress. However, because the size of the region of convexity in equity value is larger than the region of convexity in debt value (by Proposition 4), shareholders have incentives to increase asset risk even when this is suboptimal for debtholders. Consistently, Figure 5 shows that increasing asset volatility leads to an increase in yield spreads for intermediate levels of cash reserves—i.e. when the value of debt is concave. The increase in yields is greater when debt maturity is shorter (because, as shown in Table 2, the agency region is larger) and amplified for larger values of $\Gamma$.

Insert Figure 6 Here

Figure 6 further investigates how the correlation between assets-in-place and the risk-taking asset (i.e., the futures contracts) influences the effects described above. If the two types of assets are more correlated, risk-taking can lead to a larger increase in equity value when the firm is in the rollover trap. However, these strategies also lead to a substantial increase in credit spreads when the firm holds intermediate levels of cash reserves—that is, when the firm is outside distress. The higher the correlation between assets-in-place and the risk-taking instruments, the more likely the firm will face sizable positive shocks (that may help the firm escape the trap) but also significant negative shocks (that can lead the firm into financial distress more quickly). This second, negative effect is one of the factors that increased the severity of the recent financial crisis, as the correlation among portfolios held by financial institutions translated into massive realized losses at the peak of the crisis.

# 5 Robustness to alternative model specifications

## *5.1 Time-varying financing conditions*

Having explained the effects of short-term debt on corporate policies and incentives for risk-taking in a model in which firms do not have access to outside equity, we now analyze a more general environment in which funding conditions are time-varying, as described in Section 2. In such an environment, the firm still finds it optimal to hold cash reserves, but the target level of cash reserves is state-dependent, denoted by $W_i^*$. Notably, because financial frictions are more severe in state $B$ than in state $G$, we expect the target level of cash reserves to be larger in state $B$. That is, we expect $W_B^* > W_G^*$. Another key difference with the model presented in Section 3 is that the firm can raise equity at a cost in state $G$.

To solve for equity value, we first consider the region in $(0, \infty)$ over which it is optimal for firm shareholders to retain earnings. In this region, the firm does not deliver any cash flow to shareholders and equity value satisfies for $i = G, B$, $i \neq j$:

$$rE_i(w) = [(1-\theta)((r-\lambda)w + \mu - C) + m((1-\kappa)D_i(w) - P)]E_i'(w) + \frac{1}{2}\left((1-\theta)\sigma\right)^2 E_i''(w) + \pi_i\left[E_j(w) - E_i(w)\right]. \tag{12}$$

Equation (12) is solved subject to the following boundary conditions. First, when cash reserves exceed $W_i^*$, the firm places no premium on internal funds, and it is optimal to make a lump sum payment $w - W_i^*$ to shareholders. As a result, we have

$$E_i(w) = E_i(W_i^*) + w - W_i^*$$

for all $w \geq W_i^*$. Subtracting $E_i(W_i^*)$ from both sides of this equation, dividing by $w - W_i^*$, and taking the limit as $w$ tends to $W_i^*$ yields the condition:

$$E_i'(W_i^*) = 1.$$

The equity-value-maximizing payout threshold $W_i^*$ is then the solution to:

$$E_i''(W_i^*) = 0.$$

When the firm makes losses, its cash buffer decreases. If its cash buffer decreases sufficiently, the firm may be forced to raise new equity or to liquidate. Consider first state $G$ in which refinancing is possible. In this state, the firm may raise funds before its cash buffer gets completely depleted to avoid that financing conditions worsen when cash reserves are close to zero (as in Bolton, Chen, and Wang (2013)). We denote the issuance boundary in state $G$ by $W_L \in [0, W_G^*)$ and the level of cash reserves after equity issuance by $W_H \in [W_L, W_G^*)$. For any $w \leq W_L$ in state $G$, the firm raises new equity and resets its cash buffer to $W_H$. This implies that

$$E_G(w) = E_G(W_H) - (1 + p_G)(W_H - w) - \phi_G, \quad \forall w \leq W_L.$$

If $W_L$ is strictly greater than zero, the firm effectively taps the equity markets before its cash reserves are depleted. In this case, it must be that the condition

$$E_G'(W_L) = E_G'(W_H) = 1 + p_G$$

holds. Indeed, management delays equity issues until the marginal value of cash to shareholders equals the marginal cost of refinancing, given by $1 + p_G$.

Consider next state $B$. In that state, the firm has no access to outside funding and defaults as soon as its cash reserves are depleted. As a result, the condition

$$E_B(0) = \max\{\ell - P; 0\} = 0$$

holds at zero and the liquidation proceeds are used to repay debtholders.

Note that the cash reserves process evolves in $[0, W_B^*]$ in the bad state and in $[W_L, W_G^*]$ in the good state. This implies that if the financing state switches from bad to good while the firm's cash reserves are in $(0, W_L]$, the firm immediately taps the equity market to raise its cash reserves to $W_H$. In these instances, the value of equity jumps from $E_B(w)$ to $E_G(W_H) - (1 + p_G)(W_H - w) - \phi_G$ for any $w \in [0, W_L]$. If, instead, the financing state switches from bad to good when $w \in [W_G^*, W_B^*]$, the firm makes a lump sum payment to shareholders and cash reserves go down to $W_G^*$.

To solve for the value of total short-term debt $D_i(w)$, we also first consider the region in $(0, \infty)$ over which the firm retains earnings. In this region, $D_i(w)$ satisfies for $i = G, B$, $i \neq j$:

$$
\begin{aligned}
(r+m)D_i(w) = {} & [(1-\theta)((r-\lambda)w + \mu - C) + m((1-\kappa)D_i(w) - P)]D_i'(w) \\
& + \frac{1}{2}\left((1-\theta)\sigma\right)^2 D_i''(w) + C + mP + \pi_i \left[D_j(w) - D_i(w)\right].
\end{aligned}
$$

This system of equations is solved subject to the following boundary conditions. First, the firm is liquidated the first time that the cash buffer is depleted in the bad state. The value of short-term debt at this point is equal to the liquidation value of assets:

$$D_B(0) = \min\{\ell, P\} = \ell.$$

In the good state, management raises new equity up to $W_H$ whenever cash reserves are at or below $W_L$. Since the net proceeds from the issue are stored in the cash reserve, the value of short-term debt satisfies:

$$D_G(w) = D_G(W_H), \qquad \text{for } w \leq W_L.$$

Lastly, the value of short-term debt does not change when dividends are paid out, because dividend payments accrue to shareholders. We thus have:

$$D_i'(W_i^*) = 0, \qquad \text{for } i = G, B.$$

To fully characterize the value of short-term debt, note that if the state switches from bad to good when $w \in (0, W_L]$, shareholders raise new funds to set cash reserves to $W_H$ and the value of short-term debt jumps from $D_B(w)$ to $D_G(W_H)$. In addition, if the state switches from bad to good when $w \in (W_G^*, W_B^*]$, the firm makes a payment $w - W_G^*$ to shareholders, leading to a jump in the value of debt from $D_B(w)$ to $D_G(W_G^*)$.

We first analyze how time-varying financing conditions affect the price at which short-term debt is rolled over and the magnitude and sign of rollover imbalances. Consider first the bad state $B$. In that state, the firm may be forced into default after a series of negative shocks because it is too costly to raise new equity if it runs out of funds. Thus, the bad state displays a pattern that is analogous to the case analyzed in Section 3. Specifically, there exists a level of cash reserves $\overline{W}_B$ such that $(1-\kappa)D_B(\overline{W}_B) = P$, i.e. such that the net proceeds from the debt issue are equal to the principal repayment. Rollover imbalances are negative (respectively positive) below (above) the threshold $\overline{W}_B$. Moreover, as in Section 3, rollover imbalances decrease as debt maturity increases.

Consider next the good state. In this state, default never occurs because the firm can always raise capital by paying the costs of equity issuance, and the value of newly-issued debt is greater than in the bad state. As noted by Acharya, Krishnamurthy, and Perotti (2011): "Creating exposure to liquidity risk is profitable in good times, but creates vulnerability to massive losses when the risk perception changes." Figure 7 (where we use the parameters in Table 1) illustrates these results. In line with this intuition, Figure 7 (top panel) shows that short-term debt financing may be attractive to shareholders in the good state, because the market value of debt is relatively larger and so are the proceeds from debt rollover, which increases equity value (middle panel). However, short-term debt leads to rollover losses in the bad state, which increases default risk and decreases the value of equity.

Insert Figure 7 Here

The analysis in Section 3 has shown that the value of equity can be locally convex when rollover losses are large. As shown by Figure 7, this pattern is preserved in the bad state when financing conditions are time-varying. The value of equity can also be locally convex in the good state, but for a different reason (Figure 7, bottom panel). In the good state, this convexity is related to the possibility to time the market by issuing securities when the cost of external finance is low, as in Bolton, Chen, and Wang (2013). Overall, Figure 7 demonstrates that short-term debt generates incentives for risk-taking in this alternative financing environment too. That is, our main result is not specific to the way financing frictions are modeled.

### *5.2 Additional robustness checks*

In the Supplementary Appendix, we show the robustness of our results to alternative model setups. First, we consider the possibility for the firm to acquire additional debt via a credit line. We show that when credit lines are senior to market debt (as is typically the case), rollover losses are larger when the firm approaches distress, which magnifies shareholders' incentives for risk-taking (this applies more generally when short-term debt is subordinated to other claims). Second, we show that our results are not driven by the specific assumption about the stochastic process governing firm cash flows, but rather by the shareholders' inability to freely optimize their default timing. To do so, we relax the assumption that shareholders have deep pockets in a setup à la Leland (1994b, 1998) and confirm our result that short-term debt generates risk-taking incentives when firm profitability is time-varying. Finally, as an alternative way to model time-varying firm profitability, we extend the setup analyzed in Section 5.1 and assume that the cash flow drift $\mu$ is state-contingent (i.e., $\mu_G > \mu_B$). Again, we find that short-term debt increases incentives for risk-taking.

## 6 Conclusion

A commonly-accepted view in corporate finance and banking is that short-term debt can discipline management and curb moral hazard, thereby improving firm value. This view seems to be challenged, however, by the available evidence. This paper shows that, for firms facing financing frictions or other constraints that prevent shareholders from freely maximizing their default decisions, short-term debt does not decrease but, instead, increases incentives for risk-taking. To demonstrate this result and examine its implications for corporate policies, we develop a model in which firms are financed with equity and risky short-term debt and face taxation, financing frictions, and default costs. In this model, firms own a portfolio/operate a set of risky assets. They have the option to build up cash reserves and can take positions in zero-NPV investments that increase the volatility of assets. Firms maximize shareholder value by choosing their cash buffers as well as their financing, risk taking, and default policies.

With this model, we show that when a firm has short-term debt outstanding and debt is fairly priced, negative operating shocks lead to a drop in cash reserves and cause the firm to suffer losses when rolling over short-term debt, thereby amplifying the effects of operating shocks. This amplification mechanism leads to an increase in default risk, that gets more pronounced for firms financed with shorter-term debt as these firms need to roll over a larger fraction of debt on each time interval. When firms are close to distress and debt maturity is short enough, rollover losses can be larger than expected operating profits, dragging the firm closer to default. In contrast with extant models with long-term debt financing and financing frictions or with short-term debt but without financing frictions (or other frictions that prevent shareholders from freely optimizing their default decision), our model demonstrates that in such instances short-term debt provides shareholders with incentives for risk-taking. That is, we show that financing frictions combined with fair debt pricing imply behavior that is in sharp contrast with the long-standing idea that short-term debt has a disciplinary role and reduces agency costs.

## Appendix

### *A.1 Deriving the value of short-term debt*

We start by deriving the value of total short-term debt, denoted by $D(w)$. Since the firm keeps a stationary debt structure, $D(w)$ receives a constant payment rate $C + mP$ that is independent of $t$. Following standard arguments, the function $D(w)$ satisfies:

$$(r+m)D(w) = [(1-\theta)((r-\lambda)w + \mu - C) + (1-\kappa)d(w) - mP]D'(w) \\ + \frac{1}{2}\left((1-\theta)\sigma\right)^2 D''(w) + C + mP$$

where $d(w)$ is the value of currently-issued short-term debt. For any given time $t$, we denote by $d(w,\tau)$ the value of the outstanding debt of generation $\tau \leq t$, with $\tau \in [-\infty, 0]$. Therefore, $d(w,0) = d(w)$ represents the value of currently-issued short-term debt (i.e., $\tau = 0$ at the current time), and we have the following relation

$$d(w,\tau) = \; e^{m\tau} d(w) \,.$$

All remaining units of short-term debt from prior issues have the same value per unit, as units of all vintages pay the same coupon, and the remaining units of all vintages will be retired at the same fractional rate. However, there are fewer outstanding units of debt of older generations due to accumulated debt retirement. Integrating $d(w,\tau)$ over $\tau \in [-\infty, 0]$ gives the total value of short-term debt outstanding $D(w)$, and then the following important relation

$$D(w) = d(w) \int_{-\infty}^{0} e^{\tau m} d\tau = \frac{d(w)}{m}$$

holds. Using this relation, together with the ODE describing the dynamics of $D(w)$, we finally get the ODE for currently issued short-term debt, given by

$$rd(w) = [(1-\theta)((r-\lambda)w + \mu - C) + (1-\kappa)d(w) - mP]d'(w) \\ + \frac{1}{2}((1-\theta)\sigma)^2 d''(w) + mC + m\left[mP - d(w)\right].$$

The third term on the right-hand side implies that the short-term debt issued today promises a coupon payment $mC$ on any time interval. Recall that exponential repayment of debt with average maturity $1/m$ implies that debt matures randomly at the jump times of a Poisson process with intensity $m$. The fourth term on the right-hand side then represents the payoff obtained by the debtholders when the debt randomly matures times the probability of this occurrence.

## *A.2 Proof of Proposition 1*

The left-hand side of condition (8) in Proposition 1 represents the expected change in cash reserves on each time interval of length $dt$. This expression is negative if rollover losses (the second term in this expression) more than offset the sum of interests on cash and expected operating cash flows net of the coupon payment on outstanding debt and taxes (the first term, which is positive as the firm is fundamentally solvent):

$$\underbrace{(1-\theta)((r-\lambda)w+\mu-C)}_{>0}+\underbrace{m\left((1-\kappa)D(w)-P\right)}_{<0}\leq 0.$$

The second-term in this expression is zero if the firm is financed with infinite maturity debt (as $M \to \infty$ implies $m \to 0$). It is negative if $m \neq 0$ and the firm is facing rollover losses.

The left-hand side of condition (8) enters the valuation equation of equity (see equation (5)). As equity value increases with cash reserves ($E'(w) > 0$), the first term on the right-hand side of equation (5) is negative when condition (8) holds. Thus, because shareholders have limited liability (meaning that $E(w) \geq 0$), equity value becomes locally convex whenever condition (8) holds. (Equation (9) helps to see this result). In the next subsection, we prove that there is only one region of convexity that is located in a right interval of $w = 0$.

### *A.2.1 Characterizing the convexity in equity value*

We now show that there cannot be multiple regions of convexity for the equity value function. Denote by $W_1$ the lowest non-negative cash level at which $E''(w) = 0$. We want to show that $E(w)$ is concave for any $w \in (W_1, W^*)$, meaning that equity value does not have multiple regions of convexity. To do so, we start by establishing the following result:

**Lemma 7** *The function $D'(w)$ decreases (i.e. $D''(w) < 0$) over the cash interval in which the expected change in cash reserves on a small time interval*

$$\Phi(w) \equiv (1-\theta)((r-\lambda)w + \mu - C) + m((1-\kappa)D(w) - P)$$

*is positive.*

**Proof.** Differentiating the ODE for the value of debt, we have

$$\begin{aligned}(r+m)D'(w) =& [(1-\theta)((r-\lambda)w + \mu - C) + m((1-\kappa)D(w) - P)]D''(w) \\ & [(1-\theta)(r-\lambda) + m(1-\kappa)D'(w)]D'(w) + \frac{1}{2}\left((1-\theta)\sigma\right)^2 D'''(w).\end{aligned}$$

Rearranging, we have

$$\begin{aligned}&[\theta r + \lambda(1-\theta) + m - m(1-\kappa)D'(w)]\, D'(w) = \\ &[(1-\theta)((r-\lambda)w + \mu - C) + m((1-\kappa)D(w) - P)]D''(w) + \frac{1}{2}\left((1-\theta)\sigma\right)^2 D'''(w).\end{aligned} \tag{A1}$$

which can also be written as

$$\begin{aligned}[\theta r + \lambda(1-\theta) + m]\, D'(w) =& [(1-\theta)((r-\lambda)w + \mu - C) + m((1-\kappa)D(w) - P)]D''(w) \\ & + \frac{1}{2}\left((1-\theta)\sigma\right)^2 D'''(w) + m(1-\kappa)(D'(w))^2.\end{aligned} \tag{A2}$$

Equation (A2) implies that there cannot be a negative local minimum for $D'(w)$. In fact, the existence of a negative local minimum would imply that $D'(w) < 0$, $D''(w) = 0$, and $D'''(w) > 0$, and equation (A2) would not hold. Because $D'(W^*) = 0$ at the payout boundary, the fact that a negative local minimum cannot exist implies that $D'(w)$ is always positive.[14] As a result, $D(w)$ is monotonically increasing with cash reserves. This implies that $\Phi(w)$ is increasing with cash reserves (i.e., $\Phi'(w) > 0$), which in turn implies that there is at most one level of cash reserves at which $\Phi(w)$ changes sign from negative to positive. We denote this cash level by $\underline{W}$ (i.e., $\Phi(\underline{W}) = 0$). Similarly, there is at most one cash level at which the rollover imbalance is zero—using the notation in the paper, there is only one $\overline{W}$ such that $(1-\kappa)D(\overline{W}) = P$.

Now, we want to show that $D(w)$ is concave (i.e. that $D'(w)$ monotonically decreases) in the region over which $\Phi(w)$ is positive (i.e., over $(\underline{W}, W^*]$). Consider the value of debt at $W^*$ and at a generic cash level $w \in [\underline{W}, W^*]$ in the region over which $\Phi(w)$ is positive. At the payout threshold $W^*$, debt value satisfies:

$$(r+m)D(W^*) = \frac{1}{2}\left((1-\theta)\sigma\right)^2 D''(W^*) + C + mP.$$

because $D'(W^*) = 0$. At the generic cash level $w \in [W_1, W^*]$, debt value satisfies:

$$\begin{aligned}(r+m)D(w) = [(1-\theta)((r-\lambda)w + \mu - C) + m((1-\kappa)D(w) - P)]D'(w)\\ + \frac{1}{2}\left((1-\theta)\sigma\right)^2 D''(w) + C + mP.\end{aligned}$$

Subtracting the two equations, we have

$$\begin{aligned}(r+m)\left[D(W^*) - D(w)\right] = -\left[(1-\theta)((r-\lambda)w + \mu - C) + m((1-\kappa)D(w) - P)\right]D'(w)\\ + \frac{1}{2}\left((1-\theta)\sigma\right)^2\left[D''(W^*) - D''(w)\right]\end{aligned}$$

As shown above, $D(w)$ is increasing ($D'(w)$ is always positive), so the left-hand side of this equation is positive. The first term on the right-hand is just $-\Phi(w)D'(w)$. Since both the drift and $D'(w)$ are positive, this term is negative. So, the term $D''(W^*) - D''(w)$ needs to be positive or, equivalently, $D''(W^*) > D''(w)$ needs to hold. At $w = W^*$, equation (A1) boils down to

$$\begin{aligned}0 = [(1-\theta)((r-\lambda)w + \mu - C) + m((1-\kappa)D(W^*) - P)]D''(W^*)\\ + \frac{1}{2}\left((1-\theta)\sigma\right)^2 D'''(W^*). \qquad \text{(A3)}\end{aligned}$$

[14] Recall that $D'(w)$ is positive in a right neighborhood of zero (i.e., $D'(0) > 0$), otherwise the value of debt would go below the liquidation value in this neighborhood.

Because $D'(w)$ cannot go negative and $D'(W^*) = 0$, we have that $D'(w)$ is decreasing in a left neighborhood of $W^*$, which in turn implies that $D''(W^*) < 0$ and, for equation (A3) to balance, that $D'''(W^*) > 0$. Thus, $D''(w)$ has to be negative (and greater than $D''(W^*)$ in absolute value). As a result, we have that $D''(w) < 0$ on the interval in which the drift is positive, meaning that $D'(w)$ is decreasing on this interval. ■

We next turn to the main result of this section, which shows that there is at most one region of convexity for the value of equity.

**Lemma 8** *There is at most one region of convexity for the function* $E(w)$ *on* $[0, W^*]$.

**Proof.** Recall that $W_1$ denotes the lowest non-negative cash level at which the function switches from convex to concave. Our goal is to show that this threshold is unique and so $W_1 \equiv W_E$ (using the notation in the main body of the paper). At $W_1$, the following relation

$$rE(W_1) = [(1-\theta)((r-\lambda)W_1 + \mu - C) + m((1-\kappa)D(W_1) - P)]E'(W_1)$$

holds (simply by equation (5)). By definition of the threshold $W_1$, $E'(w)$ is positive and increasing below this cash level and decreasing in, at least, a right neighborhood of $W_1$. Because $E(W_1)$ is positive, the term $\Phi(w)$ (the term in square brackets on the right-hand side) needs to be positive at $W_1$ for the equation to balance. Using the result that there is at most one cash level at which $\Phi$ is equal to zero, we have that $\Phi(w)$ has to be positive for any $w \in [W_1, W^*]$. Differentiating equation (5), we obtain:

$$\begin{aligned} rE'(w) = {} & [(1-\theta)((r-\lambda)w + \mu - C) + m((1-\kappa)D(w) - P)]E''(w) \\ & [(1-\theta)(r-\lambda) + m(1-\kappa)D'(w)]E'(w) + \frac{(1-\theta)^2}{2}\sigma^2 E'''(w). \end{aligned}$$

Rearranging

$$\begin{aligned} & [\theta r + \lambda(1-\theta) - m(1-\kappa)D'(w)]\, E'(w) = \\ & [(1-\theta)((r-\lambda)w + \mu - C) + m((1-\kappa)D(w) - P)]E''(w) + \frac{(1-\theta)^2}{2}\sigma^2 E'''(w). \end{aligned} \tag{A4}$$

At $w = W^*$, equation (A4) satisfies:

$$[\theta r + \lambda(1-\theta)] = \frac{(1-\theta)^2}{2}\sigma^2 E'''(W^*)$$

Since $r > \lambda$ (as the return on cash is smaller than the risk-free rate $r$), the left hand side of this equation is positive. As a result, the right hand side is also positive, meaning that $E'''(W^*) > 0$. This in turn implies that $E'(w)$ is decreasing at least in a left neighborhood of $W^*$ (as $E''(W^*) = 0$, $E'' < 0$ in such a neighborhood).

We now show that $E''(w) < 0$ (i.e., $E'(w)$ is decreasing) in $w \in [W_1, W^*]$. Toward a contradiction, suppose that this is not true. This implies that there should be at least two cash levels, $W_2 \leq W_3$ in $(W_1, W^*)$ so that $E'$ is increasing in $(W_2, W_3)$ (i.e., $E''(w) > 0$ in this interval). $W_2$ would represent a positive local minimum and $W_3$ is a (second) positive local maximum for the function $E'(w)$.[15] In fact, because $E'(W^*) = 1$ holds and $E'(w)$ is decreasing in a left neighborhood of $W^*$ (as shown above), the existence of a positive local minimum implies that a second local maximum needs to exist in $[W_2, W^*]$.

At the positive local maxima, $W_1$ and $W_3$, we have $E' > 0, E'' = 0$, and $E''' < 0$. For equation (A4) to balance, we would need to have

$$\theta r + \lambda(1-\theta) - m(1-\kappa)D'(w) < 0 \quad \Rightarrow D'(w) > \frac{\theta r + \lambda(1-\theta)}{m(1-\kappa)}.$$

Conversely, at the positive local minimum $W_2$, we have $E' > 0, E'' = 0$, and $E''' > 0$. For equation (A4) to hold, it means that

$$\theta r + \lambda(1-\theta) - m(1-\kappa)D'(w) > 0 \quad \Rightarrow D'(w) < \frac{\theta r + \lambda(1-\theta)}{m(1-\kappa)}.$$

which is the opposite inequality compared to the one above.

Because $D'(w)$ is monotonically decreasing over the region in which $\Phi(w)$ is positive (as shown in Lemma 7), it cannot be that $D'(W_2) < \frac{\theta r+\lambda(1-\theta)}{m(1-\kappa)}$ and $D'(W_3) > \frac{\theta r+\lambda(1-\theta)}{m(1-\kappa)}$. Thus, there can be at most one positive local maximum for $E'(w)$ and at most one region of convexity. $W_1$ is then the unique threshold $W_E$ separating the region of convexity and concavity. ■

[15] A *negative* local minimum in the region in which the drift is positive would not exist, as shareholders would be better off paying out cash at a cash level lower than $W_2$ (i.e., the target cash level would be lower than $W_2$).

### *A.3 Proof of Proposition 2*

We start by showing that rollover losses at $w = 0$ are monotonically wider as maturity becomes shorter. As shown in Appendix A.2.1 (see the proof of Lemma 7), $D(w)$ is monotonically increasing in $w$. This implies that the firm faces the maximum rollover losses at $w = 0$, which are given by the following expression:

$$m((1-\kappa)D(0) - P) \equiv \frac{1}{M}((1-\kappa)\ell - P).$$

This expression implies that the magnitude (in absolute value) of rollover losses *monotonically* is smaller if debt maturity is longer. When $M \to \infty$ (i.e., $m \to 0$), rollover losses tend to zero. A straightforward implication of this result is that $\Phi(0) = (1-\theta)(\mu - C) + m((1-\kappa)\ell - P)$ is monotonic in debt maturity (as the first term in this expression does not change with maturity).

Consider equation (5) evaluated at $w = 0$. Because $E(0) = 0$, we have:

$$0 = [(1-\theta)(\mu - C) + m((1-\kappa)\ell - P)]E'(0) + \frac{(1-\theta)^2}{2}\sigma^2 E''(0).$$

Denote by $\overline{M}$ the debt maturity such that $E''(0) = 0$ (i.e., such that the inflection point for equity value $W_E = 0$). We have:

$$0 = [(1-\theta)(\mu - C) + \overline{m}((1-\kappa)\ell - P)]E'(0).$$

Because $E'$ is positive, the maturity $\overline{M} = 1/\overline{m}$ solves the following equation $(1-\theta)(\mu - C) + \overline{m}((1-\kappa)\ell - P) = 0$. This equation has the following unique solution:

$$\overline{M} = \frac{P - \ell(1-\kappa)}{(1-\theta)(\mu - C)}.$$

Because $\Phi(0)$ monotonically becomes more negative as $M$ decreases below $\overline{M}$, the following results hold:

1. For $M > \overline{M}$ ($m < \overline{m}$): At $w = 0$, the expected change in cash reserves is positive ($\Phi(0) > 0$). $E''(w)$ is negative at $w = 0$ and, thus, for any $w \in [0, W^*)$ (as shown in Appendix A.2.1). As a result, there is no inflection point in $[0, W^*)$ and shareholders have no incentives for risk-taking.

2. For $M = \overline{M}$ ($m = \overline{m}$): At $w = 0$, the expected change in cash reserves is zero ($\Phi(0) = 0$). $E''(0) = 0$ (so that $W_E = 0$), and $E''(w) < 0$ for any $w \in (0, W^*)$.

3. For $M < \overline{M}$ ($m > \overline{m}$): At $w = 0$, the expected change in cash reserves is negative ($\Phi(0) < 0$). As a result, $E''(0) > 0$ for any such $M < \overline{M}$ and the inflection point $W_E$ at which the function goes from convex to concave lies in $w \in (0, W^*)$. That is, for any maturity shorter than $\overline{M}$, shareholders' risk-taking incentives arise.

### *A.4 Proof of Proposition 3*

Let us consider debt maturities $j$ such that $M_j < \overline{M}$. For these maturities, let us consider the degree of shareholders' (pseudo) absolute risk aversion:

$$A_j(w) = -\frac{E_j''(w)}{E_j'(w)}.$$

For any maturity, equity value satisfies:

$$rE_j(w) = \Phi_j(w)E_j'(w) + \frac{(1-\theta)^2}{2}\sigma^2 E_j''(w). \tag{A5}$$

At $w = 0$, equation (A5) simplifies to:

$$0 = \Phi_j(0)E_j'(0) + \frac{\sigma^2(1-\theta)^2}{2}E_j''(0)$$

for any maturity $j$, because shareholders receive nothing in liquidation (i.e., we have $E_j(0) = 0$ for any $j$). This equation implies that:

$$A_j(0) \equiv -\frac{E_j''(0)}{E_j'(0)} = \frac{2\Phi_j(0)}{\sigma^2(1-\theta)^2}. \tag{A6}$$

By Proposition 2, we know that $\Phi_j(0) < 0$ for any $M_j < \overline{M}$, so that shareholders are effectively risk-loving in a right neighborhood of $w = 0$ for this range of maturities. In addition, $\Phi_j(0)$ becomes monotonically more negative (i.e., it is greater in absolute value) as debt maturity $M_j$ decreases below $\overline{M}$. As a result, $A_j(0)$ is more negative for smaller $M_j$ by equation (A6). Thus, shareholders are effectively more risk-loving for shorter maturity close to $w = 0$, and the claim follows.

## *A.5 Proof of Proposition 4*

As in the main text, we denote by $W_D$ the level of cash reserves that separates the region of concavity and of convexity in debt value (i.e., such that $D''(W_D) = 0$). Now, recall that debt value is non-negative ($D(w) \geq 0$) and non-decreasing in cash reserves ($D'(w) \geq 0$), as shown in Appendix A.2.1 (see the proof of Lemma 7). Lemma 7 together with Lemma 8 imply that debt is indeed concave for $w \in [W_E, W^*]$, which in turn implies that if $W_D$ is positive, it cannot exceed $W_E$. The intuition for this result is as follows. The periodic payment to debtholders $C+mP \geq 0$ is non-negative. Therefore, when condition (8) in Proposition 1 is satisfied, $D''(w)$ can be positive (and, thus, the value of debt is convex) if condition (8) is sufficiently negative, as illustrated in equation (10). That is, condition (8) is necessary but not sufficient for debtholders' risk-taking incentives to arise. Because debtholders receive the periodic payment $C + mP$ (which increases the right-hand side of the above ODE), the region of convexity in debt value is smaller than the region of convexity in equity value.

## *A.6 Proof of Proposition 5*

We derive the optimal risk-taking policy and the value of the firm's securities under the assumptions in Section 4. Assuming frictionless trading in futures contracts, standard arguments imply that, in the earnings retention region, the value of equity satisfies the Hamilton-Jacobi-Bellman equation reported in Section 4, equation (11). By simply differentiating the term in the maximum operator with respect to the control $\gamma$, we get $2(\rho\sigma + \gamma)E''(w)$. Because $\gamma \in [0, \Gamma]$ and $\rho \geq 0$, it follows that management takes on the maximum position $\Gamma$ in the future contract if $E''(w) > 0$, i.e. if the value of equity is convex. Conversely, it is suboptimal for shareholders to take positions in the future contract if $E''(w) < 0$, i.e. if the value of equity is concave. We denote by $W_E(\Gamma)$ the cash level that separates the convex and the concave region, i.e. such that

$$E''(W_E(\Gamma)) = 0.$$

The optimal risk-taking policy is thus of a bang-bang type:

$$\gamma = \begin{cases} \Gamma & \text{if } 0 \leq w < W_E(\Gamma), \\ 0 & \text{if } W_E(\Gamma) \leq w < W^*(\Gamma). \end{cases}$$

That is, if risk-taking is optimal, it happens at the maximal rate. It is worth noting that the size of the increase in asset volatility upon risk-taking is not exogenous but, rather, it is the endogenous choice of shareholders being able to increase volatility in $[0, \Gamma]$. The target level of cash holdings is denoted by $W^*(\Gamma)$ in this environment.

In analogy to Section 3, management finds it optimal to pay out dividends to shareholders when the cash reserves exceed $W^*(\Gamma)$, and the value of equity is linear above this target level. Differently, the optimal risk-taking policy means that, when $W_E(\Gamma) \in (0, W^*(\Gamma))$, the cash retention region $[0, W^*(\Gamma))$ is characterized by a risk-taking region, $[0, W_E(\Gamma))$, and a no-risk-taking region, $[W_E(\Gamma), W^*(\Gamma))$ (Appendix A.2.1 proves that there cannot be multiple regions of convexity). In the risk-taking region $[0, W_E(\Gamma))$, the value of equity satisfies the following differential equation

$$\begin{aligned} rE(w) = [(1-\theta)((r-\lambda)w + \mu - C) &+ m((1-\kappa)D(w) - P)]E'(w) \\ &+ \frac{1}{2}(1-\theta)^2\left(\sigma^2 + 2\rho\sigma\Gamma + \Gamma^2\right)E''(w). \end{aligned}$$

In the no-risk-taking region $[W_E(\Gamma), W^*(\Gamma))$, the value of equity satisfies

$$\begin{aligned} rE(w) = [(1-\theta)((r-\lambda)w + \mu - C) &+ m((1-\kappa)D(w) - P)]E'(w) \\ &+ \frac{1}{2}\left((1-\theta)\sigma\right)^2 E''(w). \end{aligned}$$

The system of ODEs for the value of equity is solved subject to the following boundary condition at the default/liquidation threshold, $E(0) = 0$, and the boundary conditions at the target cash level, $\lim_{w \uparrow W^*(\Gamma)} E'(w) = 1$ and $\lim_{w \uparrow W^*(\Gamma)} E''(w) = 0$. These boundary conditions are similar to those derived in Section 3 and admit an analogous interpretation. In addition, we now need to impose continuity and smoothness at $W_E(\Gamma)$,

$$\lim_{w \uparrow W_E(\Gamma)} E(w) = \lim_{w \downarrow W_E(\Gamma)} E(w) \quad \text{and} \quad \lim_{w \uparrow W_E(\Gamma)} E'(w) = \lim_{w \downarrow W_E(\Gamma)} E'(w),$$

to ensure that the risk-taking region and the no-risk-taking regions are smoothly pasted.

Since debtholders have rational expectations, the value of short-term debt reflects this risk-taking policy and satisfies in the risk-taking region $[0, W_E(\Gamma))$:

$$(r+m)D(w) = [(1-\theta)((r-\lambda)w + \mu - C) + m((1-\kappa)D(w) - P)]D'(w) \\ + \frac{1}{2}(1-\theta)^2\left(\sigma^2 + 2\rho\sigma\Gamma + \Gamma^2\right)D''(w) + C + mP.$$

In the no-risk-taking region $[W_E(\Gamma), W^*(\Gamma))$, $D(w)$ satisfies

$$(r+m)D(w) = [(1-\theta)((r-\lambda)w + \mu - C) + m((1-\kappa)D(w) - P)]D'(w) \\ + \frac{1}{2}\left((1-\theta)\sigma\right)^2 D''(w) + C + mP.$$

On top of the boundary conditions at 0 and $W^*(\Gamma)$ as in Section 3, respectively $D(0) = \ell$ and $D'(W^*(\Gamma)) = 0$, we impose continuity and smoothness at $W_E(\Gamma)$, i.e.

$$\lim_{w\uparrow W_E(\Gamma)} D(w) = \lim_{w\downarrow W_E(\Gamma)} D(w) \quad \text{and} \quad \lim_{w\uparrow W_E(\Gamma)} D'(w) = \lim_{w\downarrow W_E(\Gamma)} D'(w).$$

As noted in Proposition 4, the inflection point separating the regions of concavity and convexity in debt value, $W_D$, is always smaller than the inflection point in equity value, $W_E$ (when these thresholds exist and are non-trivial). As a result, shareholders have incentives to increase asset risk when it is suboptimal to debtholders (then leading to a decrease in the market value of debt and to an increase in yield spreads).

## References


Acharya, V., A. Krishnamurthy, and E. Perotti, 2011, "A Consensus View on Liquidity Risk," *Vox*, September.

Acharya, V., H.S. Shin, and T. Yorulmazer, 2010, "Crisis Resolution and Bank Liquidity," *Review of Financial Studies* 24, 2166-2205.

Admati, A., P. DeMarzo, M. Hellwig, and P. Pfleiderer, 2013, "Fallacies, Irrelevant Facts, and Myths in the Discussion of Capital Regulation: Why Bank Equity is Not Socially Expensive," Working paper, Stanford University.

Admati, A., P. DeMarzo, M. Hellwig, and P. Pfleiderer, 2018, "The Leverage Ratchet Effect," *Journal of Finance* 73, 145-198.

Admati, A., and M. Hellwig, 2013, "Does Debt Discipline Bankers? An Academic Myth about Bank Indebtedness," Working paper, Stanford University.

Altinkilic, O., and R. Hansen, 2000, "Are There Economies of Scale in Underwriting Fees? Evidence of Rising External Financing Costs, "*Review of Financial Studies* 13, 191-218.

Asvanunt, A., M. Broadie, and S. Sundaresan, 2011, "Managing Corporate Liquidity: Strategies and Pricing Implications,"*International Journal of Theoretical and Applied Finance* 14, 369-406.

Babenko, I., and Y. Tserlukevich, 2017, "Embracing Risk: Hedging Policy for Firms with Real Options," Working paper, Arizona State University.

Barnea, A., R. Haugen, and L. Senbet, 1980, "A Rationale for Debt Maturity Structure and Call Provisions in the Agency Theoretic Framework," *Journal of Finance* 35, 1223-1234.

Becker, B., and P. Strömberg, 2012, "Fiduciary Duties and Equity-debtholder Conflicts," *Review of Financial Studies* 25, 1931-1969.

van Binsbergen, J., J. Graham, and J. Yang, 2010, "The Cost of Debt," *Journal of Finance* 65, 2089-2136.

Black, F., and J. Cox, 1976, "Valuing Corporate Securities: Some Effects of Bond Indenture Provisions," *Journal of Finance* 31, 351-367.

Black, F., and M. Scholes, 1973, "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy* 81, 637-659.

Bolton, P., H. Chen, and N. Wang, 2011, "A Unified Theory of Tobin's $q$, Corporate Investment, Financing, and Risk Management," *Journal of Finance* 66, 1545-1578.

Bolton, P., H. Chen, and N. Wang, 2013, Market Timing, Investment, and Risk Management, *Journal of Financial Economics* 109, 40-62.

Bolton, P., H. Chen, and N. Wang, 2015, "Debt, Taxes, and Liquidity," Working paper, Columbia University.

Calomiris, C., and C. Kahn, 1991, "The Role of Demandable Debt in Structuring Optimal Banking Relationships," *American Economic Review* 81, 497-513.

Chen, H., R. Cui, Z. He, and K. Milbradt, 2018, "Quantifying Liquidity and Default Risks of Corporate Bonds over the Business Cycle," *Review of Financial Studies* 31, 852-897.

Chen, Z., and R. Duchin, 2019, "Do Nonfinancial Firms Use Financial Assets to Risk-Shift? Evidence from the 2014 Oil Price Crisis," Working paper, University of Washington.

Cheng, I.H., and K. Milbradt, 2012, "The Hazards of Debt: Rollover Freezes, Incentives, and Bailouts," *Review of Financial Studies* 25, 1070-1110.

Dangl, T., and J. Zechner, 2016, "Debt Maturity and the Dynamics of Leverage," Working paper, University of Vienna.

Décamps, J.P., S. Gryglewicz, E. Morellec, and S. Villeneuve, 2017, "Corporate Policies with Permanent and Transitory Shocks," *Review of Financial Studies* 30, 162-210.

Décamps, J.P., T. Mariotti, J.C. Rochet, and S. Villeneuve, 2011, "Free Cash Flow, Issuance Costs, and Stock Prices," *Journal of Finance* 66, 1501-1544.

DeMarzo, P., Fishman, M., He, Z., and N. Wang, 2012, "Dynamic Agency and the q-Theory of Investment," *Journal of Finance* 67, 2295-2340.

DeMarzo, P., and Z. He, 2018, "Leverage Dynamics without Commitment," Working paper, Stanford University.

DeMarzo, P., and Y. Sannikov, 2006, "A Continuous Time Agency Model of Optimal Contracting and Dynamic Capital Structure," *Journal of Finance* 61, 2681-2724.

Derman, E., and I. Kani, 1996, "The Ins and Outs of Barrier Options: Part 1," *Derivatives Quarterly*, 55-67.

Diamond, D., 1991, "Debt Maturity Structure and Liquidity Risk," *Quarterly Journal of Economics* 106, 709-737.

Diamond, D., 2004, "Committing to Commit: Short-term Debt When Enforcement is Costly," *Journal of Finance* 59, 1447-1479.

Diamond, D., and R. Rajan, 2001, "Liquidity Risk, Liquidity Creation and Financial Fragility: A Theory of Banking," *Journal of Political Economy* 109, 287-327.

Dumas, B., 1991, "Super Contact and Related Optimality Conditions," *Journal of Economic Dynamics and Control* 15, 675-685.

Eisdorfer, A., 2008, "Empirical Evidence of Risk-Shifting in Financially Distressed Firms," *Journal of Finance* 63, 609-637.

Eisenbach, T., 2017, "Rollover Risk as Market Discipline: A Two-Sided Inefficiency," *Journal of Financial Economics* 126, 252-269.

Ellul, A., and V. Yerramilli, 2013, "Stronger Risk Controls, Lower Risk: Evidence from U.S. Bank Holding Companies," *Journal of Finance* 68, 1757-1803.

Ericsson, J., and O. Renault, 2006, "Liquidity and Credit Risk," *Journal of Finance* 61, 2219-2250.

Faulkender, M., and R. Wang, 2006, "Corporate Financial Policy and the Value of Cash," *Journal of Finance* 61, 1957-1990.

Favara, G., E. Morellec, E. Schroth, and P. Valta, 2017, "Debt Enforcement, Investment, and Risk-Taking Across Countries," *Journal of Financial Economics* 123, 22-41.

Financial Crisis Inquiry Report, 2011, "Final Report of the National Commission on the Causes of the Financial and Economic Crisis in the United States,"

Flannery, M., 1986, "Asymmetric Information and Risky Debt Maturity Choice," *Journal of Finance,* 41, 19-37.

Flannery, M., 1994, "Debt Maturity and the Deadweight Cost of Leverage: Optimally Financing Banking Firms," *American Economic Review,* 84, 320-331.

Gan, J., 2004, "Banking Market Structure and Financial Stability: Evidence from the Texas Real Estate Crisis in the 1980s," *Journal of Financial Economics* 73, 567-601.

Glover, B., 2016, "The Expected Cost of Default," *Journal of Financial Economics* 119, 284-299.

Gormley, T., and D. Matsa, 2011, "Growing Out of Trouble? Corporate Responses to Liability Risk," *Review of Financial Studies* 24, 2781-2821.

Graham, J., 2000, "How Big are the Tax Benefits of Debt," *Journal of Finance* 55, 1901-1941.

Graham, J., and C. Harvey, 2001, "The Theory and Practice of Corporate Finance: Evidence from the Field," *Journal of Financial Economics* 60, 187-243.

Graham, J., and M. Leary, 2018, "The Evolution of Corporate Cash," *Review of Financial Studies* 31, 4288-4344.

Graham, J., M. Leary, and M. Roberts, 2015, "A Century of Capital Structure: The Leveraging of Corporate America," *Journal of Financial Economics* 118, 658-683.

Gryglewicz, S., 2011, A Theory of Corporate Financial Decisions with Liquidity and Solvency Concerns, *Journal of Financial Economics* 99, 365-384

Hackbarth, D., J. Miao, and E. Morellec, 2006, "Capital Structure, Credit Risk, and Macroeconomic Conditions," *Journal of Financial Economics* 82, 519-550.

Harford, J., S. Klasa, and W. Maxwell, 2014, "Refinancing Risk and Cash Holdings," *Journal of Finance* 69, 975-1012.

He, Z., and K. Milbradt, 2014, "Endogenous Liquidity and Defaultable Debt," *Econometrica* 82, 1443-1508.

He, Z., and W. Xiong, 2012a, "Rollover Risk and Credit Risk," *Journal of Finance* 67, 391-429.

He, Z., and W. Xiong, 2012b, "Dynamic Debt Runs," *Review of Financial Studies* 67, 1799-1843.

Hilberink, B., and L.C.G. Rogers, 2002, "Optimal Capital Structure and Endogenous Default," *Finance and Stochastics* 6, 237-263.

Huberman, G., and R. Repullo, 2015, "Moral Hazard and Debt Maturity," Working paper, Columbia University.

Hugonnier, J., S. Malamud, and E. Morellec, 2015a, "Capital Supply Uncertainty, Cash Holdings, and Investment," *Review of Financial Studies* 28, 391-445.

Hugonnier, J., S. Malamud, and E. Morellec, 2015b, "Credit Market Frictions and Capital Structure Dynamics," *Journal of Economic Theory* 157, 1130-1158.

Hugonnier, J., and E. Morellec, 2017, "Bank Capital, Liquid Reserves, and Insolvency Risk," *Journal of Financial Economics* 125, 266-285.

Jensen, M., and W. Meckling, 1976, "Theory of the Firm: Managerial Behavior, Agency Costs, and Ownership Structure," *Journal of Financial Economics* 3, 305-360.

Jiménez, G., S. Ongena, J.-L Peydro, and J. Saurina, 2014, "Hazardous Times for Monetary Policy: What do 23 Million Loans Say About the Impact of Monetary Policy on Credit Risk-Taking?," *Econometrica* 82, 463-505.

Jiménez, G., S. Ongena, J.-L. Peydro, and J. Saurina, 2017, "Macroprudential Policy, Countercyclical Bank Capital Buffers and Credit Supply: Evidence from the Spanish Dynamic Provisioning Experiments," *Journal of Political Economy* 125, 2126-2177.

Keeley, M., 1990, "Deposit Insurance, Risk, and Market Power in Banking," *American Economic Review* 80, 1183-1200.

Laeven, L., and R. Levine, 2009, "Bank Governance, Regulation and Risk Taking," *Journal of Financial Economics* 93, 259-275.

Krishnamurthy, A., 2010, "How Debt Markets Have Malfunctioned in the Crisis," *Journal of Economic Perspectives* 24, 3-28.

Leland, H., 1998, "Agency Costs, Risk Management, and Capital Structure," *Journal of Finance* 53, 1213-1243.

Leland, H., 1994a, "Corporate Debt Value, Bond Covenants, and Optimal Capital Structure," *Journal of Finance* 49, 1213-1252.

Leland, H., 1994b, "Bond Prices, Yield Spreads, and Optimal Capital Structure," Working paper, University of California Berkeley.

Leland, H., and K.-B. Toft, 1996, "Optimal Capital Structure, Endogenous Bankruptcy and the Term Structure of Credit Spreads," *Journal of Finance* 51, 987-1019.

Malamud, S., and F. Zucchi, 2019, "Liquidity, Innovation, and Endogenous Growth," *Journal of Financial Economics* 132, 519-541.

Toft, K., and B. Prucyk, 1997, "Options on Leveraged Equity: Theory and Empirical Tests," *Journal of Finance* 52, 1151-1180.

Whited, T., and G. Wu, 2006, "Financial Constraints Risk," *Review of Financial Studies* 19, 531-559.

Table 1: Baseline parametrization.

| A. Parameter values | | |
|---|---|---|
| Parameter | Symbol | Value |
| Mean cash flow rate | $\mu$ | 0.09 |
| Cash flow volatility | $\sigma$ | 0.08 |
| Risk-free rate | $r$ | 0.035 |
| Carry cost of cash | $\lambda$ | 0.01 |
| Liquidation cost | $\varphi$ | 0.45 |
| Tax rate | $\theta$ | 0.30 |
| Coupon on debt | $C$ | 0.036 |
| Principal on debt | $P$ | 0.75 |
| Average debt maturity | $M$ | 1 |
| Debt issuance cost | $\kappa$ | 0.01 |
| Correlation between assets-in-place and futures contracts | $\rho$ | 0.5 |
| Fixed equity issuance cost (good state) | $\phi_G$ | 0.012 |
| Proportional equity issuance cost (good state) | $p_G$ | 0.06 |
| Switching intensity (good to bad state) | $\pi_G$ | 0.20 |
| Switching intensity (bad to good state) | $\pi_B$ | 0.60 |
| **B. Implied variables in one-state model** | | |
| Variable | Symbol | Value |
| Target level of cash reserves | $W^*$ | 0.369 |
| Equity value at $W^*$ | $E(W^*)$ | 1.317 |

Table 2: RISK-TAKING THRESHOLDS.

The table reports the inflection points for debt ($W_D$) and equity ($W_E$), the size of the agency region ($AR$), and the target cash level ($W^*$) for different debt maturities ($M$) (top panel). Fixing $M = 1$, we also investigate the effects of varying the cash flow drift ($\mu$), cash flow volatility ($\sigma$), liquidation costs ($\varphi$), and debt issuance costs ($\kappa$).

| | $W_D$ | $W_E$ | $AR$ | $W^*$ |
|---|---|---|---|---|
| $M = 1$ | 0.073 | 0.106 | 0.033 | 0.369 |
| $M = 2$ | 0.028 | 0.053 | 0.025 | 0.314 |
| $M = 3$ | 0.008 | 0.028 | 0.020 | 0.296 |
| $M = 5$ | n.a. | 0.004 | 0.004 | 0.283 |
| $M = \infty$ | n.a. | n.a. | 0.000 | 0.280 |
| $\mu = 0.08$ | 0.080 | 0.123 | 0.043 | 0.401 |
| $\mu = 0.09$ | 0.073 | 0.106 | 0.033 | 0.369 |
| $\mu = 0.10$ | 0.065 | 0.092 | 0.027 | 0.340 |
| $\mu = 0.11$ | 0.057 | 0.079 | 0.022 | 0.313 |
| $\mu = 0.12$ | 0.050 | 0.068 | 0.018 | 0.289 |
| $\sigma = 0.06$ | 0.081 | 0.097 | 0.016 | 0.273 |
| $\sigma = 0.08$ | 0.073 | 0.106 | 0.033 | 0.369 |
| $\sigma = 0.10$ | 0.062 | 0.118 | 0.056 | 0.470 |
| $\sigma = 0.12$ | 0.048 | 0.130 | 0.082 | 0.571 |
| $\sigma = 0.14$ | 0.032 | 0.144 | 0.112 | 0.671 |
| $\varphi = 0.30$ | n.a. | 0.014 | 0.014 | 0.292 |
| $\varphi = 0.35$ | 0.008 | 0.043 | 0.035 | 0.310 |
| $\varphi = 0.40$ | 0.038 | 0.072 | 0.034 | 0.336 |
| $\varphi = 0.45$ | 0.073 | 0.106 | 0.033 | 0.369 |
| $\varphi = 0.50$ | 0.111 | 0.144 | 0.033 | 0.405 |
| $\kappa = 0$ | 0.066 | 0.092 | 0.026 | 0.339 |
| $\kappa = 0.005$ | 0.069 | 0.099 | 0.030 | 0.354 |
| $\kappa = 0.01$ | 0.073 | 0.106 | 0.033 | 0.369 |
| $\kappa = 0.015$ | 0.076 | 0.114 | 0.038 | 0.385 |
| $\kappa = 0.02$ | 0.079 | 0.123 | 0.044 | 0.402 |

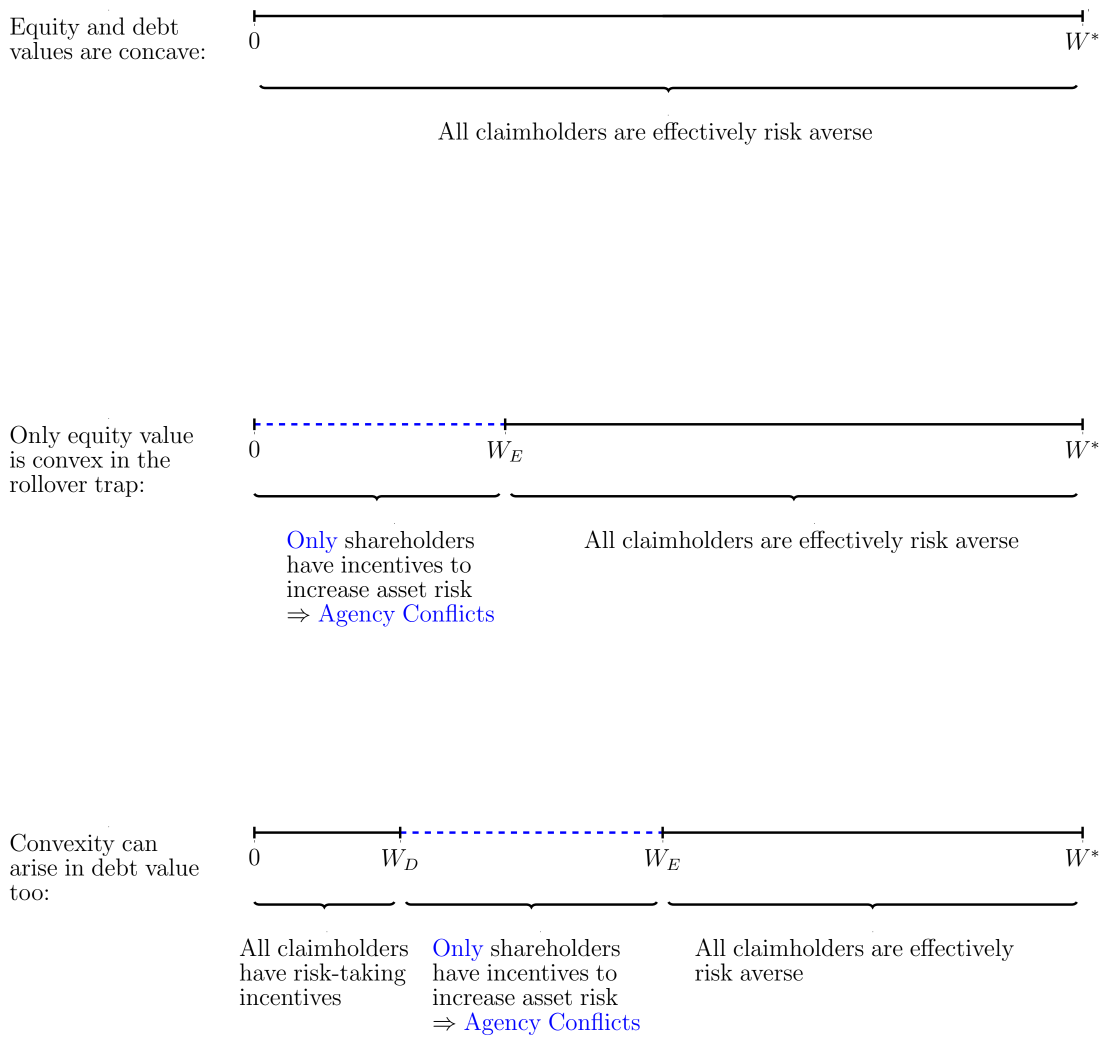


Figure 1: Short-term debt and agency conflicts.

The figure illustrates shareholders' and debtholders' risk-taking incentives as a function of cash holdings $[0, W^*]$.

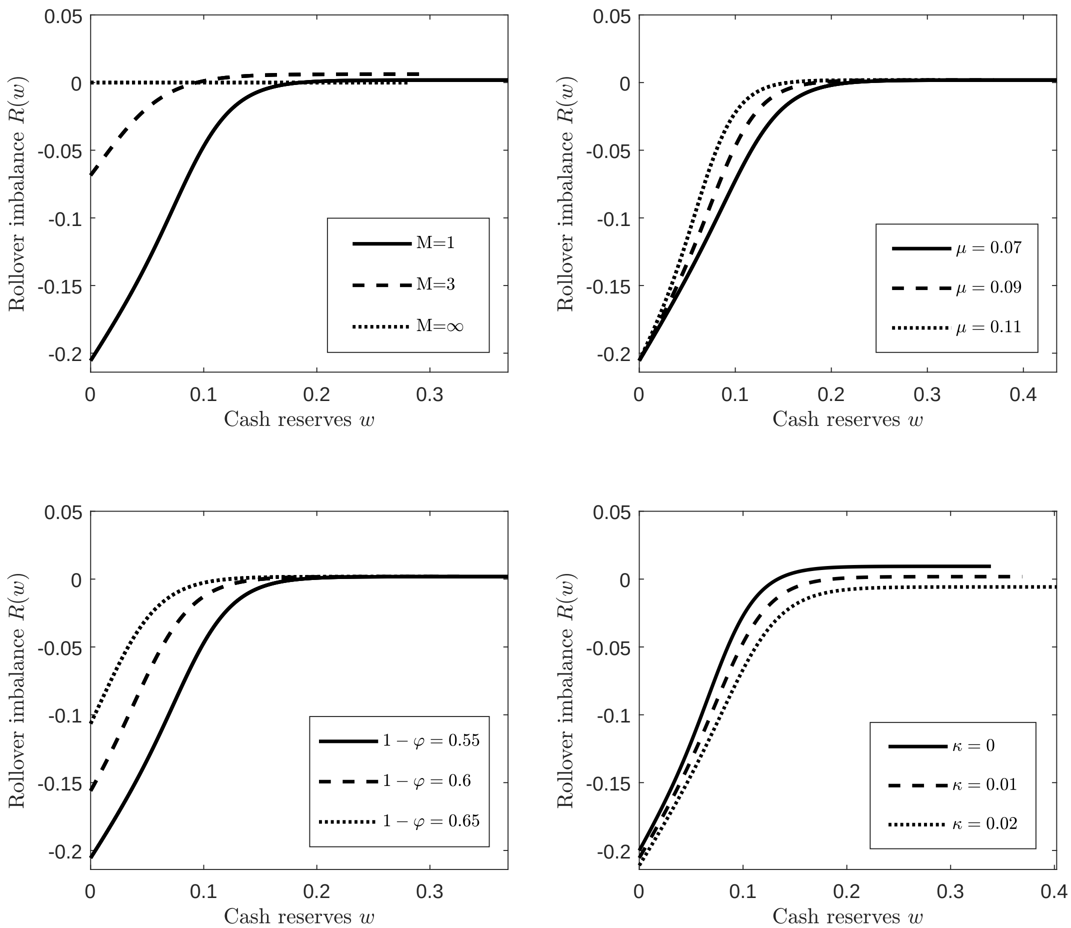


Figure 2: Rollover imbalances.

The figure plots the rollover imbalance $R(w)$ as a function of cash reserves $w \in [0, W^*]$ for different values of average debt maturity $M$, asset profitability $\mu$, asset recovery $1-\varphi$, and debt issuance cost $\kappa$.

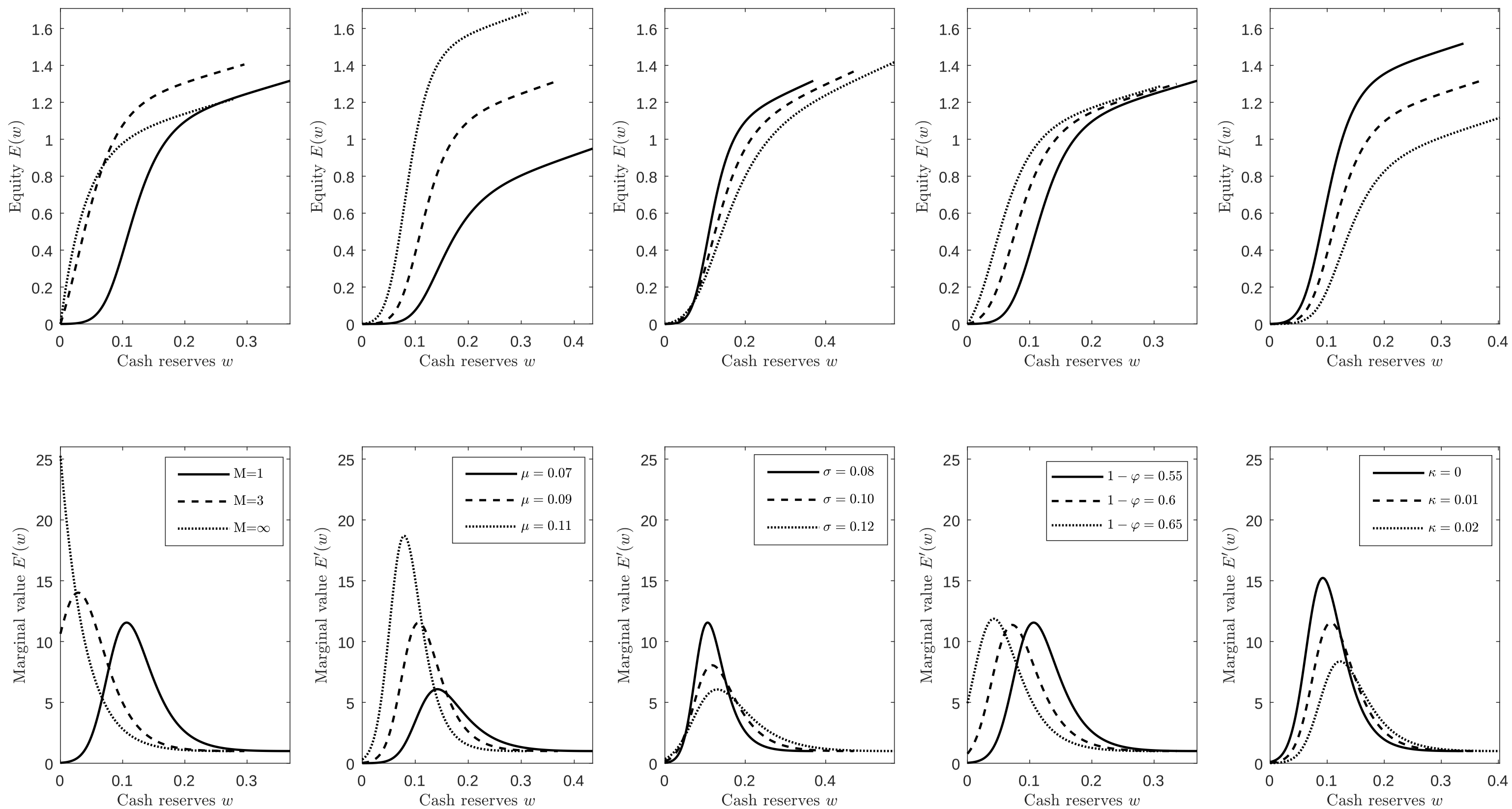


Figure 3: VALUE OF EQUITY AND THE ROLLOVER TRAP.

The figure plots the value of equity (top panel) and the marginal value of cash for shareholders (bottom panel) as functions of cash reserves $w \in [0, W^*]$, for different values of average debt maturities $M$ (first panel), asset profitability $\mu$ (second panel), asset volatility $\sigma$ (third panel), asset recovery $1-\varphi$ (fourth panel), and debt issuance cost $\kappa$ (fifth panel).

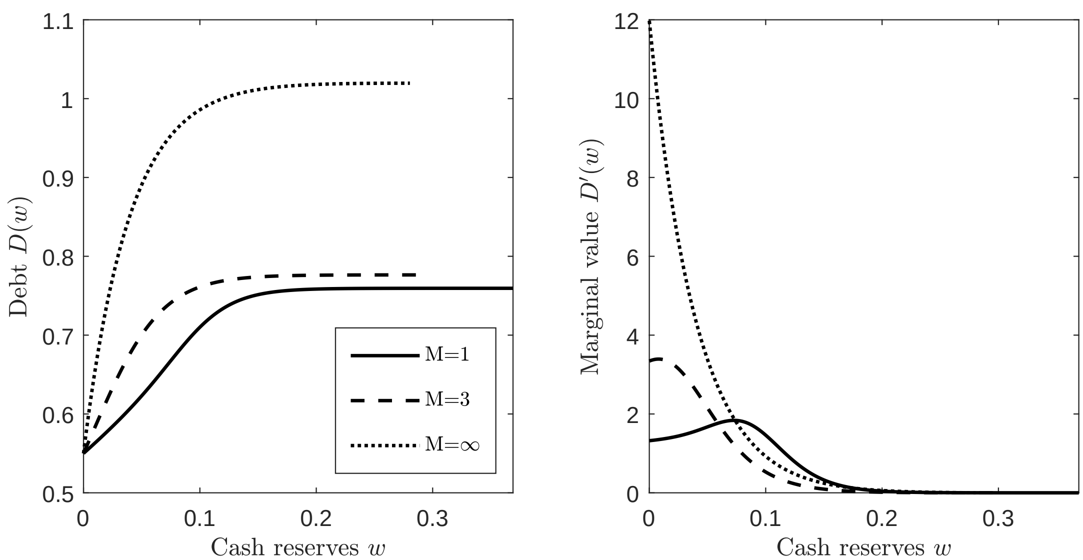


Figure 4: VALUE OF DEBT.

The figure plots the aggregate value of debt $D(w)$ and the marginal value of cash for debtholders $D'(w)$ as a function of cash reserves $w \in [0, W^*]$ and for average debt maturities $M$ of 1 year (solid line), 3 years (dashed line), and infinite (dotted line).

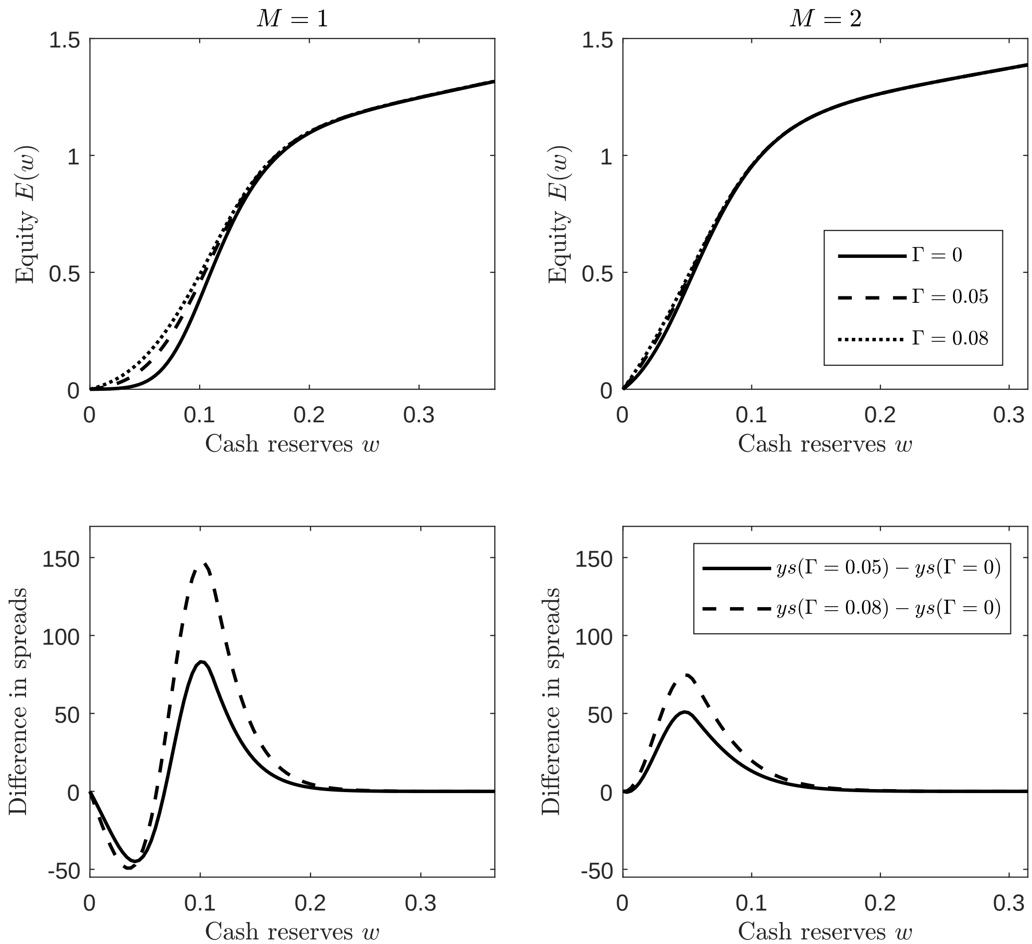


Figure 5: Risk-taking and debt maturity.

The figure plots the value of equity $E(w)$ (top panel) and the difference in yield spreads when shareholders do and do not engage in risk-taking strategies (bottom panel) as a function of cash reserves $w$ under different risk-taking strategies and for maturity $M = 1$ (left panel) and $M = 2$ (right panel).

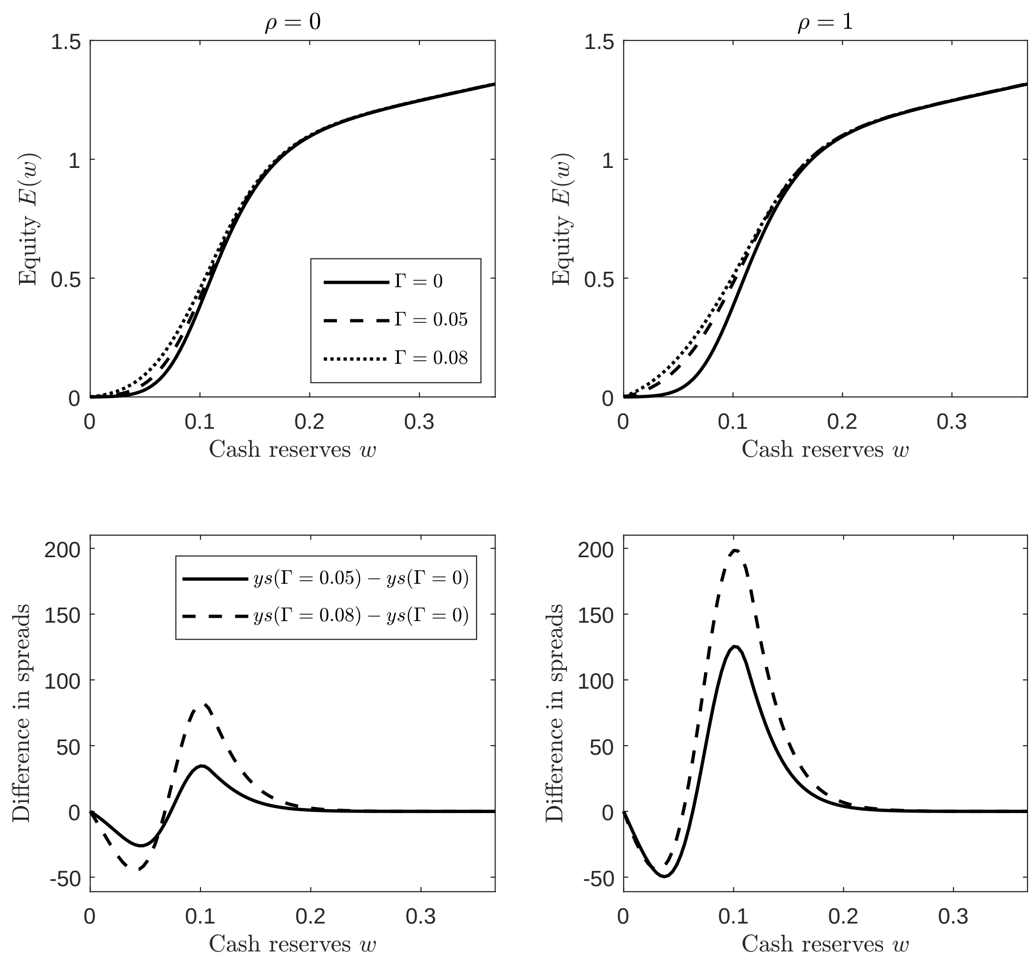


Figure 6: Risk-taking, debt maturity, and correlation among assets.

The figure plots the value of equity $E(w)$ (top panel) and the difference in yield spreads when shareholders do and do not engage in risk-taking (bottom panel) as a function of cash reserves $w$ under different risk-taking strategies and when the correlation between the assets-in-place and the risk-taking asset is zero ($\rho = 0$, left panel) and one ($\rho = 1$, right panel).

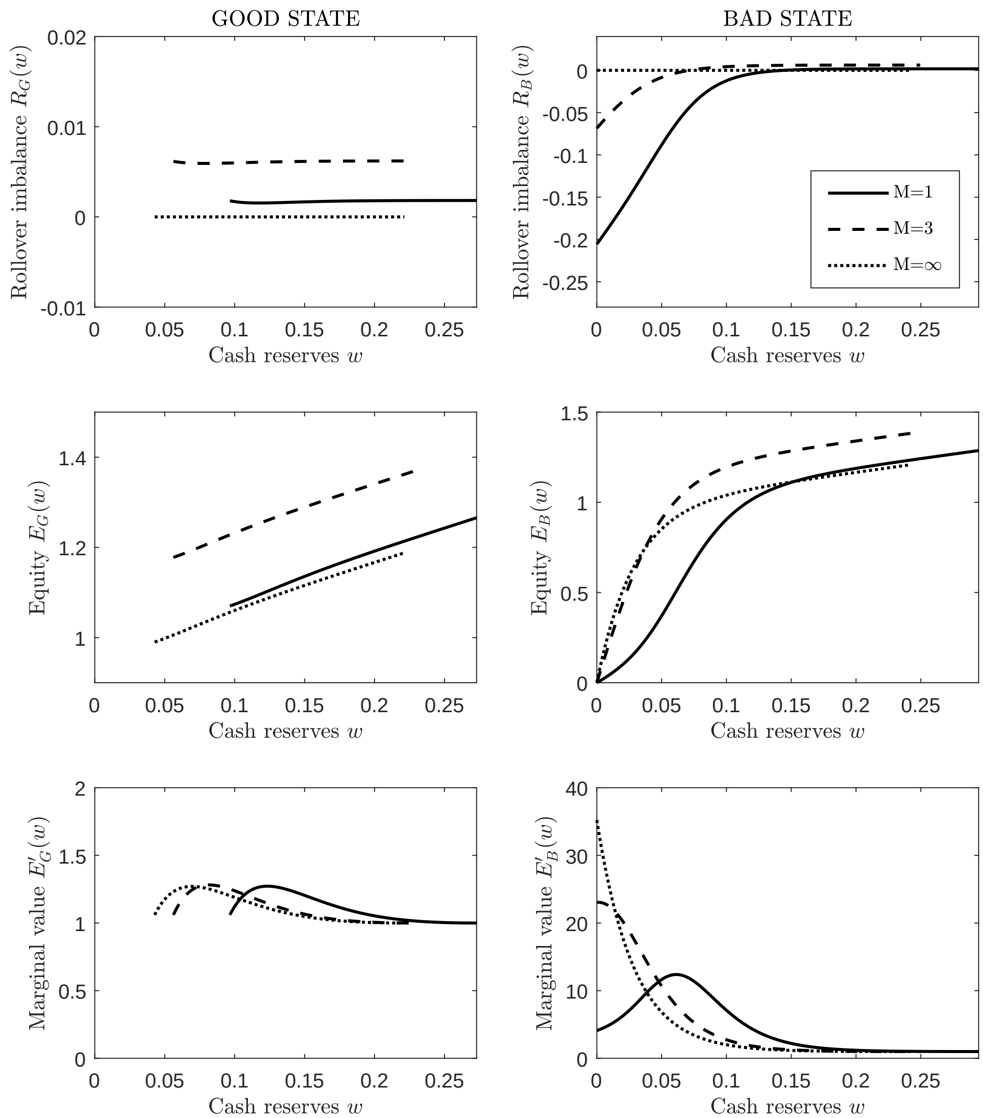


Figure 7: TIME-VARYING FINANCING CONDITIONS.

The figure plots the rollover imbalance $R_i(w)$, the value of equity $E_i(w)$, and the marginal value of cash for shareholders $E_i'(w)$ as a function of cash reserves $w \in [0, W_i^*]$ in the good state (left panel) and in the bad state (right panel) for average debt maturities $M$ of 1 year (solid line), 3 years (dashed line), and infinite (dotted line).