Ex 1 a) $I = \int_0^2 \left(\int_0^1 x^3 + 3x^2y + y^3 \, dy \right) dx$

$= \int_0^2 \left(x^3y + \frac{3x^2y^2}{2} + \frac{y^4}{4} \Big|_0^1 \right) dx$

$= \int_0^2 x^3 + \frac{3x^2}{2} + \frac{1}{4} \, dx = \frac{x^4}{4} + \frac{x^3}{2} + \frac{x}{4} \Big|_0^2$

$= 4 + 4 + 1/2 = 8\,1/2.$

b) $I = \int_1^2 \left(\int_1^2 \frac{x}{x^2+y^2} \, dx \right) dy$

$= \int_1^2 \left(\frac{1}{2} \ln(x^2+y^2) \Big|_1^2 \right) dy = \frac{1}{2} \int_1^2 \ln(y^2+4) - \ln(y^2+1) \, dy$

$= \frac{1}{2} \int_1^2 y' \left(\ln(y^2+4) - \ln(y^2+1) \right) dy$

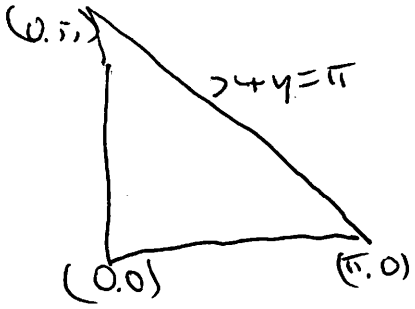
INTEGRATION BY PARTS $\frac{1}{2} y \left(\ln(y^2+4) - \ln(y^2+1) \right) \Big|_1^2 - \frac{1}{2} \int_1^2 y \left(\frac{2y}{y^2+4} - \frac{2y}{y^2+1} \right) dy$

$= \frac{1}{2} 2 \ln(8/5) - \frac{1}{2} \ln(5/2) - \int_1^2 \frac{y^2}{y^2+4} - \frac{y^2}{(y^2+1)} \, dy$

$= \ln 8 - \frac{3 \ln 5}{2} + \frac{1}{2} \ln(2) - \int_1^2 \frac{1}{1+y^2} - \frac{4}{y^2+4} \, dy$

$= \ln 8 - \frac{3 \ln 5}{2} + \frac{1}{2} \ln 2 - (\arctan(2) - \arctan(1)) + 2(\arctan(1) - \arctan(1/2))$

1 c)



$$I = \int_0^{\pi} \left(\int_0^{\pi - x} x \cos(x+y)\, dy \right) dx$$

$$= \int_0^{\pi} \left(x \sin(x+y) \Big|_0^{\pi - x} \right) dx = -\int_0^{\pi} x \sin(x)\, dx.$$

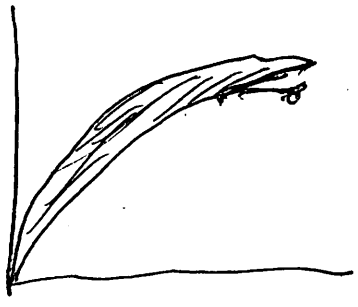
INTEGRATE BY PARTS

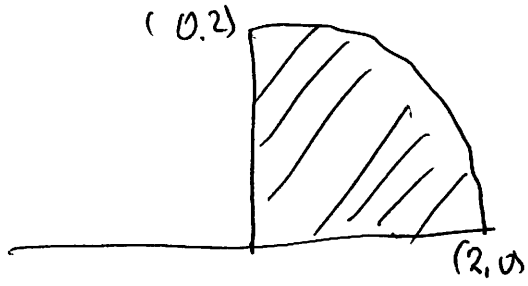
$$= x\cos x \Big|_0^{\pi} - \int_0^{\pi} \cos x\, dx$$

$$= -\pi - 0 = -\pi.$$

Ex 2

(a)





(i) $D = \{0 \le y \le 1 : y^3 \le x \le y^2\}$

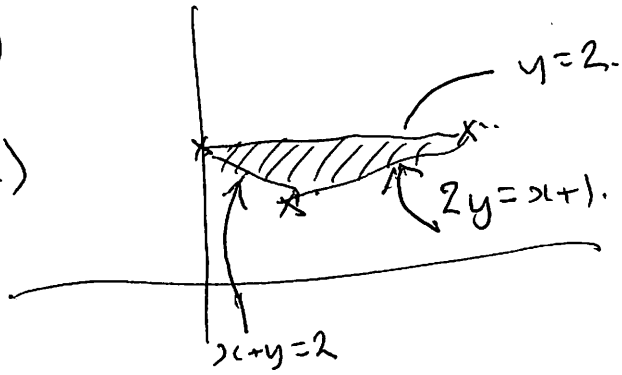
(ii) $D = \{0 \le y \le 2 \quad 0 \le x \le \sqrt{4-y^2}.$

b) (i) $\int_0^1 \left(\int_{y^3}^{y^2} f(x,y)\, dx \right) dy$

(ii) $\int_0^2 \int_0^{\sqrt{4-y^2}} f(x,y)\, dx\, dy$

3)

a)



$$D = \{0 \le x \le 3 : 2-x \vee \frac{x+1}{2} \le y \le 2\}$$

(1 max)

$$= \{1 \le y \le 2 : 2-y \le x \le 2y-1\}$$

$$\iint_D y^3 \, dx\,dy = \int_1^2 \left(\int_{2-y}^{2y-1} y^3 \, dx \right) dy$$

$$= \int_1^2 y^3(3y-3)\,dy = \frac{3y^5}{5} - \frac{3y^4}{4}\Big|_1^2$$

$$= \left(\frac{3}{5}\,32 - \frac{3}{5}\right) - \left(12 - 3/4\right).$$

b)

$$\iint_D f\,dx\,dy = \int_0^1 \left(\int_0^{x^2} x\cos y \, dy \right) dx$$

$$= \int_0^1 \left(x \sin x^2 \right) dx.$$

$$= \frac{1}{2} - \cos(x^2)\Big|_0^1$$

$$= \frac{1}{2}\left(1 - \cos(1)\right)$$

4) $D = \{0 \le x \le 2,\ 0 \le z \le 8-4x,\ 0 \le y \le \frac{8-4x-z}{2}\}$

$$\iiint_D z+x-y \, dx\,dy\,dz = \int_0^2 \left(\int_0^{8-4x} \left(\int_0^{\frac{8-4x-z}{2}} (z+x-y)\,dy \right) dz \right) dx$$

$$= \int_0^2 \left(\int_0^{8-4x} \left(zy + xy - \frac{y^2}{2} \right) \Big|_0^{\frac{8-4x-z}{2}} dz \right) dx.$$

$$= \int_0^2 \left(\int_0^{8-4x} \left(\frac{z+x}{2}\right)(8-4x-z) - \frac{(8-4x-z)^2}{8} dz \right) dx.$$

COFV LET $u = 8-4x-z$

$$= \int_0^2 \left(\int_0^{8-4x} \left(\frac{8-3x-u}{2}\right) u - \frac{u^2}{8} \, du \right) dx.$$

$$= \int_0^2 \left(\int_0^{8-4x} \frac{-5u^2}{8} + \frac{8-3x}{2} \cdot u \, du \right) dx.$$

$$= \int_0^2 -\frac{5}{24}(8-4x)^3 + \frac{8-3x}{2}\,\frac{(8-4x)^2}{2} \, dx.$$

$$= \int_0^2 (8-4x)^3 \left(\frac{1}{4} - \frac{5}{24}\right) + \frac{(8-4x)^2}{4} \, dx.$$

$$= \int_0^2 \frac{(8-4x)^3}{24} + (4-2x)^2 \, dx. = \int_0^2 \frac{(4-2x)^3}{3} + \quad = 16$$