

Exercise Session, May 30, 2016

1. Find the general solution of the following systems of differential equations and specify the shape of the phase portrait.

(a)

$$\frac{d}{dt}\mathbf{u} = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix} \mathbf{u}$$

(b)

$$\frac{d}{dt}\mathbf{u} = \begin{pmatrix} -3 & \sqrt{2} \\ \sqrt{2} & -2 \end{pmatrix} \mathbf{u}$$

(c)

$$\frac{d}{dt}\mathbf{u} = \begin{pmatrix} -1/2 & 1 \\ -1 & -1/2 \end{pmatrix} \mathbf{u}$$

2. Solve the following Cauchy problem:

(a) $y''(t) + 2y'(t) + y(t) = 0$, and $y(0) = 1, y'(0) = 0$

(b) $y''(t) + 2y'(t) + 5y(t) = 0$, and $y(0) = 3, y'(0) = 1$

(c) $y''(t) + 2y'(t) + 0.36y(t) = 0$, and $y(0) = 1, y'(0) = 0$

3. For each of the following, write the general solution of the differential equation by solving the homogeneous equation and finding the particular solution.

(a) $y''(t) + y'(t) - 2y(t) = e^{3t}$

(b) $y''(t) - 3y'(t) = e^{3t}$

(c) $y''(t) - 2y'(t) + 2y(t) = e^{-t} \cos(t)$

(d) $y''(t) + 2y'(t) + y(t) = 8e^{-t}$

4. **Bifurcation.** Consider the parametric linear system of differential equations:

$$\frac{d\mathbf{u}}{dt} = \begin{pmatrix} -1 & -1 \\ -P & -1 \end{pmatrix} \mathbf{u}$$

(a) As P varies in the interval $]-\infty, \infty[$, how many times does the phase portrait change? Identify the types of the phase portrait.

(b) Write the general solution for $P = 0$ and draw the phase portraits.

5. A tank contains 1000 liters of brine (salty water) with 15kg of dissolved salt. A stream of salty water enters the tank with salt density of 0.025 kg/L at a rate of 10 L/min. The solution is kept thoroughly mixed and drains from the tank at the same rate (10 L/min). How much salt is in the tank after t minutes?