Analysis II
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Spring Semester 2015–2016

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Exercise Session, May 23, 2016

1. Calculate
$$\iint_{\mathbb{R}^2} \frac{1}{(1+x^2+(y-x)^2)^2}\, dxdy.$$

2. Calculate
$$\iiint_{\mathbb{R}^3} e^{-x^2-2y^2-3z^2}\, dxdydz.$$

3. Let
$$E = \{(x,y,z) \in \mathbf{R}^3 : x \in [0,1], y^2+z^2 \leq x^2\}$$
Describe E and give $|E| = \mathrm{Vol}(E)$.

4. For the differential equation:
$$dy/dx = 9x^2 y$$
find the general solution.
   (a) $y(x) = Ae^{3x^3}$
   (b) $y(x) = Ae^{3x^4}$
   (c) $y(x) = Ae^{x^2}$
   (d) $y(x) = Ae^{x^3}$

5. Suppose that y_0 satisfies:
$$(x^2+9)dy/dx = xy, \qquad y(0) = 3.$$
Find the value of $y_0(9)$.
   (a) $y_0(9) = 4\sqrt{10}$
   (b) $y_0(9) = 3\sqrt{10}$
   (c) $y_0(9) = 40$
   (d) $y_0(9) = 3\sqrt{17}$

6. Suppose that y_0 satisfies:
$$(x+3)dy/dx = y-1, \qquad y(1) = 2.$$
Find the value of $y_0(4)$.
   (a) $y_0(4) = 7/2$
   (b) $y_0(4) = -1$
   (c) $y_0(4) = 3$
   (d) $y_0(4) = 11/4$

7. For the differential equation:

$$dy/dx = \frac{e^{5x}}{6y^5}$$

find the general solution.

(a) $y(x) = \pm\sqrt[5]{e^{5x}/5 + C}$

(b) $y(x) = \pm\sqrt[5]{e^{5x}/5} + C$

(c) $y(x) = \pm\sqrt[6]{e^{5x}/5 + C}$

(d) $y(x) = \pm\sqrt[6]{e^{5x}/5} + C$

8. Find the general solution of the equation:

$$2ydy/dx = 9x.$$

(a) $y = \pm\sqrt{\frac{9}{2}x^2 + C}$

(b) $y = \pm\sqrt{\frac{9}{2}x^2} + C$

(c) $y = \pm\sqrt{\frac{9}{2}x^2}$

(d) $y = \pm\sqrt{\frac{2}{9}x^2 + C}$

9. The solution $y(x)$ of the differential equation $(x^2 + 9)y' + xy - xy^2 = 0$ for $x \in \mathbb{R}$ with the initial condition $y(0) = 1/4$ also satisfies:

(a) $y(4) = 1/6$

(b) $y(4) = -1/4$

(c) $y(4) = 6$

(d) $y(4) = 1$

10. Find the general solution of the following equations

(a) $y' - \frac{3y}{x+1} = (x + 1)^4$

(b) $\cos(x)y' + \sin(x)y = 2\cos^3(x)\sin(x) - 1$

11. For each of the following differential equations check if the solution exists and is unique.

(a) $y' = 1 + y^2$, $y(0) = 0$

(b) $y' = \frac{2y}{x}$, $y(a) = b$

solve the differential equation (b) and sketch the family of solutions for some initial conditions $y(a) = b$. What happens when $a = 0$ or $b = 0$? Compare this with the existence-uniqueness theorem.