Analyse II - Prof. Mountford

Series 11:

Multiple Integrals

8 May 2017

Exercice 1:
Calculate the multiple integrals $\iint_D f(x,y)\,dydx$ where $f:\mathbb{R}^2\to\mathbb{R}$ et $D\subset\mathbb{R}^2$ is given by :

a) $f(x,y) = x^3 + 3x^2y + y^3$ and $D = [0,2]\times[0,1]$.

b) $f(x,y) = \dfrac{x}{x^2+y^2}$ and $D = [1,2]\times[1,2]$.

c) $f(x,y) = x\cos(x+y)$ et D is the triangle with vertices $(0,0)$, $(\pi,0)$, $(0,\pi)$.

For each integral check that, if one permutes the order of integration, one obtains the same result.

Exercice 2:
For each integral:

$$(1)\ \int_0^1\int_{\sqrt{x}}^{x^{1/3}} f(x,y)\,dydx \quad (2)\ \int_0^2\int_0^{\sqrt{4-x^2}} f(x,y)\,dydx$$

a) Draw the domain of integration.

b) Express the domain as a domain in terms of y and reverse the order of integration .

Exercice 3:
Calculate the integrals $\iint_D f(x,y)\,dydx$ where $f:\mathbb{R}^2\to\mathbb{R}$ et $D\subset\mathbb{R}^2$are given by :

a) $f(x,y) = y^3$ and D is thetriangle defined by points $(0,2)$, $(1,1)$, $(3,2)$.

b) $f(x,y) = x\cos(y)$ and $D$is the region bounded by $y=0$, $y=x^2$ and $x=1$.

Exercice 4:
Let D be the tetrahedron enclosed by the planes $x=0$, $y=0$, $z=0$ and $4x+2y+z=8$.

a) Express this domain in terms of variable x.

b) Calculate the integral:

$$\iiint_D (z+x-y)dx\ dy\ dz.$$