Analysis II Series 11:

Integrals

15 Mai 2017

Exercice 1:
Let $D = \{(x,y) \in \mathbb{R}^2 \ : \ 1/2 < x+y < 1, \ x > 0, \ y > 0\}$.

a) Say if the function $f(x,y) = \mathrm{e}^{\frac{x-y}{x+y}}$ is integrable on D.

b) Draw the domain of integration.

c) Doa change of variables
$$x = \frac{u+v}{2} \quad y = \frac{u-v}{2},$$
Transform the doamin of integration, draw the transformed domain and calculate the integral
$$\iint_D f(x,y)dxdy$$
by change of variables.

Exercice 2:
Let $D = \{(x,y) \in \mathbb{R}^2 \ : \ x < y < 2x, \ x < y^2 < 2x\}$.

a) Draw D.

b) By change of variables
$$u = \frac{x}{y} \quad v = \frac{y^2}{x},$$
transform the domain D.

c) Calculate
$$\iint_D \frac{y}{x} dxdy.$$

Exercice 3:
Soit $D = \{(x,y) \in \mathbb{R}^2 : \ x,y \geq 0, \ y \leq x, \ x^2+y^2 \geq 1, \}$.

a) Draw the domain.

b) Calculate $\iint_D \mathrm{e}^{-(x^2+y^2)} \ dxdy$by using the polair coordinates.

c) Say if the function $f(x,y) = \mathrm{e}^{-(x^2+y^2)}$is integrable on D and if so, calculate the l'integral $\iint_D |\mathrm{e}^{-(x^2+y^2)}| \ dxdy$.

Exercice 4:

For $D_\epsilon = \{(x,y) \in \mathbb{R}^2 : \; y \geq 0, \; \epsilon^2 \leq x^2 + y^2 \leq 4\}$ and

$$f(x,y) = \frac{2xy}{(x^2+y^2)^2}$$

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a) Draw D_ϵ.

b) Calculate integrals:

$$I_\epsilon^1 = \iint_{D_\epsilon} f(x,y) dx \; dy, \quad I_\epsilon^2 = \iint_{D_\epsilon} |f(x,y)| dx \; dy,$$

for each $\epsilon > 0$fixed using a transformation to polar coordinates.

c) Calculate the limits $\lim_{\epsilon \to 0} I_\epsilon^1$and $\lim_{\epsilon \to 0} I_\epsilon^2$. What does this say of the integrability of f on $D = \lim_{\epsilon \to 0} D_\epsilon$?

Exercice 5:

For $D = \left\{(x,y,z) \in \mathbb{R}^3 \; : \; x^2 + y^2 + z^2 - x \leq 0\right\}$.

a) Express the domainusing spherical coordinates (r, θ, φ).

b) Calculate the triple integral

$$\iiint_D \sqrt{x^2 + y^2 + z^2} dxdydz.$$