# Multiple Choice Questions 9

1. Let
$$E = \left\{(x,y,z) \in \mathbb{R}^3 : x^2 + \frac{3}{2}y^2 + 3z^2 + 2x - 6z \leq \frac{3}{2}\right\}.$$
We are interested in the extrema of the function $f : E \longrightarrow \mathbb{R}$ defined by
$$f(x,y,y) = -2x + 3y + 6z + 42.$$
Say if the following assertions are correct or not :
   - $f$ reach it's extremums on $E$ at $(-2,1,2)$ and at $(0,-1,0)$,
   - The minimum of $f$ on $E$ is $-3$ and it's maximum is 19.

   (i) TT,
   (ii) TF,
   (iii) FT,
   (iv) FF.

2. What are the values of the minimum $m$ and of the maximum $M$ of the function $f : \overline{\mathcal{B}((0,0),1)} \longrightarrow \mathbb{R}$ defined by
$$f(x,y) = e^{(x-1)y} \quad ?$$
   (i) $m = e^{-\frac{1}{4}}$ and $M = e^{\frac{1}{4}}$,
   (ii) $m = e^{-\frac{1}{4}}$ and $M = e^{\frac{3}{4}}$,
   (iii) $m = e^{-\frac{3\sqrt{3}}{4}}$ and $M = e^{\frac{1}{4}}$,
   (iv) $m = e^{-\frac{3\sqrt{3}}{4}}$ and $M = e^{\frac{3\sqrt{3}}{4}}$.

3. What are the values of the minimum $m$ and the maximum $M$ of the function $f : \overline{\mathcal{B}((0,0),1)} \longrightarrow \mathbb{R}$ defined by
$$f(x,y) = \frac{1}{4}\left(x^2 + y^2\right) - xy \quad ?$$
   (i) $m = -\frac{10+5\sqrt{5}}{10+4\sqrt{5}}$ and $M = -\frac{10-5\sqrt{5}}{10-4\sqrt{5}}$,
   (ii) $m = -\frac{1}{2}$ and $M = \frac{10-5\sqrt{5}}{10-4\sqrt{5}}$,

(iii) $m = -\frac{1}{2}$ and $M = \frac{1}{2}$,

(iv) $m = -\frac{1}{4}$ and $M = \frac{3}{4}$.

4. We are interested in the extrema of the function $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ defined by

$$f(x, y) = \int_x^y te^{-t^2} dt,$$

subject to the constraint

$$e^{x^2} + e^{y^2} - 4 = 0.$$

Say if the following assertions are correct or not :

- Those extrema are reached at $(\pm\sqrt{\ln(3)}, 0)$ and at $(0, \pm\sqrt{\ln(3)})$,
- The minimum is given by $-1/3$ and the maximum is given by $1/3$.

(i) TT,

(ii) TF,

(iii) FT,

(iv) FF.

5. What are the values of the minimum $m$ and of the maximum $M$ of the function $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ defined by

$$f(x, y) = x^2 + y^2 + z^2,$$

subject to the constraints

$$x^2 + y^2 + 2z^2 - 4 = 0 \quad \text{and} \quad xyz - 1 = 0.$$

(i) $m = -3$ and $M = 3$,

(ii) $m = \frac{5-\sqrt{5}}{2}$ and $M = \frac{5+\sqrt{5}}{2}$,

(iii) $m = 3$ and $M = \frac{5+\sqrt{5}}{2}$,

(iv) $m = \frac{5-\sqrt{5}}{2}$ and $M = 3$.

6. Let $n \in \mathbb{N}^*$ and $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ the function defined by

$$f(x_1, ..., x_n) = \prod_{k=1}^{n} x_k^2.$$

What is the value of the maximum of $f$ subject to the constraint

$$\sum_{k=1}^{n} x_k^2 = 1 \ \ ?$$

(i) $\frac{1}{4}$,

(ii) $\frac{1}{n^2}$,

(iii) $\frac{1}{2n}$,

(iv) $\frac{1}{n^n}$.

7. We want to minimize the distance between $P$ and $Q$ where $P$ is a point of the ellipsoid of equation $x^2 + y^2 + \frac{10}{9}z^2 - 10^{10} = 0$ and $Q$ is a point of the plane of equation $x + y + z - 10^8 = 0$. Say if the following assertions are correct or not. *Hint : Compute the normal vector of the plane.*

   - There is severals point where $f$ reach it's minimum,
   - The distance is given by $d(P,Q) = \frac{10^4}{\sqrt{3}}\left(10^4 - \sqrt{290}\right)$.

   (i) TT,

   (ii) TF,

   (iii) FT,

   (iv) FF.

8. Using Lagrange multipliers, find the minimum $m$ of the function defined by

$$f(x,y) = \sqrt{x^2 + 3y^2},$$

   subject to the constraint

$$x + y = 3.$$

   (i) $m = \frac{3}{2}\sqrt{3}$,

   (ii) $m = 3\sqrt{3}$,

   (iii) $m = 3$,

   (iv) The minimum doesn't exist.

9. If we use Langrange multipliers to maximize the function

$$f(x,y) = 3xy,$$

   subject to the constraint

$$x + y = 8,$$

   which of the following assertions are corrects ?

   A) The maximum is smaller or equal at 49,

B) The maximum is reached at $x = 8$,

C) The maximum is reached at $y = 4$.

(*i*) A, B and C,

(*ii*) A and C only,

(*iii*) B and C only,

(*iv*) A only.

10. Let consider a field such that its bound is an ellipse of equation $\frac{x^2}{36} + \frac{y^2}{9} - 1 = 0$. We want to extend a rectangular flag so that it doesn't exceed the top right quarter of the field (i.e. the upper right quadrant of the plane). What is the maximal area of this flag ?

(*i*) 8,

(*ii*) 7,

(*iii*) 5,

(*iv*) 9.