Multiple Choice Questions 8

1. Let $f : \mathbb{R}^2 \to \mathbb{R}$ the function defined by

$$f(x, y) = \sin(x) - \cos(y).$$

(*i*) f has a unique absolute maximum,

(*ii*) f has a unique saddle point but a non-finite number of stationary point,

(*iii*) f has a non-finite number of saddle point, local minima and local maxima,

(*iv*) f has none stationary point.

2. In the lecture, we defined local extremum in open subset of $\mathbb{R}^n$. We can extend this concept to a closed set E and also to it's boundary : a point $a \in E$ is a local maximum (resp. local minimum) if there is a ball $\mathcal{B}(a, \delta)$ such that $f(x) \leq f(a)$ (resp. $f(x) \geq f(a)$) for all $x \in \mathcal{B}(a, \delta) \cap E$. We say that it's a strict maximum (resp. minimum) is the previous inequality are strict for $x \neq a$.

Let $f : [-1, 1] \times [-1, 1] \longrightarrow \mathbb{R}$ the functino defined by $f(x, y) = (x - x_0)^2 + (y - y_0)^2 + 4$, où $(x_0, y_0) \in \mathcal{B}(0, 1/2)$. Say if the following assertions are corrects or not.

- f has a strict minimum for all $(x_0, y_0) \in B(0, 1/2)$,
- For all $(x_0, y_0) \in \mathcal{B}(0, 1/2)$, the function f has at least two local maximum.

(*i*) TT,

(*ii*) TF,

(*iii*) FT,

(*iv*) FF.

3. Let $f : \mathbb{R}^2 \to \mathbb{R}$ the function defined by

$$f(x, y) = x^2 + 4y^2 - 8x + 24y - 6.$$

The function f has :

(*i*) a local minimum at $(1, 1/2)$,

(*ii*) a saddle point at $(1, 1/2)$,

(*iii*) a local minimum at $(4, -3)$,

(*iv*) a local maximum at $(4, -3)$.

4. Let $f : \mathbb{R}^2 \to \mathbb{R}$ the function defined by

$$f(x, y) = x^2 + y^3 - 3xy^2.$$

(*i*) f has two local maxima,

(*ii*) f has a local maximum and a saddle point,

(*iii*) f has two saddles points,

(*iv*) f has a unique saddle point.

5. Let $f : \mathbb{R}^* \times \mathbb{R}^* \to \mathbb{R}$ the function defined by

$$f(x, y) = xy + \frac{2}{x} - \frac{2}{y}.$$

(*i*) f has a local maximum at $(2^{2/3}, -2^{2/3})$,

(*ii*) f has a local maximum at $(-2^{1/3}, 2^{1/3})$,

(*iii*) f has a saddle point at $(-2^{1/3}, 2^{1/3})$,

(*iv*) f has no stationary point.

6. Let $f : \mathbb{R}^2 \to \mathbb{R}$ the function defined by

$$f(x, y) = 8x^3 + 2y^3 + xy.$$

Say if the following assertions are corrects or not.

- f has a unique stationary point,
- f has a unique local maximum and a unique local minimum.

(*i*) TT,

(*ii*) TF,

(*iii*) FT,

(*iv*) FF.

7. Let $\mathcal{T}$ the closed triangle of $\mathbb{R}^2$ with vertices $(1, 0)$, $(1, 5)$ and $(6, 0)$. Let $f : T \to \mathbb{R}^2$ defined by

$$f(x, y) = 7 + xy - x - 3y.$$

Then, the value of $\max_{(x,y)\in\mathcal{T}} f(x, y)$ is given by :

(i) 7,

(ii) 9,

(iii) 5,

(iv) 6.

8. Let $(1,1)$ a stationary point of a function $f \in \mathcal{C}^2$ such that :

$$\partial_{xx} f(1,1) = 2, \quad \partial_{xy}(1,1) = 4a, \quad \text{and} \qquad \partial_{yy}(1,1) = -2.$$

Therefore f has :

(i) a local minimum if $a > 0$,

(ii) a local maximum if $a < 0$,

(iii) a saddle point for all $a \in \mathbb{R}$,

(iv) Answers (i) and (ii) are corrects.

9. Let consider the surface defined by

$$z^2 = xy + 64.$$

The closest point of the origine are :

(i) $(4,-7,6)$ and $(-4,7,-6)$,

(ii) $(-4,-7,-6)$ and $(4,7,6)$,

(iii) $(-2,14,-6)$ and $(2,-14,6)$,

(iv) $(0,0,-8)$ and $(0,0,8)$.

10. We want to build a rectangular box in adamantiumn, but we can provide only 600cm^2 of this precious metal. What must be the dimension of the box to maximize the volume ?

(i) $10 \times 10 \times 10$,

(ii) $2 \times 2 \times 74$,

(iii) $1 \times 1 \times 149.5$,

(iv) $5 \times 10 \times 20$.