Multiple Choice Questions 7

1. Consider the function $f(x, y) = e^{x^2} + \sin(y)$ defined over $U = \mathcal{B}(\overline{0}, \frac{\pi}{4})$.

   (*i*) $\nabla f(x, y) \neq 0$ for all $x, y \in U$,

   (*ii*) The function is one-to-one on U,

   (*iii*) Both are corrects,

   (*iv*) None of (i) and (ii) are corrects.

2. Let
$$\overline{v}(x, y) = \left(\cos\left(\frac{x}{\sqrt{x^2+y^2}} + \cos(\sqrt{x^2+y^2})\right), e^{\frac{1}{\sqrt{x^2+y^2}}}\right).$$
   Then v is locally injective in a neighbourhood of $(1, 1)$.

   (*i*) Yes but not over $\mathbb{R}^2/\{\overline{0}\}$,

   (*ii*) Yes and also on $\mathbb{R}^2/\{\overline{0}\}$,

   (*iii*) No,

   (*iv*) Yes, but $J_{\overline{v}}(1, 1)$ is not invertible.

3. Let $\overline{v}(x, y) = (e^{xy}, x+x^2+y^2) \equiv (s, t)$ in a neighborhood of $(1, 1)$, and let $f(x, y) = xy$. If $g(s, t) = f(\overline{v}^{-1}(s, t))$ then $\nabla_{s,t}\ g(e, 3)$ is given by :

   (*i*) $(e^{-1}, 0)^T$,

   (*ii*) $(e^{-1}, \frac{1}{3})^T$,

   (*iii*) $(-1, 0)^T$,

   (*iv*) $(-1, 1)^T$.

4. Let (x, y) such that $f(x, y) = 0$ where
$$f(x, y) = x^2 + 2e^y + \sin(xy) - 2$$
   in a neighborhood of $(0, 0)$. Say if the following assertions are correct or not.

   (*i*) There is an implicit function $y = \phi(x)$ but ϕ is not derivable,

   (*ii*) There is an implicit function $y = \phi(x)$ with $\phi'(0) = 0$.

   (*iii*) There is an implicit function $x = \phi(u)$ with $\phi'(0) = 0$.

   (*iv*) There is an implicit function $x = \phi(y)$ such that ϕ' is locally $\mathcal{C}^1$.

5. Let (x, y, z) such that $f(x, y, z) = 0$ where

$$f(x, y, z) = x^2 + y^2 - 3x + y^3 + xe^y + xze^y + z^4 e^{xz} - 1.$$

The equation of the tangent plane whenever $x = y = 0$ is :

(i) $z = 1$,

(ii) $-2x + 4(z - 1) = 0$,

(iii) $-2x + y + 4(z - 1) = 0$,

(iv) Undefined.

6. Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ whenever

$$2x^3 - y^3 + 9xy + 1 = 0.$$

(i) $\frac{dy}{dx} = \frac{2x^2 - 3y}{y^2 + 3x}$,

(ii) $\frac{dy}{dx} = \frac{3y + 2x^2}{y^2 + 3x}$,

(iii) $\frac{dy}{dx} = \frac{2x^2 + 3y}{y^2 - 3x}$,

(iv) $\frac{dy}{dx} = \frac{3y - 2x^2}{y^2 - 3x}$.

7. Find $\dfrac{\partial z}{\partial x}$ whenever

$$xe^{2y} - yz + ze^{3x} = 0.$$

(i) $\frac{\partial z}{\partial x} = \frac{e^{3x} + y}{e^{2y} + 3ze^{3x}}$,

(ii) $\frac{\partial z}{\partial x} = -\frac{e^{2y} + 3ze^{3x}}{e^{3x} - y}$,

(iii) $\frac{\partial z}{\partial x} = -\frac{e^{2y} + 3ze^{3x}}{e^{3x} + y}$,

(iv) $\frac{\partial z}{\partial x} = -\frac{e^{3x} - y}{e^{2y} + 3ze^{3x}}$.

8. If $z = f(x, y)$ and $x = r\cos 2\theta$, $\quad y = 2r\sin\theta$, gives $\frac{\partial z}{\partial \theta}$ in function of $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

(i) $\frac{\partial z}{\partial \theta} = 2r\left(\frac{\partial z}{\partial y}\cos\theta - \frac{\partial z}{\partial x}\sin 2\theta\right)$,

(ii) $\frac{\partial z}{\partial \theta} = \frac{\partial z}{\partial x}\cos 2\theta + \frac{\partial z}{\partial y}\sin\theta$,

(iii) $\frac{\partial z}{\partial \theta} = r\left(2\frac{\partial z}{\partial y}\cos\theta - \frac{\partial z}{\partial x}\sin 2\theta\right)$,

(iv) $\frac{\partial z}{\partial \theta} = 2r\left(\frac{\partial z}{\partial y}\cos\theta + \frac{\partial z}{\partial x}\sin 2\theta\right)$.

9. Let

$$g(x, y) = x^3 - y^9 = 0$$

and $(x_0, y_0) = (0, 0)$. Say if the following assertions are corrects or not :

a) It's not possible to find a function $y = \phi_1(x)$ such that $y_0 = \phi_1(x_0)$, but we can find a function $x = \phi_2(y)$ such that $x_0 = \phi_2(y_0)$,

b) It's not possible to find a function $x = \phi_2(y)$ such that $x_0 = \phi_2(y_0)$ but we can find a function $y = \phi_1(x)$ such that $y_0 = \phi_1(x_0)$,

c) $g_x(x_0, y_0) = g_y(x_0, y_0) = 0$ and thus none of both variable can be find,

d) We can find a function $y = \phi_1(x)$ such that $y_0 = \phi_1(x_0)$ and $\phi_1'(x_0) = 0$.

10. Let

$$f(x, y) = e^{x - \alpha y^2} - 1.$$

Say if the following assertions are corrects or not :

a) $f(x, y) = 0$ define in a neighborhood of $(0, 1)$ an implicit function $y = \phi(x)$, for all $\alpha \in \mathbb{R}$.

b) $f(x, y) = 0$ define in a neighborhood of $(1, \frac{1}{\sqrt{\alpha}})$ an implicit function $x = \phi(y)$, for all $\alpha \in \mathbb{R}$.

c) $f(x, y) = 0$ define in a neighborhood of $(0, 0)$ an implicit function $x = \phi(y)$ et $\phi'(0) = 1$, for all $\alpha \in \mathbb{R}$.

d) $f(x, y) = 0$ define in a neighborhood of $(0, 0)$ an implicit function $x = \phi(y)$ with $\phi'(0) = 0$, for all $\alpha \in \mathbb{R}$.