Multiple Choice Questions 6

1. Let define the vector field $\vec{v}(x,y) = (xy^2, xy^3, x)^T$. The Jacobian matrix $J_{\vec{v}}(\vec{x})$ is given by

   (i) $\begin{pmatrix} y^2 & y^3 & 1 \\ 2yx & 3y^2x & 0 \end{pmatrix}$,

   (ii) $\begin{pmatrix} 2y & 3y^2 & 0 \\ 2 & 6y & 0 \end{pmatrix}$,

   (iii) $\begin{pmatrix} 2xy & y^2 \\ 3y^2x & y^3 \\ 0 & 1 \end{pmatrix}$,

   (iv) None of the previous choices.

2. Let F the function defined by $F(x,y,z) = (x, y^2, z^3)^T$. It's rotational, $\nabla \times F(x,y,z)$, is given by :

   (i) $\vec{0}$,

   (ii) 0,

   (iii) $\begin{pmatrix} 3z^2y^2 & - & 2z^3y \\ z^3 & - & 3xz^2 \\ 2yx & - & y^2 \end{pmatrix}$,

   (iv) None of the previous choices.

3. Let F the function defined by $F(x,y,z) = (xy, yz, zx)^T$. Compute $\nabla \times (\nabla F)(1,1,1)$.

   (i) $\nabla \times (\nabla F)(1,1,1) = \vec{0}$,

   (ii) $\nabla \times (\nabla F)(1,1,1) = 0$,

   (iii) $\nabla \times (\nabla F)(1,1,1) = 3$,

   (iv) It's undefined.

4. Let $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ a function $\mathcal{C}^2(\mathbb{R}^n)$. What is

$$\sum_{i=1}^{n} \frac{\partial^2 f}{\partial x_i^2}(\vec{x}) = \sum_{i=1}^{n} D_{x_i x_i}(f)(\vec{x}) \ \ ?$$

(i) $\mathrm{Tr}(\mathrm{Hess}(f)(\vec{x}))$,

(ii) $\nabla \cdot \nabla f(\vec{x})$,

(iii) $\Delta f(\vec{x})$,

(iv) All of the previous choices.

5. Let $n = 3$ and $\vec{v}, \vec{w} \in \mathcal{C}^1(\mathbb{R}^n)$. Then $\mathrm{div}(\vec{v} \times \vec{w})$ is given by :

(i) $\langle \nabla \times \vec{v}, \vec{w} \rangle + \langle \vec{v}, \nabla \times \vec{w} \rangle$,

(ii) $\langle \nabla \times \vec{v}, \nabla \times \vec{w} \rangle$,

(iii) $\langle \nabla \cdot \vec{v}, \vec{w} \rangle + \langle \vec{v}, \nabla \cdot \vec{w} \rangle$,

(iv) None of the previous choices.

6. Let $\vec{v}, \vec{w} : \mathbb{R}^n \to \mathbb{R}^n$ two vectors fields such that $\vec{v}, \vec{w} \in \mathcal{C}^1(\mathbb{R}^n)$. Then $\nabla(\langle \vec{v}, \vec{w} \rangle)$ is given by :

(i) $J_{\vec{v}}\vec{w} + J_{\vec{w}}\vec{v}$,

(ii) $J_{\vec{v}} \cdot \vec{w} - J_{\vec{w}} \cdot \vec{w}$,

(iii) $J_{\vec{v}} J_{\vec{w}}$,

(iv) $(\vec{w}^T J_{\vec{v}} + \vec{v}^T J_{\vec{w}})^T$.

7. Let $\vec{v}$ the vector field defined by

$$\vec{v}(x, y) = \begin{pmatrix} x^3 - y^2 \\ y^3 - x^2 \end{pmatrix}.$$

In a neighborhood V of $(1, -1)$, we consider $\vec{v} : V \to W$ and $\vec{w} : W \to V$ it's inverse. The matrix $J_{\vec{w}}(0, -2)$ is given by :

(a) $\begin{pmatrix} 3 & 2 \\ -2 & 3 \end{pmatrix}$,

(b) $\frac{1}{13} \begin{pmatrix} 3 & 2 \\ -2 & 3 \end{pmatrix}$,

(c) $\begin{pmatrix} 3 & -2 \\ 2 & 3 \end{pmatrix}$,

(d) $\frac{1}{13}\begin{pmatrix} 3 & -2 \\ 2 & 3 \end{pmatrix}$.

8. Are the following assertions correct or not ?

   - For $\vec{v}(x,y) = (x^2 - 2xy, 2xy - y^2)^T$, we have that $\det(J_{\vec{v}}(1,1)) = 4$.
   - Let $\vec{v} : \mathbb{R}^2 \to \mathbb{R}^2$ such that $\vec{v} \in C^1(\mathbb{R}^2)$. If $\det(J_{\vec{v}}(\vec{x_0})) = 0$, then $\vec{v}$ is not locally invertible.

   (i) TT,
   (ii) TF,
   (iii) FT,
   (iv) FF.

9. Find the quadric approximation of $f(x,y) = \cos(x-y) + 2\sin(x-y)$ at $(0,0)$.

   (i) $Q(x,y) = 2 + 2x - 2y - \frac{1}{2}x^2 + xy + \frac{1}{2}y^2$,
   (ii) $Q(x,y) = 1 + 2x - 2y - \frac{1}{2}x^2 - xy - \frac{1}{2}y^2$,
   (iii) $Q(x,y) = 2 - 2x + 2y - \frac{1}{2}x^2 + xy + \frac{1}{2}y^2$,
   (iv) $Q(x,y) = 1 + 2x - 2y - \frac{1}{2}x^2 + xy - \frac{1}{2}y^2$.

10. Find the quadric approximation of $f(x,y) = \sqrt{1 - 4x - 2y}$ at $(0,0)$.

   (i) $Q(x,y) = 1 - 2x - y - 2x^2 - 2xy - \frac{1}{2}y^2$,
   (ii) $Q(x,y) = 1 + 2x + y + 2x^2 + 2xy - y^2$,
   (iii) $Q(x,y) = 1 + 2x + y + 2x^2 - 2xy + y^2$,
   (iv) $Q(x,y) = 1 - 2x - y - 2x^2 + 2xy - \frac{1}{2}y^2$.