Multiple Choice Questions 5

1. Let consider the function $w = xe^{\frac{y}{z}}$. If $x = t^2,\ y = 1 - t$ and $z = 1 + 3t$ compute $\frac{dw}{dt}$.

   (i) $\left(t + \frac{x}{z} + \frac{xy}{z}\right) e^{\frac{y}{z}}$,

   (ii) $\left(t + \frac{x}{z} + \frac{xy}{z^2}\right) e^{\frac{y}{z}}$,

   (iii) $\left(2t - \frac{x}{z} - 3\frac{xy}{z}\right) e^{\frac{y}{z}}$,

   (iv) $\left(2t - \frac{x}{z} - 3\frac{xy}{z^2}\right) e^{\frac{y}{z}}$.

2. Let $z = f(x, y)$ and f a function such that

$$f(-3, 2) = 2, \quad D_1 f(-3, 2) = 1 \quad \text{and} \quad D_2 f(-3, 2) = -2.$$

   The equation of the hyperplane of the graph of f at $(-3, 2)$ is given by :

   (i) $z = 2 + x + 2y$,

   (ii) $z = 2 + x - 2y$,

   (iii) $z = 2 - x + 2y$,

   (iv) $z = 9 + x - 2y$.

3. Let $f(x, y) = \sqrt{7 - x^2 - 2y^2}$. The linear approximation of f at $(2, -1)$ is given by

   (i) $7 - x + 2y$,

   (ii) $7 - 2x + 2y$,

   (iii) $-5 - 2x + 2y$,

   (iv) None of the previous choices.

4. Hess$(\frac{1}{r})(3,4)$ for $r = \sqrt{x^2 + y^2}$ is given by

(i)
$$\begin{pmatrix} \frac{12}{5^5} & \frac{3}{5^5} \\ \frac{3}{5^5}, & \frac{17}{5^5} \end{pmatrix},$$

(ii)
$$\begin{pmatrix} \frac{12}{5^5} & \frac{5}{5^5} \\ \frac{3}{5^5}, & \frac{17}{5^5} \end{pmatrix},$$

(iii)
$$\begin{pmatrix} \frac{2}{5^5} & \frac{36}{5^5} \\ \frac{36}{5^5}, & \frac{23}{5^5} \end{pmatrix},$$

(iv)
$$\begin{pmatrix} \frac{2}{5^5} & \frac{3}{5^5} \\ \frac{3}{5^5}, & \frac{23}{5^5} \end{pmatrix}.$$

5. Let $z = f(x,y) = x\ln(x+11y)$. Let define $h(t) = f(x,y)$ where $x = \sin(t)$ and $y = \cos(t)$. What is $h'(t)$?

(i) $\ln(x+11y)\cos(t) + \frac{x(\sin(t)-\cos(t))}{x+11y}$,

(ii) $\ln(x+11y)\cos(t) - \frac{11x\sin(t)}{x+11y}$,

(iii) $\ln(x+11y)\sin(t) + \frac{x(\sin(t)-11\cos(t))}{x+11y}$,

(iv) $\ln(x+11y)\cos(t) + \frac{x(\cos(t)-11\sin(t))}{x+11y}$.

6. Let $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x,y) = \begin{cases} \dfrac{x^3 y}{x^2+y^2} & \text{si } (x,y) \neq (0,0) \\ 0 & \text{si } (x,y) = (0,0) \end{cases} .$$

- $D_x f(0,0) \neq D_y f(0,0)$,
- $D_{xy} f(0,0) \neq D_{yx} f(0,0)$,

(i) TT,
(ii) TF,
(iii) FT,
(iv) FF.

7. Let f as in question 6.

- The orientation of the normal vector $\mathbf{n}$ of the graph of f at $(0,0,0)$ is $(1,1,-1)$,
- The equation of the tangent plane at $(0,0,0)$ is given by $z = p(x,y) = 6x + y$.

(i) TT,
(ii) TF,
(iii) FT,
(iv) FF.

8. Let $U \subset \mathbb{R}^n$ an open and $f, g : U \longrightarrow \mathbb{R}$ two functions that are $\mathcal{C}^2(U)$. Then $\text{Hess}(fg)$ in U is given by :

(1) $g\text{Hess}(f) + f\text{Hess}(g)$,
(2) $g\text{Hess}(f) + f\text{Hess}(g) + \nabla(f)\nabla(g)^T + \nabla(g)\nabla(f)^T$,
(3) $g\text{Hess}(f) + f\text{Hess}(g) + \nabla(f)^T\nabla(g) + \nabla(g)^T\nabla(f)$,
(4) Doesn't necessarily exist.