Multiple Choice Questions 4

1. All directional derivatives exists for $u : \mathbb{R}^2 \to \mathbb{R}$ at the point $\vec{0}$ if :

   (i) The gradient exist,

   (ii) The partial derivatives $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$ are continuous at $\vec{0}$,

   (iii) All directional derivatives exists on $\mathbb{R}^2 \backslash \{(0,0)\}$,

   (iv) None of the previous choices.

2. If all directional derivatives exists for u at $\vec{0}$:

   - The gradient exist,
   - They are continuous at $\vec{0}$.

   (i) TT,

   (ii) TF,

   (iii) FT,

   (iv) FF.

3. For $f(x, y) = e^{x^2 y}$, the derivative in $(1, 1)$ along the vector $(1, 1)$ is

   (i) 3,

   (ii) e^3,

   (iii) $3e$,

   (iv) None of the previous choices.

4. If $z = f(x, y)$ and Let $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ a function such that

$$\frac{\partial f}{\partial x}(3,5) = 4 \quad \text{and} \quad \frac{\partial f}{\partial y}(3,5) = -7.$$

   Let define $z(t) = f\big(g(t), h(t)\big)$ where the functions g and h are such that

$$g(5) = 3, \quad g'(5) = 4$$
$$h(5) = 5, \quad h'(5) = 2.$$

   What is $z'(5)$?

(i) 10,

(ii) 4 ,

(iii) 2,

(iv) 6.

5. Compute

$$\lim_{(x,y)\to(12,4)} xy\cos(x-3y).$$

(i) 4,

(ii) 48,

(iii) 0,

(iv) -4.

6. What is

$$\lim_{(x,y)\to(0,0)} \frac{5xy^2}{x^2+y^2} \quad ?$$

(i) 10,

(ii) 0,

(iii) 5,

(iv) The limit doesn't exist.

7. Let $f(x,y) = x\cos(x+y) + \sin(x+y)$. Compute $\dfrac{\partial f}{\partial x}$.

(i) $x\cos(x+y)$,

(ii) $2\cos(x+y) - x\sin(x+y)$,

(iii) $-2x\sin(x+y)$,

(iv) $-x\sin(x+y)$.

8. Let

$$f(x,y) = \frac{1}{2}x\arctan\left(\frac{y}{x}\right).$$

Compute $D_{xy}f$.

(i) $f_{xy} = \frac{xy^2}{(x^2+y^2)^2}$,

(ii) $f_{xy} = \frac{x^2y}{2(x^2+y^2)}$,

(iii) $f_{xy} = -\frac{xy^2}{(x^2+y^2)^2}$,

(iv) $f_{xy} = -\frac{xy^2}{2(x^2+y^2)}$.

9. Let

$$z = \frac{y}{x} f\left(\frac{y}{x}\right).$$

Compute $\dfrac{\partial z}{\partial x}$.

(i) $\frac{\partial z}{\partial x} = \frac{1}{y^2}\left\{yf\left(\frac{x}{y}\right) + xf'\left(\frac{x}{y}\right)\right\}$,

(ii) $\frac{\partial z}{\partial x} = yf\left(\frac{x}{y}\right) + xf'\left(\frac{x}{y}\right)$,

(iii) $\frac{\partial z}{\partial x} = -\frac{1}{x}\left\{f\left(\frac{x}{y}\right) + xyf'\left(\frac{x}{y}\right)\right\}$,

(iv) $\frac{\partial z}{\partial x} = -\frac{1}{x^2}\left\{yf\left(\frac{x}{y}\right) - xf'\left(\frac{x}{y}\right)\right\}$.