Multiple choice test 3

1. For $f(x,y,z) = e^{xy^2z^3}$, $(\nabla f)^T =$

   (i) $(e^{xy^2z^3}, 2ye^{xy^2z^3}, 3z^2e^{xy^2z^3})$

   (ii) $(y^2z^3e^{y^2z^3}, 2xyz^3e^{2xyz^3}, 3xy^2z^2e^{3xy^2z^2})$

   (iii) $(e^{xy^2z^3}, e^{xy^2z^3}, e^{xy^2z^3})$

   (iv) None of the choices (i)-(iii).

2. Let $f : \mathbb{R} \to \mathbb{R}^2$ with $f_1(t) = \frac{e^t}{\sqrt{2}}$ and $f_2(t) = \frac{e^{-t}}{\sqrt{2}}$. The length of the graph of f between $t = 1$ and $t = 2$ is equal to :

   (i) $\frac{e^2-1}{e^1-1}$

   (ii) $\int_1^2 \frac{1}{2}e^t(1+e^{-4t})^{1/2}dt$

   (iii) $\int_1^2 \frac{1}{\sqrt{2}}e^t(1+e^{-4t})^{1/2}dt.$

   (iv) $\frac{e^2-e^1+e^{-1}-e^{-2}}{\sqrt{2}}$

   (RECALL: The graph of f is the curve $\binom{t}{f(t)}$ in $\mathbb{R}^3$.)

3. Let f be defined on $\mathbb{R}^2$ as :

$$f(x,y) = \begin{cases} 0, & \text{if } (x,y) = 0 \\ x^2\ln(|x|), & \text{if } y = 0 \\ y^2\ln(|y|), & \text{if } x = 0 \\ x^2\ln(|x|) + y^2\ln(|y|), & \text{otherwise.} \end{cases}$$

   (i) $D_{xy}f = D_{yx}f$ but $D_{xx}f$ is not continuous.

   (ii) $D_{xy}f \neq D_{yx}f$ but they are both continuous.

   (iii) None of the functions $D_{xy}f$, $D_{yx}f$ or $D_{xx}f$ is continuous at $(0,0)$.

   (iv) All the alternatives (i)-(iii) are true.

4. $f : \mathbb{R}^n \mapsto \mathbb{R}$ is differentiable at $\vec{0}$ if and only if there exists a vector $\vec{b}$ such that :

$$\lim_{\bar{h}\to\vec{0},\bar{h}\neq\vec{0}} \frac{f(\vec{h}) - f(\vec{0}) - <\vec{h},\vec{b}>}{||\vec{h}||_\infty} = 0.$$

   $f : \mathbb{R}^2 \to \mathbb{R}$ is differentiable if and only if $f \in C^1(\mathbb{R}^2)$.

   (i) TT

   (ii) TF

   (iii) FT

   (iv) FF

5. What is

$$\lim_{(x,y)\to(0,0)} \frac{x^2-9y^2}{x^2+y^2} \quad ?$$

- (i) 0
- (ii) 1
- (iii) -9
- (iv) The limit does not exist.

6. Let $f : \mathbb{R}^2 \mapsto \mathbb{R}$ the function defined by :

$$f(x,y) = \begin{cases} \dfrac{x^2}{y}, & y \neq 0 \\ 0, & y = 0. \end{cases}$$

We can assert that :
a) f is continuous;
b) f is differentiable.

- (i) TT
- (ii) TF
- (iii) FT
- (iv) FF

7. Consider two curves $f, g : [0,1] \to \mathbb{R}$ defined as :

$$f(t) = \begin{pmatrix} 1-4(t-\frac{1}{2})^2 \\ 0 \end{pmatrix} \text{ and } g(t) = \begin{pmatrix} 1/2 \\ (t-\frac{1}{4})^2 \end{pmatrix},$$

and also $h = f - g$. We can assert that :

a) f and g are regular ;
b) h is regular.

- (i) TT
- (ii) TF
- (iii) FT
- (iv) FF

8. Let $g : \mathbb{R} \mapsto \mathbb{R}$ and $f : \mathbb{R}^2 \mapsto \mathbb{R}$ the function defined as :

$$f(x,y) = \begin{cases} \dfrac{xe^x - ye^y}{x-y}, & \text{si } x \neq y \\ g(x), & \text{si } y = x. \end{cases}$$

This function is continuous if and only if :

(i) $g(x) = (x+1)e^x$

(ii) $g(x) = 0$

(iii) $g(x) = 1$

(iv) None of the three above.