Multiple Choice Questions 2

1. Let θ the angle of intersection between the line of equation $\sqrt{3}y + 2x = 5/2$ and the circle of equation $x^2 + y^2 = 1$ at $(\frac{1}{2}, \frac{\sqrt{3}}{2})$. What is $\cos\theta$?

   (i) $\frac{\pi}{3}$,
   (ii) $\frac{1}{2}$,
   (iii) $\frac{3}{7}$,
   (iv) $\frac{5}{\sqrt{28}}$.

2. $f : \mathbb{R} \longrightarrow \mathbb{R}$ is continuous at x_0 if :

   (i) $\forall \varepsilon > 0, \exists \delta > 0 : |f(x) - f(x_0)| < \delta \implies |x - x_0| < \varepsilon$,
   (ii) $f(x_0) = \lim_{n\to\infty} f\left(x_0 + \frac{1}{n}\right)$,
   (iii) $\nexists x_n \longrightarrow x_0$ such that $f(x_n)$ doesn't converge to $f(x_0)$,
   (iv) For all sequence $(x_n)_{n\in\mathbb{N}}$ that converge to x_0 such that $\big(f(x_n)\big)_{n\in\mathbb{N}}$ converge, $f(x_n) \longrightarrow f(x_0)$.

3. Let consider the functions F_1 and F_2 on $E = \mathcal{C}([0,1])$ defined by

$$F_1(f) = \int_0^1 f(x)\mathrm{d}x \quad \text{and} \quad F_2(f) = f\left(\frac{1}{1}\right).$$

   - F_1 is continuous with refer to the norm $\|\cdot\|_\infty$,
   - F_2 is continuous with refer to the norm $\|\cdot\|_2$.

   (i) TT,
   (ii) TF,
   (iii) FT,
   (iv) FF.

4. The curves

$$f_1(t) = (t, 2t, t^2) \quad \text{and} \quad f_2(t) = (1, 3-t, t^3),$$

   for $t \in [0,1]$ intersect at θ. What is $\cos(\theta)$?

   (i) $\frac{4}{3\sqrt{10}}$
   (ii) $\frac{2}{5\sqrt{3}}$
   (iii) $\frac{\sqrt{3}}{2}$
   (iv) $\frac{\pi}{3}$

5. Say if the following assertion are corrects or not :

   - The space $(E, \|\cdot\|_4)$ with $E = \mathcal{C}([0,1])$ and

   $$\|f-g\|_4 = \left(\int (f(x)-g(x))^4 \mathrm{d}x\right)^{1/4}$$

   is complete.

   - The metric space $(E, \|\cdot\|_2)$ with $E = \overline{\mathcal{B}\big((0,0),1\big)}$ is complete.

   (i) TT,
   (ii) TF,
   (iii) FT,
   (iv) FF.

6. Say if the following assertion are corrects or not :

   - Let $f : [0,\pi] \to \mathbb{R}^2$ defined by $f(t) = (\sin^2(t), \cos^2(t))$. Is f regular ?
   - Let $g : [0,1] \to \mathbb{R}^2$ defined by

   $$g(t) = \begin{cases} (t,t) & t \in [0, \frac{1}{2}] \\ \big(1-t, (1-t)^2\big) & t \in (\frac{1}{2}, 1]. \end{cases}$$

   Is g a curve ?

   (i) TT,
   (ii) TF,
   (iii) FT,
   (iv) FF.

7. Let $f : \mathbb{R}^d \to \mathbb{R}$ a bounded function (i.e. there is $M > 0$ such that $|f(x)| \leq M$ for all $x \in \mathbb{R}^d$). Then, there are $x \in \mathbb{R}^d$, $y \in \mathbb{R}$ and a sequence $(x_n)_{n\in\mathbb{N}}$ such that $x_n \neq x$ for all n but

   $$x_n \longrightarrow x \quad \text{and} \quad f(x_n) \longrightarrow y.$$

   (i) T,
   (ii) F.