Multiple Choice Questions 13

1. Let consider the following equation

$$\frac{dy}{dx} = \frac{y(x)}{x^2}.$$

- For all $a, b \in (0, +\infty)$, the equation has a unique solution on $[a, b]$ such that $y(a) = y_0$, for all $y_0 \in (0, +\infty)$ fixed,
- There is a non-zero solution over $\mathbb{R} \setminus \{0\}$.

(*i*) TT,
(*ii*) FT,
(*iii*) TF,
(*iv*) FF.

2. Let consider the following equation :

$$\frac{dy}{dt} = \cos(t) \cdot e^{-y(t)}.$$

- The general solution is given by $\ln(\sin(t) + C)$, $C \in \mathbb{R}$,
- If $y(0) = -1$, the unique solution is defined on all bounded interval of $\mathbb{R}$.

(*i*) TT,
(*ii*) FT,
(*iii*) TF,
(*iv*) FF.

3. Let consider the following equation

$$\frac{2x}{1 + (x^2 + y)^2} + \frac{y'}{1 + (x^2 + y)^2} = 0, \quad y(0) = 0.$$

- There is a function H such that y given by $H(x, y(x)) = H(0, 0)$ is a solution of the equation,
- $\arctan(x^2 + y)$ is solution of the equation.

(*i*) TT,
(*ii*) FT,
(*iii*) TF,
(*iv*) FF.

4. What is the general solution of the following equation

$$y'(t) - \frac{1}{t}y(t) - (y(t))^2 = 0 \quad ?$$

Hint: make the substitution $y(t) = -1/z(t)$.

(i) $y(t) = -\frac{2t}{C+t^2}$, $C \in \mathbb{R}$.

(ii) $y(t) = -\frac{2tC_1}{C_2+t^2}$, $C_1, C_2 \in \mathbb{R}$.

(iii) $y(t) = -\frac{2tC}{t^2}$, $C \in \mathbb{R}$.

(iv) None of the previous choices.

5. Let consider the differential equation

$$ty'' + (1-2t)y' + (t-1)y = 0,$$

over $(0, +\infty)$.

- $t \longmapsto e^t$ is a solution.
- The general solution is given by $t \longmapsto (\lambda \ln(t) + \mu)e^t$, $\lambda, \mu \in \mathbb{R}$.

(i) TT,

(ii) FT,

(iii) TF,

(iv) FF.

6. Let $u : (0, +\infty) \longrightarrow \mathbb{R}$ the solution of

$$u'(t) = \sqrt{u(t) + \sin(t)} - \cos(t),$$

with $\lim_{t \to 0^+} u(t) = 0$. Find u. *Hint: make an judicious substitution.*

(i) $u(t) = \frac{1}{4}t^2 + \sin(t)$,

(ii) $u(t) = \frac{1}{4}t^2C - \sin(t)$, $C \in \mathbb{R}$,

(iii) $u(t) = \frac{1}{4}t^2 - C\sin(t)$, $C \in \mathbb{R}$,

(iv) None of the previous choices.

7. Solve the following differential equation :

$$y'' - 4y' + 4y = 0.$$

The solutions are :

(i) $y(t) = ae^{2t} + be^{-2t}$, with $a, b \in \mathbb{R}$,

(ii) $y(t) = (a + bt)e^{2t}$, with $a, b \in \mathbb{R}$,

(iii) $y(t) = a\cos(2t) + b\sin(2t)$, with $a, b \in \mathbb{R}$,

(iv) $y(t) = a\cos(2t)e^{-t} + b\sin(2t)e^{-t}$, with $a, b \in \mathbb{R}$.

8. Solve the following differential equation :

$$3y'' + 6y' + 12y = 12t.$$

The solutions are :

(i) $y(t) = (a + bt)e^{-t+t\sqrt{3}} + t - \frac{1}{2}$, with $a, b \in \mathbb{R}$,

(ii) $y(t) = ae^{-t}\cos(t\sqrt{3}) + be^{-t}\sin(t\sqrt{3})$, with $a, b \in \mathbb{R}$,

(iii) $y(t) = ae^{-t}\cos(\frac{\pi}{3}t) + be^{-t}\sin(\frac{\pi}{3}t) + t - \frac{1}{2}$, with $a, b \in \mathbb{R}$,

(iv) $y(t) = ae^{-t}\cos(t\sqrt{3}) + be^{-t}\sin(t\sqrt{3}) + t - \frac{1}{2}$, with $a, b \in \mathbb{R}$.

9. Solve the following differential equation :

$$x^2y'' + xy' - y = x^2.$$

The solutions are : *Hint : make the substitution* $z(t) = y(e^t)$.

(i) $y(x) = ax + \frac{x^2}{3}$, with $a \in \mathbb{R}$,

(ii) $y(x) = \frac{a}{x} + \frac{x^2}{3}$, with $a \in \mathbb{R}$,

(iii) $y(x) = \frac{a}{x} + bx + \frac{x^2}{3}$, with $a, b \in \mathbb{R}$,

(iv) None of the previous choices.

10. Solve the following differential equation :

$$y'' + 4y' + 3y = 0.$$

The solutions are :

(i) $y(t) = ae^{-3t} + be^{-t}$, with $a, b \in \mathbb{R}$,

(ii) $y(t) = (a + bt)e^{-4t}$, with $a, b \in \mathbb{R}$,

(iii) $y(t) = a\cos(-3t) + b\sin(-t)$, with $a, b \in \mathbb{R}$,

(iv) None of the previous choices.