Multiple Choice Questions 11

1. If $f : I \to \mathbb{R}$ is continuous and bounded, a solution of the equation

$$\begin{cases} \frac{dy}{dx} = f(x), \\ y(t_0) = y_0, \quad t_0 \in I^0 \end{cases}$$

always exist.

(i) T,

(ii) F.

2. If $f : I \to \mathbb{R}$ is continuous and bounded, there is always a unique local solution to

$$\frac{dy}{dx} = f(y).$$

(i) T,

(ii) F.

3. A sufficient condition to have the existence of a unique solution to

$$\frac{d^3y}{dx^3} = \frac{d^2y}{dx^2} f_2(x) + f_0(y)$$

on a bounded interval $I \subseteq \mathbb{R}$ with

$$y(0) = y_0, \quad y'(0) = y_0', \quad \text{and} \quad y''(0) = y_0''$$

is

(i) f_2 and f_0 are continuous,

(ii) f_2 is Lipschitz and f_0 is continuous,

(iii) f_2 is continuous and f_0 is Lipschitz,

(iv) None of the previous choice.

4. Let $f \in \mathcal{C}^1(\mathbb{R}^2, \mathbb{R})$.

   The equation

$$\begin{cases} \frac{dy}{dt} = f(t, y) \\ y(0) = 0, \end{cases}$$

   has

   (i) a local solution but not necessarily unique,
   (ii) a unique global solution
   (iii) we don't have enough informations to conclude,
   (iv) Has not necessarily a global solution.

5. For the following differential equation

$$\frac{dy}{dx} = 9x^2y,$$

   the general solution is :

   (i) $y(x) = Ae^{3x^2}$,
   (ii) $y(x) = Ae^{3x^4}$,
   (iii) $y(x) = Ae^{x^2}$,
   (iv) $y(x) = Ae^{3x^3}$.

6. If y_0 satisfy

$$\begin{cases} (x^2 + 9)\frac{dy}{dx} = xy \\ y(0) = 3 \end{cases}$$

   the value of $y_0(9)$ is :

   (i) $y_0(9) = 17^{\frac{1}{2}}3$,
   (ii) $y_0(9) = 10^{\frac{1}{2}}3$,
   (iii) $y_0(9) = 10^{\frac{1}{2}}4$,
   (iv) $y_0(9) = 40$.

7. If y satisfy

$$\begin{cases} (x + 3)y' = y - 1 \\ y(1) = 2 \end{cases}$$

   then, the value of $y(4)$ is :

(i) $y(4) = -1$,

(ii) $y(4) = 3$,

(iii) $y(4) = \frac{11}{4}$,

(iv) $y(4) = \frac{7}{2}$.

8. Solve the following differential equation

$$y' = \frac{5x^4 y}{\ln y}.$$

The solutions are given by :

(i) $y = e^{\pm\sqrt{x^5+C}}$,

(ii) $y = e^{\pm\sqrt{2x^5+C}}$,

(iii) $y = e^{\sqrt{2x^5+C}}$,

(iv) $y = e^{\pm\sqrt{2x^4+C}}$.

9. Solve the following differential equation :

$$\frac{du}{dt} = 42 + 6u + 7t + ut.$$

The solutions are given by :

(i) $u = -7 + Ce^{\frac{1}{2}t^2+6t}$,

(ii) $u = -7 + Ce^{\frac{1}{2}t+6t^2}$,

(iii) $u = -7 + Ce^{\frac{1}{2}t^2-6t}$,

(iv) $u = -7 + Ce^{t^2+6t}$.