Multiple Choice Questions 11

1. Let $\mathcal{D} = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 - x \leq 0\}$.
   What is the volume of $\mathcal{D}$?

   (a) 0,
   (b) $\frac{\pi}{6}$,
   (c) $\frac{\pi}{3}$,
   (d) $\frac{2\pi}{9}$.

2. Using the following substitution :

$$(x, y) = \left(u^2, \frac{v}{u}\right),$$

   compute

$$\iint_{\mathcal{D}} \frac{\mathrm{d}x\mathrm{d}y}{(1+x)(1+xy^2)},$$

   where $\mathcal{D} = [0, 1]^2$. The result is :

   (i) $\frac{\pi^2}{68}$,
   (ii) $\frac{\pi}{16}$,
   (iii) $\frac{\pi^2}{16}$,
   (iv) $\frac{\pi^2}{32}$.

3. Compute the double integral

$$I = \iint_{\mathcal{D}} \frac{y}{x^2+1}\mathrm{d}x\mathrm{d}y$$

   for

$$\mathcal{D} = \{(x, y) : 0 \leq x \leq 1,\ 0 \leq y \leq \sqrt{x}\}.$$

   The result is :

   (i) $I = \frac{1}{2}\ln(2)$,
   (ii) $I = \frac{1}{4}\ln(2)$,
   (iii) $I = \frac{1}{2}$,
   (iv) $I = 1$.

4. What is the volume $\mathcal{V}$ under the graph of the function defined by

$$f(x,y) = \frac{1}{x+y+3},$$

over the triangle generated by the lines

$$x = 1, \quad x = 7, \quad x + y = 4, \quad \text{and} \quad y + 3 = 0,$$

in the plane Oxy. The result is :

(i) $V = 6 + \ln(7)$,
(ii) $V = 7 + \ln(7)$,
(iii) $V = 7 - \ln(7)$,
(iv) $V = 6 - \ln(7)$.

5. Compute the following integral :

$$I = \iint_{\mathcal{D}} \left\{ (3\pi - 2\tan^{-1}\left(\frac{y}{x}\right) \right\} \mathrm{d}x\mathrm{d}y$$

where $\mathcal{D}$ is the first quadrant of the circle of equation $x^2 + y^2 = 16$. The result is

(i) $\frac{5}{4}\pi$,
(ii) 10π,
(iii) $10\pi^2$,
(iv) $20\pi^2$.

6. Compute the following integral

$$I = \int_0^1 \int_0^{\sqrt{1-y^2}} 4\cos(\pi x^2 + \pi y^2)\mathrm{d}x\mathrm{d}y,$$

using polar coordinate. The result is :

(i) $I = 1$,
(ii) $I = \frac{2}{\pi}$,
(iii) $I = 0$,
(iv) $I = 2$.

7. Using cylindric coordinates, compute

$$I = \iiint_{\mathcal{W}} y \mathrm{d}V$$

where $\mathcal{W}$ is the solid over the plane $z = 0$, between the cylinders

$$x^2 + y^2 = 4 \quad \text{and} \quad x^2 + y^2 = 6,$$

and under the plane of equation $z = x + 3$. The result is :

(i) $I = 0$,
(ii) $I = 152\pi$,
(iii) $I = 304$,
(iv) $I = 304\pi$.

8. Using spherical coordinates, compute

$$I = \iiint_{\mathcal{B}} (x^2 + y^2 + z^2) \mathrm{d}x\mathrm{d}y\mathrm{d}z$$

where $\mathcal{B}$ is the ball of equation

$$x^2 + y^2 + z^2 \leq 4.$$

The result is :

(i) $I = 32\pi$,
(ii) $I = \frac{128\pi}{5}$,
(iii) $I = \frac{32\pi}{3}$,
(iv) $I = 128\pi$.

9. Find the range of the rectangle

$$\mathcal{D} = \{(u, v) : 0 \leq u \leq 2, 0 \leq v \leq 3\}$$

by the application

$$\begin{aligned} \Phi : \mathbb{R}^2 &\longrightarrow \mathbb{R}^2 \\ (u, v) &\longmapsto (2u - 3v, u + 2v). \end{aligned}$$

The result is :

(i) a square,
(ii) a triangle,
(iii) a parallelogram, but not a rectangle,
(iv) a disc.