Multiple Choice 1

1. When does one have $\partial(E \cup F) = \partial E \cup \partial F$?

   (i) $\mathring{E} \subset F$
   (ii) $\bar{E} \cap \bar{F} = \emptyset$
   (iii) $\mathring{E} \cap \mathring{F} = \emptyset$
   (iv) $E \cap F = \emptyset$

2. We denote $\|\cdot\|_2$ for the Euclidien norm on $\mathbb{R}^n$ define the function

$$\begin{array}{rcl} N : \mathbb{R}^n & \mapsto & \mathbb{R} \\ x & \mapsto & \dfrac{\|x\|_2}{1+\|x\|_2} \end{array}$$

   as well as the function d which, for each pair $(x, y) \in \mathbb{R}^n \times \mathbb{R}^n$, gives value $N(x - y)$. Which of the following are true?

   (i) N is a norm and d is a metric.
   (ii) N is a norm but d is not a metric.
   (iii) N is not a norm but d is a metric.
   (iv) N is not a norm and d is not a metric.

3. We define the set $E = \{(x, y) \in \mathbb{R}^2 : x^2 < y < x\}$. Which of the following statements are true?

   (i) E is closed and $\partial E = \{(x, 0) : x \in [-1, 1]\} \cup \{(x, x) : x \in (-1, 1)\}$.
   (ii) E is closed and open.
   (iii) E is open and$\partial E = \{(x, x) : x \in [0, 1]\} \cup \{(x, x^2) : x \in [0, 1]\}$.
   (iv) E is open and $\partial E = \emptyset$.

4. We consider a subset E of $\mathbb{R}^n$ such that every sequence in E which converges has its limit in E. Which of the following is always true?

   (i) E is open and closed
   (ii) E is open
   (iii) E is closed
   (iv) E is neither open nor closed.

5. We define the following subset of $\mathbb{R}^2$: $E = \left\{ \left(\frac{1}{n}, e^{-(\log n)^{\frac{3}{2}}} \right), n \in \mathbb{N}^* \right\}$. Which of the following statements are true?

   (i) E is open and closed
   (ii) E is open and but not closed
   (iii) E is closed but not open.
   (iv) E is neither open nor closed.