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of the Virtual Seminar on
Stochastic Analysis, Random Fields and Applications

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Abstracts of the talks

Sandra Cerrai (University of Maryland)

Incompressible viscous fluids in the plane and SPDEs on graphs, in the presence of fast advection and non-smooth noise

The asymptotic behavior of a class of stochastic reaction-diffusion-advection equations in the plane is studied. We show that as the divergence-free advection term becomes larger and larger, the solutions of such equations converge to the solution of a suitable stochastic PDE defined on the graph associated with the Hamiltonian. Firstly, we deal with the case that the stochastic perturbation is given by a singular spatially homogeneous Wiener process taking values in the space of Schwartz distributions. As in previous works, we assume here that the derivative of the period of the motion on the level sets of the Hamiltonian is not 0. Then in the second part, without this assumption on the derivative of the period, we study a weaker type convergence of the solutions to a class of linear SPDEs.

Carsten Chong (EPFL and Columbia University)

High-frequency analysis of parabolic stochastic PDEs with multiplicative noise

We consider the stochastic heat equation in $\mathbb{R}^d$ driven by a Gaussian noise that is white in time and colored in space with the Riesz kernel of order $\alpha \in (0, 2 \wedge d)$ as spatial correlation function. We find that the normalized power variations of order p of the solution, computed along regular partitions in time at a fixed spatial point, exhibit some interesting behavior. While they satisfy a central limit theorem (CLT) for all values of $\alpha \in (0, 2 \wedge d)$ in the case of additive noise, three different situations arise under multiplicative noise: if $\alpha \in (0, 1)$ or $p = 2$, an analogous CLT holds true; if $\alpha = 1$ and $p \neq 2$, we still have a CLT but the asymptotic mean is no longer zero; if $\alpha \in (1, 2)$ and $p \neq 2$, we have to subtract certain renormalization terms, whose number tends to infinity as $\alpha \to 2$, before we obtain the CLT. As in the analysis of singular stochastic PDEs, these counterterms are most conveniently represented via a family of directed acyclic graphs. If time permits, we discuss how these results relate to the statistical estimation of parabolic SPDEs

Aurélien Deya (Université Henri Poincaré, Nancy)

A nonlinear Schrödinger equation with fractional noise

We will focus on the following *nonlinear Schrödinger equation with space-time fractional perturbation*:

$$\begin{cases} \imath\partial_t u - \Delta u = \rho^2 |u|^2 + \dot{B}\,, & t \in [0,T]\,,\ x \in \mathbb{R}^d\,, \\ u_0 = \phi\,, \end{cases} \tag{1}$$

where $1 \leq d \leq 3$, $\rho : \mathbb{R}^d \to \mathbb{R}$ is a smooth cut-off function in space, and $\dot{B} = \partial_t \partial_{x_1} \cdots \partial_{x_d} B$ stands for the derivative of a space-time fractional Brownian motion B of Hurst index $H = (H_0, \ldots, H_d) \in (0,1)^{d+1}$.

First, it turns out that when $2H_0 + \sum_{i=1}^d H_i > d+1$ (the so-called *regular* case), the interpretation and local well-posedness of (1) can be derived from quite direct arguments, based on a first-order expansion and the use of Strichartz inequalities. In brief, the equation here is shown to be locally well-posed in some suitable Sobolev space $\mathcal{H}^\beta(\mathbb{R}^d)$ ($\beta > 0$), almost surely.

The model behaves much less favorably as soon as $2H_0 + \sum_{i=1}^d H_i \leq d+1$ (the *rough* case). Indeed, due to the irregularity of $\dot{B}$, the solution u is now expected to take values in a Sobolev space $\mathcal{H}^{-\beta}(\mathbb{R}^d)$ of negative order. The interpretation of the nonlinearity in (1) must then go through a *Wick-type renormalization* procedure. The equation can finally be solved with a fixed-point argument, thanks to the specific *local regularization properties* of the Schrödinger equation in $\mathbb{R}^d$.

These considerations are based on a recent joint work with Nicolas Schaeffer and Laurent Thomann.

Franco Flandoli (Scuola Normale Superiore, Pisa)

Three examples at the boundary of mean field theory

The first example comes from environmental noise. In a joint paper with Michele Coghi and in other works of other authors it has been proved that the presence of a common noise in an interacting particle system leads, in the mean field limit, to an SPDE with transport type noise. When the space-covariance of the environmental noise is close to a delta Dirac, however, the system is intuitively close to the case of particles perturbed by independent noises, which in the mean field limit lead to PDEs of parabolic type. We have detected two regimes, starting with environmental noise, with proper scaling of the covariance of the noise, where we get parabolic PDEs in the limit; with noise surviving, or without noise, depending on the regime. These are works respectively with Dejun Luo and with Lucio Galeati and Luo.

The second example is when the interacting potential is rescaled, in order to describe interactions with different degrees of locality. This goes in the direction of hydrodynamic limits, widely understood when starting from discrete models with quite explicit and simple invariant measures, much less understood when the interacting system is made of stochastic differential equations. Some results conjectured recently in collaboration with Marta Leaocata and Cristiano Ricci will be illustrated.

Partially related to the latter is the case of virus diffusion in a human population with both small social clouds and global contacts. Also here rescaled interactions appear, of different nature and the limit PDE have new terms. We have understood rigorously this scaling limit under certain assumptions, in collaboration with Francesco Grotto, Andrea Papini and Cristiano Ricci. An interesting open problem on this topic, detected some time ago also in a work with Enrico Priola and Giovanni Zanco, is the fact that traveling waves disappear in these PDE limit models, opposite to the case of a finite number of particles, a phenomenon that will be illustrated.

Jean-Pierre Fouque (University of California, Santa Barbara)

Linear-quadratic stochastic differential games on Êdirected chain networks

We present linear-quadratic stochastic differential games on directed chains inspired by the directed chain stochastic differential equations introduced by Detering, Fouque, and Ichiba in a previous work. We solve explicitly for Nash equilibria with a finite number of players and we study more general finite-player games with a mixture of both directed chain interaction and mean field interaction. We investigate and compare the corresponding games in the limit when the number of players tends to infinity.Ê The limit is characterized by Catalan functions and the dynamics under equilibrium is an infinite-dimensional Gaussian process described by a Catalan Markov chain, with or without the presence of mean field interaction. This is joint work with Yichen Feng and Tomoyuki Ichiba.

Ludovic Goudenège (CentraleSupélec)

Stochastic α-Navier-Stokes model

We consider a stochastic perturbation of the α-Navier-Stokes model in dimension 2 or 3. The stochastic perturbation is an additive space-time noise of trace class. The deterministic version is a Large Eddy Simulation model which can be interpreted as a perturbation of the Leray regularization to restore frame invariance. It possesses a lot of physcial properties and it is also suitable for numerical simulation. We recover the Navier-Stokes model in the limit $\alpha \to 0$.

Under a natural condition about the trace of operator Q in front of the noise, we prove the existence and uniqueness of strong solution, continuous in time in classical spaces of L^2 functions with L^p estimates of non-linear terms. It is based on a priori estimates of solutions of finite-dimensional systems, and tightness of the approximated solutions.

Moreover, by studying the derivative of the solution with respect to the initial data, we can prove exponential moment of the approximated solutions, which is enough to obtain Strong Feller property and irreducibility of the transition semigroup. This leads naturally to the existence and uniqueness of an invariant measure with exponential moment.

Davar Khoshnevisan (University of Utah)

Ergodicity and central limit theorems for stochastic partial differential equations

In the past few years, in collaboration with Le Chen, David Nualart, and Fei Pu, we have established a series of ergodicity and central limit theorems for stochastic partial differential equations. Different aspects of this theory require different sets of ideas. Instead of concentrating on the differences, we present the central, relatively non-technical, ideas that are common to all of these works. Specifically, we present optimal conditions for spatial ergodicity and central limit theorem for parabolic stochastic PDEs, and their connections to Malliavin calculus, Clark-Ocone type representations of L^2 random variables and Poincaré inequalities on Gauss spaces.

Terry Lyons (University of Oxford)

Computing the full signature kernel as the solution of a Goursat problem

Recently there has been an increased interested in the development of kernel methods for sequential data. An inner product between the signatures of two paths can be shown to be a reproducing kernel and therefore suitable to be used in the context of data science. Oberhauser and Kiralyi observed that there were efficient algorithms to compute the truncated signature kernel that is subsequently used to develop a framework for variational inference based on Gaussian processes with (truncated) signature covariance. The algorithms and methods there are outlined in the case of two time-series with equal length and is mainly focused on sampled continuous paths of bounded variation. We show that the untruncated signature kernel is the solution of a Goursat problem, can be efficiently computed using finite different schemes, and applied to time-series of unequal length (python code: https://github.com/crispitagorico/SignatureKernel). Furthermore, a density argument extends the analysis to the space of geometric rough paths; one can prove using classical theory of integration of one-forms along rough paths that the full signature kernel solves a rough integral equation analogous to the PDE derived for the bounded variation case. This is joint work with Thomas Cass, Cristopher Salvi and Weixin Yang.

Abstracts of the contributions to the round table discussion

The outbreak of COVID-19 called for a mobilization of experts in mathematical models for infectious diseases, statisticians, data analysts, etc. Evaluation of transmission risk, prediction of the outbreak path, and assessment on the seemingly most effective strategies are in high demand by state governments. In this round table discussion, some of these questions will be addressed by three experienced researchers in stochastic mathematical modeling.

René Carmona (Princeton University)

Economic contract theory and mean field games to inform epidemic models

We shall introduce recent works on Contract Theory and Mean Field Games and illustrate on a simple model how they can inform regulator decision making in the control of population movement in an epidemic.

Josselin Garnier (École Polytechnique)

Uncertainty quantification for the modelling of the Covid-19 outbreak

Mathematical (SEIR-type) models have been used to predict the evolution of the Covid-19 outbreak and to justify non-pharmaceutical interventions by political decision makers. The models, however, contain a lot of free parameters and they are based on strong hypotheses on the homogeneity of the population for instance. They can be very efficient to fit the available data but they may fail to make accurate predictions as can be revealed by simple sensitivity analysis. We will address different questions: How is it possible to quantify the uncertainties of the models? What types of data should be collected to reduce the uncertainties? What strategies could be implemented to collect such data?

Etienne Pardoux (Aix-Marseille Université)

Mathematical models of epidemics: theory and practice

It is not an issue right now in early summer 2020 to insist upon the threat that infectious diseases represent for humanity, and the need of mathematical models to understand the type of danger that it represents, even if it is much easier to explain afterwards what has happened, rather than predict the fate of the epidemic while it started.

In this short talk, I want to present the main mathematical models, both stochastic and deterministic, and the relations between them, see [2], [1]. I will insist upon the non–Markovian stochastic models and their deterministic counterparts, following the recent work [7]. I will discuss some recent work relating mathematical models to the data of the Covid–19 epidemic, see in particular [5], [6], [3] and [4], in particular how the transient behavior depends upon the type of model.

We will also discuss how some poor information or uncertainties can affect dramatically the predictions.

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