Modelling and Managing Credit Risk

Séminaire du Cycle Postgrade, EPFL, 2004

Jean-Frédéric Jouanin

Groupe de Recherche Opérationnelle
Crédit Agricole – Crédit Lyonnais

Agenda

- Modelling Credit Risk
- Measuring Credit Risk: the Basle Approach
- Managing Credit Risk with Credit Derivatives.

1 Modelling Credit Risk

Here we deal with **a single** issuer. We can identify three main risk factors to be modelled:

- **the default time** τ : the time when a company goes into bankruptcy or at least fails to deliver some cash or security it was committed to.
- **the recovery rate** R : what fraction of the notional can be recovered by the bank in case of default.
- **the 'spread' risk** default probabilities are not deterministic : they may move at random given the survival of the firm. Math. speaking we have to model: $\mathbb{P}(\tau > t + h | \tau > t)$ for all $t, h > 0$. This is connected with the additional spread a firm has to pay for a credit compared to a sovereign entity.

Two main approaches to the default time :

- **the structural approach** originated with Merton's (1974) firm value model. It tries to describe the total value of the assets of the firm. It is the most famous approach but suffers from severe shortcomings in practice.
- **the reduced-form approach** does not attempt to explain the occurrence of a default in terms of a more fundamental economic process. Instead the emphasis is laid on the probability distribution of the default. It was initiated by Jarrow & Turnbull (1995) and embodies intensity and Markov chain models.

1.1 The First-Passage Time Model

The passage time mechanism was introduced by Black & Cox (1976). Let $(V(t), t \geq 0)$ be the stochastic process modelling the economic value of the firm and H a deterministic barrier.

$$\tau := \inf \{t \geq 0 : V(t) \leq H\}.$$

When interest rates and the recovery rate are deterministic we have for the issuer zero-coupon bond $B^d(t,T)$ on $\{\tau > t\}$

$$B^d(t,T) = B^0(t,T) - (1-R)B^0(t,T)\mathbb{P}(\tau \leq T|\mathcal{F}_t),$$

where $B^d(t,T)$ is the default-free zero-coupon price.

The credit spread is the difference between the yield of the two bond prices.

Example: the firm value is a geometric diffusion process.

$$\mathrm{d}V(t) = \mu V(t)\mathrm{d}t + \sigma V(t)\mathrm{d}B(t).$$

The conditional law of the default time τ is

$$\mathbb{P}\left(\tau \leq T|\mathcal{F}_t\right) = N\left(d_-(t,T)\right) + \left(\frac{H}{V_t}\right)^{2a} N\left(d_+(t,T)\right),$$

where

$$\begin{aligned} d_\pm(t,T) &:= \frac{\log\left(\frac{H}{V_t}\right) \pm (\mu - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}, \\ a &:= \frac{\mu - \frac{1}{2}\sigma^2}{\sigma^2}. \end{aligned}$$

The model suffers from many drawbacks :

- The spread vanishes when time is close to the maturity of the bond.
- The default time is then predictable.
- The calibration of the model on balanced-sheet data is not easy.

1.2 The Intensity Framework

- the Intensity model allows to add some randomness to the default threshold, in such a way that the default occurs as a complete surprise.
- this model loses the micro-economic interpretation of the default time (the model comes from reliability theory), but has much flexibility to produce a large set of distributions.
- The intensity model was initiated by Duffie (1997) and Lando (1998).

The **default time** of a firm is often defined by

$$\tau := \inf\left\{t \geq 0 : \int_0^t \lambda(s)\mathrm{d}s \geq \theta\right\}, \qquad \theta \perp\!\!\!\perp \mathcal{F}_\infty$$

- λ a nonnegative, continuous, $\mathcal{F}$-adapted process called the **intensity process**. It contains the information on the credit quality of the issuer. Here, for simplicity, we will suppose it to be deterministic in the examples.
- θ is a random threshold (an exponential r.v. of parameter 1), independent of the intensity.

The zero-coupon bond of the issuer is given by (when $\tau > t$) by the same formula as before with

$$\mathbb{P}\left(\tau \leq T | \mathcal{F}_t \vee \sigma(\tau > t)\right) = \mathbb{E}\left[e^{-\int_t^T (r(s)+\lambda(s))\mathrm{d}s} | \mathcal{F}_t\right]$$

If we choose a deterministic intensity, we can calibrate it on Credit Default Swaps market prices. When the term structure is flat $s(T) = s$, a good approximation of the intensity is:

$$\lambda = \frac{s}{1 - R}$$

We can identify the intensity process with the spread margin of the issuer.

2 Measuring Credit Risk : the Basle Approach

To face the possible high cost of a default, the bank is required to reserve some capital by the rules defined by the Basle Committee. We review two main methods :

- The Standardized Approach (Cooke, 1988) where the capital requirement is a fraction of the total credit exposure of the bank.
- The Internal Rating Based Approach which takes into account the dependence of the defaults.

2.1 The Standardized Approach

We consider a portfolio of I loans with notional $N_i, \quad i = 1 \ldots I$.

$$\text{Risk} = \sum_{i=1}^{I} \text{RW}_{\text{i}} N_i.$$

The risk weights are based on external ratings:

Rating		AAA/AA-	A+/A-	BBB+/BBB-	BB+/B-	B-/C	non rated
Sovereign		0%	20%	50%	100%	150%	100%
Bank	1	20%	50%	100%	100%	150%	100%
	2	20%	50%	50%	100%	150%	50%
	2 (–3M)	20%	20%	20%	50%	150%	20%
Corporate				BBB+/BB-		B+/C	
		20%	50%	100%		150%	100%

2.2 The Basle Internal Rating Based Approach

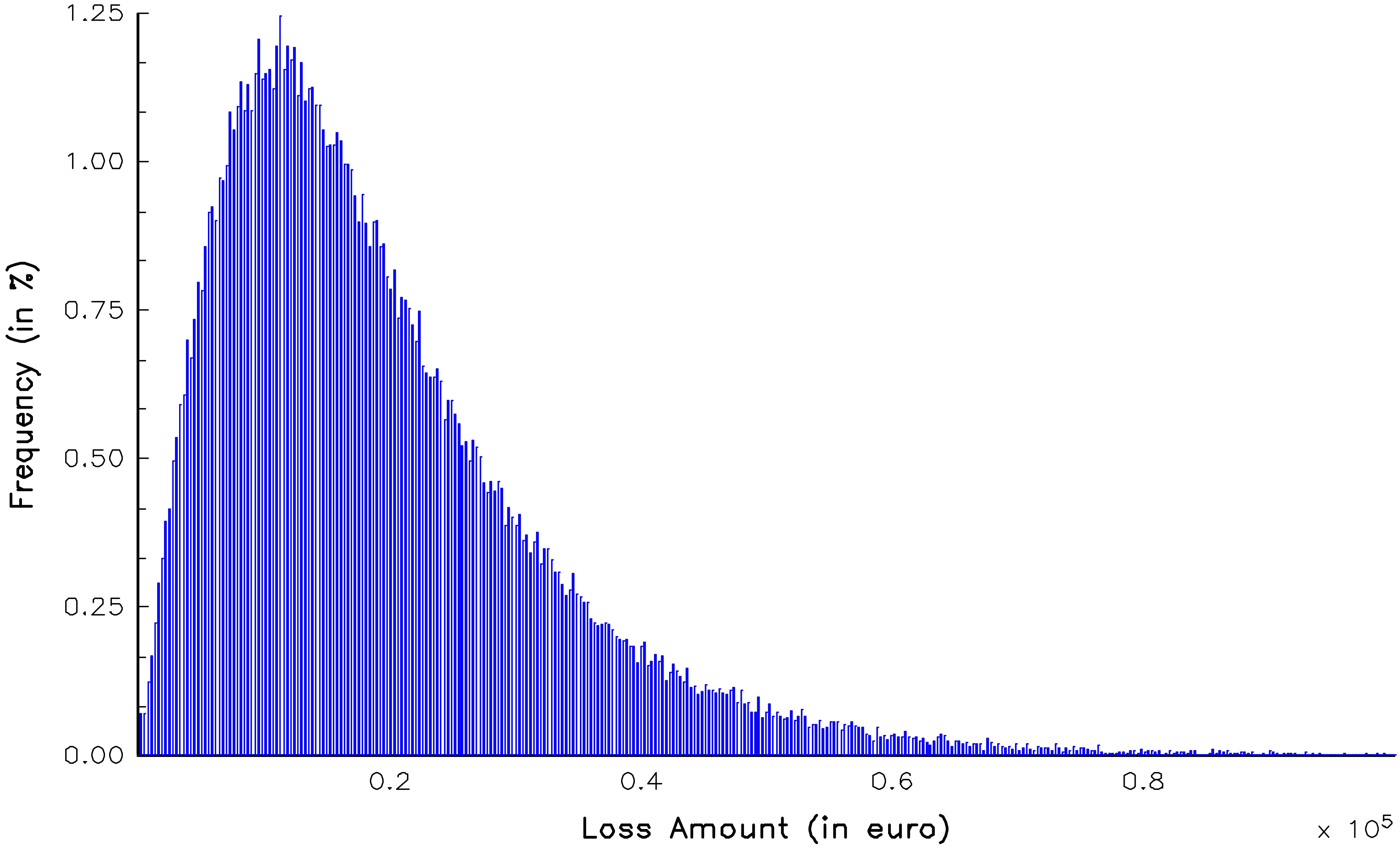
We want to measure the risk on a large portfolio of loans. We consider I firms labelled $i = 1 \ldots I$. We assume each firm has contracted one loan with the bank. We consider the loss $L_I(T)$ within time T on the portfolio :

$$L_I(T) := \sum_{i=1}^{I} (1 - R_i) N_i \mathbf{1}_{\{\tau_i \leq T_i \wedge T\}},$$

- τ_i and R_i the default time and recovery rate of the issuer i
- N_i and T_i the notional and maturity of the loan

As the risk measure we choose a quantile of the loss distribution:

$$\text{CreditVar}_\beta := \inf \{x \geq 0 : \mathbb{P}(L_I(T) \leq x) > \beta\}.$$



Frequency of the Loss Distribution
(Normal Copula/Sector Correlations)

Dependence Mechanism between the Defaults

We model the default times with an intensity approach

$$\tau_i := -\frac{1}{\lambda_i} \log \left(1 - N(\sqrt{\rho}X - \sqrt{1-\rho}\varepsilon_i)\right),$$

- $X \sim N(0,1)$ is a systematic risk factor,
- $\varepsilon_i \sim N(0,1)$ is the idiosyncratic risk factor for issuer i,
- λ_i is such that $\mathbb{P}(\tau_i \leq T_i \wedge T|X) = p_i$ the observed default probability at time $T_i \wedge T$ (in fact $p_i = 1 - \exp(-\lambda_i(T_i \wedge T))$),
- ρ is a correlation parameter e.g. $\rho = 20\%$.

Notation:

$$P_i(X) := \mathbb{P}(\tau_i \leq T_i \wedge T|X) = N\left(\frac{N^{-1}(p_i) + \sqrt{\rho}X}{\sqrt{1-\rho}}\right)$$

The Infinite Granularity Assumption

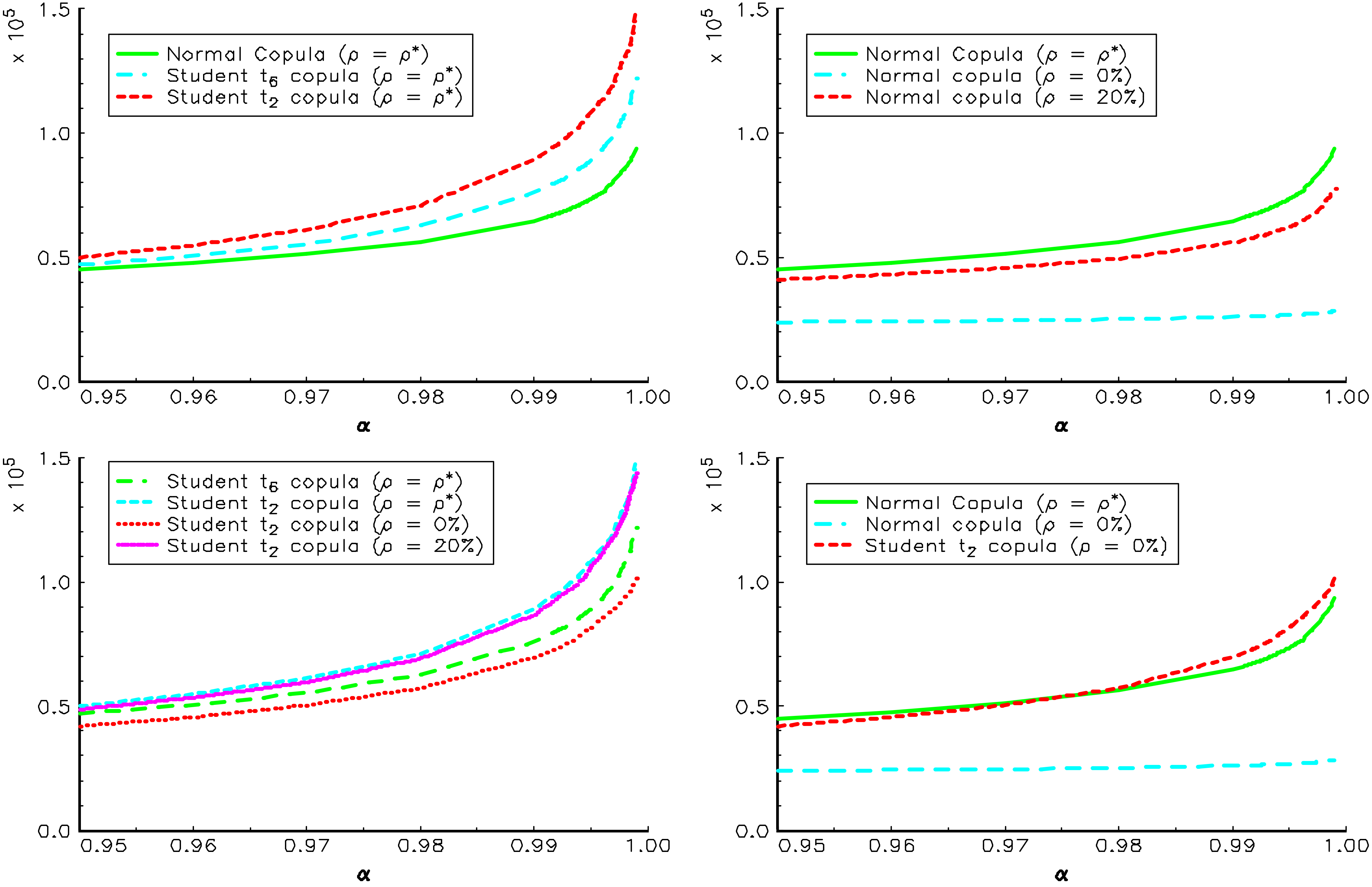
Under some simple assumptions (the recovery rates are independent from the default times, some control on the growth of $N_I / \sum_{i=1}^{I} N_i$) Gordy (2000) shows that we have

$$\frac{L_I(T)}{\sum_{i=1}^{I} N_i} - \mathbb{E}\left(\frac{L_I(T)}{\sum_{i=1}^{I} N_i} | X\right) \quad \xrightarrow[I\to\infty]{(d)} 0,$$

$$\mathbb{P}\left(\frac{L_I(T)}{\sum_{i=1}^{I} N_i} \leq \mathbb{E}\left(\frac{L_I(T)}{\sum_{i=1}^{I} N_i} | X = N^{-1}(\beta)\right)\right) \quad \xrightarrow[I\to\infty]{} \beta.$$

This explains the following Basle approximation :

$$\begin{aligned} \text{CreditVar}_\beta &\approx \mathbb{E}\left(L_I(T) \mid X = N^{-1}(\beta)\right) \\ \text{CreditVar}_\beta &\approx \sum_{i=1}^{I} \mathbb{E}(1 - R_i) N\left(\frac{N^{-1}(p_i) + \sqrt{\rho} N^{-1}(\beta)}{\sqrt{1-\rho}}\right) N_i. \end{aligned}$$



Portfolio Credit VaR

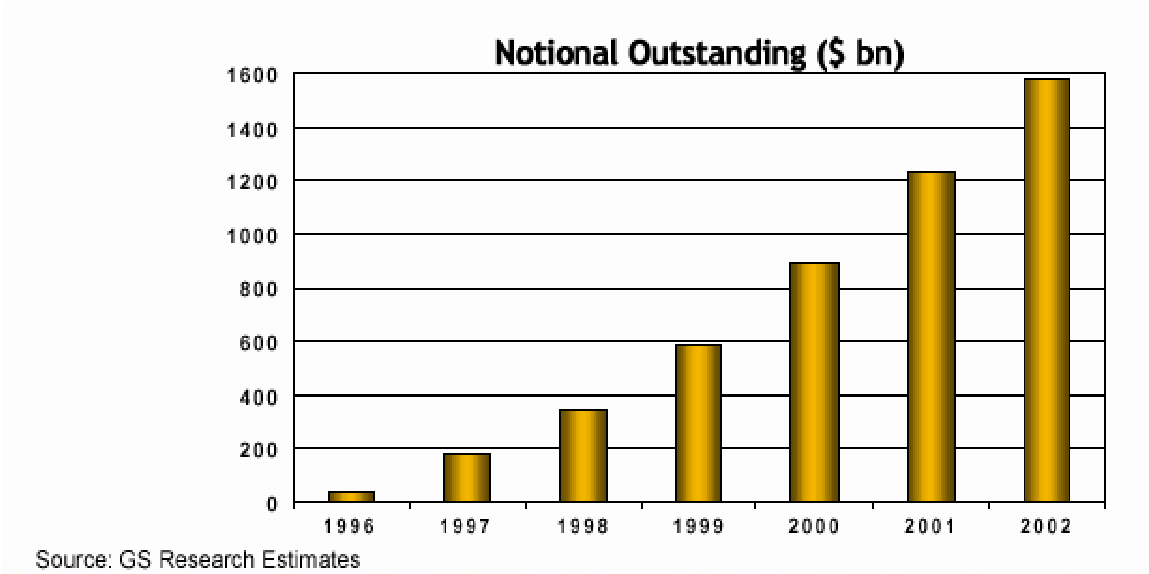
3 Managing Credit Risk with Credit Derivatives

One way to manage its credit exposure and reduce the cost of capital requirement, the bank can decide to externalize part of its credit risk. There are two main ways :

- the bank may sell the loans to another counterpart.
- the bank may keep the loans and ask its trading room to buy a credit derivative that gives some insurance in case of defaults.

We review here some of the most popular products of the credit derivatives business.

3.1 The Credit Derivatives Business



CDO in Europe

Year	1996	1997	1998	1999	2000	2001	2002
Number	1	3	3	24	50	133	144
Volume ($ bn)	5	5.7	4.5	29.2	63.2	106	143.4

Source: Moody's Investor Service

3.2 The Credit Default Swap

This is the basic credit derivatives. In counterpart of the regular payment of a fixed premium m, it provides an insurance in case of a default of a single issuer (before the maturity of the swap). The payoff is therefore (for settlement dates) $(T_1, \ldots, T_K)$:

$$\underbrace{(1-R)N\mathbf{1}_{\{\tau \le T_K\}}}_{\text{Structured Leg}} \quad - \quad \underbrace{mN \sum_{k=1}^{K} (T_k \wedge \tau - T_{k-1} \wedge \tau)}_{\text{Fixed Leg}},$$

When $\tau \sim (\lambda)$ and when R and interest rates are deterministic, it is easy to show that – when $\min_k(T_k - T_{k-1})$ is small – the margin m such that the inception price of the CDS vanishes is

$$m = \lambda(1-R).$$

3.3 The Collateralized Debt Obligation

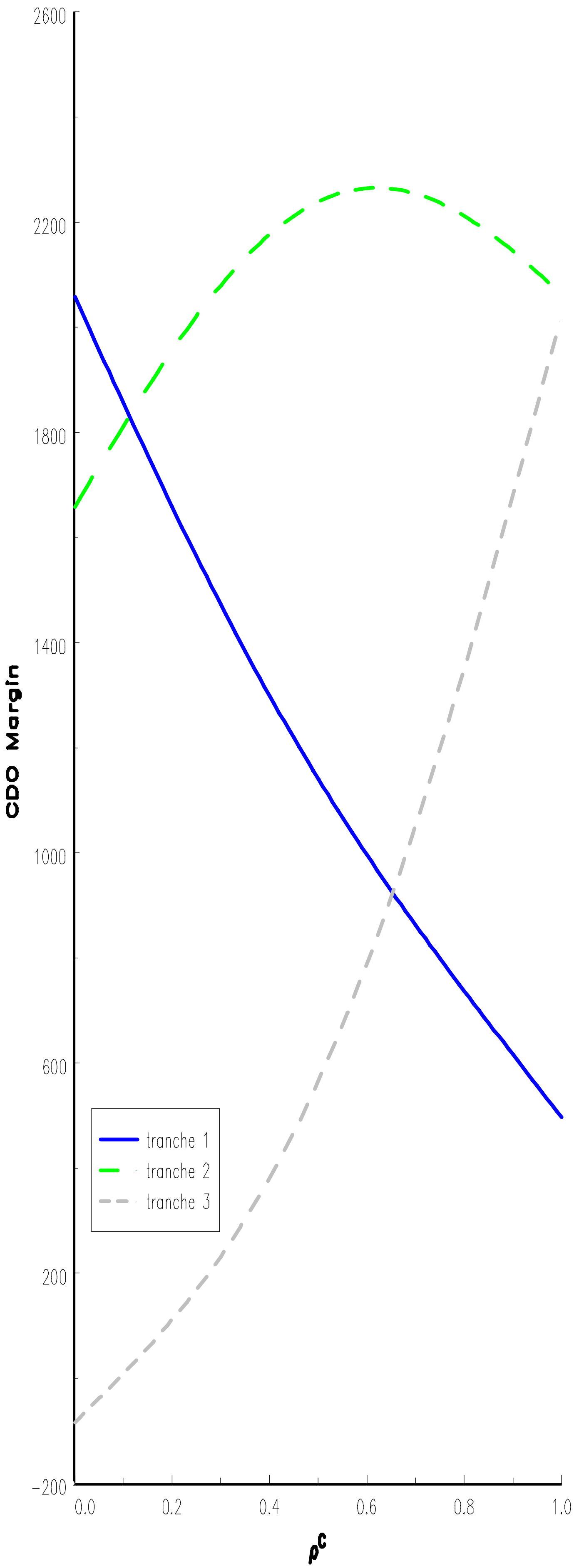
We now consider a large portfolio of I credits. Instead of buying I CDS, we can buy a single structured basket product that gives a protection on a part of the loss.

$$L_I(T) := \sum_{i=1}^{I} (1 - R_i) N_i \mathbf{1}_{\{\tau_i \le T_i \wedge T\}},$$

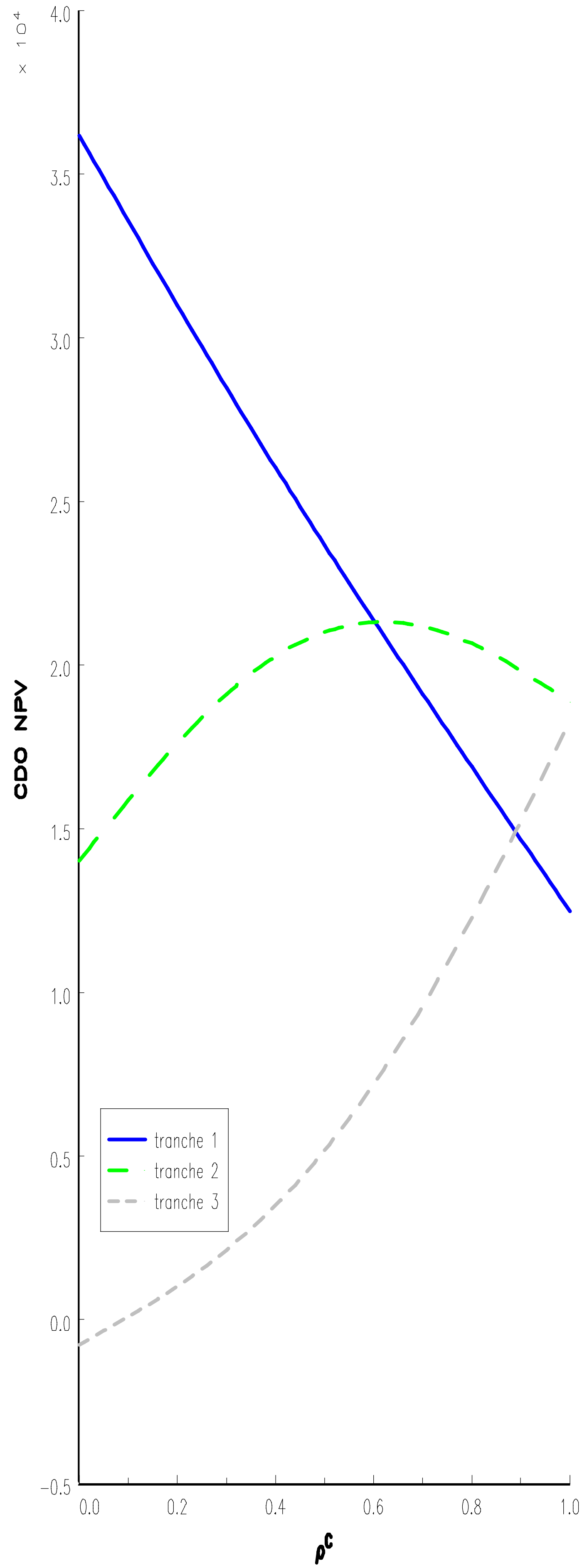
We consider some strikes
$0 =: L_0 \le L_1 \le L_2 \le \ldots \le L_{K-1} \le L_K := \sum_{i=1}^{I} N_i$ corresponding to different levels of possible loss and tranche the loss as

$$L_I(T) = \sum_{k=1}^{K} \left(L_I(T) - L_{k-1}\right)^{+} \wedge L_k$$

A CDO is a credit derivative like a CDS but pays a protection on a particular tranche of the portfolio loss.



CDO Margin
2600
2200
1800
1400
1000
600
200
−200
0.0
0.2
0.4
0.6
0.8
1.0
ρc
tranche 1
tranche 2
tranche 3



x 10⁴
CDO NPV
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
0.0
0.2
0.4
0.6
0.8
1.0
ρc
tranche 1
tranche 2
tranche 3