

# Unité de probabilités

## Séminaires programmés

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Mercredi 1er avril 2009  
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## Conférence en probabilité

Mercredi 1er avril 2009 ~~11h15~~ **16:15 (nouvel horaire)**  
~~MA3-30~~ **CM 09(changement de salle)**, EPFL, Ecublens

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### **Perpetual American options in models with default risk and random dividends**

#### **Résumé**

We present closed form solutions to the problems of pricing of perpetual American standard options in two diffusion models of financial markets with presence of default risk. The method of proof is based on reducing the initial discounted optimal stopping problems to equivalent free-boundary problems and solving the latter by means of smooth-fit conditions. Applying the recently derived change-of-variable formula with local time on surfaces, we verify that the obtained solutions of the free-boundary problems turn out to be solutions of the initial optimal stopping problems.

In the first model, it is assumed that a firm can spontaneously announce a default and change the value of the dividend rate at that time. Being inaccessible for usual investors trading at the market, such a change in the dividend rate can be actually hidden into the behaviour of the firm value under the risk-neutral measure. Aiming at explicit expressions for the rational option prices and related exercise boundaries, we assume exponential distribution for

the default time at which the dividend rate changes from one constant value to another, within the framework of a diffusion model for the firm value dynamics. We embed the initial problem into a discounted optimal stopping problem for a two-dimensional Markov process, having the firm value and conditional survival probability as components. It turns out that the latter process is equivalent to the posterior probability of the occurrence of 'disorder' in the corresponding problem of detecting a change in the drift rate of an observed Wiener process. By means of solving the equivalent free-boundary problem, we show that the exercise boundary for the firm value is constant and can be characterized as a unique solution of a transcendental equation.

In the second model, it is assumed that the default happens when the firm value falls to some random barrier, which is not observable from the market. Aiming at closed form expressions for ex-dividend prices and exercise boundaries, we assume that the barrier has exponential distribution and it is independent of the firm value, which is modelled by a geometric Brownian motion. We embed the initial problem into an optimal stopping problem for a two-dimensional Markov process having the firm value and its running minimum as components. We show that the optimal exercise boundary for the firm value can be expressed as a function of the running minimum process. By solving the equivalent free-boundary problem, where the normal reflection condition at the diagonal of the two-dimensional state space breaks down, we show that the exercise boundary can be characterized as a unique solution of a nonlinear ordinary differential equation.

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