Seminar of Probability and Stochastic Process

Tuesday, 1th April, from 16h15
MA A1 12, EPFL, Ecublens

Prof. Raluca Balan

University of Ottawa

Stochastic integration with respect to Levy colored noise, with applications to SPDEs

Abstract:

The purpose of this talk is to introduce a new type of noise for problems in stochastic analysis, which behaves in time like a finite-variance Levy process without a Gaussian component. In the space variable, the noise is a stationary random distribution (in the sense introduced in Ito, 1954), whose covariance is a non-negative definite distribution ρ, which can be viewed as the Fourier transform of a tempered measure μ. In the Gaussian case, a similar type of noise was introduced in Dalang (1999), under the assumption that the distribution ρ is induced by a tempered non-negative function f (or a tempered measure Γ). We develop a theory of stochastic integration with respect to this noise without this assumption. The same theory can be developed in the Gaussian case, the motivating example (in spatial dimension $d = 1$) being a noise which behaves like a fractional Brownian motion in space, with Hurst index $H < 1/2$.

As an application of this theory, we consider the linear stochastic wave (or heat) equation with this noise. The random field solution of

this equation exists if and only if the measure μ satisfies the condition:

$$\int_{R^d} \frac{1}{1+|\xi|^2} \mu(d\xi) < \infty,$$

introduced in Dalang (1999).

In the example mentioned above, μ has density function $c_H |\xi|^{1-2H}$, and the previous condition holds for any $H \in (0,1)$.

If $H > 1/2$, ρ is the distribution induced by the Riesz kernel $f(x) = |x|^{2H-2}$, but if $H < 1/2$, ρ is a genuine distribution which coincides with the second distributional derivative of the function $V(x) = |x|^{2H}$ (as shown in Jolis, 2010).

Studying non-linear SPDEs with this kind of noise remains an open problem, even in the Gaussian case.

Date of last change: Thu, 20 Mar 2014 08:20:14, by Carlo Ciccarella