The Economics of Sharing Markets

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What you seek is seeking you.
— Rumi

To my parents and my brother,

whom I can never thank adequately.

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Lausanne, September 2017 Maryam Razeghian.

Abstract

Sharing and redistributing assets between individuals has become a noticeable part of the economy. Ownership is no longer the sole mode of consumption and consumers have the option of choosing between ownership and access-based consumption. This change in the consumer behaviour creates both threats and opportunities for the incumbent firms. In this dissertation, techniques from microeconomics and game theory are utilized to investigate market equilibrium in presence of sharing markets. We focus on the peer-to-peer sharing of rival goods with economic motivations.

Chapter 1 provides definitions, introduces the theoretical framework, and gives an overview of the dissertation. Chapter 2 introduces a robust mathematical model that captures the equilibrium dynamics in peer-to-peer markets. We provide a closed-form solution for the S-shaped evolution of asset sharing, with price-adjustment delays and transaction costs.

Chapter 3 studies the effects of sharing markets on the purchase price and durability of new products. We show that the pricing and product design can serve as strategic tools to exploit or mitigate the impact of a peer-to-peer economy on the manufacturer's profit. Adjusting the built-in durability allows the firm to either defeat sharing (by shutting down the sharing market) or promote it (by offering durable and shareable objects).

Chapter 4 develops a model to addresses the question of the firm's optimal adaptation of business model in the presence of sharing markets. We investigate second-degree price discrimination in the form of offering rental and/or purchase options and find that the population's sharing propensity is a major determinant of the firm's optimal selling strategy. As the population's sharing propensity increases, the firm moves from unbundling (exclusive renting), via mixed bundling (selling and renting upon demand), to pure bundling (exclusive selling).

Key words: Collaborative consumption, sharing economy, peer-to-peer markets, equilibrium dynamics, durability, planned obsolescence, product design, selling vs. renting, consumption bundling, pricing, profit maximization.

Résumé

Le partage et la re-distribution des biens entre les individus représentent une part grandissante de l'économie. La propriété n'est plus le seul modèle de consommation; et les consommateurs ont aujourd'hui le choix entre la possession et une consommation basée simplement sur l'accès aux biens. Ce changement de comportement des consommateurs est autant une menace qu'une opportunité pour les compagnies bien établies sur le marché. Cette thèse a pour but de mobiliser les techniques de la Théorie de jeux et de la Micro-économie pour analyser l'équilibre du marché en présence des marchés de partage.

Le premier chapitre fourni les principales définitions, introduit le cadre théorique et offre un aperçu global de la dissertation. Le deuxième chapitre propose un modèle mathématique illustrant les dynamiques d'équilibre dans les marchés de partage. Nous introduisons une solution analytique à l'évolution "sous forme de S" du partage des biens, avec les retards occasionnés par les ajustements de prix ainsi que les coûts de transaction.

Le troisième chapitre étudie les effets des marchés de partage sur le prix d'achat et la durabilité des nouveaux produits. Nous démontrons que la tarification et la conception de nouveaux produit peuvent servir d'outil stratégique pour exploiter ou atténuer l'impact de l'économie de partage sur le profit du fabricant. L'ajustement de la durabilité prévue permet au fabricant de l'emporter sur le partage (par la fermeture du marché de partage), ou bien de le promouvoir (en offrant des biens durables et partageables).

Le quatrième chapitre développe un modèle qui adresse la question du choix optimal de stratégie commerciale pour les entreprises en présence de marchés de partage. Nous investiguons la question de la discrimination par les prix de deuxième type à travers une offre de choix entre les options de location et/ou d'achat. Ceci nous permet de conclure que la propension de partage de la population est un déterminant majeur de la stratégie de vente optimale de la compagnie. Lorsque la propension de partage de la population augmente, la compagnie étend sa stratégie, de l'offre non-groupée pure (location uniquement) à l'offre groupée pure (vente uniquement), en passant par l'offre groupée mixte (vente et location selon les demandes).

Mots clés: Consommation collaborative, économie de partage, marchés de partage, dynamique d'équilibre, durabilité, obsolescence programmée, conception de produit, vente vs. location, consommation groupée, tarification, maximisation du profit.

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1 Introduction

The emergence of the sharing culture has created a socio-economic network, in which assets and services are shared and jointly used by individuals. The economy is evolving around peer-to-peer exchanges of goods and services in the forms of reselling, renting, and swapping through online community-based platforms. Advances in technology have helped to overcome the standard problem of finding matching transaction partners, and the online rating and review systems have toned down the imperfections of sharing activities related to informational asymmetries between the different parties.

The cultural transition from ownership towards access-based consumption has been widely remarked by the researchers alongside with the press. Already in 2011, *Time* magazine included "sharing" in their list of the 10 top ideas with the potential to change the world.[1] Botsman and Rogers (2010) described "collaborative consumption" as an alternative against the modern hyper-consumerism in the form of throw-away living or self-storage of idle capacity. They argued that not only hyper-consumerism is unsustainable, but also hampers satisfaction in life. The sharing culture could potentially reverse or at least slow down this trend.

Nevertheless, despite the wide coverage of the sharing economy by academics, practitioners, and the press, there is no systematic definition of the sharing economy, and identifying clear boundaries seems nearly impossible (Schor 2014). There are various definitions that may or may not include the transfer of ownership, gift giving, service delivery, financial remunerations, and B2C models (Puschmann and Alt 2016). Benkler (2004) pointed out that shareable objects have two key characteristics in common. First, the items show systematic overcapacity (lumpiness); and second, the demand frequency justifies the purchase for a large enough pool of consumers (mid-granularity). Bardhi and Eckhardt (2012) dimensionalized access-based consumption to describe how various types of sharing distinguish themselves. They identified type of accessed object (functional vs. experimental), temporality (short term vs. long-term), market mediation, anonymity, consumer involvement, and political consumerism (promoting idealogical interests) as six main dimensions that can be used to map the realm of access-based consumption. In this

[1] "10 Ideas That Will Change the World," *Time*, March 17, 2011.

dissertation, we follow the definition of Frenken et al. (2015) and Frenken and Schor (2017), and define the sharing economy as "consumers granting each other temporary access to under-utilized physical assets ("idle capacity"), possibly for money" (Frenken and Schor 2017, pp. 4–5). Their definition excludes resale ("second-hand economy"), B2C models ("product-service economy"), and peer-to-peer service delivery ("on-demand economy").

1.1 Conceptual Framework

To help better characterize and categorize sharing markets, we use two dimensions that relate to the motives and the nature of sharing activities, namely market orientation and the object's rivalry. Schor (2016) was the first to recognize platform orientation (non-profit vs. for-profit) as a factor that influences the logics of sharing and how peer-to-peer exchanges take places. Sharing transactions could create economic incentives for both owners and non-owners. New income opportunities are created for owners, and consumption is possible at a lower cost for non-owners. Moreover, the convenience and wider choice set that sharing market offers adds additional value to the users. According to PwC (2015), 86% of the participants of such markets agree that sharing makes life more affordable.

Nevertheless, economic benefits are not the only incentive behind sharing activities. Schor and Fitzmaurice (2015) identified environmental, and social motivations as other motives of engaging in not-for-profit sharing activities with strangers. In a time of growing concern about man-made environmental changes, reusing, redistributing, and collaborative consumption of goods can lower the ecological footprint of human activities. The environmental impacts of sharing include waste reduction, mitigation of traffic congestions, decrease of emissions associated with production, and expansion of recycling. Moreover, sharing encourages social connections. Peer-to-peer activities are formed based on trust and develop social bonds between the involved parties. From a psychological and sociological viewpoint, sharing promotes altruism and responds to a sense of community (Belk 2007, 2010).

Sharing activities also distinguish themselves based on the rivalrous nature of the goods being shared. The possibility of concurrent consumption of a good by multiple users is referred to as the (non-)rival nature of the good (Cornes and Sandler, 1996). Rival goods cannot be consumed by more than one user at the same time. For example, using a power drill or a personal computer creates a hurdle for other potential consumers that are willing to use the item at the same time. In contrast, perfectly non-rival goods are not constrained in capacity and can be consumed by an unlimited number of users simultaneously. In other words, one user does not prevent others from consumption. Examples of perfectly non-rival goods include files, films, and music albums. Nevertheless, many of the goods are semi-rival. Although multiple users can simultaneously access and consume the good, the capacity is limited. Carpooling is an example of sharing semi-rival goods. It does not prevent the driver from self-consumption, however, the maximum number of passengers that can share the ride is limited. The capacity of simultaneous consumption and the degree of rivalry can significantly influence the owners' decision to join such markets.

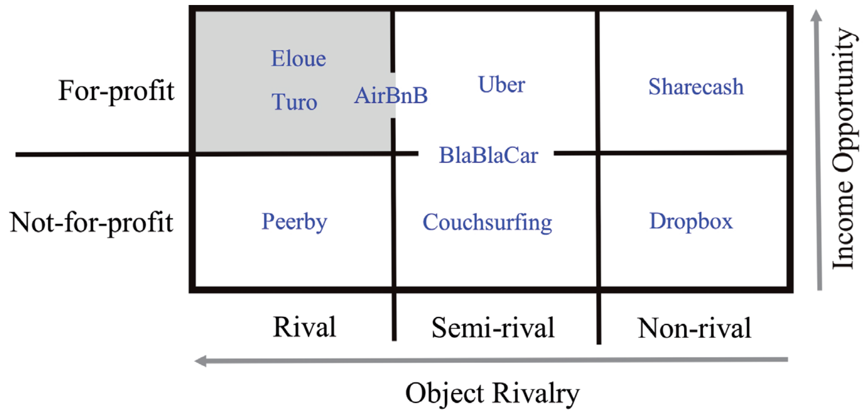

Figure 1.1: Sharing matrix describing various types of sharing markets based on the exclusivity of the shared goods and the income opportunity for the suppliers.

Figure 1.1 depicts a "sharing matrix" that categorizes peer-to-peer sharing in six distinct groups based on the two aforementioned criteria: market orientation (income opportunity for the sharers), and the object's rivalry. The matrix distinguishes between economic and non-economic sharing motives, which give rise to for-profit and not-for-profit sharing markets. For example, some platforms allow for remote interactions and the transactions could be realized without the parties meeting (e.g., Eloue and Turo). The primary motivation for the users of such platforms is the opportunity to earn more income or get access to the items at a lower cost. Non-economic sharing incentives such as the sense of community and mutuality play a less significant role in such exchanges. Some other sharing markets are formed solely on social connections where no monetary value is exchanged (e.g., Peerby). This type of sharing, described as "sharing-in" (Belk 2010) mostly takes place within family, friends, or neighbours and through such platforms strangers find an opportunity to form a bond with the community. Other platforms such as BlaBlaCar encourage financial transactions that are limited to covering the costs. In this regard, Turo and Uber are both for-profit carpooling platforms that allow the car owners to earn an extra income through sharing. However, while Uber facilitates the ride-sharing in which simultaneous consumption is shared among strangers (semi-rival), Turo focuses on car sharing where the owner is not permitted to use the shared item (rival). Sharing of non-rivals mostly revolves around information goods. Examples include Dropbox (not-for-profit file sharing) and Sharecash (uploading files to make money).[2]

This dissertation focuses primarily on sharing markets classified as rival/for-profit. In the next chapters, we aim to provide economic insights on the growth patterns of such markets, the resulting paradigm shift in consumption, and the strategic responses of the incumbent firms regarding pricing, product design, and business-model adaptation.

[2]Table A.1 in App. A provides more information about the companies mentioned in this chapter.

1.2 Contributions and Structure

In this dissertation, we utilize techniques from microeconomics and game theory to investigate various sharing-related business situations and the resulting market dynamics in equilibrium and disequilibrium. The dissertation contains three self-contained chapters based on three separate articles.

To aid in the description of the significant recent growth in sharing, Chapter 2 provides a robust mathematical model that captures the equilibrium dynamics in peer-to-peer markets with price-adjustment delays and transaction costs. Using a game-theoretic framework, we study a set of infinitely lived, heterogeneous suppliers, who take recurring decisions about entering or exiting the market. The analysis provides a closed-form solution for the nonlinear evolution of the equilibrium in the sharing economy, typically resulting in an S-curve diffusion pattern. We show that in a market that does not necessarily clear, a steady increase of the effective transaction price encourages more suppliers to join the market. Our results show that the intermediary can have two-sided effects on the dynamics of peer-to-peer markets. The intermediary's commission fee per se decreases the attainable level of participation since it takes away a fraction of the consumer surplus. However, the intermediary may improve the penetration of sharing by actively improving the price discovery mechanism. This chapter is based on the following paper:

◦ RAZEGHIAN, M., WEBER, T.A. (2015) "To Share or Not to Share: Adjustment Dynamics in Sharing Markets," Manuscript Submitted for Publication.

Chapter 3 studies the effects of sharing markets on the prices for new products and the product design regarding durability. We study a dynamic economy with overlapping generations, in which heterogeneous consumers take strategic purchasing decisions, anticipated by a durable-goods monopolist. The pricing and product design can serve as a strategic tool to exploit or mitigate the impact of a peer-to-peer economy on the manufacturer's profit. By controlling the product's design, the firm retains the option of disabling the sharing market by either reducing the durability or the retail price. We show that the possibility of "sharing shutdown" is attractive for the firm when production costs are small, or when consumers are impatient. Another key determinant is the firm's ability to commit to its price, its product design, or both. As a rule, the larger the firm's commitment power, the more rent it can extract and the stronger it feels about enabling sharing. In general, the presence of sharing never decreases the incentives to provide durability thus contributing to a sustainable product design. Chapter 3 is based on the following paper:

◦ RAZEGHIAN, M., WEBER, T.A. (2016) "The Impact of Sharing Markets on Product Durability," Manuscript Submitted for Publication.

Chapter 4 develops a model to address the question of how the firm can optimally adapt its business model in the presence of a sharing market. Empirical evidence suggests that consumers' sharing propensity towards sharing varies with culture and socio-demographic characteristics. This may create an opportunity for the firm to offer profitable short-term rentals, provided that the consumers are more willing to engage in transactions with the firm rather than their peers. In an economy with overlapping generations of heterogeneous consumers, we study dynamic selling by a durable-goods monopolist in equilibrium. Feasible pricing strategies include second-degree price discrimination in the form of offering rental and/or purchase options. We find that when the sharing propensity is low, the firm is able to shut down the sharing market by exclusively offering rentals. For intermediate sharing propensities, both the B2C rental and the peer-to-peer market coexist. As the sharing propensity further increases, only the peer-to-peer market survives and the firm offers the product only for purchase. We show that an increase in sharing propensity has an ambiguous effect on the firm's profit. This chapter is based on the following paper:

◦ RAZEGHIAN, M., WEBER, T.A. (2017) "The Advent of Sharing Culture and its Effect on Product Pricing," Working Paper.

Each chapter of this dissertation is a stand-alone research project and has dedicated sections to introduction and literature review, model description, analysis, conclusions and future research directions. Chapters 2–4 are the result of collaborations with my advisor, Prof. Thomas A. Weber.

2 Dynamic Equilibrium in Peer-to-Peer Markets with Frictions

2.1 Introduction

The collaborative consumption of durable goods, such as cars, flats, power tools, and clothes, by strangers has become a noticeable part of the economy. As of December 2014, almost half of the U.S. adult population is familiar with the sharing economy, and close to one fifth of this population has been involved in a sharing transaction (PwC 2015). More than half of those familiar with the sharing economy could imagine themselves becoming a supplier, which foreshadows a substantial growth from the currently 23% of those individuals having acted as a supplier thus far (ibid.).[1] The sharing economy has been growing enormously, and by 2014 more than 200 start-ups with an approximate two billion dollars worth of funding have been participating in the growing market for peer-to-peer sharing of physical assets (Teubner 2014).

Understanding the diffusion of sharing markets has important resource implications for platforms and regulators. In a dynamic (dis-)equilibrium setting, we investigate this question and relate it to well-known diffusion dynamics. The model rationalizes various growth patterns and allows for a robust prediction of the market evolution. While market dynamics with conventional buyers and sellers have been studied extensively, both empirically and theoretically, to the best of our knowledge the adoption of peer-to-peer transactions has remained unexplored so far. The model is tailored to the characteristics and imperfections of sharing markets in several important respects.

1. *Uncertain and heterogeneous needs.* Each owner's need for his shareable good is uncertain, and the corresponding need realizations are stochastic at each time period. Owners rationally enter and exit the sharing market based on rational expectations of their financial gain from renting or consuming their goods.
2. *Persistent adjustment costs.* Owners face costly entry and exit decisions. Indeed, an important impediment to sharing consists in getting access to the marketplace. The latter

[1] Based on survey data collected from more than 90,000 respondents, Vision Critical (2014) estimates that there are currently about 80 million sharers in the US (39%), 23 million sharers in the UK (52%), and 10 million sharers in Canada (41%).

includes systems for trust and matching, designed to alleviate information-related market imperfections from which we abstract.[2] A shareable asset, such as a flat or a car, needs to be prepared for sharing transactions by removing personal attributes, creating advertising information (e.g., an item description and listing with an intermediary), and possibly acquiring private insurance.[3] Thus, conceptually, a "conversion cost" needs to be incurred by an owner to share his item. Such a conversion cost can generally also be expected when exiting the sharing market and repossessing the item for personal use. The presence of fixed costs typically implies an inaction region in the space of decision-critical parameters (Stokey 2008), which in turn results in an adjustment-cost-induced decision inertia, also referred to as hysteresis (Dixit 1992). In particular, if expected gains from changing the sharing state of their assets are small, individuals prefer to remain in their *status quo*.[4]

3. *Limited private supply.* Each potential supplier in the sharing market can provide at most one unit of a given item. This contrasts with conventional markets where suppliers not only take entry and exit decisions but also optimize with respect to the quantity of the production units. In peer-to-peer sharing markets between non-commercial agents, the suppliers' choice is by and large limited to entry/exit decisions, with the quantity of the supplied good fixed to a single unit.

4. *Price stickiness.* Price discovery in sharing markets is usually imperfect, especially in the early diffusion stages. Whether the pricing is centrally determined by the intermediary, or decentralized such that the suppliers are allowed to announce their own prices, it is natural to expect that price adjustments are not instantaneous. This holds true well beyond the sharing economy for pretty much any market: even for high-frequency traders in financial markets, the relative stickiness of prices is a major source of revenue and thus has decisive effects on the resulting allocations (Cohen et al. 1980; Chan 1993; Lin and Rozeff 1995). In the context of sharing, while prices are often set by suppliers in a decentralized manner, it is unrealistic to assume that suppliers are continuously adjusting their posted prices to match the supply and demand. Indeed, based on data from task-rabbit, Cullen and Farronato (2014) find that prices tend to remain unchanged for long periods of time. The reasons for price stickiness in the sharing economy may be similar to macroeconomic price stickiness in various industries which may be attributed to price-adjustment costs (Rotemberg 1982), associated (perhaps) with searching for and analysing relevant data, and with the perceived negative impact of changing consumer expectations. Another explanation for price stickiness at the macro level is the staggering of prices (Taylor 1980; Calvo 1983), causing individual price adjustments by suppliers to not all happen simultaneously. Sharing intermediaries such as AirBnB recommend that sharers

[2]Informational imperfections (such as moral hazard) in sharing markets are discussed elsewhere; see, e.g., Weber (2014).

[3]Certain intermediaries (e.g., AirBnB) offer basic insurance as part of the transaction, others (e.g., HouseTrip) do not.

[4]Fixed costs distort the classical net-present-value (NPV) rule because the implied partial irreversibility of sharing decisions turn opportunities into real options: a strictly positive NPV is required to make sharing decisions attractive (Dixit and Pindyck 1994).

look at similar listings on their website and price their listings accordingly. Although each potential lender can in principle adjust the price as frequently as desired, in a reasonably competitive market suppliers care about their posted price relative to those charged by other suppliers, and this backward-orientation causes prices to adjust slowly.

5. *Intermediation.* The rents obtained by the participants in the sharing economy are diminished by an intermediary's commission rate. The effect is akin to a consumption tax, the incidence of which lies with the supplier of the good. The intermediary's take has an effect on the speed of diffusion as well as the steady-state level of the sharing economy.

Taking the sharing-specific features into account, the model produces dynamic diffusion patterns which are comparable to logistic growth (without assuming such a pattern, as in many standard estimation models; see, e.g., Peres et al. 2010). Using natural primitives the S-shaped diffusion pattern provides a dynamic-equilibrium analogue to the well-known product diffusion model by Bass (1969). Despite a decrease in the observed posted prices, the growth of sharing is driven by a steady increase of the "effective transaction price" which accounts for the posted price and the transaction probability in a market that does not necessarily clear. While the model in the main text uses simple assumptions, such as a linear demand curve, in order to obtain closed-form results for the market dynamics, the S-shaped growth pattern is remarkably robust and continues to hold for nonlinear demand specifications and is qualitatively insensitive to changes in the speed of price adjustments.[5] In the absence of macroeconomic shocks, the market equilibrates in the long run, and the supply-demand mismatch vanishes asymptotically. In the presence of shocks, for example, in the form of sudden and unexpected parameter adjustments, the market would readjust and the sharing market would track an updated S-shaped pattern. The findings also allow for statements about how the diffusion of sharing depends on an intermediary. The commission fee charged by the intermediary tends to hamper diffusion in terms of both speed and asymptotic penetration level. On the other hand, technical progress in the price discovery at the hands of an intermediary can improve the penetration of sharing.

2.1.1 Literature

Our findings relate directly to the well-known product diffusion model by Bass (1969) which fits the diffusion patterns for a wide range of products (and ideas) exceptionally well. Diffusion models have been used to estimate the adoption of products (Mahajan et al. 2000), and more recently, also to describe the spread of innovations in social networks (Peres et al. 2010). In an empirical study of product commercialization in various industries Agrawal and Bayus (2002) observe that the number of firms takes off before the actual demand takes off, possibly because the supply creates demand. Similarly, Shen and Villas-Boas (2010) argue that competition between forward-looking firms causes them to enter the market before actual demand rises exogenously. Shen (2014) explores firms' entry and exit decisions when demand is growing stochastically

[5] Sec. 2.6 provides various robustness checks for the model, with respect to demand and price-clearing assumptions, as well as in the presence of transaction costs.

over time and the demand potential co-varies with the number of firms. The incentives to enter early depend critically on early market-development cost. These findings are consistent with our model, which tends to produce a persistent disequilibrium between supply and demand, in which the supply exceeds the demand when the sharing market is growing.

The economic impact of sharing markets has been addressed both empirically and theoretically. In the context of room sharing, Zervas et al. (2014) find that sharing intermediaries such as AirBnB can have a negative effect on hotel revenues. Cervero et al. (2007) and Martin et al. (2010) observe that car sharing is associated with a significant decrease in car ownership, miles travelled, and gasoline consumption. Fraiberger and Sundararajan (2015) calibrate a stationary equilibrium model to data from the U.S. car market and observed significant consumption and surplus shifts as a result of car sharing. Weber (2015) provides a theoretical framework to analyze the impact of sharing on product sales of durable goods and showed that the impact is ambiguous, depending on the price of ownership. Weber (2016) points out that sharing markets tend to increase retail prices (even in the absence of agency effects), and that retailers or manufacturers prefer the presence of sharing markets when goods are relatively expensive to provide. Horton and Zeckhauser (2016) find that bring-to-market costs increase ownership incentives while also decreasing the volume of sharing transactions. Benjaafar et al. (2015) study the ownership decisions in the presence of matching frictions when the rental price is exogenously determined by the intermediary, and showed that depending on the rental price, the product sales may increase or decrease. Einav et al. (2015) provide a general discussion of the economics of P2P markets, including an intermediary and professional sellers, leading to a market outcome with little or no surplus for all but the intermediary. Weber (2014) introduces an analytical model for sharing with intermediation, and found that in a setting with risk-neutral parties, intermediation can in fact solve the moral hazard problem; the latter can extract surplus up to the outside option of the sharers. Jiang and Tian (2016) construct an alternative model, examining the effect of unresolved moral hazard on the transaction price; they also analyze the impact of sharing on manufacturer decisions such as quality. In this paper, we abstract from the standard transactional inefficiencies as well as ownership incentives, and focus on (rational) owners' decisions about whether or not to participate in an intermediated sharing market.

Our model finally relates to the literature on equilibrium search in labor markets with frictions (Alvarez and Veracierto 2000; Alvarez and Shimer 2011). This stream of literature was pioneered by Lucas and Prescott (1974) with a discussion about the equilibrium rate of unemployment across various industries/occupations/locations when unemployed workers can engage in costly search. Based on a system of Bellman equations for a set of heterogeneous individuals and a transversal equilibrium condition, we obtain a set of equilibrium threshold rules, which for each agent implies a decision hysteresis (Dixit 1992). The price thresholds vary between individuals and also depend on the aggregate of all individuals, whose decisions therefore exert an externality on each other.

2.1.2 Outline

The remainder of this paper is organized as follows. Sec. 2.2 introduces the model primitives, including the heterogeneous consumers and their payoffs, the demand, and the market mechanism. Sec. 2.3 characterizes the equilibrium behavior of agents in a sharing economy with adjustment costs and a (possibly persistent) supply-demand imbalance. We provide comparative statics and analyze the limiting behavior of the economy as a function of the model parameters. Sec. 2.4 examines the supply-side sharing decisions on the equilibrium path in more detail and provides a specific example to illustrate the findings. Sec. 2.5 provides some examples. Robustness issues are examined in Sec.2.6, and Sec. 2.7 concludes. Robustness issues are examined in Sec.2.6. App. A summarizes the notation, and the proofs of formal results are in App. B.

2.2 Model

2.2.1 Supply

Consider a continuum of agents, each indexed by his type $\theta \in \Theta = [0,1]$, and each of whom owns a potentially shareable durable good. Without any loss of generality, the total number of agents is normalized to 1.[6] The type $\theta \in \Theta$ of a given agent characterizes the probability with which he will need the item at any given time t.[7] More specifically, it describes the distribution of the agent's need state $\tilde{s}_t$, with realizations s_t in $\mathscr{S} = \{L, H\}$, that can be either "low need" ($s_t = L$) or "high need" ($s_t = H$), in the sense that

$$P(\tilde{s}_t = H) = \theta.$$

For simplicity we assume that the type distribution is uniform on Θ. At time $t \in \{0,1,2,\ldots\}$, any given agent can decide whether to change the state of his durable good (e.g., a flat or a car) from a "keeping" state ($x_t = 1$) to a "sharing" state ($x_{t+1} = 0$) or vice-versa, from $x_t = 0$ to $x_{t+1} = 1$. The corresponding conversion cost, to go from state $x_t = i$ to $x_{t+1} = j$, is denoted by c_{ij}, where $c_{ii} = 0$ and $c_{ij} \geq 0$, for any $i, j \in \mathscr{X} = \{0,1\}$. The conversion cost c_{10} measures the effort required to get a good ready to be shared, including cleaning, installation of sharing-specific features (e.g., robust furniture in a flat), and creating a listing for the item with a sharing intermediary. On the other hand, by expending c_{01} an agent can repossess the shared item by reintegrating it into his private possessions, de-listing it with the sharing intermediary, and so forth. The switching costs between subsequent states x_t and x_{t+1} in the binary state space $\mathscr{X}$ introduce frictions and partial irreversibilities in the agents' decisions. While for most shared goods c_{10} is likely to be significant in magnitude, the relative importance of c_{01} depends on the particularities of the goods. In some markets (e.g., car sharing) c_{01} may be small by comparison, whereas in others

[6] Accordingly, model predictions about, say, the number of agents participating in the sharing market are to be interpreted as fractions of the total number of agents; the latter is assumed to be stationary for simplicity.

[7] An agent's type θ can also be interpreted as the marginal value for an increase in the utility difference Δ between the high-need state and the low-need state.

(e.g., clothes sharing) this cost tends to be non-negligible. Agents are risk-neutral, and at each time t they all take individual binary adjustment decisions $a_t \in \mathscr{A} = \{0,1\}$, where the values of 0 and 1 correspond to sharing and keeping, respectively, such that

$$x_{t+1} = a_t.$$

This reflects the fact that in the presence of a sharing market, owners currently using their items (i.e., "keepers") rationally decide in each time period whether it is beneficial to get their items ready to be offered on the sharing market. Similarly, owners currently present on the sharing market (i.e., "sharers") take a decision about possibly converting their assets back to personal use. Adjustments in the usage between sharing and keeping are costly, and the best decision for any given agent in each period depends on his beliefs about all agents' current and future actions.[8]

An agent's per-period payoff $u(s,x)$ depends on both the current need state $s \in \mathscr{S}$ and the item availability (sharing state) $x \in \mathscr{X}$. For simplicity, we assume that in the low-need state the agent's utility vanishes, no matter if the item is being shared or not, i.e., $u(L,0) = u(L,1) = 0$. In the high-need state, the agent experiences a disutility from not having the item at his disposal, and we introduce u_0 and $u_1 = u_0 + \Delta$ such that[9]

$$u_0 = u(H,0) \le 0 < u(H,1) = u_1.$$

2.2.2 Demand

At any given time, the sharing demand for the durable good consists of individuals who do not own the item and find themselves in a high-need state. In the tradition of Mussa and Rosen (1978), a potential renter is assumed to have a marginal utility $\mu \in [0,1]$ for using a durable item of quality $\gamma > 0$ (see also Jiang and Tian 2016; Weber 2014). For simplicity, we assume that marginal utilities are distributed uniformly on $[0,1]$. If a potential renter decides not to participate in the sharing market, he resorts to his second-best (outside) option, the payoff of which is normalized to 0. Thus, a non-owner is willing to rent the item if $\mu\gamma - p \ge 0$, so that the demand on the sharing market becomes $n = \max\{0, 1 - p/\gamma\}$. Equivalently, the (inverse) demand for shared items describes the market price,

$$p(n) = \gamma \cdot (1 - n), \tag{2.1}$$

[8] As is standard in dynamic market models, we assume that expectations about the future are rational (Muth 1961).

[9] Essential for the results is only that the difference Δ between u_1 and u_0 is positive; apart from that, neither the specific values of the utilities nor their signs matter.

as a function of $n \in [0,1]$, where γ also parametrizes the elasticity of demand.[10] We concentrate on the interesting case where u_1 is sufficiently large, so

$$0 = p(1) < p(0) = \gamma \le u_1 \le \Delta. \quad (= u_1 - u_0) \tag{2.2}$$

This means that if all owners were to offer their goods, for the sharing market to clear (in a steady state), the price would have to vanish, i.e., there is enough potential supply to satisfy demand in the long run, and rationing could therefore not be persistent. The utility of an owner in a high-need state (u_1) is larger than the gross benefit (γ) a renter with maximum marginal utility $\mu = 1$ can obtain. This is plausible because any contract between owner and non-owner would necessarily be incomplete and allocate all residual claims to the owner (Grossman and Hart 1986; Hart and Moore 1990). In addition, an owner can usually disable certain features of the durable good that is being shared, such as access to spare parts, accessories, or special functionality (e.g., a locked utility closet in shared house or administrator privileges for a shared personal computer). Another desirable consequence of the assumption $u_1 \ge \gamma$ is that this excludes the (economically) degenerate situation where *all* owners would be willing to share, irrespective of their need state and price movements in the sharing market.

2.2.3 Price Formation

As noted before, the prices in the sharing market are critical for the agents' incentives to share. However, it seems unreasonable to assume that the price discovery in such markets is efficient and instantaneous, even using a centralized exchange .[11] As in any market, prices in the sharing economy are (at least somewhat) sticky: when at the beginning of time t the supply of shared goods is n_t, the corresponding market price adjusts at the end of that period,[12] so

$$p_{t+1} = p(n_t), \tag{2.3}$$

for all $t \ge 0$, where the initial supply n_0 at time $t = 0$ is given. The unit delay in the price response allows for a temporary supply-demand imbalance, as can be observed in actual sharing markets.[13] Its main effect is that price discovery in the market is not immediate but takes place over some finite time horizon. Accordingly, the supply and demand in the sharing market can fluctuate. The robustness of all results with respect to changes in the price-adjustment time scale is examined in Sec.2.6.

[10] The (price) elasticity of demand, $\varepsilon = p/(\gamma - p) = (1-n)/n$, takes on all values in $(0,\infty)$ as p varies from $p(1) = 0$ to $p(0) = \gamma$. As is shown in App. B (see La. B.1.1) the demand elasticity ε is decreasing in the (in)elasticity parameter γ. Sec.2.6 discusses an extension of the model to nonlinear demand specifications.

[11] To counter inefficiencies, sharing intermediaries may offer price-optimization tools as guidance. For example, using data-mining techniques AirBnB provides a utility to recommend a daily updated rental price as a function of the transaction probability. Furthermore, AirBnB routinely recommends that hosts check similar listings to gauge appropriate price levels.

[12] As shown in Sec.2.6, the model remains robust with respect to shortening the price-adjustment time, even in the limit.

[13] The Economist (2013) provides the example of AirBnB's renting about 40,000 out of 250,000 listed rooms in any given night, corresponding to a transaction probability of 16%.

For any time $t > 0$, a type-θ agent's current-period expected net payoff, given action a, sharing state x, and the price in the sharing market p, is

$$\bar{g}(a, x, p|\theta) = \begin{cases} p + \theta u_0, & \text{if } (a, x) = (0, 0), \\ p + \theta u_0 - c_{01}, & \text{if } (a, x) = (1, 0), \\ \theta u_1, & \text{if } (a, x) = (1, 1), \\ \theta u_1 - c_{10}, & \text{if } (a, x) = (0, 1). \end{cases} \tag{2.4}$$

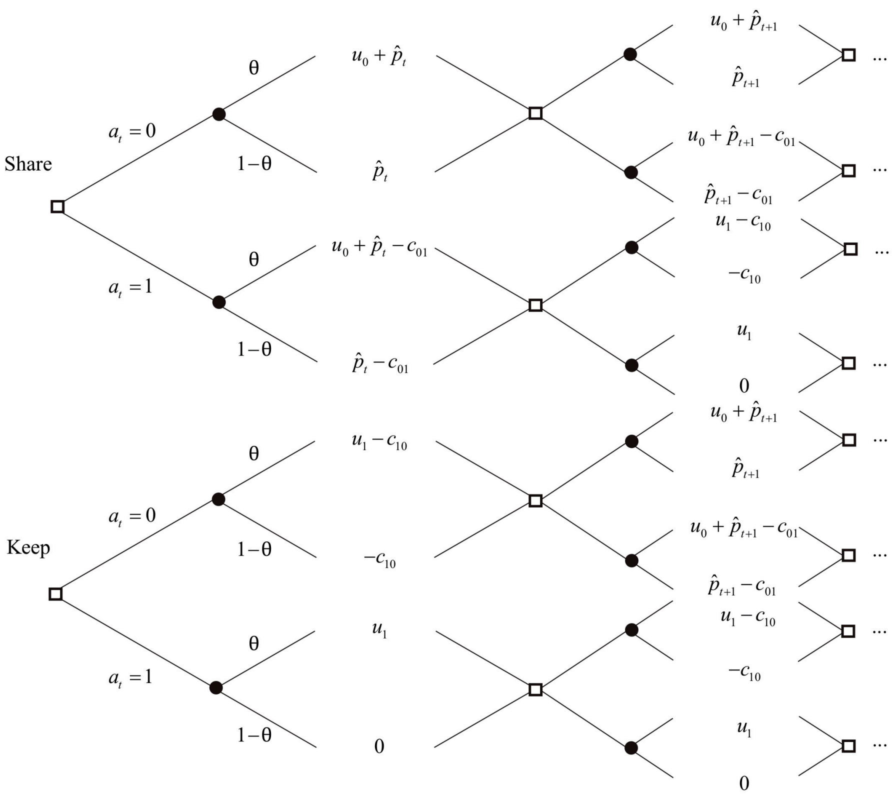

Figure 2.1: Decision process for a type-θ agent, viewed at time t.

Fig. 2.1 depicts an agent's decision process in form of a lattice, with payoffs that include the transition (conversion) costs between different sharing states and the *ex ante* expected per-period payoffs. All agents are assumed to behave optimally with respect to their objectives and the information available to them. Since the value of keeping the item for private use is increasing in an agent's type, at each time t there is a sharing threshold ϑ_t, below which low-θ agents participate in the sharing market. On the other hand, high-θ agents, above the sharing threshold, abstain from the market.[14] Hence, for any agent of type $\theta \in \Theta$, the sharing decision is

$$x_t = \xi(\theta, \vartheta_t) \triangleq \begin{cases} 1, & \text{if } \theta > \vartheta_t, \\ 0, & \text{otherwise,} \end{cases} \tag{2.5}$$

where $\xi(\theta, \vartheta_t)$ describes the sharing state of type θ, given the sharing threshold ϑ_t. Consequently, the supply in the sharing market (i.e., the total number of agents sharing the item) at time $t > 0$ is

$$n_t = \int_\Theta \mathbf{1}_{\{\xi(\theta,\vartheta_t)=0\}} \, d\theta = 1 - \int_\Theta \xi(\theta, \vartheta_t) \, d\theta = \vartheta_t, \tag{2.6}$$

where the initial size $n_0 = \vartheta_0 \in [0,1]$ of the sharing supply is given.

2.3 Supply-Demand (Im)balance

A natural consequence of imperfections in the price-formation and supply-adjustment processes is that sharing markets may not always clear.[15] Given the price $p_t = p(n_{t-1})$, the demand at time t lags behind the supply for one period, and excess demand is

$$z_t \triangleq n_{t-1} - n_t. \tag{2.7}$$

If excess demand is positive ($z_t > 0$), then some potential users willing to rent an item on the sharing market at the current price are unable to find a seller and therefore have to take their outside option instead (at zero payoff). On the other hand, if excess demand is negative ($z_t < 0$), then there is in fact excess supply of shared items. Accordingly, the *transaction probability* for any given sharer is

$$q(\vartheta_t, \vartheta_{t-1}) \triangleq \min\{1, \vartheta_{t-1}/\vartheta_t\} \quad (= \min\{1, n_{t-1}/n_t\}). \tag{2.8}$$

Moreover, let $\rho \in [0,1]$ be a sharing intermediary's *commission rate*, corresponding to the captured fraction of the posted price p.[16] Therefore, instead of considering the transaction probability and pass-through transfer after deduction of the sharing intermediary's commission explicitly, one can restrict attention to the *effective transaction price*

$$\hat{p}(\vartheta_t, \vartheta_{t-1}) \triangleq (1-\rho)\, p(\vartheta_{t-1}) \cdot q(\vartheta_t, \vartheta_{t-1}). \tag{2.9}$$

Remark 2.1. The percentage charged by the intermediary varies across different platforms. For example AirBnB charges a 3% host service for every completed booking, while TaskRabbit

[14] The optimality of a threshold rule is formally established by Prop. 2.2 below.

[15] In contrast to Weber (2015), who uses Nash-bargaining for the price discovery in potentially unbalanced sharing markets, we use the market mechanism to determine the dynamic price in a rational-expectations equilibrium.

[16] Echoing Caillaud and Jullien (2003), Weber (2014, p. 51) notes that intermediated price depends in equilibrium only on a 'commission ratio,' composed of the commissions charged by the intermediary on both sides of the market. Because of this neutrality result one can limit attention to rents extracted on the supply side, setting the buyers' commission to zero.

charges a 20% commission fee on each successful task. The degree to which the intermediary is able to syphon off the gains from trade depends on the agents' outside option (e.g., the probability of matching in the absence of the sharing intermediary; see Weber 2014).

Remark 2.2. Let $\hat{\gamma} = (1-\rho)\gamma$ be the *intermediated* demand-elasticity parameter, so the effective transaction price becomes $\hat{p}(\vartheta_t, \vartheta_{t-1}) = \hat{\gamma}(1-\vartheta_{t-1}) \cdot q(\vartheta_t, \vartheta_{t-1})$. The presence of a sharing intermediary affects the market by changing the price elasticity of demand for potential suppliers.

In the analysis that follows, we assume that all agents discount their payoffs using the same discrete-time discount factor,

$$\delta \triangleq \frac{1}{1+r} \in (0,1),$$

where $r > 0$ denotes the per-period interest (or discount) rate. The (maximum) expected present value V that a type-θ agent, in sharing state x, can obtain over an infinite horizon is the solution of a system of Bellman equations,

$$V(x, \vartheta_t, \vartheta_{t-1}|\theta) = \max_{a \in \mathscr{A}} \{\bar{g}(a, x, \hat{p}(\vartheta_t, \vartheta_{t-1})|\theta) + \delta V(x', \vartheta', \vartheta_t|\theta)\}, \quad \theta \in \Theta, \tag{2.10}$$

on $\mathscr{X} \times \Theta^2$; the agents' respective sharing states in the economy evolve according to

$$x' = a = \pi(x, \vartheta_t|\theta),$$

where $\pi(\cdot|\theta) : \mathscr{X} \times \Theta \to \mathscr{A}$ is type θ's optimal policy. All agents $\theta \in \Theta$ decide in each time period $t \geq 0$ whether or not to share. Hence, the sharing threshold evolves according to

$$\vartheta' = 1 - \int_0^1 \pi(x, \vartheta_t|\theta)\, d\theta.$$

We refer to the resulting infinite-horizon (super-)game (keeping track of the sharing threshold ϑ_t) as $\mathscr{G}(\vartheta_0)$, where the given initial value of the sharing threshold ($\vartheta_0 \in \Theta$) is common knowledge.

2.3.1 Adjustment Dynamics

The sharing-state distribution $\xi(\cdot, \vartheta_t)$ in any given period t, introduced in Eq. (2.5), is fully characterized by the sharing threshold ϑ_t. Thus, to describe the dynamic development of the sharing market, consider the sequence of sharing thresholds $(\vartheta_0, \vartheta_1, \ldots)$, where $\vartheta_0 \in \Theta$ is given. The sequence describes the evolution of the size of the sharing market, in terms of its supply-side liquidity. An agent's transformed optimal policy at time t, denoted by $\hat{\pi}(\theta, \vartheta_t)$, depends on his type θ and on current market participation, i.e., by Eq. (2.6) on the threshold ϑ_t. Provided the agent uses a threshold policy, it is of the form

$$\hat{\pi}(\theta, \vartheta_t) \triangleq \pi(\xi(\theta, \vartheta_t), \vartheta_t|\theta) = \xi(\theta, \vartheta') = \mathbf{1}_{\{\theta > \vartheta'\}}. \tag{2.11}$$

With this, next period's sharing threshold ϑ' can be expressed as a function of the current threshold ϑ_t:

$$\vartheta' = \alpha(\vartheta_t) \triangleq \inf\{\theta \in \Theta : \hat{\pi}(\theta, \vartheta_t) = 1\},$$

using the convention that $\inf \emptyset = \sup \Theta = 1$. This defines a (time-invariant) system function $\alpha(\cdot)$ which describes the updates of the current sharing threshold. The optimality of a threshold policy, formally established in Prop. 2.2, means that an agent of a type higher than θ never prefers an action strictly lower than $\hat{\pi}(\theta, \vartheta_t)$.

Nash Equilibrium

In what follows, we use the concept of subgame-perfect Nash equilibrium by Selten (1965) to derive predictions about the outcome of the dynamic game, where any potential sharer maximizes his expected utility in each period, while taking the other agents' strategies as given. Indeed, for the current sharing threshold ϑ_t, the optimal decision for an agent of type θ (who is currently in the sharing state $\xi(\theta, \vartheta_t)$) is to keep the item if and only if the expected utility of keeping exceeds the expected utility of sharing, given that all other agents follow the equilibrium policy. To determine the subgame-perfect Nash equilibrium of the infinite-horizon dynamic game $\mathscr{G}(\vartheta_0)$ for any given initial value $\vartheta_0 \in \Theta$, we introduce $\hat{V}(\theta, \vartheta_t, \vartheta_{t-1}) \triangleq V(\xi(\theta, \vartheta_t), \vartheta_t, \vartheta_{t-1}|\theta)$ and rewrite Eq. (2.10) in the form

$$\begin{aligned} \hat{V}(\theta, \vartheta_t, \vartheta_{t-1}) \quad = \quad & \bar{g}(\hat{\pi}(\theta, \vartheta_t), \xi(\theta, \vartheta_t), \hat{p}(\vartheta_t, \vartheta_{t-1})|\theta) \\ & + \delta[\underbrace{\bar{g}(\hat{\pi}(\theta, \vartheta'), \xi(\theta, \vartheta'), \hat{p}(\vartheta', \vartheta_t)|\theta) + \delta \hat{V}(\theta, \alpha(\vartheta'), \vartheta')}_{\hat{V}(\theta, \vartheta', \vartheta_t)}], \quad \theta \in \Theta. \end{aligned} \tag{2.12}$$

The time-$(t+1)$ threshold $\vartheta' = \vartheta_{t+1}$ can be obtained as a function of ϑ_t (and, as it turns out, *not* of ϑ_{t-1}) by the one-shot deviation principle (Fudenberg and Tirole 1991). The latter implies a "transversal" equilibrium condition (2.13) that links the agents' decision problems, which in turn yields ϑ'.[17]

Lemma 2.1. *For any $t \geq 0$, the next-period sharing threshold is $\vartheta' = \alpha(\vartheta_t)$, where*

$$\begin{aligned} \alpha(\vartheta_t) \quad = \quad & \inf\{\theta \in \Theta : \bar{g}(1, \xi(\theta, \vartheta_t), \hat{p}(\vartheta_t, \vartheta_{t-1})|\theta) + \delta \bar{g}(\hat{\pi}(\theta, \vartheta'), 1, \hat{p}(\vartheta', \vartheta_t)|\theta) \geq \\ & \bar{g}(0, \xi(\theta, \vartheta_t), \hat{p}(\vartheta_t, \vartheta_{t-1})|\theta) + \delta \bar{g}(\hat{\pi}(\theta, \vartheta'), 0, \hat{p}(\vartheta', \vartheta_t)|\theta)\}; \end{aligned} \tag{2.13}$$

it is such that $\hat{\pi}(\vartheta', \vartheta') = 0$.

In addition to providing an expression for the system function, the preceding result describes an

[17] The details are given in the proof of La. 2.1 in App. B.

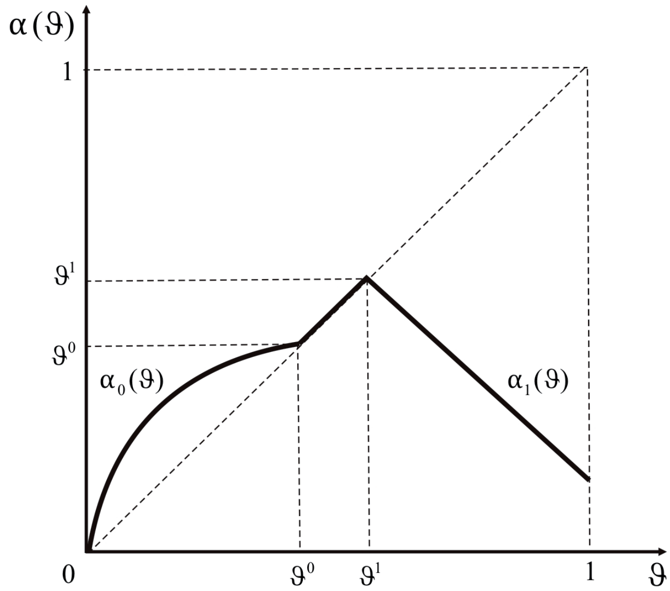

Figure 2.2: Law of motion for the sharing threshold when $0 < \vartheta^0, \vartheta^1 < 1$.

invariance property of the threshold policy in Eq. (2.11), namely that it is always optimal for a marginal type to share, i.e., $\hat{\pi}(\vartheta_t, \vartheta_t) \equiv 0$ for all $t \geq 1$. Agents who find it optimal to share will remain active as suppliers in the sharing economy for all times. This insight is helpful in pinning down the equilibrium path $(\vartheta_t)_{t=0}^{\infty}$, which depends on the "(lower) invariance threshold,"

$$\vartheta^0 \triangleq \max\left\{0, \frac{\hat{\gamma} - r c_{10}}{\hat{\gamma} + \Delta}\right\}, \tag{2.14}$$

corresponding to a marginal type, below which all agents are willing to share in the long run. It also depends on the "(upper) invariance threshold,"

$$\vartheta^1 \triangleq \min\left\{1, \frac{\hat{\gamma} + c_{10} + (1+r) c_{01}}{\hat{\gamma} + \Delta}\right\}, \tag{2.15}$$

corresponding to a marginal type, above which agents would want to exit the sharing market. Agent types in the *invariance region* $\mathscr{R} \triangleq [\vartheta^0, \vartheta^1]$ prefer to remain in their respective *status quo*, so $\vartheta' = \vartheta_t = \bar{\vartheta}$, where $\bar{\vartheta}$ implies a *stationary* sharing-state distribution $\bar{\xi}(\cdot, \bar{\vartheta})$ which stays in place for all times greater than t; see Sec. 2.3.3 for details.

Remark 2.3. For all $c_{01}, c_{10} \geq 0$, the lower invariance threshold cannot exceed the upper invariance threshold, i.e., $\vartheta^0 \leq \vartheta^1$. The two thresholds together define an interval for feasible sizes of the sharing economy in the long run. They coincide in a *frictionless economy*, where adjustment costs vanish.

The preceding discussion is now formalized in a full characterization for the equilibrium dynamics of the type threshold in a sharing economy with frictions.

Proposition 2.1 (Equilibrium Path). *The law of motion for the sharing threshold is $\vartheta' = \alpha(\vartheta)$, for all $t \geq 0$, where the system function is given by*

$$\alpha(\vartheta) \triangleq \max\{\alpha_0(\vartheta), \min\{\vartheta, \alpha_1(\vartheta)\}\} = \begin{cases} \alpha_0(\vartheta), & \text{if } \vartheta < \vartheta^0, \\ \vartheta, & \text{if } \vartheta^0 \leq \vartheta \leq \vartheta^1, \\ \alpha_1(\vartheta), & \text{if } \vartheta > \vartheta^1, \end{cases} \tag{2.16}$$

with [18]

$$\alpha_0(\vartheta) = \max\left\{0, \frac{(1-\rho)\, p(\vartheta)\, (\vartheta/\alpha_0(\vartheta)) - r c_{10}}{\Delta}\right\}, \tag{2.17}$$

$$\alpha_1(\vartheta) = \min\left\{1, \frac{(1-\rho)\, p(\vartheta) + c_{10} + (1+r) c_{01}}{\Delta}\right\}, \tag{2.18}$$

for all $\vartheta \in \Theta$; ϑ^0 and ϑ^1 are specified in Eqs. (2.14) and (2.15).

The Nash-equilibrium path $(\vartheta_t)_{t=0}^{\infty}$ in Prop. 2.1 implies the agents' equilibrium policy $\hat{\pi}$ in Eq. (2.11) for all types $\theta \in \Theta$, which in turn determines a unique subgame-perfect Nash equilibrium in the supergame $\mathscr{G}(\vartheta_0)$. In this dynamic sharing economy, each agent finds it optimal to implement a threshold-type policy $\hat{\pi}$.

Proposition 2.2 (Threshold Optimality and Uniqueness of the Equilibrium). *For any given $\vartheta_0 \in \Theta$, the unique subgame-perfect Nash equilibrium of $\mathscr{G}(\vartheta_0)$ is such that $\vartheta_{t+1} = \alpha(\vartheta_t)$ and $\hat{\pi}(\theta, \vartheta_t) = \mathbf{1}_{\{\theta > \alpha(\vartheta_t)\}}$, for all $t \geq 0$ and all $\theta \in \Theta$.*

Fig. 2.2 illustrates the law of motion of the sharing threshold, starting from any given initial number of suppliers in the economy. The equilibrium path describes the steady-state and non-steady state evolution of the sharing threshold governed by optimizing behavior of the agents in the sharing economy. In the remainder of this section, the salient characteristics of the equilibrium path are discussed in detail.

Equilibrium Dynamics

The different properties of the system function $\alpha(\cdot)$, summarized below, characterize the temporal dynamics of the sharing supply.

Lemma 2.2. *The system function $\alpha(\cdot)$ has the following properties:*

(i) $\alpha(0) = 0$ *(by continuous completion);*

[18] The lower arc of the system function, $\alpha_0(\vartheta) = \left(-r c_{10} + \sqrt{(r c_{10})^2 + 4\Delta(1-\rho)\vartheta p(\vartheta)}\right)/(2\Delta)$, is obtained in closed form as the unique nonnegative solution of the corresponding fixed-point problem, for all $\vartheta \in [0, \vartheta^0)$.

(ii) For all $\vartheta \in (0,1]$, the dynamic increment, $\alpha(\vartheta) - \vartheta$, has the (weak) single-crossing property (with respect to $\mathscr{R}$);[19]

(iii) $\alpha''(\vartheta) < 0 < \alpha'(\vartheta)$, *for all* $\vartheta \in (0, \vartheta^0)$.

The system function $\alpha(\cdot)$ corresponds to the law of motion pertaining to the size of the sharing supply, and its characteristics determine the transient dynamics of the sharing economy.

a. No sharing without sharers. By its very nature, the sharing economy needs at least *some* sharers to sustain collaborative consumption. Whenever $\vartheta_t = 0$, then also $\vartheta_{t+1} = 0$, which—by backward induction—means that the economy must have been at zero sharing at all times. In other words, the model does not provide for a driving force other than a positive number of sharers at the beginning. It is agnostic about what might have caused the existence of a positive number of sharers at the initial time.

b. Invariance region separates sharing growth from sharing decline and determines the feasible interval for the stationary market coverage. For sharing thresholds ϑ below the invariance region $\mathscr{R}$, the system function describes an upward movement of the sharing supply while for sharing thresholds above $\mathscr{R}$ the system function prescribes a downward movement. This is reflected by the single-crossing property stated in La. 2.2(ii). This property ensures that, for any nonzero initial condition, the stationary value $\bar{\vartheta}$ of the sharing threshold (and consequently the price and supply of the shared item) can lie only in the invariance region $\mathscr{R}$.

c. Downward adjustments in sharing supply do not persist unless they lead directly to a steady state. Following the intuition of "years to build, seconds to destroy," growth in the sharing economy tends to be more incremental than decline. A one-time decrease in the size of the sharing economy happens above the invariance region $\mathscr{R}$, i.e., for $\vartheta_0 > \vartheta^1$, and *either* leads to a stationary state *or* it is followed by an incremental upward re-adjustment. More specifically, for $\vartheta \in (\vartheta^1, \vartheta^2]$, with the "rest threshold"

$$\vartheta^2 \triangleq \min\{1, \alpha_1^{-1}(\vartheta^0)\} = \min\left\{1, \frac{\hat{\gamma}^2 + c_{10}(\hat{\gamma} + \Delta/\delta) + c_{01}(\hat{\gamma} + \Delta)/\delta}{\hat{\gamma}(\hat{\gamma} + \Delta)}\right\}, \tag{2.19}$$

the decline of the sharing economy is small enough such that the subsequent sharing threshold $\vartheta' = \alpha(\vartheta)$ rests in the invariance region $\mathscr{R}$ and the economy becomes stationary in finite time. For $\vartheta \in (\vartheta^2, 1]$, however, the sharing economy experiences a steeper downturn, such that the subsequent sharing threshold drops below ϑ^0, i.e., it falls below $\mathscr{R}$. The economy then starts an incremental growth. This possible nonmonotonicity in the evolution of the sharing economy is illustrated in Fig. 2.3.

[19] See, e.g., Athey (2002), p. 190; in particular, the increment vanishes on $\mathscr{R}$.

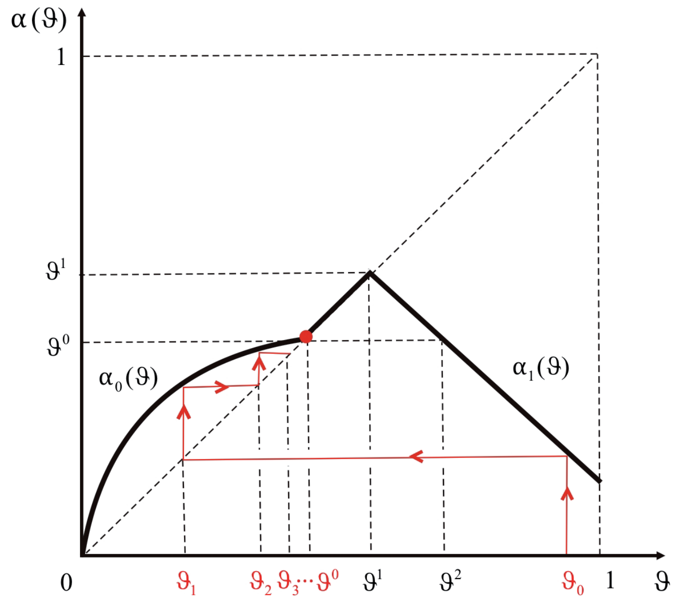

Figure 2.3: Growth path of the sharing threshold when $0 < \vartheta^0 < \vartheta^2 < \vartheta_0$.

d. Growth occurs only below the invariance region, with an S-shaped diffusion pattern. The system function $\alpha(\cdot)$ implies that the sharing economy grows if the supply of the shared item is small enough such that the sharing threshold does not exceed the lower invariance threshold ϑ^0. The sharing economy can go through growth adjustments at varying speeds. The concavity of the system function below the invariance region $\mathscr{R}$ by La. 2.2(iii) implies that the growth rate of the sharing economy must be unimodal.

Proposition 2.3 (**Maximum Diffusion**). *Let $rc_{10} < \hat{\gamma}$, so $\vartheta^0 > 0$. Then there is a unique "maximum-diffusion" (sharing) threshold, $\vartheta^\mu \in \arg\max_{\vartheta\in(0,\vartheta^0)}\{\alpha(\vartheta) - \vartheta\}$, at which the growth of the sharing economy is maximal.*

By La. 2.2(i) and the definition of the lower invariance threshold in Eq. (2.14), $\alpha(\vartheta) - \vartheta$ vanishes at the boundaries of the growth region $[0, \vartheta^0]$. The concavity of $\alpha(\cdot)$ in the interior of the growth region (by virtue of La. 2.2(iii)) therefore yields the existence of a unique maximal increment of the sharing supply. This property suggests an S-shaped growth curve of the sharing supply; a pattern of growth in which, in a sufficiently small economy, the population of the sharers increases initially in a positive acceleration phase; but then the growth continues at a decreasing rate, with the sharing economy approaching a steady state. Depending on the initial value ϑ_0 (below the maximum-diffusion threshold), the fastest growth of sharing occurs at a well-defined (finite) maximum-diffusion time

$$t^\mu \in \arg\max_{t\geq 0}\{\alpha(\vartheta_t) - \vartheta_t\}, \tag{2.20}$$

subject to the law of motion in Prop. 2.1, which is such that $\alpha_0^{-1}(\vartheta^\mu) \leq \vartheta_{t^\mu} \leq \alpha_0(\vartheta^\mu)$. At this time, the supply-demand mismatch in the economy is maximal. We can immediately infer the

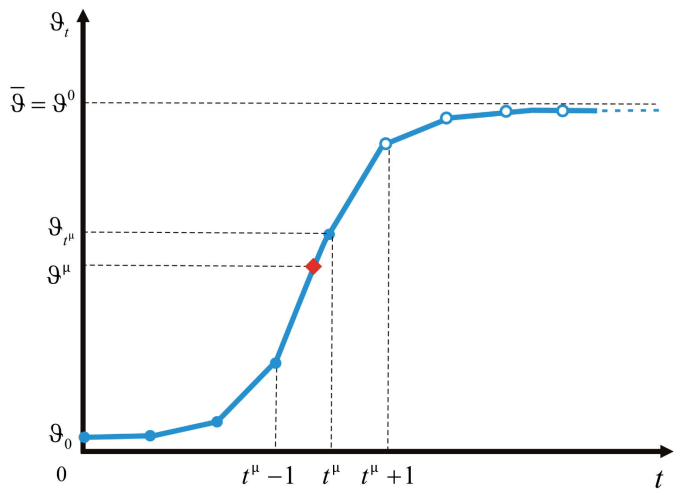

Figure 2.4: S-shaped diffusion of supply in a sharing economy.

following result (therefore stated without proof).

Corollary 2.1. *If the sharing economy starts with ϑ_0 below the invariance region $\mathscr{R}$, the absolute value of the excess demand z_t is maximized either at $t^\mu > 0$ (for $\vartheta_0 < \vartheta^\mu$) or at $t^\mu = 0$ (for $\vartheta_0 \geq \vartheta^\mu$).*

If the economy starts with a sufficiently small number $n_0 = \vartheta_0$ of sharers, for all $t < t^\mu$ the diffusion of sharing accelerates, whereas for all $t > t^\mu$ it decelerates. The resulting S-shaped diffusion pattern is shown in Fig. 2.4. This growth pattern of sharing shows the same characteristics as the well-known diffusion model by Bass (1969), which fits the empirically observed adoption paths for a wide range of products (Bass et al. 1994). The finding is remarkable because it is obtained as the equilibrium of a game played by market participants, in the absence of the standard assumptions on the sales of products. The robustness of S-shaped diffusion in sharing markets is investigated in Sec. 2.6.

2.3.2 Comparative Statics

The invariance region $\mathscr{R} = [\vartheta^0, \vartheta^1]$ determines the participation levels in the sharing economy that can be maintained in the long run. The following result describes how the invariance thresholds ϑ^0 and ϑ^1 delimiting this region depend on the interest rate r, the conversion costs (c_{01}, c_{10}), the utility gain Δ, the intermediary's commission rate ρ, and the demand-elasticity parameter γ (see footnote 10). This determines the asymptotic behavior of the sharing economy in the long run, as a function of the salient model parameters.

Proposition 2.4 (**Monotonicity of the Invariance Thresholds**). *Let $\mathscr{R} \subset (0,1)$. Then the 'interior' invariance thresholds ϑ^0 and ϑ^1 satisfy the following monotonicity properties:*[20]

[20] The invariance thresholds are interior, i.e., $\vartheta^0, \vartheta^1 \in (0,1)$, if and only if $c_{10} < \hat{\gamma}/r$ and $c_{10} + (1+r)c_{01} < \Delta$.

(i) ϑ^1 *and* ϑ^0 *are decreasing in* Δ*;*

(ii) ϑ^1 *(resp.* ϑ^0*) is increasing (resp. decreasing) in* (c_{01}, c_{10}, r)*;*

(iii) ϑ^1 *and* ϑ^0 *are increasing in* γ *and decreasing in* ρ*.*

Following an increase of the utility difference Δ the invariance region $\mathscr{R}$ shifts downwards, and its size (diameter),

$$|\mathscr{R}| = \vartheta^1 - \vartheta^0 = (1+r)\,\frac{c_{01} + c_{10}}{\hat{\gamma} + \Delta}, \tag{2.21}$$

becomes smaller. On the other hand, when the demand-elasticity parameter γ increases or the intermediary's share ρ decreases, the invariance region $\mathscr{R}$ shifts upwards. Corresponding to a smaller price elasticity of demand ε,[21] an increase in $\hat{\gamma}$ tends to expand the number of sharers in an equilibrium steady state. At the same time, a decremental move in the sharing economy is less likely. More individuals become prone to switching sharing states, thus reducing the size of the invariance region where no further adjustments take place.

The monotonicity of the invariance thresholds has practical implications for intermediaries as well as regulators. Indeed, in order to achieve a higher market coverage in the long run and achieve this improved steady state faster, a sharing intermediary can lower the commission rate ρ, especially in markets with elevated conversion-cost levels. Prop. 2.4 also implies that the size of $\mathscr{R}$ increases both in the magnitude of the conversion cost and in the per-period interest rate. As the conversion costs increase, switching becomes less attractive for a larger fraction of the agents. Another interesting feature is a substitution effect between conversion costs and interest rate, in the sense that for the invariance thresholds to remain unchanged, at a lower interest rate, conversion costs need to be higher. Since switching decisions are taken one period ahead of the actual adjustment of the agents' sharing states, if the potential sharers do not care sufficiently about their future payoffs (because of a high discount rate), they willingly forego the expected future benefits of sharing and remain inactive.

2.3.3 Steady States

Stationary Sharing-State Distribution

For initial values ϑ_0 in the invariance region $\mathscr{R} = [\vartheta^0, \vartheta^1]$, the sharing economy remains at rest for all times $t \geq 0$, i.e., a steady state $\bar{\vartheta} \in \mathscr{R}$ is attained immediately. In the case of oversharing where the initial value lies above $\mathscr{R}$ but does not exceed the rest threshold, i.e., when $\vartheta_0 \in (\vartheta^1, \vartheta^2]$, the sharing economy experiences a downward adjustment to a steady state in $\mathscr{R}$. In all other cases (except for $\vartheta_0 = 0$ according to Sec. 2.3.1.a), the sharing economy never attains a steady state in finite time, i.e., it remains in *dis*equilibrium.

[21] For details, see La. B.1.1 in App. B.

When in a steady state, the sharing market clears and each agent is locked into an 'acceptable' sharing state, given the conversion costs c_{10} and c_{01}, and the collective externality of the other agents' choices. Agents of types $\theta \le \bar{\vartheta}$ act as suppliers in the sharing economy, for all $t \ge \tau$, where $\tau \in \{0,1\}$. The stationary transaction volume is $\bar{n} = \bar{\vartheta}$.

Remark 2.4. For any steady state $\bar{\vartheta} \in \mathscr{R}$, the stationary sharing-state distribution $\bar{\xi} : \Theta \to \mathscr{X}$ is such that $x_t(\theta, \vartheta_t) \equiv \bar{\xi}(\theta, \bar{\vartheta})$, for all $t \ge \tau$. This distribution encapsulates the agents' equilibrium policy:

$$\bar{\xi}(\theta, \bar{\vartheta}) = \pi(\bar{\xi}(\theta, \bar{\vartheta}), \bar{\vartheta}|\theta),$$

for all agent types $\theta \in \Theta$.

Remark 2.5. If the sharing threshold reaches its steady state $\bar{\vartheta} \in \mathscr{R}$ at time τ, then by Eq. (2.3) the price reaches its stationary value $\bar{p} = p(\bar{\vartheta})$ at most one period later; the transaction probability for suppliers is always 1.

Stationary Equilibrium Payoffs

In steady state from time τ, a type-θ agent in sharing state $x \in \{0,1\}$ obtains the "terminal value" $\bar{V}^x(\theta|\bar{\vartheta})$, which is equal to the discounted sum of the per-period rewards in a stationary equilibrium with sharing threshold $\bar{\vartheta} \in \mathscr{R}$. Taking into account the lack of switching in steady-state, the type-θ agent's payoff in Eq. (2.4), the stationary sharing-state distribution in Remark 2.4, and the geometric-series formula together imply that in equilibrium $\bar{V}(\theta|\bar{\vartheta}) \triangleq V^{\xi(\theta,\bar{\vartheta})}(\theta|\bar{\vartheta})$, so that

$$\bar{V}^0(\theta|\bar{\vartheta}) = \frac{\bar{g}(0,0,\bar{p}|\theta)}{1-\delta} = \frac{\bar{p} + \theta u_0}{1-\delta}, \quad \theta \in [0, \bar{\vartheta}],$$

and

$$\bar{V}^1(\theta|\bar{\vartheta}) = \frac{\bar{g}(1,1,\bar{p}|\theta)}{1-\delta} = \frac{\theta u_1}{1-\delta}, \quad \theta \in (\bar{\vartheta}, 1],$$

where $\bar{p} = p(\bar{n})$ and $\bar{n} = \bar{\vartheta}$.

Remark 2.6. In a frictionless economy, a steady state is never attained, unless the sharing economy starts (and then remains) at $\vartheta_0 = \bar{\vartheta}$. Even a small amount of randomness perturbing the initial state of the sharing economy would reduce a steady-state sharing economy to a zero-probability event (at least as long as $\bar{\vartheta} > 0$). For any ϑ_0 different from $\bar{\vartheta}$, the sharing economy remains in perpetual *dis*equilibrium.

Lemma 2.3. *The steady state $\bar{\vartheta}$ lies in the invariance region $\mathscr{R}$ and satisfies*

$$(\bar{p} - rc_{10})/\Delta \le \bar{\vartheta} \le (\bar{p} + (1+r)c_{01} + c_{10})/\Delta. \tag{2.22}$$

The preceding result formalizes the observation that an expected per-period utility gain $\bar{\vartheta}\Delta$ for a marginal type must lie between the price $\bar{p}$ in the sharing market plus or minus the relevant per-period rents from the conversion costs, depending on whether the agent is currently sharing (when the price remains above a lower bound, so as to make repossessing the item unattractive) or keeping (when the price remains below an upper bound, so that the required investment c_{10} does not warrant entering the sharing market). As can be verified using Eqs. (2.1), (2.14) and (2.15), inequality (2.22) is in fact equivalent to $\bar{\vartheta} \in \mathscr{R}$, i.e., it fully characterizes the set of possible steady-state sharing thresholds.

2.3.4 Limiting Behavior

For initial values $\vartheta_0 \notin [\vartheta^0, \vartheta^2] \cup \{0\}$, the sharing economy is growing. A steady state is not reached in finite time, but becomes the asymptotic limit of the equilibrium path of the sharing economy for $t \to \infty$. The subgame-perfect equilibrium of $\mathscr{G}(\vartheta_0)$ (in Prop. 2.2) and the evolution of the sharing threshold ϑ_t on the equilibrium path (in Prop. 2.1) together describe the rise of the sharing economy for $t > 0$. We now establish a one-to-one mapping from the initial condition to the asymptotic limit of the sharing economy.

Proposition 2.5 (Sharing Asymptotics). *For any given $\vartheta_0 \in \Theta$ the equilibrium path $(\vartheta_t)_{t=0}^{\infty}$ of sharing thresholds converges to a stationary value $\bar{\vartheta} = \varphi(\vartheta_0) \in \mathscr{R} \cup \{0\}$, such that*

$$\vartheta_0 \mapsto \varphi(\vartheta_0) = \begin{cases} \vartheta_0, & \text{if } \vartheta_0 \in \mathscr{R} \cup \{0\}, \\ \alpha_1(\vartheta_0), & \text{if } \vartheta_0 \in (\vartheta^1, \vartheta^2], \\ \vartheta^0, & \text{otherwise}. \end{cases} \tag{2.23}$$

Correspondingly, the time when the sharing threshold reaches its steady state is

$$\tau(\vartheta_0) = \begin{cases} 0, & \text{if } \vartheta_0 \in \mathscr{R} \cup \{0\}, \\ 1, & \text{if } \vartheta_0 \in (\vartheta^1, \vartheta^2], \\ \infty, & \text{otherwise}. \end{cases} \tag{2.24}$$

Note that the price attains its steady state at time $t = 2\tau(\vartheta_0)$.

Remark 2.7. In a frictionless economy (where $c_{01} = c_{10} = 0$), there is a unique stationary sharing threshold; $\bar{\vartheta} = p(\bar{\vartheta})/\Delta = \gamma/(\gamma + \Delta) \in (0, 1/2]$, and a unique steady-state price $\bar{p} = \gamma\Delta/(\gamma + \Delta)$ in the sharing market.

Fig. 2.5 depicts the eventual market coverage and the price starting from any initial condition. If the economy is initially *undersharing* with the sharing threshold below $\mathscr{R}$, it converges to the lower invariance threshold ϑ^0 via an infinite sequence of incremental adjustments. In an *oversharing* economy starting from above $\mathscr{R}$, the sharing supply goes through a rapid phase of decline, followed by incremental adjustment, unless it drops to a value in the invariance region $\mathscr{R}$, where the conversion costs prevent all agents from changing their states. In times when the sharing economy gradually expands, the excess supply—as the negative of excess demand in Eq. (2.7)—is positive. Any positive excess supply is persistent, but diminishes over time.

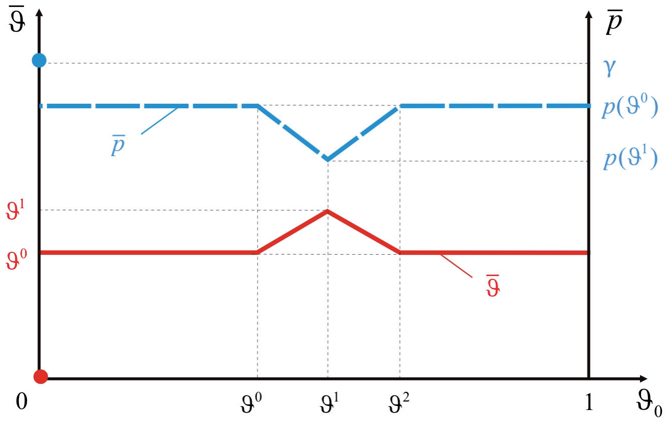

Figure 2.5: Steady-state values of price and supply as a function of the initial value $\vartheta_0 \in \Theta$.

Corollary 2.2. *Any excess demand (resp. supply) vanishes asymptotically, i.e.,* $\lim_{t\to\infty} z_t = 0$.

In the absence of macroeconomic shocks, the demand-supply imbalance in the sharing market equilibrates in the long run, reflecting asymptotic market stability. In the presence of shocks, for example, in the form of sudden and unexpected parameter adjustments, the market would readjust and the sharing state would track the updated invariance region.

2.4 Frictions and Sharing Inertia

2.4.1 Persistence of Disequilibrium

A disequilibrium in a sharing economy is persistent resulting in perpetual adjustments, as long as the conversion costs are not so large as to render an economic adjustment of their respective sharing states impossible for the agents. This applies *a forteriori* also to a frictionless economy. With high adjustment costs, on the other hand, the sharing economy would attain a steady state quickly, in finite time.

Although all agent types $\theta \in [0, \vartheta^0]$ eventually prefer sharing to keeping, higher-type agents in this growth regime prefer to delay their switching, possibly for very long, waiting for the *effective* transaction price $\hat{p}$ in Eq. (2.9) to increase. That is, because of the demand-supply mismatch in the early periods, the transaction probability q in Eq. (2.8) is fairly low, so that despite the relatively high market price (corresponding to a scarce supply) the expected revenue is too low to persuade higher-need types to participate in sharing. In other words, agents who are more likely to need the item are more reluctant to incur both the conversion cost and the opportunity cost of sharing, when the risk of not transacting is too high. The following result establishes that the effective transaction price is indeed increasing in the growth portion of the sharing thresholds, below the invariance region $\mathscr{R}$.

Lemma 2.4. *For all* $\vartheta \in (0, \vartheta^0)$, *the effective transaction price* $\hat{p}(\alpha(\vartheta), \vartheta)$ *increases in the sharing* *threshold ϑ.*

It turns out that for almost any initial condition, the sharing economy experiences a positive growth diffusion, provided only that conversion costs are low enough, which means that frictions are not too large. The cost c_{01} of repossessing an item plays a role solely in the (somewhat hypothetical) case where the initial number of sharers is very high, exceeding the steady state of sharers in a frictionless economy.

Proposition 2.6 (Persistence of Sharing Growth). *For any positive initial sharing threshold $\vartheta_0 \in \Theta \setminus \{\hat{\gamma}/(\hat{\gamma}+\Delta)\}$, the sharing economy does not converge in finite time if the conversion costs are low enough, such that either*

$$c_{10} < \frac{\delta(\hat{\gamma}+\Delta)}{1-\delta}\left(\frac{\hat{\gamma}}{\hat{\gamma}+\Delta} - \vartheta_0\right),$$

for $\vartheta_0 \in (0, \hat{\gamma}/(\hat{\gamma}+\Delta))$, or

$$\begin{bmatrix} c_{01} & c_{10} \end{bmatrix} \begin{bmatrix} 1 & \Delta/\hat{\gamma} \\ \delta & \Delta/\hat{\gamma} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} < \delta(\hat{\gamma}+\Delta)\left(\vartheta_0 - \frac{\hat{\gamma}}{\hat{\gamma}+\Delta}\right),$$

for $\vartheta_0 \in (\gamma/(\gamma+\Delta), 1]$.

Note that $\bar{\vartheta} = \hat{\gamma}/(\hat{\gamma}+\Delta)$ is the long-run sharing threshold in a frictionless economy with intermediary (see Remark 2.7), and is therefore always in the invariance region $\mathscr{R}$. The impossibility of adjustments when starting at the frictionless threshold explains the dichotomy in Prop. 2.6. It implies that the economy reaches the stationary regime almost immediately, usually when conversion costs are significant.

2.4.2 Critical Sharing Thresholds

The three critical sharing thresholds (ϑ^0, ϑ^1, and ϑ^2), discussed in Sec. 2.3.1 create four different regimes, depending on the current number of sharers in the economy $n = \vartheta$.

- $\vartheta \in (0, \vartheta^0)$ ("Growth"): The sharing economy is growing, the excess demand is negative, and the transaction probability is less than 1.
- $\vartheta \in [\vartheta^0, \vartheta^1] \cup \{0\}$ ("Stagnation"): The sharing economy is in the invariance region (or non-existent), with no adjustment at all. The market clears with probability 1, and the excess demand vanishes.
- $\vartheta \in (\vartheta^1, \vartheta^2]$ ("Terminal Adjustment"): The economy declines for one period. The excess demand is positive for one period and vanishes afterwards. The transaction probability equals 1 for the sharers, at all times.

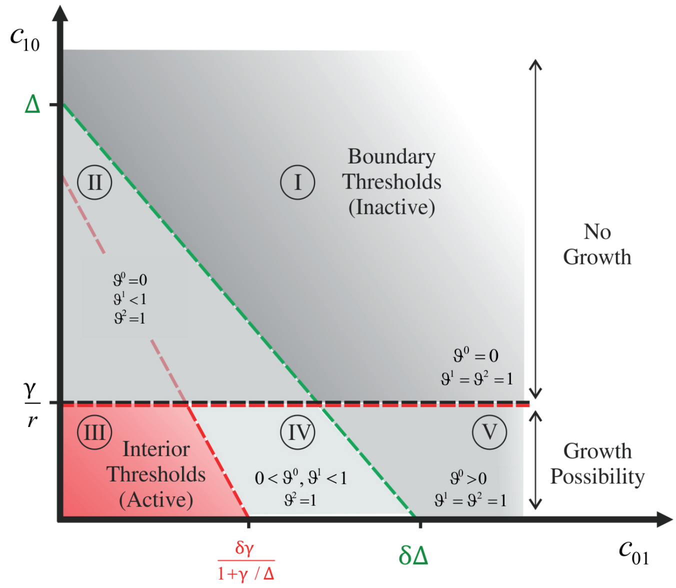

Figure 2.6: Critical sharing thresholds as a function of conversion costs (c_{01}, c_{10}).

- $\vartheta \in (\vartheta^2, 1]$ ("Growth Adjustment"): The economy declines for one period, followed by an incremental diffusion. The excess demand is positive for the initial period with $q = 1$, and negative afterwards with $q < 1$.

As the conversion costs become large, one or several of these regimes may disappear.

Lemma 2.5. *It is* $\vartheta^0 \in [0, 1/2]$ *and* $0 < \vartheta^1 \leq \vartheta^2 \leq 1$. *Furthermore,*

(i) $\vartheta^0 > 0$ *if* $c_{10} < \delta\hat{\gamma}/(1-\delta)$*;*

(ii) $\vartheta^1 < 1$ *if* $(c_{01}/\delta) + c_{10} < \Delta$*;*

(iii) $\vartheta^2 < 1$ *if* $c_{10} < \delta\hat{\gamma}/(1-\delta)$ *and* $(1 + \frac{\delta\hat{\gamma}}{\Delta})\, c_{01} + (1 + \frac{\hat{\gamma}}{\Delta})\, c_{10} < \delta\hat{\gamma}$.

The interiority conditions for the critical sharing thresholds ϑ^0, ϑ^1, and ϑ^2 in La. 2.5 partition the conversion-cost space—with points (c_{01}, c_{10})—into five different regions, denoted by Roman numerals (I through V) in Fig. 2.6. If the cost c_{10} exceeds $\hat{\gamma}/r$, then $\vartheta^0 = 0$: the sharing economy cannot grow, as no agent is willing to convert his item to sharing, irrespective of the equilibrium price (regions I and II).[22] This can be useful for manufacturers who may find it in their self-interest to either enable or disable sharing markets. If they can design the item, so that de-personalizing it is sufficiently expensive, then sharing cannot happen at all. Conversely, manufacturers can encourage sharing by subsidizing the conversion cost for prospective sharers. Note also that

[22]In regions I and II, the rest threshold $\vartheta^2 = 1$. Therefore, if the economy is "oversharing" with initial condition $\vartheta_0 > \vartheta^1$ in region II, then all agent types between ϑ_0 and $\bar{\vartheta}$ switch from $x = 0$ to $x = 1$, and by Eq. (2.24) the steady state $\bar{\vartheta}$ is attained at time $\tau = 1$.

the viability of a sharing market (i.e., the positivity of ϑ^0) implies an upper bound for a sharing intermediary's commission rate. That is,

$$\rho \leq 1 - r c_{10}/\gamma. \tag{2.25}$$

As a result, an intermediary's take is limited by the sharer's initial conversion cost as well as their level of impatience (measured by the discount rate r). On the other hand, if c_{10} remains sufficiently small with a value less than $\hat{\gamma}/r$ (regions III through V), then the economy can grow with dynamics that also depend on the cost c_{01} of repossessing the item. If c_{01} is small enough, then frictions in the sharing economy are minor (region III), and one obtains the dynamics with asymptotic behavior described by Prop. 2.5 and the mapping in Eq. (2.23). As the cost of switching from keeping to sharing increases, the rest threshold ϑ^2 and the upper invariance threshold ϑ^1 increase, and it becomes more unattractive to exit the sharing market. In region IV, the rest threshold hits the upper boundary, and in region V both the rest threshold and the upper invariance threshold saturate, indicating a somewhat utopian situation where all agents are keen sharers.

2.4.3 Externalities and Aggregate Switching Behavior

In the interesting case where the conversion costs are small enough, so the system can exhibit all types of transitional behavior (see region III in Fig. 2.6), the critical sharing thresholds are *interior*, i.e., $0 < \vartheta^0, \vartheta^1, \vartheta^2 < 1$. The sharing behavior for each agent type depends on the externality exerted by the agents in the economy who are already sharing. When seen over the entire equilibrium path, there are five possible behavioral patterns,[23] as a function of the initial condition, of which usually two or three coexist for a given initial value ϑ_0. Fig. 2.7 shows the agents' aggregate switching behavior as a function of the initial condition. The red line highlights the sharing threshold in equilibrium, corresponding to the red line in Fig. 2.5, above which all agents choose to keep the item for personal use ($\bar{x} = 1$), and below which all agents share the item on the market ($\bar{x} = 0$); see Prop. 2.5.

If the sharing economy starts below the invariance region $\mathscr{R}$, all agents with $\vartheta_0 \leq \theta \leq \vartheta^0$, switch once from keeping to sharing. However, all switches do not occur at the same time, and at each time t, only agents with $\theta \in [\vartheta_t, \vartheta_{t+1}]$ change their sharing state. As pointed out in Sec. 2.3.4, this is directly related to the fact that the effective transaction price in the market is increasing (see La. 2.4). If the economy starts above the invariance region $\mathscr{R}$ where $\vartheta_0 > \vartheta^1$, then it experiences a temporary decline, and subsequent incremental growth. Agent types $\theta \in [\vartheta_1, \vartheta_0]$ switch from keeping to sharing at $t = 0$. Agent types $\theta \in [\vartheta_1, \bar{\vartheta})$ therefore switch twice, eventually converting back from keeping to sharing; in the long run, all of these agent types participate in the sharing market. Lastly, all other types remain in their *status quo*: agents with $\theta < \vartheta_0 < \vartheta^0$ share at all times, while agents with $\theta > \vartheta_0 > \vartheta^0$ always hold on to their assets for personal use.

[23] The possible patterns are: "Share," "Keep → Share," "Keep," "Share → Keep," and "Share → Keep → Share."

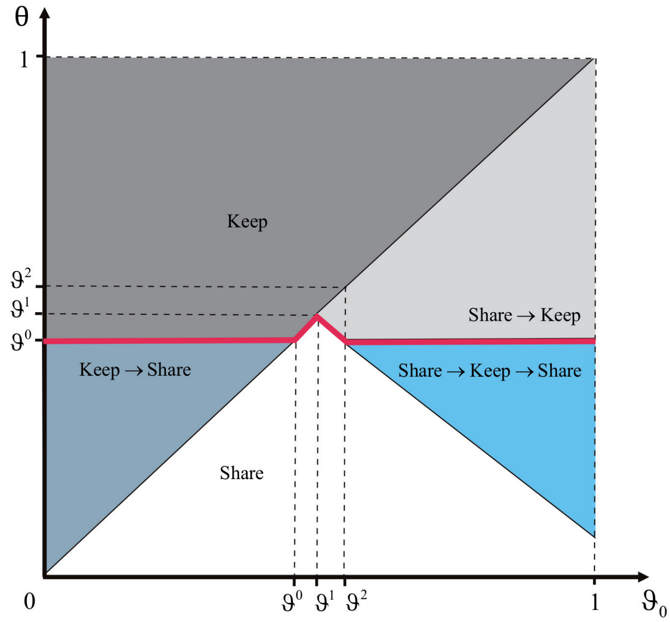

Figure 2.7: Aggregate switching behavior as a function of the initial condition.

2.4.4 Decision Hysteresis

An agent of type θ prefers to start sharing his item if the effective price $\hat{p}$ in the sharing market is sufficiently high, i.e., exceeds $\hat{p}^0(\theta)$. Since sharing is eventually preferable as long as $\theta < \vartheta^0$, one obtains a (type-specific) *critical sharing price*,

$$\hat{p} > \hat{p}^0(\theta) \triangleq \min\{\hat{\gamma}, \Delta\theta + rc_{10}\}.$$

Similarly, the agent prefers to abandon sharing and switch to using the item if the effective price in the sharing market drops below a critical price $\hat{p}^1(\theta)$. Since keeping proves preferable for $\theta > \vartheta^1$, we obtain a (type-specific) *critical keeping price*,

$$\hat{p} < \hat{p}^1(\theta) \triangleq \max\{0, \Delta\theta - c_{10} - (1+r)c_{01}\}.$$

The two critical prices bracket the agent's expected (incremental) utility of keeping the item, from above and below, respectively:

$$\hat{p}^1(\theta) \le \Delta\theta \le \hat{p}^0(\theta).$$

In a frictionless economy the two thresholds coincide, and agents switch exactly when the expected utility $\Delta\theta$ (in positive or negative direction) is at least offset by the effective transaction price in the sharing market. With conversion costs the gap between the thresholds gives rise to inertia in either direction, also referred to as decision hysteresis (Dixit 1992); see Fig. 2.8.

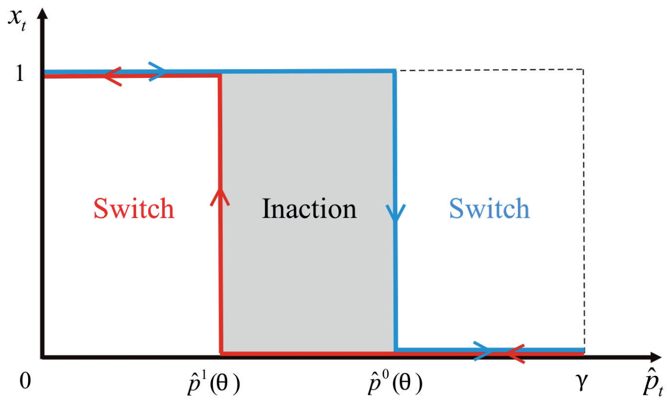

Figure 2.8: Decision hysteresis for a type-θ agent, with inaction region $\mathscr{P}(\theta) = [\hat{p}^1(\theta), \hat{p}^0(\theta)]$.

For effective transaction prices between the price thresholds, i.e., when

$$\hat{p} \in \mathscr{P}(\theta) \triangleq [\hat{p}^1(\theta), \hat{p}^0(\theta)], \tag{2.26}$$

a type-θ agent remains inactive. Even though the price in the sharing market might not reflect exactly his optimal sharing state, the conversion costs prevent the agent from taking action. Similar to the behavior of the size of the invariance domain $\mathscr{R}$ in Eq. (2.21), the width of the agent's inaction region $\mathscr{P}(\theta)$ in the price space,[24]

$$|\mathscr{P}(\theta)| = \hat{p}^0(\theta) - \hat{p}^1(\theta) = \min\{\hat{\gamma}, (1+r)(c_{01}+c_{10})\}, \tag{2.27}$$

increases in the conversion costs and the discount rate; it is independent of the agent's type (i.e., his probability of need θ). Yet, it is important to note that while the inaction region $\mathscr{P}(\theta)$ does not change its size, it shifts upwards with increasing θ, at the rate Δ.

The sharing economy attains its steady state once $\hat{p}_t \in [\hat{p}^0(\vartheta^1), \hat{p}^1(\vartheta^0)]$. Then the effective transaction price is bracketed by the critical prices relevant for the types that define the invariance thresholds of the sharing economy in Eqs. (2.14) and (2.15), respectively. In particular, for all agent types $\theta \in [0, \vartheta^0)$ the effective transaction price is higher than their sharing thresholds, i.e., all the lower-type agents share. Conversely, for all agent types $\theta \in (\vartheta^1, 1]$ the effective price is less than their keeping thresholds, i.e., all higher-type agents keep. Intermediate agent types in the invariance region $\mathscr{R}$ are in their respective inaction regions; see Fig. 2.9.

2.5 Examples

To illustrate the results, the equilibrium model of the sharing economy is implemented numerically. The assumed conversion-cost vector $(c_{01}, c_{10}) = (0.01, 0.05)$ reflects the empirical regularity that usually the cost of preparing the item to be shared is greater than the cost of converting it back

[24] Attention is restricted to the interesting case where both thresholds are interior, i.e., in the interval $(0, 1)$.

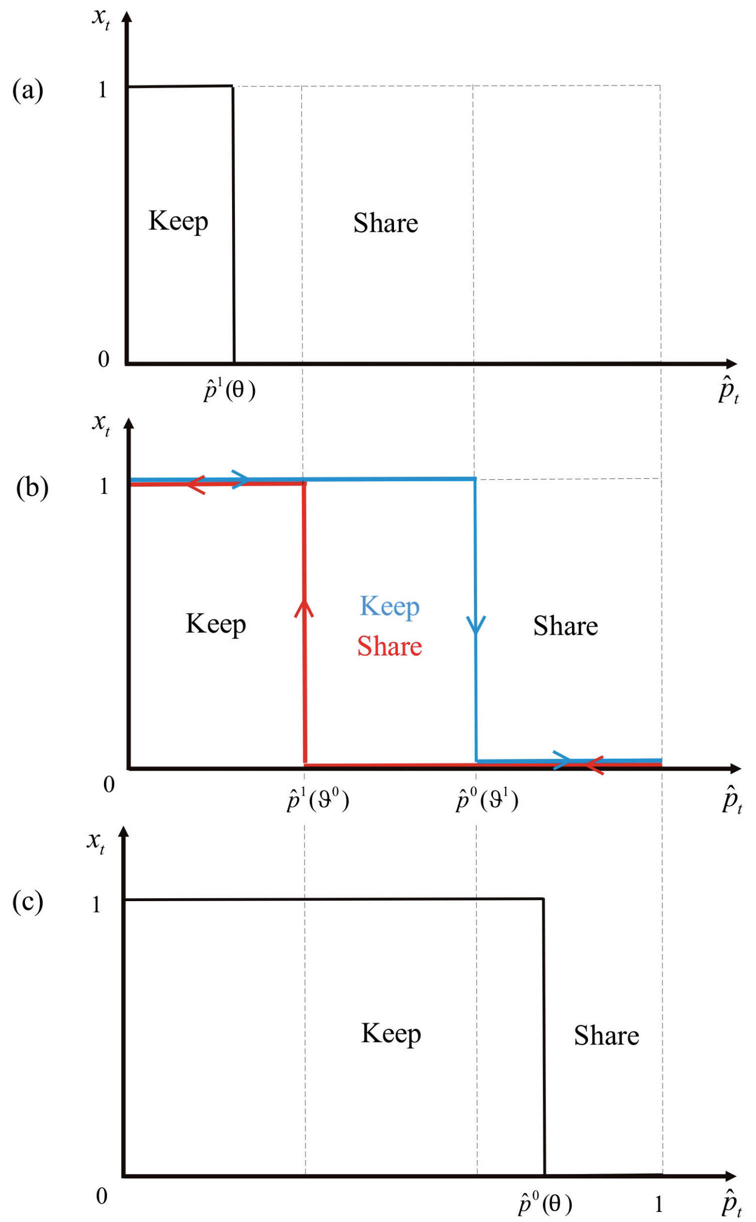

Figure 2.9: Agents' threshold decisions: (a) for $\theta < \vartheta^0$; (b) for $\vartheta^0 \leq \theta \leq \vartheta^1$; (c) for $\theta > \vartheta^1$.

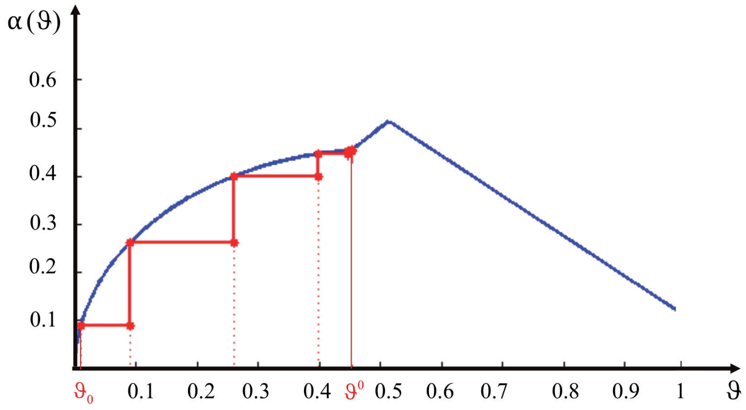

Figure 2.10: Price dynamics starting from the initial condition $\vartheta_0 = 0.01$.

to one's personal assets. For a unit demand-elasticity parameter ($\gamma = 1$) and in the absence of a sharing intermediary ($\rho = 0$), a per-period interest of $r = 20\%$ (corresponding to $\delta = 0.83$),[25] and utilities $(u_0, u_1) = (-0.6, 0.6)$ (corresponding to $\Delta = 1.2$), Eqs. (2.14), (2.15), and (2.19) yield the critical sharing thresholds,

$$(\vartheta^0, \vartheta^1, \vartheta^2) = (0.450, 0.482, 0.522),$$

as discussed in Sec. 2.3.1. Depending on the initial fraction of sharers ($\vartheta_0 \in (0, 1]$), market participation will eventually attain a fraction of sharers in the invariance region $\mathscr{R} = [45.0\%, 48.2\%]$. Fig. 2.10 depicts the law of motion (based on Prop. 2.1) and a particular trajectory $(\vartheta_t)_{t=0}^{\infty}$ of the sharing threshold for the initial value $\vartheta_0 = 0.01$, i.e., when the economy starts with $\vartheta_0 = 1\%$ of sharers.

We now examine the model behavior for two archetypical scenarios, "undersharing," when the economy starts below the invariance region (i.e., when $\vartheta_0 < \vartheta^0$), and "oversharing," when the economy begins with a fraction of sharers above the invariance region (i.e., when $\vartheta_0 > \vartheta^1$).

2.5.1 Undersharing

Suppose that the sharing economy initially starts with only 1% of the population actively sharing. Fig. 2.11 shows the resulting incremental diffusion. As the sharing supply increases, the market price drops to about half of its initial level, whereas the effective transaction price monotonically increases to about five times its initial level. This great discrepancy is due to the strong variation of the transaction probability, from around 10% initially to close to 100% near the steady state. Despite the very small number of initial sharers, the lag in the price-formation process (see

[25] This is consistent with a 6-month interest rate in a high-risk environment.

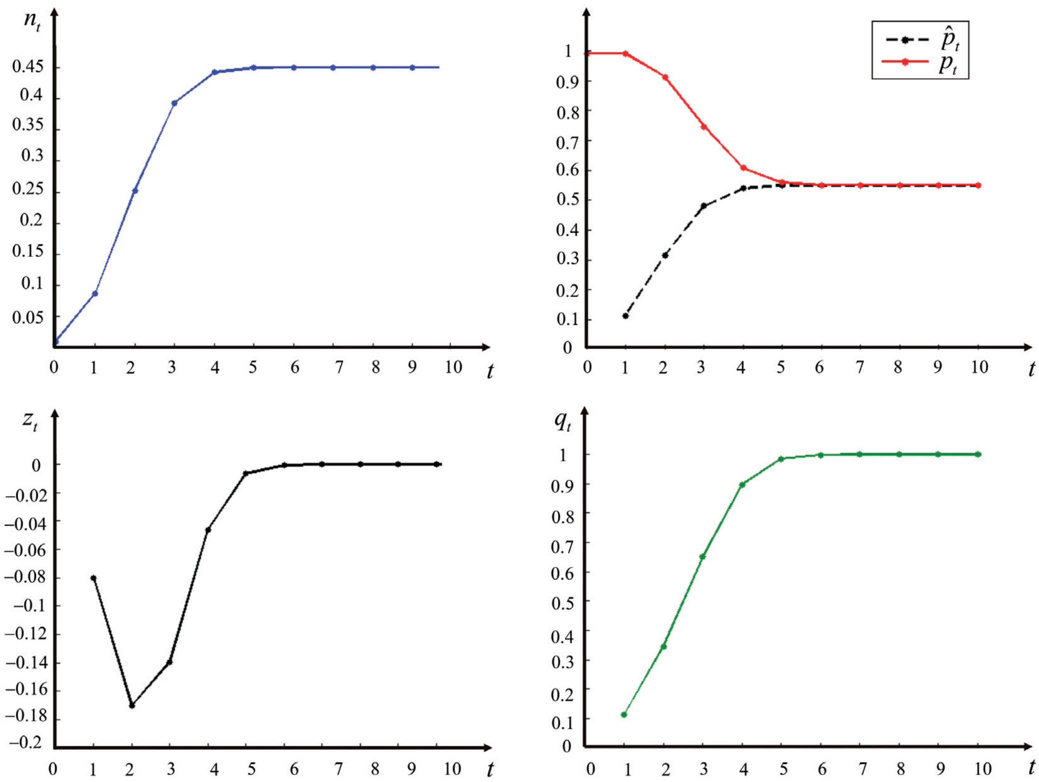

Figure 2.11: Adjustment dynamics starting from the initial condition $\vartheta_0 = 0.01$.

Eq. (2.3)) causes the price to be too high for all the suppliers to be matched, so that only about 1/10 of the initial population of sharers actually transact, while 9/10 of them incur the disutility of having converted the asset to a sharing state which makes private use difficult. In the long run, supply is matched with demand, and by Eq. (2.23) participation in the sharing market approaches the steady-state level of $\bar{\vartheta} = \vartheta^0 \approx 45.0\%$. The transient excess supply (minus the excess demand in Eq. (2.7)) is positive but nonmonotonic. Overall, the diffusion of sharing exhibits an S-shaped growth pattern, driven mainly by the (quasi-)concavity of the law of motion (by Prop. 2.3 and Corollary 2.1). The latter features a maximum-diffusion sharing threshold, $\vartheta^\mu \approx 13.0\%$.[26] Correspondingly, the maximum-diffusion time (when the sharing economy grows fastest) is by Eq. (2.20) at $t^\mu = 2$, i.e., after about 1 year (equal to 2 six-month periods).

2.5.2 Oversharing

Fig. 2.12 depicts the dynamics of an oversharing economy, where $\vartheta_0 = 80\% \in (\vartheta^2, 1]$. The sharing supply declines immediately, where in the first period approximately 75% of the initial sharers leave the market. It is then followed by an incremental diffusion; by Eq. (2.23) participation in the sharing market approaches the steady-state level of $\bar{\vartheta} = \vartheta^0 = 45.0\%$. Following the massive

[26] An S-shaped growth generally obtains for initial values ϑ_0 below the maximum-diffusion threshold ϑ^μ.

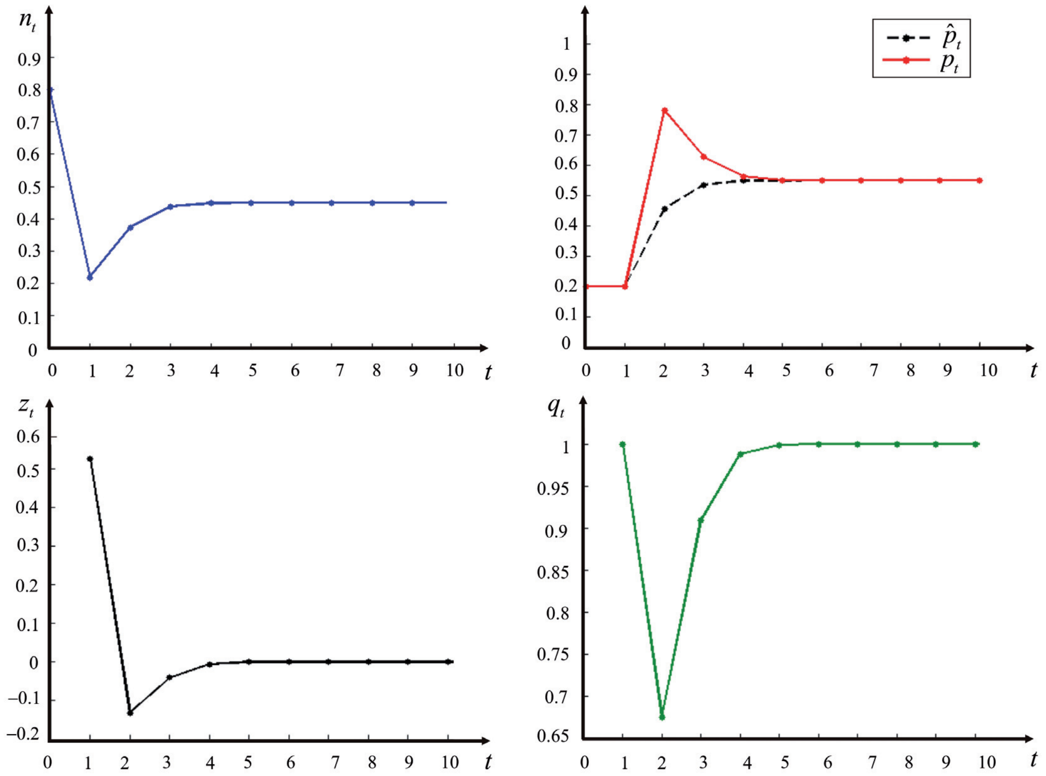

Figure 2.12: Adjustment dynamics starting from the initial condition $\vartheta_0 = 0.8$.

decline of the sharing economy in the first period, the transaction price increases to about 4 times its initial value. It then decreases monotonically towards its steady-state level, which amounts to more than twice the initial price.

The effective transaction price starts at the same initial value as the actual market price, but monotonically increases towards its steady-state level. The transients depend on the substantial variations of the transaction probability. In the first period, where the transaction price is low, the transaction probability for suppliers equals 1. Following the sudden price increase in the sharing market, the transaction probability falls below 70%, where approximately 1/3 of the sharers do not get the chance to actually transact. The transaction probability increases monotonically afterwards and approaches 100% again in the long run. Due to the initial oversharing, there is a positive excess demand in the first period. From $t = 2$ onwards, the transient excess supply (minus the excess demand in Eq. (2.7)) becomes positive and decreases monotonically towards 0. The monotonicity follows from the fact that the minimum amount of supply (at $t = 1$) corresponds to about 20% market participation, which is still greater than the maximum-diffusion sharing threshold, $\vartheta^{\mu} \approx 13.0\%$. Correspondingly, by Eq. (2.20) the maximum-diffusion time is $t^{\mu} = 2$, i.e., the fastest growth of the sharing economy takes place immediately after its sharp decline, resulting in marked volatility.

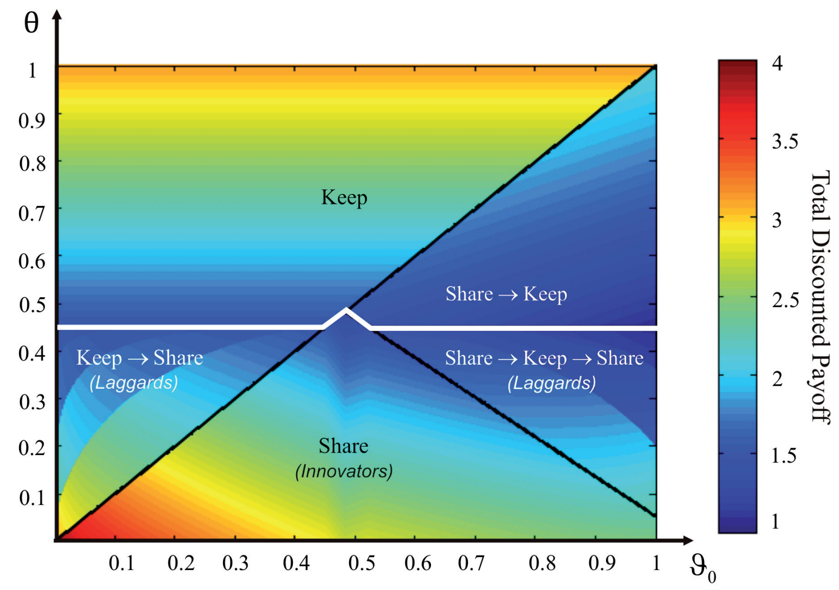

Figure 2.13: Equilibrium value $\hat{V}(\theta|\vartheta_0)$ for all $(\theta, \vartheta_0) \in \Theta \times \Theta$.

2.5.3 Agents' Payoffs

The discounted payoff (value) for a type-θ agent can be computed, based on his externality- and initial-condition-induced switching behavior, depending on the initial sharing state x_0. The latter is fully determined by θ and the initial value ϑ_0, resulting in five possible lifetime switching patterns (Fig. 2.13).

(i) *Initial keepers: $x_0 = 1$, i.e., $\theta > \vartheta_0$.* For types greater than the lower invariance threshold ϑ^0, keeping is always best; otherwise there will be one switch to sharing at the type-dependent sharing-switch time, after which an agent stays a sharer for the future,

$$\bar{t}(\theta) = \sup\{t \in \mathbb{N} : \theta > \vartheta_t\}, \tag{2.28}$$

where we set, as is customary, $\sup \emptyset \triangleq \infty$. Thus,

$$V(\theta|\vartheta_0) = \begin{cases} \sum_{t=0}^{\infty} \delta^t \theta u_1, & \text{if } \max\{\vartheta_0, \vartheta^0\} < \theta \leq 1, \\ \sum_{t=0}^{\bar{t}(\theta)} \delta^t \theta u_1 - \delta^{\bar{t}(\theta)} c_{10} + \sum_{t=\bar{t}(\theta)+1}^{\infty} \delta^t (\hat{p}_t + \theta u_0), & \text{if } \vartheta_0 < \theta \leq \vartheta^0, \end{cases}$$

where $\hat{p}_0 \triangleq (1-\rho)\, p(n_0) = \hat{\gamma}(1-\vartheta_0)$ (with $n_0 = \vartheta_0$).

(ii) *Initial sharers: $x_0 = 0$, i.e., $\theta \leq \vartheta_0$.* The situation is more intricate when agents start out as sharers. Types above the lower invariance threshold ϑ^0 switch to own use immediately and never switch back, whereas types below $\alpha_1(\vartheta_0)$ will always participate in the sharing market. Lastly, an intermediate type θ first switches to keeping and then, at the sharing-switch time $\bar{t}(\theta)$

in Eq. (2.28), back to sharing, so

$$\hat{V}(\theta|\vartheta_0) = \begin{cases} \sum_{t=0}^{\infty} \delta^t (\hat{p}_t + \theta u_0), & \text{if } 0 \le \theta \le \alpha_1(\vartheta_0), \\ \hat{p}_0 + \theta u_0 - c_{01} + \sum_{t=1}^{\bar{t}(\theta)} \delta^t \theta u_1 - \delta^{\bar{t}(\theta)} c_{10} + \sum_{t=\bar{t}(\theta)+1}^{\infty} \delta^t (\hat{p}_t + \theta u_0), & \text{if } \alpha_1(\vartheta_0) < \theta \le \vartheta^0, \\ \hat{p}_0 + \theta u_0 - c_{01} + \sum_{t=1}^{\infty} \delta^t \theta u_1, & \text{if } \vartheta^0 < \theta \le 1. \end{cases}$$

It is remarkable that the value function on the equilibrium path,

$$\hat{V}(\theta|\vartheta_0) = \bar{g}(\hat{\pi}(\theta,\vartheta_0), \xi(\theta,\vartheta_0), p(\vartheta_0)|\theta) + \delta \hat{V}(\theta, \alpha(\vartheta_0), \vartheta_0),$$

as a solution to the system of Bellman equations (2.12) and the transversal equilibrium condition (2.13) for all θ, ϑ_0 in Θ, exhibits generic discontinuities at the boundaries of the switching regions. In standard optimization problems, by the Berge maximum theorem (Berge 1963, p. 116), the continuity of the objective functions in parameters implies the continuity of the value function. The discontinuities here are driven by the fact that $\hat{V}$ depends not only on the agent's own type θ but also on the aggregate switching behavior in the market. The latter externality is encapsulated by the sharing threshold ϑ_t, i.e., the number of sharers in the market at any given time t.

Fig. 2.13 depicts the value function as a function of (θ, ϑ_0). The black lines partition the space according to the aggregate switching behavior of the agents, as in Fig. 2.7. For agents above the white line, who always prefer keeping to sharing, the total discounted payoffs are increasing in their respective types, whereas for the eventual sharers below the white line, the payoffs are decreasing in their respective types. For a given initial value, the *innovators* of the sharing economy are the agents of types $\theta \in (0, \min\{\vartheta_0, \vartheta_1\})$; they are present on the sharing market from the very beginning, never switch to keeping, and tend to gain the highest discounted payoffs, which may amount to a value up to 4 times greater than the discounted payoffs for the *laggards*, i.e., agents who prefer to share but would rather delay participating in the market as much as possible. Each agent's decision affects other agents' payoffs, which in general creates payoff-discontinuities in the (θ, ϑ_0)-space, driven by the fulfilled-expectations character of the equilibrium.

2.6 Robustness

The empirical relevance of the model depends on the sensitivity of its properties on the key assumptions. In particular, the Nash equilibrium and the characteristic S-curve diffusion pattern of sharing depend on the validity of La. 2.1 and Prop. 2.3 because the latter guarantee the transversal equilibrium condition and the unimodality of the growth increments in the sharing supply, respectively. We now show that these results are robust with respect to relaxation of the assumptions about the price formation.[27] Thus, the main results exhibit structural stability, as they continue to hold away from the nominal model assumptions which were used to obtain closed-

[27] La. 2.1 implies that Eqs. (2.17) and (2.18) continue to hold, ensuring that the law of motion in Eq. (2.16) provided by Prop. 2.1 (without the explicit expressions for α_0, α_1) and the subgame-perfect Nash equilibrium in Prop. 2.2 of $\mathscr{G}(\vartheta_0)$ remain valid.

form expressions of the transitory sharing equilibrium. Specifically, the qualitative behavior of the model remains unchanged when demand is nonlinear but sufficiently elastic and/or prices can adjust at a different time scale. In Sec. A.3, we discuss how to incorporate an expected per-period servicing cost for remaining in the sharing market into the model without materially affecting the results.

2.6.1 Nonlinear Demand

The first model relaxation to consider is that instead of the linear (inverse) demand $p(\vartheta)$ in Eq. (2.1) we allow for any downward-sloping demand curve $D(p)$ (i.e., any downward-sloping inverse demand $p(\vartheta)$) which is elastic at the lower invariance threshold ϑ^0; for such a nonlinear demand,[28] the lower and upper invariance thresholds are (uniquely) defined in terms of fixed points, as shown below.

Lemma 2.6. *Let $y \in \mathbb{R}$. There exists a unique $\varphi = \varphi(y)$ such that $\varphi = (p(\varphi) + y)/\Delta$.*

As for the results in the main text, all proofs are provided in App. B. By La. 2.6, the fixed-point mapping $\varphi(\cdot)$ is well-defined as a single-valued function. For nonlinear demand, the lower and upper invariance thresholds in Eqs. (2.14) and (2.15) become

$$\vartheta^0 \triangleq \varphi(-rc_{10}) \tag{2.14'}$$

and

$$\vartheta^1 \triangleq \varphi(c_{10} + (1+r)c_{01}), \tag{2.15'}$$

respectively. With this, we can formalize the aforementioned demand-elasticity condition,

$$\varepsilon(\vartheta) = -\frac{p(\vartheta)}{\vartheta} \cdot \frac{\partial D(p)}{\partial p} = -\frac{p(\vartheta)}{\vartheta^0 p'(\vartheta)} \geq 1, \quad \vartheta \in (0, \vartheta^0). \tag{A}$$

Requirement (A) is key to extending the results establishing the subgame-perfect Nash equilibrium in Sec. 2.3.1 to nonlinear demand functions.

Lemma 2.7. *Provided the demand-elasticity condition (A) holds, the conclusion of La. 2.1 remains valid, i.e., for any $t \geq 0$, the next-period sharing threshold $\vartheta' = \alpha(\vartheta_t)$ is such that $\hat{\pi}(\vartheta', \vartheta') = 0$.*

Persistent growth in the sharing economy is a robust result given any (downward-sloping) demand which is elastic in an undersharing economy.

[28] If the minimum possible payoff of joining the market exceeds the owner's utility difference, i.e., $p(1) \geq \Delta$, then $\vartheta^0 = 1$ and all agents are willing to share in steady state (regardless of the need state) such that full sharing penetration is attained in finite time. As stipulated by Eq. (2.2), we concentrate on the interesting case where $p(1) \leq \Delta$.

Remark 2.8. Important classes of demand specifications satisfy the elasticity condition (A), as illustrated by the following three representatives:[29]

- *constant-elasticity demand:* $p(\vartheta) = \gamma\vartheta^{-1/\eta}$, for $\eta \geq 1$ and $\gamma > 0$;
- *semi-logarithmic demand:* $p(\vartheta) = \gamma_0 - \gamma_1 \ln(\vartheta)$, for $\gamma_0 \geq \gamma_1 > 0$;
- *quasi-affine demand:* $p(\vartheta) = \gamma_0 - \gamma_1\vartheta^{1/\eta}$, for $\eta \geq (\gamma_1/\gamma_0)/(1-(\gamma_1/\gamma_0))$ and $\gamma_0 > \gamma_1 > 0$.

It is easy to check that all of the preceding demand curves are globally elastic, i.e., such that $\varepsilon(\vartheta) \geq 1$, and the relevant inequalities in Eq. (2.2) can readily be satisfied. Note also that these classes of demand specifications have been widely used in practice; see, e.g., Bulow and Pfleiderer (1983).

Remark 2.9. The elasticity requirement in Eq. (A) means that a sharing equilibrium is attained in an elastic part of the demand curve. This is reminiscent of the monopoly pricing rule (Tirole 1988, p. 66) that a monopolist's relative markup must be equal to the inverse elasticity (or Lerner index), which implies that the demand elasticity must exceed 1 at the optimal monopoly price. This also means that important for the results is not the linearity of demand but the fact that demand is elastic, i.e., responsive to price movements, when only small quantities are available on the sharing market.

The following result establishes that for these three fairly important classes of demand curves, the diffusion speed in the sharing economy is unimodal (i.e., quasi-concave) in the penetration. Fig. 2.14 provides simulation results,[30]

Lemma 2.8. *For constant-elasticity, semi-logarithmic, and quasi-affine demand, as specified in Remark 2.8, the conclusion of Prop. 2.3 remains valid, i.e., there is a unique "maximum-diffusion" (sharing) threshold ϑ^μ in the interval $(0,\vartheta^0)$, at which the growth of the sharing economy is maximal.*

The finding also obtains for any inverse demand which is concave.

2.6.2 Change of the Time Scale for Price Adjustments

The second model relaxation concerns the intertemporal price-adjustment process in Eq. (2.3), where we allow for any positive price-adjustment time scale,

$$dt \equiv t_{k+1} - t_k > 0, \qquad \text{(B)}$$

for $k \in \{0,1,2,\ldots\}$, instead of the normalized interval length with $t_k \equiv k$ and $dt = 1$ in the main text. Without loss of generality, we can restrict attention to a *decrease* of the price-adjustment

[29]Without loss of generality, set $\rho = 0$; for $\rho \in (0,1)$, it is sufficient to replace $(\gamma,\gamma_0,\gamma_1)$ by $(1-\rho)(\gamma,\gamma_0,\gamma_1)$.
[30]$c_{01} = c_{10}/5 = 0.01$, $\Delta = 1.2$, $r = 20\%$ (as in the main text); $\gamma_0 = 1.2$, $\gamma_1 = 1$, and $\eta = 8$.

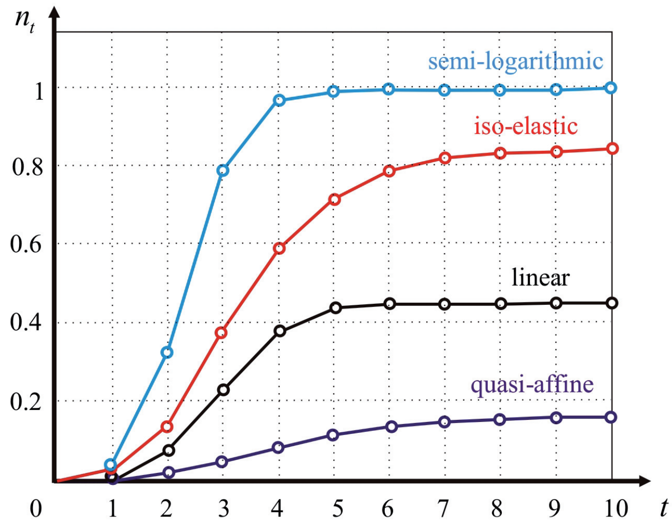

Figure 2.14: Sharing-diffusion S-curves for standard nonlinear demand specifications, compared to an S-curve generated by the nominal model with linear demand.

time scale because the unit of the original time scale is arbitrary. By reducing dt, the per-period rate interest also drops. Intuitively, as dt decreases, adjustments take place more frequently, but the variation of size of these increments is not so easy to guess. To understand the change in the intertemporal model behavior in response to faster price updates, one needs to assess the effect of the time-scale compression on the system function *and* on the invariance threshold.

Lemma 2.9. *By compressing the price-adjustment time scale in (B) to $dt = \lambda \in (0,1)$, the growth arc of the system function becomes $\hat{\alpha}_0(\vartheta)$ and the lower invariance threshold becomes $\hat{\vartheta}^0$ which are such that*

$$0 < \hat{\alpha}_0(\vartheta) - \alpha_0(\vartheta) < \left(r - \frac{(1+r)^{\lambda} - 1}{\lambda} \right) \frac{c_{10}}{\Delta} < (r - \ln(1+r)) \frac{c_{10}}{\Delta},$$

for all $\vartheta \in (0, \vartheta^0)$, and

$$0 < \hat{\vartheta}^0 - \vartheta^0 = \left(r - \frac{(1+r)^{\lambda} - 1}{\lambda} \right) \frac{c_{10}}{\hat{\gamma} + \Delta} < (r - \ln(1+r)) \frac{c_{10}}{\hat{\gamma} + \Delta}.$$

An important implication of the preceding result is that a faster time scale for price adjustment does *not* imply that adjustments in the economy become smaller or that they disappear altogether. While the faster adjustments tend to become larger, the asymptotic limits in the sharing market also become larger. In other words, it is true that the adjustment steps are larger (but always finite, bounded uniformly) and take place more frequently, but also the distance from a given initial value ϑ_0 (below $\mathscr{R}$) to the time-compressed invariance threshold $\hat{\vartheta}^0$ is larger than the original adjustment distance from ϑ_0 to ϑ^0. The latter effect somewhat balances out the former effect. This remains true in the limit, for $\lambda \to 0^+$. Thus, even with close-to-infinite adjustment speed, the economy remains in perpetual disequilibrium, so that the effects discussed in this paper for $dt = 1$ are robust with respect to the choice of the length of the price-adjustment period. The latter may be subject to the effects of technological change which may, possibly with the aid of an intermediary, improve the price discovery on the sharing market.

Remark 2.10. The upper arc $\alpha_1(\cdot)$ of the system function becomes equal to 1 when the time-scale compression factor λ is sufficiently small. This also applies to the invariance threshold ϑ^1. Thus, in a regime with rapid price adjustments a sharing economy tends to either grow or stagnate; technology improvements which lead to faster price adjustments therefore bypass negative supply shocks and the concomitant (deterministic) price volatility.

2.6.3 Servicing Cost

Suppose that in addition to the conversion cost c_{10}, there is also a per-period servicing cost $\kappa \geq 0$ associated with a continued presence in the sharing market as a supplier, which includes expected outlays for cleaning, maintenance, repair, and transportation. Transportation costs may include sending the item through mail services, or bringing it personally to an agreed place of exchange. In this context, Caillaud and Jullien (2003) showed that in intermediated markets, such costs are usually borne by the supplier, similar to the intermediary's commission rate. Note that an agent incurs the per-period servicing cost only if a transaction takes place. Thus, κ changes the effective transaction price seen from the perspective of any supplier. Incorporating the fixed cost in the model, the effective transaction price, including the effect of intermediation, becomes

$$\hat{p}(\vartheta_t, \vartheta_{t-1}) = \left(\hat{\gamma} - \kappa - \hat{\gamma}\vartheta_t\right) q(\vartheta_t, \vartheta_{t-1}).$$

Thus, the results in the main part of the paper are unaffected, in particular the shape of the market growth curve.

Lemma 2.10. *For all $\kappa \geq 0$, the conclusion of Prop. 2.3 remains valid, i.e., the sharing economy exhibits an S-shaped diffusion pattern.*

The main effect of the servicing cost is to reduce the intermediated demand-elasticity parameter $\hat{\gamma}$, which also determines the maximal payoff from a sharing transaction for a potential supplier.

Remark 2.11. As in the base case without servicing cost, the lower invariance threshold,

$$\vartheta_0 = \frac{\hat{\gamma} - \kappa - rc_{10}}{\hat{\gamma} + \Delta},$$

is located in the elastic part of the demand curve.

For a sharing market to be viable, the lower invariance threshold ϑ^0 must be positive, so $\gamma > (\kappa + rc_{10})/(1-\rho)$. This limits the sharing intermediary's commission to be strictly less than $\bar{\rho} \triangleq 1 - (\kappa + rc_{10})/\gamma$. For this commission to be positive, necessarily $\gamma > \kappa + rc_{10}$, i.e., the per-period value (as measured by the quality parameter γ) must exceed the per-period "total servicing cost," which includes all transaction and maintenance costs *and* the cost of servicing the capital needed for converting the item to its sharing state.

2.7 Conclusion

The infinite-horizon equilibrium model for the diffusion of sharing in an economy with heterogeneous agents and intermediary introduced in this paper is the first to analyze transient sharing-market dynamics in the presence of frictions. It makes three main contributions. First, the closed-form expressions of the dynamic market growth path (as a unique subgame-perfect Nash equilibrium in Prop. 2.1 and 2.2) provide structural insights into the behavior of agents and the dependence of this behavior on key parameters. For example, as the price elasticity of demand increases,[31] the overall participation in the sharing market decreases, whereas an increase in the discount rate tends to have the opposite effect (see Prop. 2.4). The participation in the sharing market also decreases in the intermediary's commission rate. The complex dynamics captured in the equilibrium path feature generally nonmonotonic growth behavior. Second, the model allows for a persistent disequilibrium between supply and demand, which manifests itself usually in the form of excess supply, driven by a realistic lag of price-adjustments in the sharing market. The supply-demand mismatch produces a phenomenon also observed in finite-horizon models of sharing markets (Weber 2015). Agents' incentives to incur costs when switching the status of their possessions (from keeping to sharing or vice-versa) depend on the *effective* transaction price, as the product of transaction probability and intermediated market price. A positive diffusion of sharing is driven by an increase in the effective transaction price (see La. 2.4). Third, the model generates S-shaped diffusion patterns endogenously, using natural primitives.[32] The characteristic change in growth rates is obtained via the (quasi-)concavity of the system function, which is a robust model property that does not depend on particular parameter values. It continues to hold for nonlinear demand specifications and is qualitatively insensitive with respect to changes in the speed of price adjustments (as shown in Sec. 2.6). Thus, whenever the economy starts with an initial number of sharers below a "maximum-diffusion threshold" (see Prop. 2.3), the expansion path of the sharing economy is S-shaped.

From a managerial (and possibly regulatory) viewpoint, the model clearly indicates that while an intermediary may enable sharing transactions through its provision of a platform, including trust, transaction, and matching technologies, it also slows down the diffusion of sharing in two ways: firstly, the intermediary's commission decreases the speed of adjustment (as determined by the law of motion in Prop. 2.1); secondly, it also decreases the maximum level of sharing in the economy (as determined by the relevant invariance threshold and by Prop. 2.4). This echoes earlier findings for search intermediaries, who pass on efficiency improvements to market participants—only in much reduced form—by creating an endogenous obfuscation, knowingly

[31] This corresponds to a decrease in the demand-elasticity parameter $\hat{\gamma}$; see footnote 21.

[32] This is in contrast to the extremely successful diffusion model by Bass (1969), where the right-hand side of a differential equation of the change of an installed base is essentially pre-assumed to be quadratic, resulting in an S-shaped growth curve.

displaying suboptimal search results to increase the number of clicks and thus their own revenues (Weber and Zheng 2007). Similarly, by increasing the commission rate, sharing becomes more difficult and evolves more slowly. This is also reflected in the model by Einav et al. (2015) where, in a P2P market with free entry, the intermediary is the only one to come away with a positive surplus. Finally, our model implies a simple viability condition for sharing markets: the per-period value created by a sharing transaction minus the intermediary's take must exceed the "total servicing cost," which includes all expected transaction and maintenance expenses in addition to the current interest on the capital required to convert the item to a sharing state.

Regarding limitations, the model rests on a number of simplifying assumptions. Firstly, all agents are assumed to be risk-neutral and all items on the sharing market are homogeneous. A model of a dynamic sharing economy with differentiated goods and/or risk-averse agents is left for future research. Secondly, the diffusion dynamics represent a supply-side response to a stationary demand function, and it reflects the partial-equilibrium notion that the demand function is not in itself affected by supply. Relaxing the stationarity assumption would pose challenges to being able to derive a closed-form solution, since the law of motion would need to carry an explicit time dependence. Thus, the behavior of the sharing economy over time would reflect the imposed time dynamics. This raises the question of how time-dependencies are transmitted through the economy, for instance in terms of the impulse response of a linearized version of the model to unit demand shocks. This type of analysis is common for dynamic general equilibrium models, which are notoriously difficult to solve, even computationally. Lastly, the full universe of available buyer and seller decisions was assumed to be extremely simple. In each period sellers could decide to enter or exit the sharing market, and buyers could decide whether to rent or not. Realistically, some buyers may decide to become owners, thus increasing the potential supply, and decreasing demand. This in turn would trigger a dynamic and strategic optimization by retailers of the prices at which goods are sold, which can be expected to give rise to complex dynamics, the exact nature of which will depend on the particular assumptions made. We leave it for future research to tackle the purchase/sale decision as well as the dynamic price optimization and possible short-term capacity planning by intermediaries to match supply with demand; these deserve their own theoretical investigations.

3 Strategic Durability in the Presence of Sharing Markets

3.1 Introduction

Sharing markets are becoming increasingly attractive to individuals. Peer-to-peer asset sharing now includes a vast variety of products such as vehicles, tools, computers, electronics, home appliances, leisure equipment, and clothing. Thanks to such markets, consumers are now enabled to realize economic gains from their underutilized resources, and certain consumer segments have begun to forgo ownership for the sake of access-based consumption.

Peer-to-peer markets are (at least partially) competing with conventional sellers, and they are becoming big and disruptive enough for the incumbent companies to strategically react to their presence. Although previous studies have attempted to capture the effect of sharing on product sales and purchase price (Weber 2015, 2016; Jiang and Tian 2015), to the best of our knowledge, the effectiveness of planned durability as a manufacturer's strategy in the presence of sharing markets has remained unexplored. Planned durability aims at engineering the failure or expiry of a product. The strategic obsolescence can result in an increase in the number of repeating purchases, and generate more revenue for the manufacturer. Moreover, low quality and non-durable products may discourage peer-to-peer sharing activities. Hence, manufactures' long-term sales and profits heavily depend on the business decisions that determine durability. In this paper, we study how manufacturers may plan durability to exploit or mitigate the effect of sharing markets on their profitability.

Durability can be embedded in the product design. Intentional use of key components that are less durable shortens the life time of the product and intrinsically reduces durability. Nevertheless, obsolescence is not necessarily a natural characteristic of the product, and can also be achieved extrinsically. Supplying complementary products that are not compatible with the older models, software upgrades that do not support former hardware versions, stopping after-sales services, and not replacing damaged or lost parts of the product after a certain time are examples of such action plans.

Product durability is an important purchase criterion for the consumers, as well. From a purchaser's perspective, acquiring a durable good resembles an investment in a bundle of present and future consumptions (Orbach, 2004). When a sharing market is present, product durability not only partially determines the amount of future sales-volume, but also the future supply on the sharing market. In this respect, when investing in a purchase of a durable good, consumers also take potential future payoffs (from usage or participation in the market) into account.

The impact of the peer-to-peer activities on the strategic durability design is ambiguous. On the one hand, manufacturers may be more prone to provide durability, as it allows them to capitalize on the "shareability" of the product and increase the prices. On the other hand, they may find it optimal to reduce durability, disincentivize sharing, and increase the revenue thanks to repeating purchases.

To provide insights on this conundrum, we study the equilibrium behavior of an overlapping generations in the presence of sharing markets. The model allows for the coexistence of heterogeneous individuals in different phases of their lives, where they can engage in peer-to-peer transactions and exchange temporary usage rights. We analyze how the strategic deployment of planned durability in the presence of sharing markets can significantly affect the market structure and consumer behavior.

By manipulating the durability, the firm can fully deactivate the sharing market, if desired. By offering non-durable options, the supply on the sharing market vanishes, and the only way of consumption is throughout the purchase. We show that this is an optimal strategy when the production cost is low, and the consumers are myopic. Nevertheless, when sharing shut-down is not optimal to pursue, peer-to-peer markets have a positive effect on durability. Compared to a benchmark case, where there is no sharing, manufacturers allow the products to be more durable in the presence of such markets. We show that the cost threshold, at which the firm shifts the business strategy from sharing shut-down to strategic coexistence varies with her ability to commit to his actions. Manufacturers that are highly committed to their price and product design, the transition to sharing at a lower cost and are more profitable than those who have less commitment power.

3.1.1 Literature Review

Recent studies have examined the sharing markets from an economic perspective. Bardhi and Eckhardt (2012) identified market-mediation as one of the main dimensions of access-based consumption and highlighted the economic motives of sharing transactions in the context of car-sharing. Weber (2014) showed that intermediation could solve the moral hazard problem of sharing markets. Fraiberger and Sundararajan (2015) used a calibrated dynamic programming model to show the shifts from car ownership to access-based consumption for below-median income consumers. Zervas et al. (2013) found that the presence of AirBnB has negatively effected the local hotels' revenues. Other studies have aimed to model and quantify the impacts of sharing markets on consumer behavior, ownership incentives, and sales volume. Razeghian and Weber (2015) provided a dynamic model to describe the intertemporal behavior of sharing suppliers. They showed that the long-term penetration of sharing decreases in transactions costs. Horton and Zeckhauser (2016) also confirmed this result and showed that in the presence of bring-to-market costs, ownership incentives increase, and the quantity of peer-to-peer transaction decreases. Benjaafar et al. (2015) studied the ownership decisions in the presence of matching frictions when the rental price is exogenously determined by the intermediary. They showed that the rental price determines whether the sales volume increases or decrease, compared to the conventional case without sharing.

This paper builds on the overlapping-generations model by Weber (2016) which shows that an active sharing market tends to increase the purchase price by positive sharing premium. However, there the focus is solely on pricing, and the product design remains entirely exogenous. Jiang and Tian (2016) study a manufacturer's pricing and quality decisions in the presence of a sharing market using a two-period model. In that setting, the sharing market has a positive effect on the firm's price and product quality, and the manufacturer enjoys a positive added profit in the presence of sharing. Our results differ in two ways: first, controlled durability (or induced obsolescence) is different from product quality as it includes product failure, thus potentially preventing future use for any consumer generation. Second, we obtain the critical role of durability as a strategic tool to choke off the sharing market, which has not been examined in the literature before. The firm's possible desire to shut down sharing means in fact that its profits without sharing may well be higher than with sharing, in contrast to the earlier findings about product quality.

This study also contributes to the extant literature on the economics of durable goods, which has been of particular interest to researchers for decades. An early stream of the literature suggested that a profit-maximizing monopolist is only interested in cost minimization and will not be motivated to decrease the durability (Swan 1971, Swan 1972). Follow-up studies suggested that this conclusion is not necessarily valid and providing full durability is not an optimal decision for manufacturers. A *durapolist* cannot exert full-market power, despite the monopolistic position, and they may be willing to employ business practices to prevail over the challenges they face (Orbach 2004). Coase (1972) conjectured that, as the market gets exhausted, a monopolist of durable-goods is in constant competition with its future self and ends up with a price equal to its marginal cost. Bulow (1986) demonstrated that the time-consistency issue provides the monopolists with second incentives to lower durability and reduce future secondary market competition. In this regard, Fudenberg and Tirole (1998) and Chen et al. (2013) among others have studied the firms pricing and durability decisions when owners can resell their item, and non-owner can decide to buy a second-hand item.[1]

Although a sharing market is effectively a secondary market, it has specific characteristics of its own. First, in a sharing transaction, the ownership rights are not transferred. As opposed to

[1] Waldman (2003) and Swan (2006) have provided exhaustive reviews on a variety of issues related to durability.

the traditional secondary markets, the transaction is not a one-time settlement, and the sharing decision is contingent on the suppliers need state and is recurring. Moreover, conventionally, the literature assumes that the goods available on the secondary market are vertically inferior to those on the secondary market. However, this is not necessarily the case in the sharing activities. The quality may or may not be inferior, and there are social and environmental benefits that increase the consumption value. In this study, we assume that regarding the value obtained from a single consumption, there is no vertical differentiation between the two markets and the usage right is transferred only for one period.

3.1.2 Outline

The remainder of this paper is organized as follows. Sec. 3.2 introduces the model primitives: a monopolist's design choices and overlapping generations of heterogeneous consumers. Sec. 3.3 provides an equilibrium analysis of the consumers' dynamic choice behavior, the firm's preferred product design, and the resulting equilibrium in a sharing market—provided it exists. The company's choice depends on its ability to commit, and it may include the deliberate shutdown of sharing markets. Sec. 3.4 examines the aggregate impact of the peer-to-peer economy on consumers and on society, for the different commitment regimes. Sec. 3.5 concludes with a discussion of the strategic nature of product design in view of the peer-to-peer economy, as well as model limitations together with managerial and societal implications of the results.

3.2 Model

We begin by considering a monopolist who is maximizing profits over an infinite time horizon. The monopolist has a constant unit production cost $c \in [0, \bar{c}]$, and at each time period $t \in \{0, 1, 2, \ldots\}$, the firm chooses both the product's price r_t and its durability $q_t \in [0, 1]$.[2] Over its maximum lifetime of two periods, which corresponds to the lifetime of any given consumer in our model, failure after a first consumption period t occurs at the rate $1 - q_t$. That is, of n_t units available at the beginning of time t, at the end of this period $(1 - q_t) \cdot n_t$ units fail, and $q_t n_t$ units remain intact in period $t + 1$. As long as the units are functional, consumers derive full utility, as they would from newly purchased units.[3]

As in Weber (2016), we use an overlapping-generations model to describe the consumers' life cycles. This is illustrated in Fig. 3.1. At any time $t \geq 0$, a new generation of consumers (agents) is born and lives for two periods. Correspondingly, time t is referred to as the *early consumption phase* of generation t (denoted by $\mathscr{C}_0^t$), while $t + 1$ is called the *late consumption phase* of generation t (denoted by $\mathscr{C}_1^t$). Consequently, total product sales at time $t \geq 1$ is the sum of sales

[2] To avoid biased results, it is assumed that the product's durability can be changed at no cost. In reality, depending on the base version of the product, it may be costly to either increase or decrease the product's durability from its nominal value. The former is a standard product upgrade, while the latter would amount to a "damaged good" in the sense of Deneckere and McAfee (2005) without the benefit of product differentiation implied there.

[3] This (classical "one-hoss shay") assumption is regularly used in the durable-goods literature (see, e.g., Swan 1972; Stokey 1981; Bulow 1986; Fethke and Jagannathan 2002; Goering 2007).

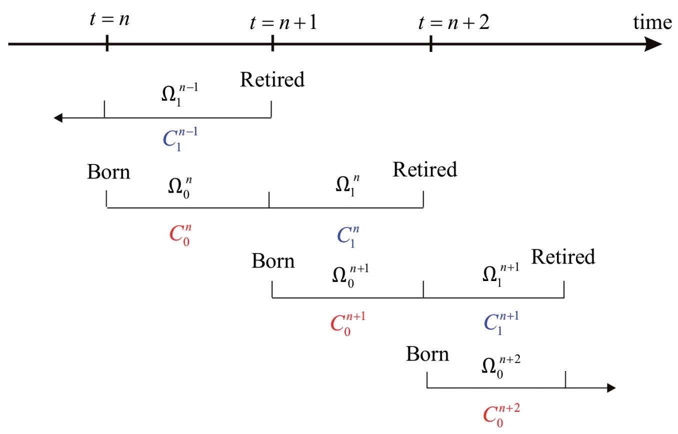

Figure 3.1: Overlapping-generations model.

Ω_0^t to generation t in its early consumption phase $\mathscr{C}_0^t$, and sales Ω_1^{t-1} to generation $t-1$ in its late consumption phase $\mathscr{C}_1^{t-1}$. For simplicity, we assume that no later generation inherits products from its ancestors.

The agents in the economy have heterogeneous preferences. Any consumer ("he") is characterized by a type (θ, v) in the unit square $\mathscr{Q} = \Theta \times \mathscr{V}$. The *likelihood* $\theta \in \Theta \triangleq [0,1]$ specifies the agent's (subjective) probability with which he will find himself in a state of high need in any future time period. More specifically, it describes the distribution of the agent's need state $\tilde{s}_j^t$ in his consumption phase $\mathscr{C}_j^t$, for $j \in \{0,1\}$, with realizations s_j^t in $\mathscr{S} = \{0,1\}$; the need state can be either "low" ($s_j^t = 0$) or "high" ($s_j^t = 1$), and *ex ante*:

$$P(\tilde{s}_j^t = 1) = \theta.$$

The *value* $v \in \mathscr{V} = [0,1]$ describes the payoff the agent derives from having access to the item in a high-need state. For simplicity we assume that the type distribution is uniform on $\mathscr{Q}$, that each consumer's type is persistent, and that the need states are uncorrelated. Given a stationary population trend, the total number of agents in a given generation is normalized to 1, without loss of generality.

At any time t, an agent's indirect utility $u_i(y, s|v)$ depends on his income level y, the realization of his need state s, and whether he has the item at his disposal or not (corresponding to $i = 1$ or $i = 0$, respectively). As in Razeghian and Weber (2015) we assume that in the low-need state the agent's utility vanishes, no matter if the item is available for use or not, so

$$u_i(y, s|v) = \begin{cases} y + iv, & \text{if } s = 1, \\ y, & \text{if } s = 0. \end{cases}$$

Agents are risk-neutral, and they are rational so as to maximize their objectives given all subjectively available information. In the early consumption phase, a newly-born agent realizes his type and observes his early need state (s_t^0). He then decides about purchasing the item at the price r_t. In the next period, the agent observes his late need state (s_t^1) and realizes whether his item has failed (conditional on being an owner). He can then renew his purchase decision.

In the presence of an active peer-to-peer market, all non-owners have the option of renting the item from the sharing market at a clearing price p_t. At the same time, any owner may decide to *either* keep and use the item *or* offer it on the sharing market to others in view of enjoying the additional income $y_t = p_t$. We thereby assume that the item cannot be shared on the market in the very same period in which it was acquired.[4]

Remark 3.1. The fact that the different overlapping consumer generations co-exist induces stationary model behavior which allows us to focus on the effect of the sharing economy on product durability, abstracting from complications arising from nonstationarities in combination with the Coase problem (Coase 1972; Gul et al. 1986). Still, in our setting the firm's commitment ability matters for the equilibrium, and this dependence is examined in detail (see Sec. 3.3).

3.3 Equilibrium Analysis

To derive predictions for the firm's and the agents' behavior in the underlying dynamic game of complete information, with and without sharing markets, we use the concept of subgame-perfect Nash equilibrium by Selten (1965). Due to the limited lifetime and coexistence of the two consumer generations in every period, the dynamic equilibrium of the supergame will be stationary. Indeed, the firm faces the same distribution of agents in each period and therefore will find the same choice for its durability and price optimal. While consumption choices are nonstationary for each generation, in aggregation they are stationary. Thus, the time index is omitted for convenience.

3.3.1 Consumption and Product Design without Sharing

Isolated Consumption Decisions

In the absence of a sharing market, ownership is the sole mode of consumption. The agents' optimal consumption decisions are obtained by backward induction.

In the late consumption phase $\mathscr{C}_1$, an owner of type (θ, v) (with a functioning item) obtains the payoff

$$\hat{V}_{s_1} = v s_1, \tag{3.1}$$

[4]Getting an item ready for the sharing market (including cleaning, preparation and installation of sharing-specific features, and creating a listing for the item with a sharing intermediary) takes time and usually comes at a cost; see Razeghian and Weber (2015) for a model which considers such transaction costs explicitly.

which is positive only in the high-need state. A non-owner of type (θ, v), on the other hand, can decide to acquire the item or else forgo use at a zero payoff. Thus, a purchase transaction takes place if and only if $vs_1 - r \geq 0$, resulting in the non-owner's optimal payoff,

$$\hat{U}_{s_1} = \max\{0, vs_1 - r\}. \tag{3.2}$$

In the early consumption phase ($\mathscr{C}_0$), the purchase decisions are based on each agent's rational assessment of his present and future prospects. The latter correspond to the agent's expected payoffs in the late consumption phase ($\hat{V}_{s_1}$ and $\hat{U}_{s_1}$) in the anticipated role of owner or non-owner, respectively. To compute the agent's possible payoffs, we first note that in a low-need state he would never purchase the item because of the standing option to buy the item in the next consumption phase at the same stationary price, should a high-need state realize then. In other words, without a present high need no agent would be willing to invest in uncertain future consumption. Hence, purchasers are necessarily in their high-need state. An agent of type (θ, v), in the high-need state, would acquire the item in his early consumption phase if the expected discounted payoff of ownership exceeds the expected payoff of non-ownership,

$$v - r + \delta\theta\left((1-q)\hat{U}_1 + q\hat{V}_1\right) \geq \delta\theta\hat{U}_1, \tag{3.3}$$

where $\delta \in (0, 1]$ is a discrete-time discount factor, common for all agents. The preceding relation yields a purchasing threshold in terms of the likelihood of future need:

$$\theta \geq \theta_0(v|q, r) \triangleq \frac{\max\{0, r - v\}}{\delta q v}. \tag{3.4}$$

Agent types with sufficiently high θ purchase as long as $v \geq r/(1+\delta q)$; an agent type with any θ purchases if $v \geq r$. As a result, sales to consumers in the early consumption phase are

$$\hat{\Omega}_0(q, r) = \int_{\mathscr{Q}} \mathbf{1}_{\{\theta \geq \theta_0(v|q,r)\}}\, d(\theta, v).$$

In the late consumption phase ($\mathscr{C}_1$), the demand for ownership consists of two groups of agents in their high-need state (both with values $v \geq r$): (i) first-time purchasers who were in the low-need state in their early consumption phase; (ii) second-time purchasers who were owners in $\mathscr{C}_0$ but ended up with a defective item at the end of their early consumption phase. Sales to this generation are

$$\hat{\Omega}_1(q, r) = \int_{\mathscr{Q}} \mathbf{1}_{\{v \geq r\}} \left[(1-\theta)\theta + (1-q)\theta^2\right] d(\theta, v). \tag{3.5}$$

In any given period, the aggregate demand for ownership ($\hat{\Omega}$) is the sum of the respective demands by the young generation ($\hat{\Omega}_0$) and by the mature generation ($\hat{\Omega}_1$).

Proposition 3.1 (Demand for Ownership without Sharing). *Let* $q \in [0, 1]$ *and* $r \in [0, 1 + \delta q]$.

In the absence of sharing, the demand for ownership is $\hat{\Omega}(q,r) \triangleq \hat{\Omega}_0(q,r) + \hat{\Omega}_1(q,r)$,[5] *where*

$$\hat{\Omega}_0(q,r) = \frac{1}{2} + \frac{\max\{0, r^2-1\}}{2(\delta q)^2} - \frac{r}{\delta q}\left(1 - \frac{1}{\delta q}\ln\left(\frac{1+\delta q}{\max\{1,r\}}\right)\right), \tag{3.6}$$

and

$$\hat{\Omega}_1(q,r) = \max\{0, 1-r\}\left(\frac{1}{2} - \frac{q}{3}\right). \tag{3.7}$$

When durability goes up, then *ceteris paribus* the demand by young consumers increases, whereas the demand by mature consumers decreases.

Corollary 3.1. *For any* $(q,r) \in [0,1] \times \mathbb{R}_+$, *the generational demands for product ownership,* $\hat{\Omega}_0$ *and* $\hat{\Omega}_1$, *are such that*

$$\frac{\partial \hat{\Omega}_1(q,r)}{\partial q} \leq 0 \leq \frac{\partial \hat{\Omega}_0(q,r)}{\partial q}.$$

The reason for the somewhat counterintuitive negative dependence of $\hat{\Omega}_1$ on q is that a higher durability tends to deplete the pool of second-time purchasers.

Remark 3.2. We concentrate on the interesting case where the unit production cost is low enough, such that the monopolist is able to offer a unit purchase price $r \leq 1$ for all $q \in [0,1]$. This case is particularly interesting because the choice of durability affects the sales in both consumption phases (early and late). If the purchase price exceeds unity, for each generation sales occur only in the early consumption phase.

Isolated Product-Design Decisions

The firm maximizes its expected discounted profit with respect to durability and price. In any given period, the (stationary) profit $\hat{\Pi}(q,r)$ is the product of the markup $(r-c)$ and the sales $\hat{\Omega}(q,r)$ to both generations, so

$$\hat{\Pi}(q,r) = (r-c)\,\hat{\Omega}(q,r) = (r-c)\left(1 - \frac{q}{3} - r\rho(q)\right), \tag{3.8}$$

where $\rho(q) \triangleq 1/2 - q/3 + [1 - \ln(1+\delta q)/(\delta q)]/(\delta q)$ and $c \in [0, \bar{c}]$.[6] While simultaneous optimization with respect to both q and r is possible, more insight is obtained by considering the optimal pricing problem and the optimal durability problem sequentially which ultimately leads to the same result. Yet, arguably it is easier for the firm to adjust the retail price than to change the durability characteristics of its product. Durability, as a product-design decision that affects the

[5] One can verify that $\hat{\Omega}(q,r)$ is twice continuously differentiable on the interior of its domain.

[6] For the purchase price to not exceed the unity given all $\delta \in (0,1]$, the production cost needs to be in the interval $[0, (12\ln(2) - 10)/(6\ln(2) - 7)] \approx [0, 0.59]$.

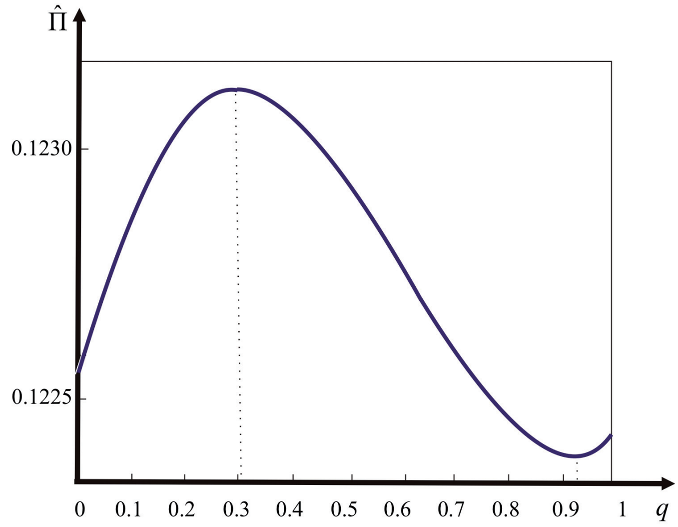

Figure 3.2: Isolated profit as a function of durability, for $(\delta, c) = (0.6, 0.3)$.

hardware of the product, tends to be more sticky than the posted price. A sequential solution approach thus also reveals, in addition to the optimal product design, the optimal price for any given durability level. Hence, via backward induction, we begin by determining the optimal product price $\hat{r}(q)$ for all $q \in [0,1]$ and subsequently determine the optimal durability $\hat{q}$. The resulting optimal product design $(\hat{q}, \hat{r})$ with $\hat{r} = \hat{r}(\hat{q})$ is the same as if both instruments (price and durability level) had been optimized simultaneously.

Optimal Pricing Problem. The first-order necessary optimality condition for the maximization of the firm's profit in Eq. (3.8) with respect to r yields the optimal price as a function of a given durability level q:

$$\hat{r}(q) = \frac{c}{2} + \frac{1}{2\rho(q)}\left(1 - \frac{q}{3}\right). \tag{3.9}$$

This price is increasing in both the marginal cost c and the provided level of durability q. Furthermore, it is remarkable that in absolute terms the firm's price is more sensitive to changes in the agents' discount factor δ than to changes in the durability q, even though the consumption decisions depend only on their product (δq), as shown in Sec. 3.3.1.

Lemma 3.1. *For any* $\delta, q \in (0,1)$*, the optimal price in Eq.* (3.9) *is such that*

$$0 \le \hat{r}'(q) \le \frac{\partial \hat{r}(q)}{\partial \delta}.$$

An exogenous change in consumers' level of patience has a more profound impact on the price than an increase in the durability, which is moderated by the consumers' patience.

Optimal Durability Problem. For any given price, the firm's optimal level of durability maximizes the demand $\hat{\Omega}$ for the product.[7] Given that the monopolist uses the optimal price in Eq. (3.9) for any feasible $q \in [0,1]$, the firm's profit in Eq. (3.8) becomes

$$\hat{\Pi}(q, \hat{r}(q)) = \frac{\left(1 - c\rho(q) - q/3\right)^2}{2\rho(q)}. \tag{3.10}$$

Since the latter is generally nonconcave in q (see Fig. 3.2), and the objective function involves a transcendental function, the optimal durability problem,

$$\hat{q} \in \arg\max_{q \in [0,1]} \hat{\Pi}(q, \hat{r}(q)), \tag{3.11}$$

cannot be solved in closed form. A more detailed analysis reveals that for small production costs, the firm has an incentive to provide low levels of durability. When the unit production cost is high enough, then the firm finds it optimal to produce a fully durable product (i.e., $\hat{q} = 1$).

Proposition 3.2 (Optimal Durability without Sharing). *Let $(\delta, c) \in (0,1] \times [0, \bar{c}]$ and set $\check{c} \triangleq (1-\delta)/(1+\delta)$. Then there exists $\hat{c} \in (\check{c}, \bar{c})$ such that (i) $\hat{q} = 0$ for $c \le \check{c}$; (ii) $\hat{q} \in (0,1)$ for $c \in [\check{c}, \hat{c}]$; (iii) $\hat{q} = 1$ for $c > \hat{c}$.*

In other words, it is optimal for the monopolist to make low-cost products *disposable*, and in stark contrast, to make high-cost products *perfectly durable*.[8] The thresholds in Prop. 3.2 can also be formulated in terms of the discount factor. When consumers heavily discount future payoffs, the firm has an incentive to produce disposable products ($\hat{q} = 0$). At the other end of the spectrum, when consumers are very patient because δ is close to 1, then the firm finds it optimal to produce a fully durable product ($\hat{q} = 1$).

Corollary 3.2. *Let $c \in [0, \bar{c}]$ and set $\check{\delta} \triangleq (1-c)/(1+c)$. Then there exists $\hat{\delta} \in (\check{\delta}, 1]$ such that (i) $\hat{q} = 0$ for $\delta \le \check{\delta}$; (ii) $\hat{q} \in (0,1)$ for $\delta \in [\check{\delta}, \hat{\delta}]$; (iii) $\hat{q} = 1$ for $\delta > \hat{\delta}$.*

The following result can be used to compute the optimal durability level $\hat{q}$. The corresponding optimal price $\hat{r} = \hat{r}(\hat{q})$ follows from Eq. (3.9).

Corollary 3.3. *Let $(\delta, c) \in (0,1] \times [0, \bar{c}]$. For $c \in (\check{c}, \hat{c})$, or equivalently $\delta \in (\check{\delta}, \hat{\delta})$, the optimal durability $\hat{q}$ satisfies*

$$-\left(1 + c\rho(\hat{q}) - \frac{\hat{q}}{3}\right)\frac{\rho'(\hat{q})}{\rho(\hat{q})} = \frac{2}{3}, \tag{3.12}$$

and it is intermediate (i.e., $0 < \hat{q} < 1$).

---

[7] By La. 3.1 this means that it is (locally) optimal to increase the durability as long as the resulting marginal increase of $\hat{\Omega}_0$ outweighs the marginal decrease of $\hat{\Omega}_1$.

[8] The cost thresholds $\check{c}$ and $\hat{c}$ in Prop. 3.2 are such that $\check{c} \le \left.\frac{\sigma - 2/3}{(1-\sigma)\sigma}\right|_{\sigma = \sqrt{\rho(1)}} \le \hat{c}$. The algebraic details are in App. B.

In line with the intuition provided by Prop. 3.2, the optimal durability depends monotonically on the consumers' patience and the firm's production cost.

Proposition 3.3 (Durability Drivers). *The optimal durability $\hat{q}$ is increasing the discount factor δ and the production cost c, for all $(\delta, c) \in (0,1] \times [0, \bar{c}]$.*

More patient consumers expect better-quality products and are willing to pay more at the margin, thus leading the firm to increase durability. Similarly, to sustain a substantial markup over an increasing production cost, a firm would find it best to also increase the level of durability. Consider now the firm's optimal profit $\hat{\Pi}^* \triangleq \hat{\Pi}(\hat{q}, \hat{r})$.

Lemma 3.2. *The optimal profit $\hat{\Pi}^*$ is increasing in the consumers' discount factor δ and decreasing in the production cost c, for all $(\delta, c) \in (0,1] \times [0, \bar{c}]$.*

The fact that the firm's optimal profit increases in the consumers' level of patience has some interesting business implications if there are indirect ways for the monopolist to decrease the implied interest rate for the customer. By offering financial services (e.g., loans) to facilitate the purchase, the consumers' discount rate may effectively decrease, implying a higher discount factor. Depending on the firm's cost of capital, this may result in a net profit increase.[9] The optimal durability and profit are shown in Fig. 3.3, as a function of (δ, c).

Remark 3.3. Maximizing profits with respect to durability for a *given* price r is equivalent to maximizing the demand for ownership $\hat{\Omega}(q, r)$ with respect to q. An interior solution is such that

$$\hat{r}(q) = -\frac{1}{3\rho'(q)}. \tag{3.13}$$

Hence, by combining Eqs. (3.9) and (3.13), one obtains a fixed-point problem satisfied by an interior optimal durability $\hat{q} \in (0,1)$:

$$\hat{q} = 3\left(1 + c\rho(\hat{q})\right) + 2\left(\rho(\hat{q})/\rho'(\hat{q})\right). \tag{3.14}$$

This condition is equivalent to Eq. (3.12).

3.3.2 Consumption and Product Design with Sharing

In the presence of an active sharing market, the monopolist faces consumers that can choose between ownership and mere access to the products. In addition, owners have the option to rent out their items when they are not needed. As pointed out by Weber (2016), the value created by this flexibility creates an incentive for the firm to increase the retail price, at least for perfectly durable goods. The question answered here is how durability interacts with the firm's preferred retail price and how much durability is optimal to provide for the monopolist when consumers can share their goods.

[9] While the firm's per-period profit is stationary, the net present value of the monopolist's operations depends on its own discount factor δ'. The latter is generally different from the consumers' δ; usually $\delta' < \delta$.

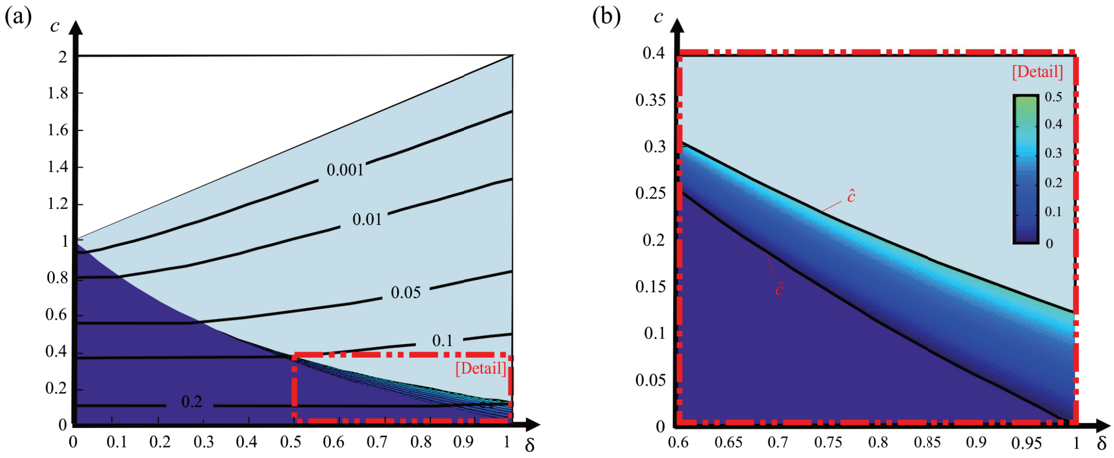

Figure 3.3: Optimal durability and profit without sharing as a function of (δ, c).

Consumption Decisions: Access versus Ownership

For transactions on the sharing market to exist, the price p of renting a product for one period cannot exceed the (stationary) purchase price r set by the firm.[10] In other words, for sharing to take place it is necessary that

$$p < r. \tag{3.15}$$

Remark 3.4. In case of a tie between the price of ownership r and the rental price p, we assume that consumers prefer ownership, for a variety of possible reasons. The latter may include avoiding informational problems inherent to rental markets and the resulting potential for ex-post disputes, as well as the enjoyment of residual claims (e.g., the unmodelled possibility of resale).

Hence, for a *non-owner* in his late consumption phase, the sharing market is more attractive than purchasing, independent of his type (θ, v); his resulting payoff is

$$U_{s_1} = \max\{0, vs_1 - p\}, \tag{3.16}$$

given the realized need state $s_1 \in \{0, 1\}$. By contrast, an *owner* in the late consumption phase can choose whether to use his item or become a supplier on the sharing market. Because of the vanishing use value in the low-need state, he would participate in the sharing market whenever $s_1 = 0$. On the other hand, in the high-need state he would be willing to share, as long as the clearing price p exceeds his own utility when using the item privately. This implies an owner's payoff of the form

$$V_{s_1} = \max\{vs_1, p\}, \tag{3.17}$$

[10]If $p \geq r$, then there is no sharing, and the results in Sec. 3.3.1 do apply.

contingent on the realization of his need state $s_1 \in \{0,1\}$. Ownership decisions are based on an evaluation of the lifecycle payoffs, analyzed next.

In the early consumption phase, each agent observes his need state s_0, evaluates his expected future payoff, and decides whether it is best to purchase, to rent, or to do nothing. For an agent of type (θ, v), the state-dependent total discounted payoff is therefore

$$T_{s_0} = \max\left\{\delta \bar{U}, v s_0 - p + \delta \bar{U}, v s_0 - r + \delta(q\bar{V} + (1-q)\bar{U})\right\},$$

where $\bar{U} \triangleq (1-\theta)U_0 + \theta U_1$ and $\bar{V} \triangleq (1-\theta)V_0 + \theta V_1$. Purchasing is best when the expected payoff of ownership outweighs the expected return from non-ownership in the high-need state:

$$v - p + \delta q \bar{U} \leq v - r + \delta q \bar{V}. \tag{3.18}$$

Substituting Eqs. (3.16)–(3.17) in Eq. (3.18) and then rearranging the terms yields the ownership criterion

$$r \leq \min\{v, p\} + \delta p q. \tag{3.19}$$

In the presence of an active collaborative economy, an agent's purchase decision depends only on his value v, not on the likelihood of future need θ. This underlines the fact that sharing markets allow for hedging of need-state-specific payoff variations, thus providing the consumer base with the possibility of mutual insurance.

The ownership criterion in Eq. (3.19) implies that no agent is willing to make a purchase if the retail price strictly exceeds the effective net present value of the item when accessed via the sharing market in consecutive periods, i.e., if $r > (1+\delta q)p$, so that necessarily

$$\frac{r}{1+\delta q} \leq p. \tag{3.20}$$

Remark 3.5. Only agents in the high-need state become owners. In the early consumption phase, no agent in a low-need state is willing to purchase. The latter can be shown by contradiction. Suppose an agent who observed his need state $s_0 = 0$ makes a purchase. This can be optimal only if $-p + \delta\bar{U} \leq -r + \delta(q\bar{V} + (1-q)\bar{U})$, or equivalently if $r \leq \delta p q$,[11] which (for $\delta q < 1$) violates liquidity requirement (3.15). This condition also implies that no purchases are made in the late consumption phase as long as there is a functioning sharing market.

Based on the preceding remark and the ownership criterion (3.19), early-generation agents in the high-need state with values $v \geq r - \delta p q$ are willing to buy the product, resulting in the steady-state demand for ownership,

$$\Omega(q, r; p) = (1 - (r - \delta p q))\bar{\theta} = \frac{1 + \delta p q - r}{2}, \tag{3.21}$$

[11] This inequality is equivalent to the ownership criterion (3.19) when $v = 0$ (as the benefit vanishes in the low-need state).

where $\bar{\theta} \triangleq \int_0^1 \theta \, d\theta = 1/2$. As will become clear below, the motivation for ownership differs across agents, depending on v. Consumers with high values $v \in [p, 1]$ buy the item primarily for their personal use, and they act as suppliers only if they end up in a low-need state during their late consumption phase. Supra-marginal consumers with intermediate values $v \in [r - \delta pq, p]$ purchase the item to benefit from the additional income they can gain by offering it on the sharing market in their late consumption phase, regardless of the need state.

Equilibrium in the Sharing Market

By combining the requirements (3.15) and (3.20) the clearing price p of a functioning sharing market for a product with characteristics (r, q) must satisfy the *liquidity condition*

$$\frac{r}{1 + \delta q} \leq p < r. \tag{L}$$

Of the buyers in their early consumption phase, only the fraction q are expected to still own a functioning item at the beginning of their late consumption phase. Of these residual owners, all could in principle act as a supplier, except those who are in a high-need state for the second time in a row and whose valuation v exceeds the rental price p in the sharing market. Aggregate supply is therefore

$$\begin{aligned} S(q, r; p) &= q\left((1-p)\int_0^1 \theta(1-\theta)\, d\theta + (p - (r - \delta pq))\int_0^1 \theta \, d\theta\right) \\ &= \frac{q}{2}\left(\frac{1}{3} + \left(\frac{2}{3} + \delta q\right)p - r\right), \end{aligned} \tag{3.22}$$

where the number of owners $\Omega(q, r; p)$ is specified in Eq. (3.21). Conversely, the demand for the shared item consists of all non-owners with value $v \geq p$ in their late consumption phase:

$$D(q; p) = (1-p)\left(\int_0^1 (1-\theta)\theta \, d\theta + (1-q)\int_0^1 \theta^2 \, d\theta\right) = \frac{1-p}{2}\left(1 - \frac{2q}{3}\right), \tag{3.23}$$

independent of the purchase price r. The first term in middle of Eq. (3.23) captures the non-owners in $\mathscr{C}_0$ who find themselves in a high-need state in $\mathscr{C}_1$. The second term reflects the additional demand created due to product failure in $\mathscr{C}_1$, as experienced by the fraction $1 - q$ of initial owners. The sharing market clears if supply equals demand, i.e., if

$$S(q, r; p) = D(q; p), \tag{3.24}$$

which in turn determines the price p in the sharing market as a function of the firm's product design (q, r). In this context, it is important to note that the liquidity condition (L) guarantees a functioning sharing market by bracketing the clearing price p. It can also be stated in terms of

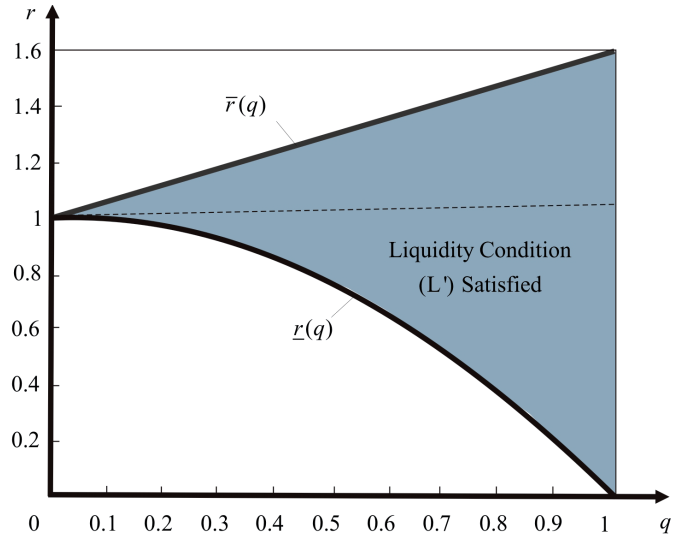

Figure 3.4: Retail-price liquidity thresholds in Eq. (L') as a function of durability ($\delta = 0.6$).

bounds on the retail price r, as follows:

$$\underline{r}(q) < r \leq \bar{r}(q), \tag{L'}$$

where $\underline{r}(q) \triangleq (1-q)/(1-q+\delta q^2)$ and $\bar{r}(q) \triangleq 1+\delta q$; see Fig. 3.4. For maximum durability ($q=1$), the liquidity condition (L') is automatically satisfied, as $\underline{r}(1)=0$ and $\bar{r}(1)=1+\delta$.

Proposition 3.4 (Equilibrium in the Sharing Market). *Given a product design (q,r) which satisfies the liquidity condition* (L')*, the equilibrium price in the sharing market is*

$$p(q,r) = \frac{1-(1-r)q}{1+\delta q^2}, \tag{3.25}$$

resulting in the equilibrium transaction volume

$$Q(q,r) = \frac{q}{2}\left(1-\frac{2q}{3}\right)\left(\frac{\bar{r}(q)-r}{1+\delta q^2}\right), \tag{3.26}$$

for any given discount factor $\delta \in (0,1]$.

By construction, the transaction volume can only be nonnegative if the liquidity condition (L') holds. The latter can be broken by the firm's product design, leading to a deliberate "sharing shutdown," further examined in Sec. 3.3.3. The sensitivity of the transaction volume in the sharing market to the product design is illustrated by the next result.

Lemma 3.3. *Consider an active sharing market for product designs (q,r) that satisfy the liquidity condition* (L')*. (i) The clearing price $p(q,r)$ is decreasing in the durability q and increasing in the retail price r. (ii) The transaction volume $Q(q,r)$ is unimodal in q and decreasing in r.*

For lower levels of durability, the scarcity arising from the concomitant planned obsolescence produces a higher price in the sharing market, which in turn dissuades some agents from participating in the peer-to-peer economy. A higher retail price also tends to increase the clearing price in the sharing market, thus reducing the transaction volume. Perhaps the most interesting point of La. 3.3 is that the volume of the sharing transactions is maximal for items of intermediate durability. When products are disposable, there is nothing to share. Perfectly durable goods, on the other hand, ultimately decrease demand because of the inherent lack of product failure and replacement, thus resulting in a somewhat lower sharing volume than the maximally attainable transaction volume on the peer-to-peer market.

Market Responsiveness and Commitment Regimes

The monopolist's best actions depend on the responsiveness of the sharing market. An isomorphic and more classical way to view this dependency, is that the firm's optimal product design is influenced by its ability to commit to its choice. To understand the importance of commitment, let $\Omega(q,r;p)$ denote the demand for ownership, given a price p in the sharing market and a product-design choice (q,r).

1. *Stackelberg play (SP)*: With full ability to commit to a product design, the firm can capitalize on the anticipated market response $p(q,r)$ as specified in Prop. 3.4, and maximize its "Stackelberg profits"

$$\Pi_{\text{SP}}(q,r) = (r-c)\,\Omega_{\text{SP}}(q,r), \tag{3.27}$$

   where $\Omega_{\text{SP}}(q,r) \triangleq \Omega(q,r;p(q,r))$ is the demand for ownership in this sequential-move Stackelberg play. This commitment regime is relevant unless the sharing market is very responsive to changes in the retail price or product durability or if the firm has no adjustment cost for design changes.[12]

2. *Simultaneous-move play (SMP)*: Without commitment ability and fast adjustment, the sharing market can be viewed as a player all by itself who *reacts* to the firm's product design so as to maximize the welfare of its participants. Indeed, by the first fundamental welfare theorem an exchange economy produces an efficient outcome for its participants, and thus in aggregate can be viewed as a single rational "player" interacting repeatedly with the firm, at each time $t \geq 0$ in simultaneous moves. In this (lack-of-)commitment regime, the firm maximizes its "simultaneous-move profits"

$$\Pi_{\text{SMP}}(q,r;p) = (r-c)\,\Omega_{\text{SMP}}(q,r;p), \tag{3.28}$$

   so as to determine its best response $(q(p),r(p))$ to any viable market price p that satisfies

[12] Usually an additional costly production run and design changes are required to change the product durability. Price adjustments can also be costly because of the expense to update catalogues, relabeling of physical products, communication in the retail network and delays.

the liquidity condition (L).

3. *Simultaneous-move play with Durability Commitment (DC)*: While the SP and SMP commitment regimes can be viewed as extremes on a continuum of possible commitment levels, the DC regime presents a perhaps more realistic intermediate situation. Durability is a built-in product feature, arguably much more difficult to adjust in most cases than the product price. The simultaneous-move play with durability commitment regime allows the firm to partially commit to its design feature q and maximize its "durability-commitment profits,"

$$\Pi_{\mathrm{DC}}(q,r;p) = (r-c)\,\Omega_{\mathrm{DC}}(q,r;p). \tag{3.29}$$

The induced durability level q may or may not be equal to the best-response product design of the SMP game.

We compare and contrast the analysis for the three regimes. Overall, similar to the prediction of the Coase conjecture (Coase 1972; Gul et al. 1986), the firm does best with full commitment, i.e., with Stackelberg play. The authors feel that this regime reflects reality fairly well because the price adjustment on sharing markets is arguably limited (Razeghian and Weber 2015). The following auxiliary result specifies the demand for ownership with and without a sharing market.

Lemma 3.4 (Demand for Ownership). *For any admissible product design (q,r), the demand for ownership is $\Omega(q,r;p) \triangleq \Omega_0(q,r) + \Omega_1(q,r)$, where*

$$\Omega_0(q,r;p) = \begin{cases} (1-r+\delta pq)/2, & \text{if } \underline{r}(q) < r \leq \bar{r}(q), \\ \hat{\Omega}_0(q,r), & \text{otherwise}, \end{cases} \tag{3.30}$$

and

$$\Omega_1(q,r;p) = \begin{cases} 0, & \text{if } \underline{r}(q) < r \leq \bar{r}(q), \\ \hat{\Omega}_1(q,r), & \text{otherwise}. \end{cases} \tag{3.31}$$

All agents can acquire ownership in either consumption phase ($\mathscr{C}_0$ or $\mathscr{C}_1$), and the total per-period demand for ownership obtains as the sum for both coexisting generations. With active sharing markets no products are acquired in the late consumption phase. For this reason, the monopolist may resort to product designs that intentionally shut down the sharing markets, as discussed in Sec. 3.3.3.

Product Design in the Stackelberg Regime

At the beginning of each period, the monopolist selects a product design (q,r) so as to maximize the Stackelberg profits in Eq. (3.27). As in Sec. 3.3.1, we consider (without any loss of generality) the monopolist's decisions about durability and price sequentially.

Optimal Pricing Problem. The firm anticipates the sharing price in Prop. 3.4 and for a given product design q chooses a retail price so as to maximize the expected per-period profit $\Pi_{\mathrm{SP}}(q,r)$ by solving

$$r(q) \in \arg\max_{r} \left\{ \frac{1}{2}\left(1 - r + \delta q \left(\frac{qr + (1-q)}{1+\delta q^2}\right)\right)(r-c) \right\}. \tag{3.32}$$

The corresponding first-order necessary optimality condition yields the unique solution

$$r(q) = \frac{c}{2} + \frac{1+\delta q}{2}, \tag{3.33}$$

for any given level of durability $q \in [0,1]$, so that (q,r) satisfies the liquidity condition (L'). Benefiting from its commitment ability, the firm is able to syphon off the value that the sharing market adds for consumers, in the form of a "sharing premium."

Lemma 3.5. *In the Stackelberg regime, the price with sharing $r(q)$ exceeds the price without sharing $\hat{r}(q)$ by a nonnegative sharing premium $\pi(q)$. More specifically, $r(0) = \hat{r}(0) = (1+c)/2$, and*

$$\pi(q) \triangleq r(q) - \hat{r}(q) = \frac{1}{2}\left(\frac{1-q/3}{\rho(q)} - (1+\delta q)\right) \quad (\geq 0)$$

is increasing, for all $q \in [0,1]$ such that the product design $(q, r(q))$ satisfies the liquidity condition (L').

It is remarkable that the sharing premium is *independent* of the production cost. This is in contrast to the absolute price levels, both with and without a sharing market, which clearly depend on c. Thus, for low-cost products the relative weight of the sharing premium is likely to be substantially larger in relative terms than for high-cost products.

Optimal Durability Problem. Given the optimal price $r(q)$ in Eq. (3.33), the firm's Stackelberg profit becomes a function of durability only:

$$\Pi_{\mathrm{SP}}(q, r(q)) = \frac{1}{8}\frac{(1+\delta q - c)^2}{1+\delta q^2}.$$

As this function is strictly increasing in q, sharing markets provide strong incentives for companies to make their products durable.[13]

Proposition 3.5 (Optimal Product Design (SP)). *In the Stackelberg regime, the optimal product design is (q^*_{SP}, r^*_{SP}), with $q^*_{SP} = 1$ (perfect durability) and $r^*_{SP} = (1+\delta+c)/2$.*

By virtue of its perfect durability, the product design $(q^*_{\mathrm{SP}}, r^*_{\mathrm{SP}})$ always satisfies the liquidity

[13]Since durability is in general costly to provide implies that the marginal profit with respect to durability needs to equal the marginal cost of durability provision; see also Sec. 3.5 for further discussion.

condition (L'). The firm's resulting optimal profit is

$$\Pi^*_{\text{SP}} \triangleq \Pi_{\text{SP}}(q^*_{\text{SP}}, r^*_{\text{SP}}) = \frac{1}{8}\frac{(1+\delta-c)^2}{1+\delta}, \tag{3.34}$$

for all $(\delta, c) \in (0,1] \times [0, \bar{c}]$.

Remark 3.6. The firm's payoff is increasing in the consumers' level of patience (as measured by the discount factor δ), and it is decreasing in the firm's production cost c. More precisely:

$$\frac{\partial \Pi^*_{\text{SP}}}{\partial c} = -\frac{1}{4}\left(1 - \frac{c}{1+\delta}\right) < 0 < \frac{1}{8}\left(1 - \frac{c^2}{(1+\delta)^2}\right) = \frac{\partial \Pi^*_{\text{SP}}}{\partial \delta},$$

for all $(\delta, c) \in (0,1) \times (0, 1+\delta)$. Thinking creatively, the firm has therefore an interest to augment an agent's patience, e.g., by offering advantageous financing. Because of the added benefit through profit increase, the company can in principle afford offering a loan below the market rate, slightly cross-subsidizing the anticipated extra rent.

Product Design with Simultaneous-Move Play

Consider now a setting with a very efficient and fast sharing market and where the company cannot commit to its product design at the beginning of the period because it expects a quasi-instant response by the sharing market. For each sharing price, the company determines a best response in terms of its product design. Together with the price response of the sharing market (see Prop. 3.4), this determines a Nash-equilibrium outcome. The viability of the sharing market and thus the existence of the Nash equilibrium is, however, subject to the liquidity constraint.

Best-Response Product Design. Given a clearing price p in the sharing market, consider the firm's product-design problem. A solution is obtained by maximizing the simultaneous-move profit $\Pi_{\text{SMP}}(q, r; p)$ as specified in Eq. (3.29) with respect to the product design (q, r).

Lemma 3.6. *Given any (nonnegative) sharing price p, the firm's best-response product design is $(q_{SMP}(p), r_{SMP}(p))$, where*

$$r_{SMP}(p) = \begin{cases} \max\{c, (1+\delta)p\}, & \text{if } p \in [0, (1+c)/(2+\delta)], \\ (1+c+\delta p)/2, & \text{if } p \in ((1+c)/(2+\delta), (1+c)/(2-\delta)], \\ \hat{r}(\hat{q}), & \text{otherwise}, \end{cases} \tag{3.35}$$

and

$$q_{SMP}(p) = \begin{cases} 1, & \text{if } p \in [0, (1+c)/(2+\delta)], \\ 1, & \text{if } p \in ((1+c)/(2+\delta), (1+c)/(2-\delta)], \\ \hat{q}, & \text{otherwise}; \end{cases} \tag{3.36}$$

$\hat{r}(\cdot)$ is the optimal retail-price schedule without sharing specified in Eq. (3.9)*, and $\hat{q}$ is the optimal product durability without sharing as described in Prop. 3.2.*

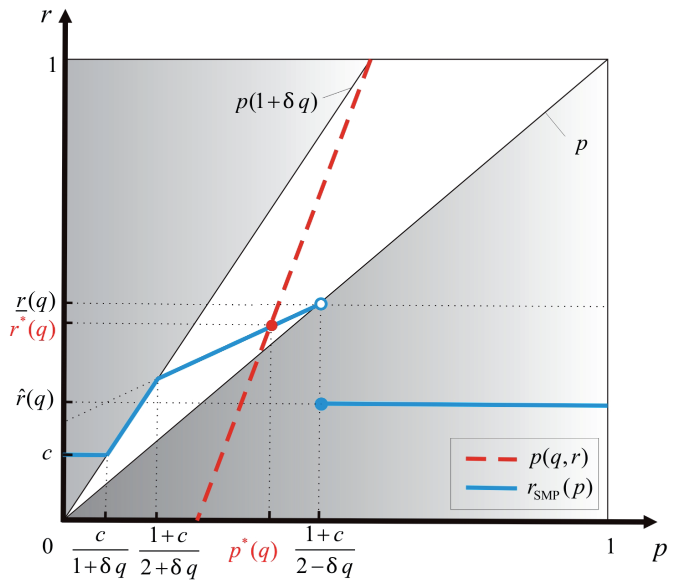

Figure 3.5: Best responses in a simultaneous-move game between firm and sharing market.

For small levels of p, the firm makes no sale. Thus, instead of charging marginal cost, any retail price is also optimal. In this case, the firm does not really care about the particular level of durability (which can therefore also be different from 1). Note also that the requirement

$$\frac{1+c}{2+\delta} < p \leq \frac{1+c}{2-\delta} \tag{L''}$$

is equivalent to the liquidity conditions (L) and (L').

Nash Equilibrium without Commitment. By intersecting the best-response product design in La. 3.6 with the market-price response in Prop. 3.4 one obtains a unique Nash equilibrium as prediction for the repeated stage-game interaction between the firm and the sharing market. Even though the firm's equilibrium design is stationary, it is fundamentally influenced by the fact that it could—if necessary—easily change this design. The players' best responses are shown in Fig. 3.5, intersecting at a unique Nash equilibrium.

Proposition 3.6 (**Nash Equilibrium (SMP)).** *The unique Nash-equilibrium sharing-price and product-design profile* $(p^*_{SMP}, (q^*_{SMP}, r^*_{SMP}))$ *are such that* $p^*_{SMP} = (1+c)/(2+\delta)$, $q^*_{SMP} = 1$ *(perfect durability), and* $r^*_{SMP} = (1+c)(1+\delta)/(2+\delta)$, *for any* $c \in [0, \bar{c}]$.

Given the product design $(q^*_{\text{SMP}}, r^*_{\text{SMP}})$ in the simultaneous-move game with the sharing market, the firm obtains the equilibrium profit

$$\Pi^*_{\text{SMP}} \triangleq \Pi_{\text{SMP}}(q^*_{\text{SMP}}, r^*_{\text{SMP}}) = \frac{1}{2}\frac{(1+\delta-c)^2}{(2+\delta)^2},$$

for any $c \in [0, \bar{c}]$. The firm's payoffs are monotonic in the salient model parameters.

Remark 3.7. The firm's profit is increasing in the consumers' discount factor δ and decreasing in the production cost c.

$$\frac{\partial \Pi^*_{\mathrm{SMP}}}{\partial c} = -\frac{1+\delta-c}{(2+\delta)^2} < 0 < \frac{(1+c)(1+\delta-c)}{(2+\delta)^3} = \frac{\partial \Pi^*_{\mathrm{SMP}}}{\partial \delta},$$

for all $\delta \in (0,1]$ and $c \in [0, \bar{c}]$, similar to the Stackelberg regime. Again, increasing the consumers' level of patience turns out to be beneficial for the firm (see Remark 3.6).

Product Design with Durability Commitment

We now provide a more general result, allowing the firm to partially commit to a durability level q which may or may not be equal to the perfect durability induced by the best-response product design of the simultaneous-move game.

Lemma 3.7. *Given any (nonnegative) sharing price p, and precommitment on durability $q \in [0,1]$, the firm's best-response retail price is*

$$r_{DC}(p;q) = \begin{cases} \max\{c, (1+\delta q)p\}, & \textit{if } p \in [0, (1+c)/(2+\delta q)], \\ (1+c+\delta pq)/2, & \textit{if } p \in ((1+c)/(2+\delta q), (1+c)/(2-\delta q)], \\ \hat{r}(q), & \textit{otherwise}, \end{cases} \tag{3.37}$$

where $\hat{r}(\cdot)$ is the optimal retail-price schedule without sharing specified in Eq. (3.9).

Intersecting the best-response price schedule $r_{\mathrm{DC}}(\cdot;q)$ in La. 3.7 with the sharing-market response $p(\cdot,q)$ in Prop. 3.4 yields the Nash equilibrium outcome with durability commitment.

Proposition 3.7 (Nash Equilibrium with Durability Commitment). *Given a durability commitment $q \in [0,1]$ and a production cost $c \geq \underline{c}(q)$, the Nash-equilibrium market-price/retail-price profile $(p_{DC}(q), r_{DC}(q))$ is such that*

$$p_{DC}(q) = \frac{2-(1-c)q}{2+\delta q^2} \quad \textit{and} \quad r_{DC}(q) = \frac{1}{2}\left(1+c+\delta q\left(\frac{2-(1-c)q}{2+\delta q^2}\right)\right), \tag{3.38}$$

whereby the cost threshold is $\underline{c}(q) \triangleq \max\{0, (1-q-\delta q)/(1-q+\delta q^2)\}$. In the case where $c < \underline{c}(q)$, the sharing market collapses.

The firm's Nash-equilibrium profit with durability commitment q becomes

$$\Pi_{\mathrm{DC}}(q) \triangleq \Pi_{\mathrm{SMP}}(q, r_{\mathrm{DC}}(q)) = \frac{1}{2}\left(\frac{1+\delta q-c}{2+\delta q^2}\right)^2.$$

It is clear that durability precommitment can only be beneficial to the monopolist, since by committing to perfect durability ($q=1$) the firm obtains the Nash-equilibrium outcome without precommitment. In this setting, it is always in the firm's best interest to commit to a level of durability which may not necessarily be equal to 1.

Proposition 3.8 (Product Design with Durability Commitment). *In the simultaneous-move regime with durability commitment $q \in [0,1]$, the optimal durability is*

$$q^*_{DC} = \min\left\{1, \frac{\sqrt{(1-c)^2 + 2\delta} - (1-c)}{\delta}\right\} \in \arg\max_{q\in[0,1]} \Pi_{DC}(q);$$

perfect durability ($q^*_{DC} = 1$) is optimal for $c \in [\delta/2, \bar{c}]$. The corresponding optimal retail price is $r^*_{DC} = r_{DC}(q^*_{DC})$.

The market response in a simultaneous-move equilibrium with durability commitment is

$$p^*_{\text{DC}} \triangleq p(q^*_{\text{DC}}, r^*_{\text{DC}}) = \max\left\{\frac{1}{2}, p^*_{\text{SMP}}\right\},$$

by virtue of Prop. 3.4, independent of (δ, c).

Comparison with the Stackelberg Regime. We now compare the solutions for the regimes with full commitment (Stackelberg) and partial/no commitment, respectively, in the simultaneous-move stage game, in terms of durability, retail price, and equilibrium profit.

1. *Durability:*

   $$q^*_{\text{DC}} \le q^*_{\text{SMP}} = q^*_{\text{SP}}.$$

   With full commitment the firm finds it optimal to provide full-durability as long as there is an active peer-to-peer market. With partial commitment and sufficiently low production costs, the product durability becomes imperfect (i.e., less than 1), making planned obsolescence a part of the profit-maximizing design.

2. *Retail Price*:

   $$\max\{r^*_{\text{SMP}}, r^*_{\text{DC}}\} < r^*_{\text{SP}} \quad \text{and} \quad r^*_{\text{SMP}} \lesseqgtr r^*_{\text{DC}}.$$

   Whereas in a Stackelberg regime the firm benefits from a positive sharing premium on each unit sold, the purchase price in a simultaneous-move play (as described in Prop. 3.6 or La. 3.6) is not necessarily higher than the retail price induced by durability commitment regime (as described in Prop. 3.7).

3. *Profit:*

   $$\Pi^*_{\text{SMP}} \le \Pi^*_{\text{DC}} < \Pi^*_{\text{SP}}.$$

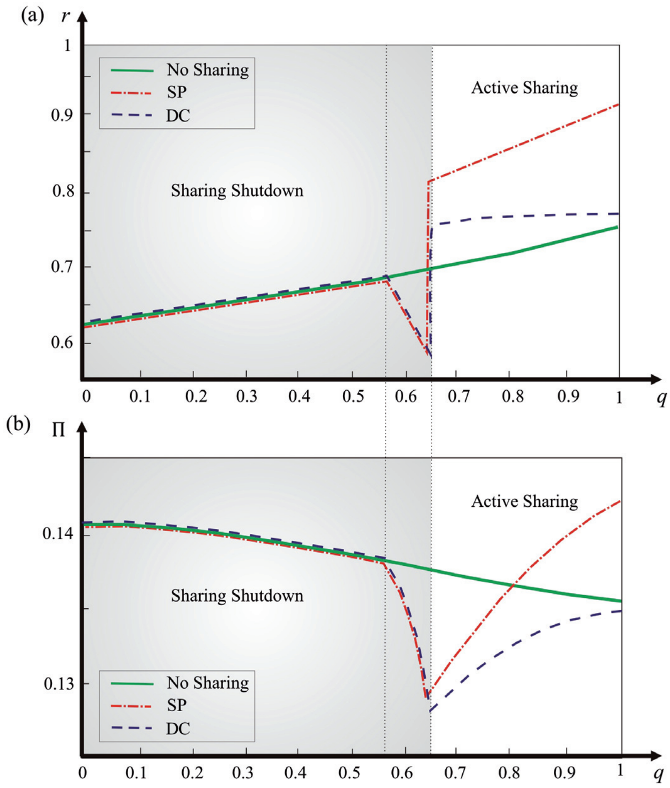

Figure 3.6: (a): Optimal price; (b) optimal profit, for $(\delta, c) = (0.6, 0.25)$.

The profit in the Stackelberg regime always exceeds the profit obtained with partial or no commitment. In terms of sensitivity to parameters, the responsiveness to changes in costs or customer patience satisfies the following inequalities:

$$\frac{\partial \Pi^*_{\mathrm{SP}}}{\partial c} < \frac{\partial \Pi^*_{\mathrm{SMP}}}{\partial c} \leq \frac{\partial \Pi^*_{\mathrm{DC}}}{\partial c} < 0 < \min\left\{\frac{\partial \Pi^*_{\mathrm{SP}}}{\partial \delta}, \frac{\partial \Pi^*_{\mathrm{SMP}}}{\partial \delta}, \frac{\partial \Pi^*_{\mathrm{DC}}}{\partial \delta}\right\},$$

for all $\delta \in (0,1]$ and $c \in [0, \bar{c}]$.

In addition to optimizing the product design in the presence of a sharing market, the firm can use product design to deliberately inhibit sharing, a possibility we examine next.

3.3.3 Sharing Shutdown

By undercutting the sharing market, the firm has the option to effectively disable the peer-to-peer economy. Specifically, if for a given level of durability $q \in [0,1]$ a retail price r is chosen below the lower bound $\underline{r}(q)$, then the liquidity condition (L') is violated and therefore the transaction volume on the sharing market drops to zero. Conditional on the resulting "sharing shutdown"

(SS), the firm's best price is

$$r_{\mathrm{SS}}(q) \triangleq \min\left\{\underline{r}(q), \hat{r}(q)\right\}, \tag{3.39}$$

where the liquidity bound $\underline{r}(q)$ and the no-sharing price $\hat{r}(q)$ appear in Eqs. (L') and (3.9), respectively.

Shutdown Pricing. We first assume that the firm is pre-committed to a certain durability level, for example, having disbanded its design team. When choosing an optimal retail price, conditional on q, the company would prefer a functioning peer-to-peer exchange to sharing shutdown if and only if

$$\hat{\Pi}(q, r_{\mathrm{SS}}(q)) \le \Pi_j(q, r_j(q)),$$

where $j \in \{\mathrm{SP}, \mathrm{DC}\}$, depending on the relevant commitment regime. Fig. 3.6 shows the optimal price and the profit, for full product-design commitment (SP), durability commitment (DC), and in the absence of sharing. In order to choke off the sharing market, the firm may need to sacrifice a portion of the optimal no-sharing profit, charging the highest possible no-sharing retail price $\underline{r}(q)$ per unit. The shaded area highlights the durability levels for which the sharing market is active. In this area, the firm may not gain from the presence of a peer-to-peer market; however, it would be too costly to drop its rate to the shutdown price $\underline{r}(q)$.

Fig. 3.7 provides an overview of the viability of a sharing market, as controlled by the firm's choice of its product design. For any given level of production cost c, it can lower the level of durability q and/or adjust its price to $r_{\mathrm{SS}}(q)$. For very small levels of durability the no-sharing price $\hat{r}(q)$ is optimal whereas for intermediate durabilities the company prefers to disable sharing at the choke-off retail rate $\underline{r}(q)$. For large durability levels, sharing is preferred—all things considered. Given the ability to at least partially commit to its product design by fixing the level of its durability, the firm is able to extract a sharing premium when its user base is faced with an active peer-to-peer economy.

Shutdown Design. We now examine the firm's option to disable the sharing market when optimizing over its full product design, including retail price and durability. Given the commitment regime $j \in \{\mathrm{SP}, \mathrm{DC}\}$, the firm finds it optimal to use a "sharing-shutdown design" $(r_{\mathrm{SS}}^*, q_{\mathrm{SS}}^*)$, with $r_{\mathrm{SS}}^* = r_{\mathrm{SS}}(q_{\mathrm{SS}}^*)$, as long as

$$\max_{q \in [0,1]} \Pi_j(q, r_j(q)) \le \max_{q \in [0,1]} \hat{\Pi}(q, r_{\mathrm{SS}}(q)) = \hat{\Pi}(r_{\mathrm{SS}}^*, q_{\mathrm{SS}}^*) \triangleq \Pi_{\mathrm{SS}}^*,$$

where $r_{\mathrm{SS}}(q)$ is the sharing-shutdown price in Eq. (3.39). Note in particular that the monopolist can always obtain the sharing-shutdown profit Π_{SS}^* by strategically reducing its product durability. Fig. 3.8 shows the optimal durability for SP and DC, as well as SS. By Prop. 3.9 in the Stackelberg regime, the most profitable way for the firm to disable sharing is to produce at zero durability. Note

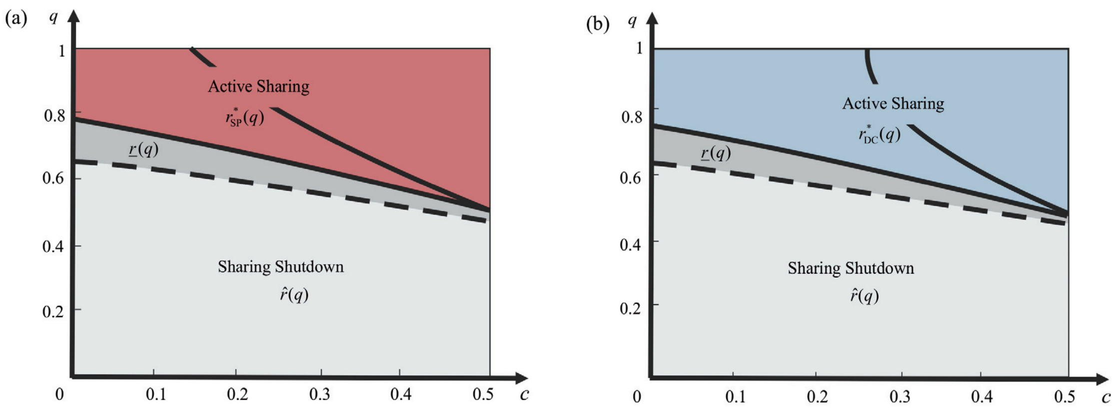

Figure 3.7: Active sharing vs. sharing shutdown in the durability-cost space ($\delta = 0.6$), for both commitment regimes: (a) SP; (b) DC.

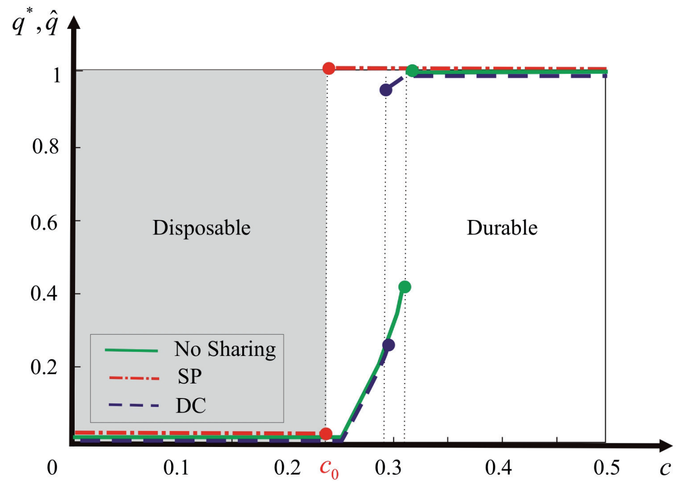

Figure 3.8: Durability as a function of production cost ($\delta = 0.6$).

that in a simultaneous-move play, the firm might be willing to choose an intermediate durability level that is small enough to disable the sharing market and yet maximizes the no-sharing profit.

Proposition 3.9 (**Extreme Durability with Full Commitment).** *In the Stackelberg regime (SP), there exists a production-cost threshold $c_0 \in [0, \check{c}]$, such that the firm's optimal durability is 0 (disposable) for $c \leq c_0$ and 1 (perfect) for $c \geq c_0$.*

Some firms have started to complement their product portfolio with sharing. Producers of durable goods such as car manufacturers have launched one-time-use products and services in response to the emergence of the sharing economy. For example, BMW's DriveNow service allows its customers fee-based access to a fleet of cars (for its BMW and MINI brands).[14] The latter can be

[14]Similarly, Audi's pilot program "Audi On Demand" offers one-day car rentals, while Ford's GoDrive introduces a pay-per-minute car-sharing program.

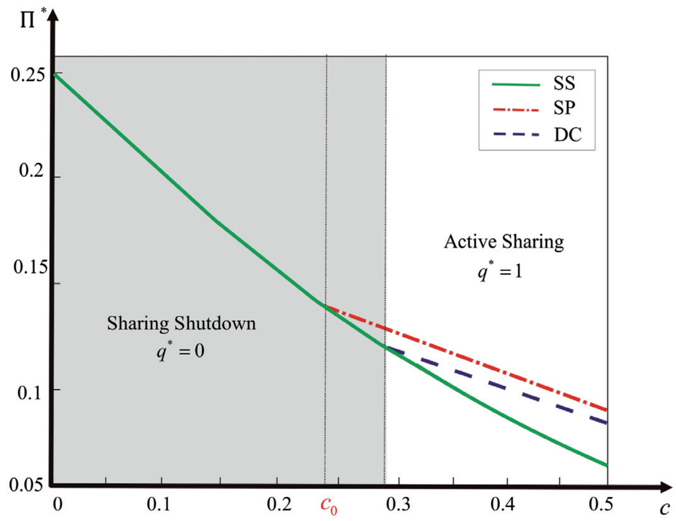

Figure 3.9: Firm's profit with and without sharing markets ($\delta = 0.6$).

viewed as a portfolio of disposable products. Consistent with the model, the high-cost models (e.g., BMW 6 or 7 series) are *not* part of the DriveNow fleet.

Remark 3.8. Making products *shareable* may be costly.[15] Let κ be the additional unit cost for producing a shareable item. In the Stackelberg commitment regime, if $\kappa \geq \check{c} - c_0$, then the optimal durability is necessarily strictly greater than 0, and the rental program is no longer an option. It might therefore become more profitable to invest in promoting the sharing market. For example, in an attempt to promote peer-to-peer markets, GM has recently invested $500 million in Lyft.

Remark 3.9. The optimal profit in Fig. 3.9 has a convex kink at $c = c_0$. The monopolist benefits from cost function uncertainty around this point. For example consider an R&D investment on cost reduction with random realizations in $\{c_0 - \Delta, c_0 + \Delta\}$, with probabilities φ and $1 - \varphi$, respectively, where $\varphi \in (0, 1)$ and $\Delta \in (0, c_0)$. The low realization encourages the firm to deactivate the sharing market, whereas the high realization requires the firm to promote sharing. The mean-preserving lottery therefore increases the monopolist's expected payoff (by Jensen's inequality). Hence, any R&D investment of $R \leq (1 - \varphi)\Pi^*(c_0 + \Delta) + \varphi\hat{\Pi}^*(c_0 - \Delta)$ is a profitable investment for a risk-neutral firm.

3.4 Consumer Surplus and Social Welfare

We now examine how social welfare depends on the presence of a sharing economy (be it active or not) and a firm that optimizes its product design in terms of retail price and durability.

[15] The control and pricing of shareability is discussed by Weber (2017).

Consumer Surplus without Sharing. At each time $t \geq 1$, the consumers' aggregate benefit is the sum of the surplus belonging to young and mature consumers currently present. At the firm's optimal no-sharing product design $(\hat{q}, \hat{r})$, total consumer surplus is

$$\widehat{\mathrm{CS}}(\hat{q}, \hat{r}) = \widehat{\mathrm{CS}}_0(\hat{q}, \hat{r}) + \widehat{\mathrm{CS}}_1(\hat{q}, \hat{r}).$$

In the early consumption phase ($\mathscr{C}_0$), only those who purchase the product obtain a positive surplus, leading to the consumer surplus

$$\begin{aligned} \widehat{\mathrm{CS}}_0(\hat{q}, \hat{r}) &= \int_0^1 \left(\int_{\hat{r}/(1+\delta\hat{q}\theta)}^1 (v - \hat{r})\, dv \right) \theta\, d\theta \\ &= \frac{1}{4} + \frac{\hat{r}}{\delta\hat{q}} \left(\frac{(3/2) + \delta\hat{q}}{1 + \delta\hat{q}} r - \frac{\delta\hat{q}}{2} \right) - \frac{3}{2} \left(\frac{\hat{r}}{\delta\hat{q}} \right)^2 \ln(1 + \delta\hat{q}). \end{aligned} \tag{3.40}$$

In the late consumption phase ($\mathscr{C}_1$), consumer surplus is

$$\begin{aligned} \widehat{\mathrm{CS}}_1(\hat{q}, \hat{r}) &= \int_0^1 \left[\hat{q} \left(\int_{\hat{r}/(1+\delta\hat{q}\theta)}^1 v\, dv \right) + (1 - \hat{q}) \left(\int_{\hat{r}}^1 (v - \hat{r})\, dv \right) \right] \theta^2 d\theta \\ &\quad + \int_0^1 \left(\int_{\hat{r}}^1 (v - \hat{r})\, dv \right) (1 - \theta)\theta\, d\theta \\ &= \frac{(3 - 2\hat{q})(1 - \hat{r})^2}{12} + \frac{\hat{q}}{6} - \left(\frac{\hat{r}}{\delta\hat{q}} \right)^2 \left(\frac{\hat{q}(2 + \delta\hat{q})}{2(1 + \delta\hat{q})} - \ln(1 + \delta\hat{q}) \right). \end{aligned} \tag{3.41}$$

The first term is the surplus of those who are in a high-need state over the full lifecycle, depending on whether they buy the item once or twice (after product failure), respectively. The second term corresponds to the surplus of first-time purchasers in their late consumption phase.

Consumer Surplus with Active Sharing Market. Similarly, when the peer-to-peer economy is liquid, for a market price p and a product design (q, r) in equilibrium, total consumer surplus at each time $t \geq 1$ is, as before, the sum of the surplus gained by both overlapping generations,

$$\mathrm{CS}(p, q, r) = \mathrm{CS}_0(p, q, r) + \mathrm{CS}_1(p, q, r).$$

Consumer surplus of the young generation (in $\mathscr{C}_0$) consists of the purchasers' ownership benefits,

$$\mathrm{CS}_0(p, q, r) = \int_0^1 \left(\int_{r - \delta q p}^1 (v - r)\, dv \right) \theta\, d\theta = \frac{(1 - r)^2 - (\delta q p)^2}{2}. \tag{3.42}$$

Consumer surplus for the mature generation (in $\mathscr{C}_1$) is

$$
\begin{aligned}
\mathrm{CS}_1(p,q,r) =& \int_0^1 \left[q\left(\int_p^1 v\,dv\right) + (1-q)\left(\int_p^1 (v-p)\,dv\right)\right]\theta^2 d\theta \\
&+ \int_0^1 \left(\int_p^1 (v-p)\,dv\right)(1-\theta)\theta\, d\theta \\
&+ qp\int_0^1 \left[\left(\int_{r-\delta qp}^{p} dv\right) + (1-\theta)\left(\int_p^1 dv\right)\right]\theta\, d\theta \\
=& \frac{(1-p)^2}{12} + \frac{qp}{2}(p-r+\delta qp) + \frac{q}{6}(1-p)(1+2p).
\end{aligned}
\tag{3.43}
$$

The first term collects the surplus of those always in high need (including owners who do not participate in sharing and former owners who borrow after product failure); the second term corresponds to the non-owners in a high-need state who borrow the item; finally, the third term contains the surplus of those who lend out items acquired in the early consumption phase.

Fig. 3.10(a) shows the consumer surplus as a function of the production cost. In the absence of sharing, the consumers' aggregate surplus is decreasing in the production cost for disposable products (where $\hat{q} = 0$). This is because the higher production costs tend to increase the purchase price, so that less consumers purchase the item at a smaller surplus. As c increases further, the optimal durability becomes strictly positive, and the consumer surplus goes up as long as the durability is less than perfect ($\hat{q} < 1$). This happens because the price increase is more than offset by a higher product quality. For perfectly durable goods, the consumer surplus decreases once again in c, for the consumers are charged more but cannot obtain compensation because the quality is already maximal. With active sharing on the other hand, consumers are never worse off in a durability-commitment regime, where $(q,r) = (q^*_{\mathrm{DC}}, r^*_{\mathrm{DC}})$ and $p = p^*_{\mathrm{DC}}$. However, in a Stackelberg regime, where $(q,r) = (q^*_{\mathrm{SP}}, r^*_{\mathrm{SP}})$ and $p = p^*_{\mathrm{SP}}$, when the firm charges a high sharing premium, consumers do not necessarily benefit from the peer-to-peer economy. In fact, when the production cost is high enough (such that the optimal durability is 1), consumers are better off without sharing. Conversely, they benefit from sharing when the active secondary exchange provides sufficient incentives for the firm to provide highly durable items.

Social Welfare. The social welfare in the economy is the sum of the consumer surplus and the firm's profit. Without sharing it is

$$\hat{W} = \hat{\Pi} + \widehat{\mathrm{CS}},$$

where—in the presence of an active sharing economy—social welfare amounts to

$$W_j = \Pi^*_j + \mathrm{CS}_j,$$

where $j \in \{\mathrm{DC}, \mathrm{SP}\}$ denotes the commitment regime. Fig. 3.10(b) shows the social welfare in the benchmark case of no sharing, as well as in the two regimes with sharing, i.e., durability-

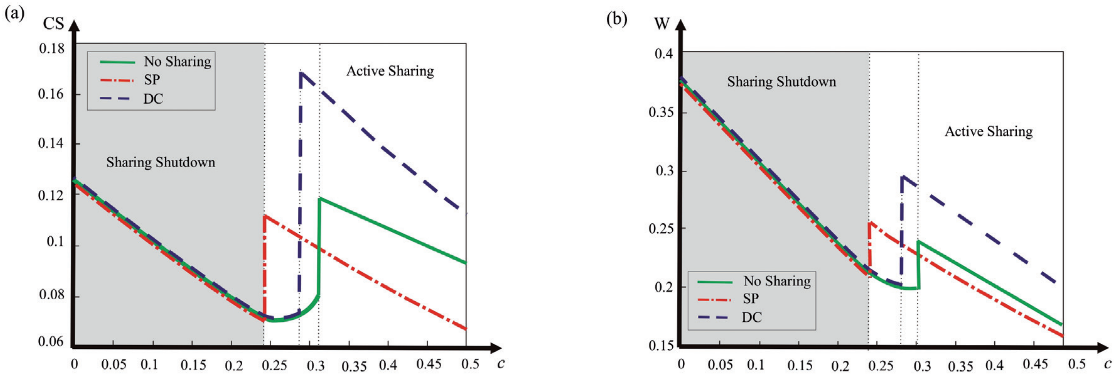

Figure 3.10: (a): Consumer surplus; (b): welfare, as a function of production cost ($\delta = 0.6$).

commitment and Stackelberg play. Social welfare follows a similar pattern as consumer surplus: in DC, when the sharing market and the firm adjust their pricing decisions fast, social welfare increases in production cost. In SP, the economy benefits from active sharing for intermediate production costs only. Sharing increases social welfare when otherwise imperfect durability would be optimal; it also provides incentives for the firm to produce perfectly durable goods.

3.5 Conclusion

The sharing economy is competing with the traditional manufacturers by attracting infrequent consumers that are interested in short term access rather than ownership. The emerging competition and the change in consumption pattern create the need for manufacturers to develop response strategies that allow them to adapt to the new environment. In this regard, pricing and product design are among the deliberate tools enabling manufacturers to influence the secondary peer-to-peer market.

The dynamic overlapping generations model with heterogeneous consumers presented in this paper is the first to analyze product durability as a strategic design choice in the presence of sharing markets. Durability determines the aggregate amount of sharing supply on the peer-to-peer market, which (in)directly affects the manufacturer's profitability. The consumer's willingness to purchase increases if the product is shareable and can generate additional revenue *or* it is rare or non-existent on the secondary market. In this regard, the firm may want to undertake either a defensive strategy by limiting the competition, or an offensive strategy by capitalizing on the sharing market and staying ahead of the competition.

The defensive strategy aims at shutting the sharing market down. There are two mechanisms that effectively allow the firm to do so. First, reducing the durability increases the failure rate of products and shrinks the sharing supply. This results in an increase of the sharing price and makes the peer-to-peer market unattractive for potential renters. The second mechanism is to lower the purchase price and play down the financial benefits of sharing. This strategy shrinks the demand pool of the sharing market and consequently discourages the supply side to participate in non-profitable sharing transactions.

Although the purchase rates are inevitably lower in the defensive strategy, the firm can compensate for the price drop by increasing the sales volume, so that the effect on the revenue is mitigated. The results show that blocking the peer-to-peer market is a profitable strategic management act where the unit production cost is small, and the consumers are not highly patient.

By employing an offensive strategy, the firm embraces the competition and exploits the existence of the peer-to-peer market. The main focus of the firm is to stay ahead of the game by promoting sharing and changing the consumption pattern. She deliberately offers more durable products that can be shared and reused during the life-span of the consumers. Strategic coexistence with the sharing market allows the manufacturer to charge a sharing premium on every unit sold, and target a finer consumer segment that has the highest willingness to pay. The price increase in this strategy compensates for the decrease in the volume. We show that this strategy is efficient when production is costly, and consumers are patient enough to care about future streams of revenue from sharing.

Our study of the strategic coexistence with the sharing market also extends to the commitment ability of the firm and the adjustment speed of the market. The commitment ability of the firm includes both the price and the design characteristics, while the sharing market can only maneuver over adjusting the sharing price. We study three commitment scenarios, namely Stackelberg game, simultaneous-move play, and an intermediate case where the firm can commit to product design, while the price adjustments happen simultaneously. In a Staclekberg regime, the firm moves first and fully commits to its decisions. The sharing market moves more slowly, observes the firm's strategy and adjusts accordingly. Simultaneous move play describes the situation where the sharing market is flexible enough to predict and respond quickly to the firm's strategic movements. The intermediate case with durability commitment is perhaps more realistic since the design characteristics are sticky, while prices are more prone to change.

We show that the firm's profit is increasing in her ability to commit. In a Stackelberg game, she is able to extract maximum surplus from the consumers and achieve the highest profit. Moreover, as the firm becomes more committed to its actions, the cost threshold at which the firm transitions from sharing shut-down to strategic coexistence decreases. This result implies that for products whose design process is lengthy and rigid, the manufacturers are more willing to embrace the sharing culture and live off the sharing premium. By seeking "commitment devices," firms could encourage themselves to stick to the optimal level of durability and the respective price, and increases their long-term profitability. Even when the sharing market is very flexible regarding pricing, not changing the built-in durability design would financially benefit the firm. Credible promises about the product features encourage consumers to pay high sharing premiums, given that the product ensures them a revenue stream in the course of their lives.

The findings also provide policy makers with a practical insight to find out which durable manufacturers need to be incentivized in order to accept the sharing culture. This is often the case if the unit production cost is small and the firm is able to completely defeat the peer-to-peer market. Otherwise, the firms do not need financial motivation to do so. It is naturally of their best interest to promote sharing and enjoy the revenue increase thanks to the sharing premium.

However, interestingly, we find that the existence of the secondary peer-to-peer market does not necessarily benefit the consumers. It is true that the consumers' choice set expands as they have the option to choose between ownership and access-based consumption, yet, the firm can extract a great portion of the gains from trade, and leave the consumers with a smaller surplus than what they would have obtained without sharing. In fact, for large production costs, providing durability is an optimal decision for the manufacturer, even without sharing. The introduction of the sharing market only allows the firm to ask for higher fees. This is especially the case where the firm has strong commitment to her business decisions. Nevertheless, when the sharing market is sophisticated enough to engage in a simultaneous price adjustment, the competition strictly benefits the consumers and increases their surplus. In this regard, the policy makers may want to help out the sharing markets in becoming more dynamics and responsive to the strategic decisions of the incumbents.

Several interesting extensions to this model may be considered as directions for future research. First, we have reduced the complexity of the problem, by abstracting from cost considerations associated with providing durability. There are a number of ways to achieve planned obsolescence, which may or may not significantly affect the production cost. Action plans such as changes in software in a way that is not compatible with older hardware versions, or stopping to supply spare parts for products of former models can be undertaken with negligible costs. On the other hand, strategies such as the use of non-durable components may notably affect the cost structure of production. In this study, we have assumed that providing durability is not costly for the firm. We conjecture that our results continue to hold qualitatively if the cost structure is quadratic. At higher levels of durability, firms prefer not to increase the production volume, and hence would embrace the sharing market. Nevertheless, it would be interesting to solve a more general case, where providing durability is costly to the producer.

Second, we have treated durability as a failure rate of products. Another interesting approach would be to define durability as the product lifetime, or equivalently, the total number of times a given item can be used. In this regard, although sharing generates revenue for the suppliers, it decreases the product value, as it is likely to stop performing in a shorter period of time. The hazard rate of the product changes with usage, and suppliers and renters may have different approaches towards sharing for items of different ages. Examining the optimal product lifetime in the presence of sharing markets with changing hazard rates could be difficult but valuable.

Finally, for simplicity, we have assumed that the manufacturer offers only one product. We have abstracted from the more complex problem where the manufacturers may want to alter their business portfolio, as the sharing culture develops. Product-line extensions, offering products at different qualities, and including both rental and purchase offers in the portfolio are examples of such business decisions. Our model offers a theoretical benchmark that can be compared to the results of such model extensions.

4 The Advent of the Sharing Culture and its Effect on Product Pricing

4.1 Introduction

Since the introduction of the peer-to-peer markets, sharing is gaining more and more in popularity. Nevertheless, the propensity towards sharing is rather product-specific and varies across cultures. Some societies are more prone to accepting the sharing culture, whereas others are more reluctant to switch from conventional modes of consumption to sharing. According to Nielsen (2014), while only 43% of the population are willing to borrow from others in the North America region, this number in China accounts for 94%. Cultural trust-attitudes, regulations and governmental support, technological development, reliability of financial transactions and the sharing intermediaries, economic status of the population, consumerism, and environmental awareness are among the various factors that affect a population's propensity towards sharing. Evidence supports that the socio-demographic data of the target customers including age, gender, income and social status can predict the consumers' propensity to share. According to PwC (2015), survey results show that consumer groups who are most excited about sharing in the US are 18 to 24-year-old adults, households with income between $50k and $75K, and those with kids in the house under age of 18. Peer-trade propensity also depends on the current consumer base of sharing markets and the resulting word-of-mouth. According to Vision Critical (2014), people are becoming increasingly willing to participate in the sharing activities, and there are roughly equal numbers of recent and prospective users.

The reluctance to engage in sharing activities can be perceived as a cost on the potential consumers. The *propensity mismatch* cost is either a mental burden or an actual adjustment fee, such as transportation cost or consumption tax. Moral-hazard costs also have a hand in the propensity mismatch. Perceived risks, credit checks, license and history checks, and insurance fees are among examples of such costs. In this regard, the peer-trade propensity is not solely an intrinsic characteristic of the individuals. Sharing intermediaries, manufacturers, and regulators can pro-actively affect the propensity mismatch, by taking actions such as adjusting the consumption tax, offering customer service of higher quality, and providing better insurance options.

The consumers' attitude towards peer-to-peer sharing is an important factor that needs to be taken into account by the incumbent firms. In a recent survey by BCG (2017), respondents from various countries were asked whether they have any strong preferences if they had the option of short-term borrowing either from peers or from a established firm. The survey confirmed that the results vary by culture. While in the US 48% of the respondents prefer professional suppliers, only 27% of the German users share the same attitude. The finding clearly indicates that some societies embrace the peer-to-peer sharing culture more than the others. This creates an opportunity for the incumbents to leverage the population's peer-trade propensity and potentially accommodate rental services in their product portfolios.

Anecdotal evidence shows that established manufacturers of durable goods are already responding to the population's positioning towards sharing by introducing their own rental programs, in addition to the classic purchase options. *Car2Go* is a subsidiary of Daimler that offers rental services since 2008. BMW is operating *DriveNow* since 2011, and Audi launched the *Audi On Demand* in 2015, allowing the consumers to rent an Audi instead of purchasing it.

To the best of our knowledge, this study is the first to analyze the firm's incentives to redesign their product portfolios in the presence of sharing markets. Using a game-theoretic approach, we show that the population's peer-trade propensity plays a significant role in the firm's consumption-bundling choice. Introducing rental markets can deliberately deactivate the sharing market if the population shows significant trust in the established brand and the peer-trade propensity is very low. For intermediate sharing propensities, the optimal strategy is to allow both the rental service and the sharing market to operate side-by-side. Although the firm's transaction volume decreases, she can compensate the loss by increasing both the rental rate and the purchase price. For higher sharing propensities, offering the rental service does not benefit the firm. The firm exclusively sells shareable goods and her activities effectively decouples from those of the sharing market. We also study the firm's optimal portfolio regarding menu design and pricing, such that each menu item targets the right consumers.

Our results further show that when the sharing market is non-functional, the firm benefits from lowering the population's peer-trade propensity. However, in the presence of an active sharing market, the firm may have incentives to invest in speeding up the cultural transition towards sharing.

Literature Review

The propensity to trust others was first identified by Mayer et al. (1995). They defined the trust propensity as the "general willingness to trust others" and pointed out that cultural background, personal characteristics as well as educational background and experiences affect people's trust propensity. The cost of mismatch to the sharing economy has been identified in various forms in the sharing literature. Razeghian and Weber (2015) construct a diffusion model for peer-to-peer markets, in a sharing economy in which entry and exit decisions are costly. Varian (2000) showed that when the "transaction cost" of sharing is less than the marginal cost of production, the firm benefits from the existence of the sharing market. Benjaafar et al. (2015) showed that "inconvenience costs" lead to higher ownership incentives, abstracting from the firm's optimal pricing decisions. Horton and Zeckhauser (2016) showed that the "bringing-to-market" costs increase the sales revenue and cause the transaction volume on the sharing market to go down. Jiang and Tian (2016) studied the effect of "moral-hazard cost" on the product design and pricing, and showed that such costs have an ambiguous effect on the firm's profit. Our notion of peer-trade propensity is more general. It not only includes the various forms of frictional costs identified in the literature before, but also highlights the community's tendency toward either provider. Furthermore, paper differs from the previous studies, by allowing the firms to act upon the changes in the peer-trade propensity and design their portfolios accordingly.

This study also relates to the streams of literature that examine the durable-goods monopolist's portfolio choice of selling and renting. Selling the product and/or offering rental services create different future competition for the firm. An early stream of literature including Coase (1972), Bulow (1982), and Stockey (1981) suggested that leasing dominates selling for a durable goods monopolist. Renting deactivates or limits the secondary markets (including sharing markets) and helps the durable-goods monopolist control the market. Desai and Purohit (1998, 1999) demonstrated that leasing is not necessarily more profitable than selling if the sold and leased items do not depreciate at the same rate, or the market is competitive. Bhaskaran and Gilbert (2005) studied the effect of complementary products on the selling and leasing strategies. They studied a hybrid selling and leasing strategys and showed that as the degree of complementarity increases, the optimal fraction of leasing in decreases and the for higher degrees the firm's optimal strategy is pure selling. In line with this result, we show that peer-to-peer transactions complement the purchase option, and as the sharing propensity grows the firm's strategy shifts from mixed bundling to pure bundling. Chien and Chu (2008) showed that selling might be more profitable than leasing when the products exhibit network effects. Our results also implicitly confirm this finding, by showing that selling dominates renting when the size of the peer-to-peer market increases.

This paper builds on Weber (2016) and Razeghian and Weber (2016) who use an overlapping generations model to study the optimal pricing and product design in the presence of sharing markets. Nevertheless, none of the previous studies address the optimal business-model choice, and the effect of the population's sharing propensity is unexplored. Abhishek et al. (2016) compared three business models, namely monopoly, selling and rental, and P2P sponsoring and showed that when the heterogeneity in the usage rate is high, offering sales and rentals at the same time is more profitable for the firm. In this study we allow the peer-to-peer market to naturally form and consider the rental service as a strategic tool to control the peer-to-peer market.

4.2 Model

Consider a profit-maximizing monopolist (“she”) who produces durable goods over an infinite time horizon, at a unit production cost $c > 0$. Each item becomes obsolete after two periods, which is identical to the life span of the consumers in this economy. Each item can be used at most once in each period.

Consumers. As in Weber (2016), we use an overlapping generations models to describe the consumers in this economy. At any period of time $t \in \{0, 1, 2, \ldots\}$, a new generation of agents is born and lives for two periods, termed early consumption phase $\mathscr{C}^0$ and late consumption phase $\mathscr{C}^1$, respectively. Agents cannot inherit units from a past generation, nor pass items to a future generation. Consumers in this economy are heterogeneous with respect to their probability of need. At each period, a given agent finds himself in either a “high” or a “low” need state for the item produced by the monopolist. Let $\tilde{s}^j$ be an agent’s need state in his consumption phase $j \in \{0, 1\}$ with realizations in $\mathscr{S} = \{0, 1\}$, where the realization $s^j = 0$ corresponds to the low-need state and s^j to the high-need state. An agent’s type $\theta \in \Theta = \{\theta_L, \theta_H\}$ describes the i.i.d. probability with which he finds himself in the high-need state at any given time,[1] i.e.,

$$P(\tilde{s}^j = 1) = \theta.$$

When an agent is born, he learns his type θ and observes his need state s^0 in the early consumption phase $\mathscr{C}^0$. For simplicity, we assume that all consumers derive the common monetary value v of a unit consumption. Hence, an agent’s utility function $u : \mathscr{S} \to \mathbb{R}$ maps the subjective realization of a need state s to a willingness-to-pay, so

$$u(s) = sv,$$

where $v > 0$ is an agent’s consumption value in case of a high need for the item.[2] The type distribution is fully described by $\lambda \in [0, 1]$, which denotes the seller’s belief that a given agent has a high probability of need:

$$P(\tilde{\theta} = \theta_H) = \lambda. \qquad \big(= 1 - P(\tilde{\theta} = \theta_L)\big)$$

Without loss of generality, the number of consumers in each generation is normalized to 1. Hence, the fraction of high-type consumers in the total population (of size 2) is 2λ. All agents have the same discount factor $\delta \in [0, 1]$; furthermore, they are risk-neutral and rational (i.e., expected-payoff-maximizing) decision-makers given all information available to them.

[1] While consumers’ types are persistent, need realizations are uncorrelated across the two consumption phases.

[2] To keep the analysis tractable, all agents have the same consumption value v; in this way, the agent’s expected utility is similar to the standard utility function used by Mussa and Rosen (1978) where the likelihood of need takes the role of marginal utility.

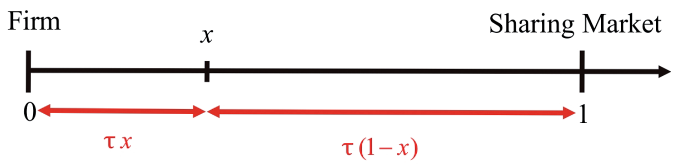

Figure 4.1: Sharing propensity and the cost of mismatch.

Firm. In an attempt to capture as much consumer surplus as possible, the monopolist provider of the good pursues a second-degree price-discrimination strategy, offering a consumption menu containing both *rental service* and *purchase* as options. Any consumption menu is characterized by a tuple $(\rho, q) \in \{(\phi, 1), (r, 2)\}$, where ρ is the price and q is the duration of service offered by the monopolist.[3] The rental service is offered at price ϕ and is valid for consumption in one period. The purchase option is available at price r and can be used over the buyer's remaining lifetime. The purchase premium

$$\pi = r - \phi,$$

is the surcharge that a consumer pays to obtain unlimited usage rights.

Sharing Market. To exchange usage rights during their lifetime, consumers can transact on a sharing market provided the latter is "active" (in the sense that total transaction volume is positive). The sharing price p matches supply with demand in this secondary peer-to-peer economy. If an active sharing market exists, non-owners can decide whether to rent from the firm, borrow from the sharing market, or purchase.

Peer-Trade Propensity ("Sharing Propensity"). Depending on the general sentiment in the agent population, consumers collectively experience a peer-trade propensity, which we refer to as "sharing propensity" in the context of our analysis. As the BCG (2017) survey revealed, peer-to-peer rentals are not always preferred to the services offered by the firm, and different cultures may lean towards one or another. The peer-trade propensity mismatch proposed in this model corresponds to a (perceived) transportation cost for agents that transact with the sharing market. Moreover, there are also cost burdens (mental or physical) on the consumers that use the firm's services. As a representation we employ the standard linear city by Hotelling (1929) where the monopolist and the sharing market are located at the two extremities of a line segment $\mathscr{X} = [0, l]$ with fixed length $l > 0$. All consumers have the common sharing propensity $x \in \mathscr{X}$ and higher values of x represent higher willingness to engage in sharing activities. To get access to the firm, agents with sharing propensity x are subject to the linear transportation cost τx, where τ is the unit peer-trade mismatch cost.[4] Similarly, accessing the sharing market costs $\tau(l - x)$ to the agents. For simplicity, we normalize the distance between the firm and the sharing market by

[3]Since consumers live for only two periods, the contract durations on offer need not exceed 2.

[4]Throughout the text we use "propensity-mismatch cost" and "transportation cost" interchangeably.

setting $l = 1$, so the firm and the sharing market are located at $x = 0$ and $x = 1$, respectively. We further assume that $v \geq \tau$, to rule out the uninteresting situations where consumers are *a priori* not interested in consumption. Figure 4.1 illustrates locational sharing-propensity representation. As alluded to Sec. 4.1, location x describes the general trend in a population's evolving peer-trade propensity.

Remark 4.1. The transportation cost can be re-interpreted differently. For example, as in Varian (2000), it could include the actual cost of travelling to each market, or alternately, the willingness to pay to avoid the perceived inconvenience from engaging in transactions with the respective counterparties. In addition, the transportation cost may include any taxes imposed by a regulator and levied on consumers for accessing either market. In this last view, the firm, the sharing market, and the regulator may all play active roles in shaping the sharing propensity x, in medium- to long-run.

4.3 Equilibrium Analysis

As described in the previous section, the consumer population is stationary, and because of the recurring nature of all choices and simultaneous presence of all generations, the consumer types' aggregate consumption choices are stationary, so that the sharing price needs to be stationary as well. At the level of an individual consumer, however, there is no reason that the choice behavior is stationary, i.e., invariant across the two consumption phases. To analyze equilibrium behavior of all parties involved, we will use the concept of subgame-perfect Nash equilibrium by Selten (1965). For this, we start by considering the agents' consumption decisions using backward induction for a given generation, born at time $t \geq 1$.

Renting vs. Borrowing in $\mathscr{C}^1$. Non-owners in their late consumption phase, conditional on a realized need, may choose any of the available options to get access to a product, including rental, purchase, or sharing. Nevertheless, the end-of-horizon effect implies that it is never optimal for non-owners to choose the purchase option $(r, 2)$ and incur the surcharge $\pi \geq 0$, given that they would not be able to take advantage of the full usage duration. Hence, a non-owner of type $\theta \in \Theta$, with sharing propensity $x \in [0, 1]$, would *either* choose to rent at the price ϕ *or* to borrow at the price p. Specifically, in the high-need state, the rental service is preferred if and only if

$$v - \phi - \tau x \geq v - p - \tau(1 - x),$$

or equivalently:

$$x \leq \frac{1}{2}\left(1 + \frac{p - \phi}{\tau}\right). \tag{4.1}$$

For any given prices p and ϕ, as long as the sharing propensity x is small enough, by condition (4.1) non-owners do prefer to rent from the firm rather than getting access to the product on a sharing market. By contrast, when the sharing propensity is high, so condition (4.1) is violated, non-ownwers prefer the sharing market as channel to satisfy their need for the product through peer-to-peer access. Hence, a type-θ non-owner's expected payoff becomes [5]

$$\bar{U}(\theta;x) = \theta\left[v - \min\{p + \tau(1-x), \phi + \tau x\}\right]_+.$$

Remark 4.2. If $\phi - p \geq \tau$, then there would be no $x \in \mathscr{X}$ at which the firm's rental service is used by the consumers. The price difference justifies costly travels to the sharing market, even if the sharing propensity is at its lowest level. When condition (4.1) is not satisfied, the rental service, even if offered by the firm, is not able to attract customers and fails to compete with the peer-to-peer market. We elaborate more on this case in Sec. 4.3.2.

Lending vs. Using in $\mathscr{C}^1$. An owner derives utility from his product in his late consumption phase only if he finds himself in need for the item ($s^1 = 1$) for the second time in his life. In the low-need state, the utility of consumption is $u(0) = 0$, however, the sharing possibility can generate revenue for the owners. For the owners, sharing is attractive only if the transportation cost is compensated by the sharing price. That is

$$p - \tau(1-x) > 0. \tag{4.2}$$

Hence, the expected indirect utility of an owner of type $\theta \in \Theta$ with sharing propensity $x \in \mathscr{X}$ in his late consumption phase is

$$V(\theta;x) = \theta v + (1-\theta)[p - \tau(1-x)]_+.$$

Note that only if $x > 1 - p/\tau$, owners supply their unused resources on the market. Otherwise, the peer-to-peer market collapses.

Remark 4.3. In equilibrium, the owner's payoff from consumption necessarily exceeds the net profit obtained from sharing, i.e.,

$$p - \tau(1-x) \leq v. \tag{4.3}$$

If Eq. (4.3) is not satisfied, then $v \leq p$, which translates into no agent being willing to borrow from the sharing market. This results in a collapse of the sharing market. Therefore, Eq. (4.3) always holds and ensures that owners participate as suppliers in the sharing market only if they find themselves in the low-need state. We assume that in the case of equality, the agents tend not to share.

As a result of Eqs. (4.1)-(4.2)-(4.3), the equilibrium sharing price is bounded from below and above, such that

$$\tau(1-x) < p \leq \min\{v - \tau(1-x), \phi + \tau(2x-1)\}. \tag{4.4}$$

[5] $[z]_+ \triangleq \max\{0, z\}$, for all $z \in \mathbb{R}$.

The lower bound on the sharing price ensures that the suppliers get a positive payoff. The upper bound prevents the renters from getting a negative payoff and ensures that they are not strictly better off by abandoning the sharing market and renting from the firm. The sharing market is shut-down, as soon as the sharing price drops to its lower bound. In what follows, we study the two cases separately and determine the conditions under which the sharing market is (in-)active.

4.3.1 Inactive Sharing Market

We start by analysing the case where the sharing market is not attractive to the consumers and hence is forced to be inactive. In equilibrium, the unpopularity of sharing brings the sharing price p down to its lower bound, at which the owners are no longer willing to share. By Eq. (4.4),

$$p \equiv p(x) = \tau(1-x), \tag{4.5}$$

at which the peer-to-peer market is effectively disabled. In the late consumption phase, non-owners in the high-need state have the sole option of renting from the firm at price ϕ. The individual rationality (IR) constraint ensures that the renters get a non-negative payoff from renting, i.e.,

$$\phi \leq v - \tau x. \tag{IR}$$

Hence, the expected indirect utility of non-owners is

$$\bar{U}(\theta; x) = \theta(v - \phi - \tau x). \tag{4.6}$$

Without sharing, owners in the late consumption phase derive utility only from self consumption. Therefore, for an owner of type $\theta \in \Theta$, the expected indirect utility is

$$\bar{V}(\theta; x) = \theta v. \tag{4.7}$$

Purchase Decision in $\mathscr{C}^0$. We now examine which consumer types are willing to invest in purchasing. The incentive compatibility (IC) constraint ensures that the purchasers benefit the most from choosing the menu-item that is specifically designed for them. That is

$$v - r - \tau x + \delta \bar{V}(\theta; x) \geq v - \phi - \tau x + \delta \bar{U}(\theta; x). \tag{IC}$$

The (IC) constraint prevents the buyers from consuming the rental option and determines which consumer types (if any) belong to the purchasers group. This is formally shown in the following lemma.

Lemma 4.1. *In the case of no-sharing, an agent of type $\theta \in \Theta$ with sharing propensity $x \in \mathscr{X}$*

opts for the purchase option $(r,2)$ *if*

$$\theta \geq \frac{\pi}{\delta(\phi+\tau x)} \triangleq \underline{\theta}(\phi,\pi;x).$$

La. 4.1 implies that by strategically setting the rental price ϕ and the purchase premium π, the firm can control whether only high-types, both types, or neither type would invest in purchasing.

Monopolist's Optimal Menu Design

The sharing market may be inactive either in a natural or an induced manner. If the population's sharing propensity is low enough, such that either the potential suppliers or borrowers get a non-positive payoff from sharing, i.e., if

$$[v-p-\tau(1-x)][p-\tau(1-x)] \leq 0,$$

the peer-to-peer market naturally does not form. By Eqs. (4.4)-(4.5), sharing market is naturally choked-off if

$$x \leq \min\left\{0, 1-\frac{v}{2\tau}\right\} \triangleq \underline{x}. \tag{4.8}$$

The sharing threshold $\underline{x}$ depends on the consumers' consumption value v as well as the unit cost of transportation τ. For a given propensity $x \in \mathscr{X}$, sharing is not feasible when the transportation cost is very high compared to the usage value. This discourages all the consumers from choosing the peer-to-peer market, regardless of the sharing price. On the other hand, if τ is sufficiently small such that $2\tau \leq v$, *a priori* the peer-to-peer market is active for all $x \in \mathscr{X}$. The transportation cost is always smaller than consumption value, which creates profitable sharing opportunities for suppliers.

If $x > \underline{x}$, the firm might be still able to strategically deactivate the sharing market. This is achievable by lowering the rental price such that Eq. (4.1) remains satisfied. The peer-to-peer market is choked-off and consumers are discouraged from engaging in sharing activities. The following lemma characterizes the propensity regions, in which natural and induced choke-off strategies are viable.

Lemma 4.2.

1. *If* $x \leq \underline{x}$*, the sharing market is "naturally inactive," and the optimal rental price is* $\phi \equiv \phi(x) = v - \tau x$.
2. *If* $\underline{x} \leq x \leq 2/3$*, then the "induced choke-off strategy" is viable, by offering the rental price* $\phi \equiv \phi(x) = \tau(2-3x)$.

La. 4.2 implies that when sharing propensity is too small, the firm is able to increase the rental price, such that the renters are left with zero surplus. However, as the sharing propensity grows, the firm's only tool to defeat the peer-to-peer market is to lower the rental price and offer a more attractive option to the consumers. In this situation, the (IR) constraint does not bind, and the renters enjoy a strictly positive gain. In other words, for intermediate sharing propensities where the consumers have strong preferences towards neither the firm nor the sharing market, the competition between the firm and the sharing market works in favor of the consumers.

By optimally setting the sharing premium π, the firm is able to pursue second-degree price discrimination in the form of three different consumption-bundling strategies, namely *pure rental*, *high-end selling & rental*, and *mass selling & rental*. The distribution of the agents together with the cost structure associated with each bundling strategy determine the monopolist's optimal menu design and pricing.

Remark 4.4. Note that renting is not associated with the same marginal cost as selling. Each item that is not sold but rented out can be offered on the market for two periods. Hence, if the demand for the rental service in steady state is $D_\phi \in [0, 2]$ units, the firm needs to produce $(D_\phi/2)$ units at each period. This is equivalent to having a unit production cost of $c/2$ for a rental service.

In what follows, we characterize the three possible pricing and menu design strategies, when sharing is not feasible.

Pure Rental. If the purchase premium is high enough, such that

$$\theta_H \leq \underline{\theta}(\phi, \pi; x), \tag{4.9}$$

then by La. 4.1 purchasing is not attractive to any consumer segment.[6] This is equivalent to eliminating the purchase option, and exclusively offering rentals to both generations when they find themselves in the high-need state. The monopolist's profit from employing this strategy is

$$\Pi_0(x) = 2((1-\lambda)\theta_L + \lambda\theta_H)\left(\phi(x) - \frac{c}{2}\right), \tag{4.10}$$

where $\phi(x) \in \{v - \tau x, \tau(2-3x)\}$ is the optimal rental price, as specified by La. 4.2.

High-end Selling & Rental. If the firm's pricing strategy is such that

$$\theta_L \leq \underline{\theta}(\phi, \pi; x) \leq \theta_H,$$

then the purchase option $(r, 2)$ is exclusively designed for the consumers of type θ_H in the early consumption phase. In equilibrium, it is always optimal for the firm to increase the purchase premium such that the (IC) constraint binds for the high-type consumers. That is

$$\pi \equiv \pi(x) = \delta\theta_H(\phi + \tau x). \tag{4.11}$$

[6] By setting $\pi = v$, Eq. (4.9) is automatically satisfied.

The high type non-owners in $\mathscr{C}^1$, as well as the low-type agents in $\mathscr{C}^0$ and $\mathscr{C}^1$ are served by the rental service. The profit obtained from employing this strategy is

$$\Pi_1(x) = \lambda\theta_H(r(x) - c) + (\lambda\theta_H(1-\theta_H) + 2(1-\lambda)\theta_L)\left(\phi(x) - \frac{c}{2}\right), \tag{4.12}$$

where by La. 4.2 and Eq.(4.11), the prices are such that

$$(\phi(x), \pi(x)) \in \{(v - \tau x, \delta\theta_H v), (\tau(2-3x), 2\delta\theta_H\tau(1-x)\}. \tag{4.13}$$

Mass Selling & Rental. When the menu design is such that

$$\underline{\theta}(\phi, \pi; x) \leq \theta_L,$$

purchasing is the optimal decision for *all* young agents of any type $\theta \in \Theta$, if they find themselves in need. The monopolist optimally increases the purchase premium such that the (IC) constraint binds for the agents of type θ_L, i.e.,

$$\pi \equiv \pi(x) = \delta\theta_L(\phi + \tau x). \tag{4.14}$$

While this pricing strategy leaves the low-type agents with zero expected surplus, the high-type buyers capture a positive *information rent*. The rental service only serves the non-owners in their late consumption phase. The firm's profit is therefore equal to

$$\begin{aligned}\Pi_2(x) = &(\lambda\theta_H + (1-\lambda)\theta_L)(r(x) - c)\\ &+ (\lambda\theta_H(1-\theta_H) + (1-\lambda)\theta_L(1-\theta_L))\left(\phi(x) - \frac{c}{2}\right),\end{aligned} \tag{4.15}$$

and by La. 4.2 and Eq.(4.14), the optimal pricing strategy requires that

$$(\phi(x), \pi(x)) \in \{(v - \tau x, \delta\theta_L v), (\tau(2-3x), 2\delta\theta_L\tau(1-x)\}. \tag{4.16}$$

One might conjecture that *a priori* under certain conditions, the monopolist might be willing to shut down the rental service and only offer a single purchase option $(r, 2)$. La. 4.3 shows that when sharing is not feasible, such strategy is always dominated and never optimal for the monopolist. The underlying reason is that the firm can always generate more revenue by renting to the non-owners among the older generation.

Lemma 4.3. *If the sharing market is inactive, disabling the rental service is a dominated strategy.*

The optimality of each strategy depends on several factors. Let

$$\{\hat{c}, \hat{v}\} \triangleq \frac{\{c, v\}}{\tau}$$

be the normalized production cost and consumption value. The "pure rental" threshold is defined as

$$x_1 = \min\left\{\hat{c}\left(\frac{1-\theta_H}{2\theta_H}\right) + \hat{v}(1-\delta), \underline{x}\right\}, \tag{4.17}$$

below which it is optimal for the firm to exclusively offer rentals. By omitting the purchase option, the firm is able to avoid the quantity discount and capture the consumer's entire surplus. This strategy is compatible with the optimal leasing strategy of the durable-good monopolists, as spotted in the stream of literature on selling vs. renting by Coase (1972) and Bulow (1982), among others. As the sharing propensity increases, the firm is compelled to lower the rental price and consequently her revenue decreases. At the threshold x_1, the firm finds mixed bundling more profitable. She opens up the sales market at x_1 and offer the rental and purchase options side by side. Note that the purchase option could be designed in a way that targets only high-types, or both groups of consumers. The results show that mixed bundling starts with high-end selling, and as the sharing culture evolves, it transitions to mass selling. Let

$$\ell \triangleq \frac{(1-\lambda)\theta_L}{\lambda\theta_H}, \tag{4.18}$$

be the likelihood-ratio of consumption, that measures how the per period aggregate consumption is divided between the high and low type agents. Furthermore, let

$$\Delta\theta \triangleq \theta_H - \theta_L \tag{4.19}$$

be the difference between the high and low-type agents. The "high-end selling" threshold is defined as

$$x_2 = \min\left\{\hat{c}\left(\frac{1-\theta_L}{2\theta_L}\right) + \hat{v}\left(1-\delta+\frac{\delta\Delta\theta}{\ell\theta_L}\right), \underline{x}\right\}, \tag{4.20}$$

below which high-end selling is the dominant strategy. As the sharing propensity passes this threshold, the firm switches to the mass selling strategy. Prop. 4.1 formalizes this result.

Proposition 4.1. *Let $\underline{x}$, x_1, and x_2 be the propensity thresholds defined by Eqs. (4.8), (4.17), and (4.20). When sharing is not feasible, then*

i. for all $x \in [0, x_1]$, the monopolist's optimal strategy is "pure-rental," where the optimal rental option is $(v - \tau x, 1)$;

ii. for all $x \in [x_1, x_2]$, the monopolist's optimal strategy is "high-end selling & rental," where the optimal consumption menu consists of $(v - \tau x, 1)$ and $((1+\delta\theta_H)v - \tau x, 2)$;

iii. for all $x \in [x_2, \underline{x}]$ the monopolist's optimal strategy is "mass selling & rental," where the optimal consumption menu consists of $(v - \tau x, 1)$ and $((1+\delta\theta_L)v - \tau x, 2)$; and

iv. for all $x \in [\underline{x}, 1]$, the induced sharing choke-off may or may not be optimal.

As $\underline{x} \to 1$, sharing is not feasible for any $x \in \mathscr{X}$. This might be the case when the transportation cost is too high compared to the consumption value. For example for highly personalized goods such as hygienic items, the high mismatch cost prevents the consumers from access-based consumption. Note that the transportation cost can also be *artificially* increased by imposing taxes on consumption. The same effect is observed if the perceived consumption-value decreases. This might happen if other imperfect substitutes for the item are introduced on the market, and the consumers' perceived value of consumption diminishes.

Remark 4.5. If $x_2 < \underline{x}$, offering a choke-off "high-end selling & rental" is a dominated strategy for all $x \in \mathscr{X}$. This is because the profit of choke-off "high-end selling & rental" is smaller than the profit of natural "high-end selling & rental" and at $x = \underline{x}$, this strategy is already no-longer optimal. Similarly, when $x_1 < \underline{x}$, offering a choke-off "pure rental" is a dominated strategy for all $x \in \mathscr{X}$. At $x = \underline{x}$, the profit of choke-off "pure rental" is smaller than the profit of natural "pure rental" and applying this strategy would not be optimal for higher values of x.

Note that the thresholds vary with model parameters. La 4.4 describes the comparative statics when the sharing market is naturally choked-off.

Lemma 4.4. *i. x_1 is non-decreasing in c, and non-increasing δ and θ_H. x_1 is first increasing and then decreasing in v, with the maximum achieved at*

$$v_0 \triangleq \max\left\{0, \frac{\theta_H(2+\hat{c})-\hat{c}}{\theta_H(3-2\delta)}\right\}\tau.$$

ii. x_2 is non-decreasing in c, θ_H, λ, and $\Delta\theta$, and non-increasing in θ_L and ℓ. It is non-decreasing in δ if

$$\ell \le \frac{\Delta\theta}{\theta_L}.$$

x_2 is first increasing and then decreasing in v, with the maximum achieved at

$$v_1 \triangleq \max\left\{0, \frac{\theta_L(2+\hat{c})-\hat{c}}{3\theta_L-\delta(2\theta_L-\Delta\theta)}\right\}\tau.$$

Assume that the intervals $[0, x_1]$, $[x_1, x_2]$, and $[x_2, \underline{x}]$ are non-degenerate. As the consumption value v increases or the unit transportation cost τ decreases, the interval $[0, x_1]$ increases in size. Customers' net willingness to pay increases, which allows the firm to extract all their surplus, by avoiding the quantity discount. Note that at the same time, $\underline{x}$ decreases. Hence, the proportion of the pure rental region to the entire natural choke-off region grows. When x_1 hits the threshold $\underline{x}$ and pure rental is the firm's only strategy in the absence of sharing, the no-sharing propensity region starts to decreases in the consumption value. An increase in the consumption value also increases x_2, the threshold for "high-end selling." This reduces the size of the interval $[x_2, \underline{x}]$ in which "mass selling & rental" is optimal. Consumers' higher willingness to pay allows the firm to serve more consumers by the rental service and offer fewer purchase options. Consequently, the firm can capture more surplus from the consumers.

The same effect exists when the production cost c increases. As it is less costly to satisfy the rental demand, the "pure rental" region increases and the "mass selling & rental" region shrinks, until the two thresholds hit $\underline{x}$. Note that the production cost c does not affect $\underline{x}$. This is because the threshold explicitly depends on the agents' willingness to share or borrow, and their sharing decisions are unaffected by the production cost.

θ_H decreases the threshold x_1 and increases x_2, i.e., the region in which "high-end selling & rental" is optimal becomes larger. The firm is able to increase her profits by augmenting the purchase premium that is adjusted for the high-type consumers. An increase in θ_L or the types difference $\Delta\theta$ does not affect the pure rental threshold, but diminishes x_2. The interval in which "mass selling & rental" is optimal grows in size. It becomes more profitable to include the low-type agents in the purchasing pool, and enjoy the revenue increase. The same effect exists when the number of low-type agents in the economy $(1-\lambda)$ increases. The low-type agents exert externalities on the other consumer segment, such that the high-type agents can benefit from a positive information rent. As λ grows, x_2 approaches $\underline{x}$, the information rent vanishes, and the firm never finds it optimal to design the purchase option in a way that is appealing to both types. The high-type consumers benefit from the existence of enough number of low-type consumers in the economy, whose type difference is not too large.

As consumers become more patient and δ increases, they are willing to invest more in purchasing, resulting in the shrinkage of the "pure rental" region. The effect of an increase in δ on the two regions of "high-end selling & rental" and "mass selling & rental" is ambiguous. Depending on the two types as well as their distribution, each region may increase or decrease in size.

4.3.2 Active Sharing

If the peer-to-peer market is active, either sharing is strictly preferred by all consumers, or the agents are indifferent between borrowing at price p, or renting at price ϕ. Otherwise, the sharing market is not attracting any consumers, and consequently, the sharing price drops to $p = \tau(1-x)$, as discussed in Sec. 4.3.1. In this section, we concentrate on the case where $p > \tau(1-x)$ and characterize the changes in the firm's optimal menu-design as the sharing propensity grows.

In the late consumption phase, non-owners in need can either borrow from the sharing market or rent from the firm. The individual rationality (IR') constraint ensures that in the presence of sharing, the borrowers not only get a non-negative payoff from borrowing but also (at least weakly) prefer to engage in a sharing transaction, i.e.,

$$v - p - \tau(1-x) \geq [v - \phi - \tau x]_+. \tag{IR'}$$

The expected utility of a non-owner is

$$\bar{U}(\theta;x) = \theta(v - p - \tau(1-x)). \tag{4.21}$$

When sharing is welcome, an owner of type $\theta \in \Theta$ consumes his asset in $s^1 = 1$ and supply it on the sharing market in $s^1 = 0$. Hence, the expected utility of an owner in $\mathscr{C}^1$ is

$$\bar{V}(\theta;x) = \theta v + (1-\theta)(p - \tau(1-x)). \tag{4.22}$$

We now examine which consumer segments are willing to choose the purchase option when sharing is possible.

Purchase Decision in $\mathscr{C}^0$. Note that if an agent prefers peer-to-peer borrowing to the rental service in $\mathscr{C}^1$, he does so in the early consumption-phase $\mathscr{C}^0$. If (IR') is satisfied, by non-negativity of the utility function $\bar{U}(\theta;x)$ we get

$$v - \phi - \tau x + \delta\bar{U}(\theta;x) \leq v - p - \tau(1-x) + \delta\bar{U}(\theta;x),$$

which implies that the expected utility of borrowing outweighs the expected utility of renting, even in the first period. In other words, the optimal choice between rental and sharing is time-invariant. It follows that potential purchasers only need to compare the costs and benefits of the purchase option with the ones of accessing the peer-to-peer market. In this regard, the incentive compatibility (IC') constraint needs to ensure that the targeted purchasers do not have any incentive to use other offers available on the market. The purchase option needs to be designed in way that expected benefits of ownership exceeds the expected gains of non-ownership for purchasers. Formally speaking

$$v - r - \tau x + \delta\bar{V}(\theta;x) \geq v - p - \tau(1-x) + \delta\bar{U}(\theta;x), \tag{IC'}$$

where $\bar{U}(\theta;x)$ and $\bar{V}(\theta;x)$ are specified in Eqs. (4.21)-(4.22). The consumer types that satisfy the (IC') constraint belong to the group of purchasers. The following lemma formulates the purchasing criteria for agents of type $\theta \in \Theta$.

Lemma 4.5. *In the presence of an active sharing market, an agent of type $\theta \in \Theta$ with sharing propensity $x \in \mathscr{X}$ chooses the menu item $(r,2)$ if*

$$\theta \geq \frac{1}{2} + \frac{r - p(1+\delta) + \tau(2x-1)}{2\delta\tau(1-x)} \triangleq \bar{\theta}(r;x).$$

La. 4.5 plays a critical role in the monopolist's optimal menu design. By adjusting the pricing strategy, she can target the consumer segments that generate her the highest possible profits.

Monopolist's Optimal Menu Design

Note that if no purchase option is offered by the firm, the sharing market is forced to be inactive, as discussed in Sec. 4.3.1. Hence, the "pure rental" strategy is not compatible with the coexistence of the sharing market and is therefore omitted from the analysis in this section. The feasible set of strategies includes "high-end selling & rental" and "mass selling", which are characterized by the following.

High-end Selling & Rental. In this strategy, the purchase option $(r,2)$ is designed in a way that

$$\theta_L \le \bar{\theta}(r;x) \le \theta_H.$$

While high-type consumers are willing to invest in buying, low-types prefer the sharing service, at all their stages of life. As a result, the total demand for sharing *a priori* equals

$$\bar{D} = 2(1-\lambda)\theta_L + \lambda(1-\theta_H)\theta_H,$$

where the first term corresponds to the low-type agents in the high-need state from both generations, and the second term represents the high-type non-owners, who happen to be in need only in their latest period of life. The sharing supply is provided by the high-type owners who find themselves in the low-need state in the second period. That is

$$S = \lambda\theta_H(1-\theta_H).$$

Clearly, $\bar{D} \ge S$ meaning that the demand for sharing always surpasses the supply. In equilibrium, the competition between the non-owners increases the sharing price up to its upper bound, specified in Eq. (4.4), i.e.,

$$p = p(x) = \min\{v - \tau(1-x), \phi + \tau(2x-1)\}.$$

The dependency of the sharing price on ϕ creates an opportunity for the firm to attract a fraction of the demand for sharing. By offering a rental price ϕ low enough such that

$$p(\phi;x) = \phi + \tau(2x-1), \tag{4.23}$$

the monopolist makes the consumers indifferent between borrowing and renting, given the cost of transportation.

Remark 4.6. If $x \ge 1/2$, then the nominal sharing price is greater than the nominal rental price, and yet peer-to-peer borrowing is (at least weakly) preferred by the consumers. There is anecdotal evidence that in real life the nominal price of sharing services is not the only determinant of consumption. For example, it happens often-time that the Uber's multipliers are high enough such that the price of sharing surpasses the price of a taxi, and yet the sharing service is still steadily used by the consumers.

Knowing that in equilibrium, no agent is strictly better off by borrowing or renting, the firm optimally increases the rental price up to the point where the entire renters' surplus is squeezed out. That is

$$v - p(\phi; x) - \tau(1-x) = v - \phi - \tau x = 0.$$

This determines the optimal rental price

$$\phi \equiv \phi(x) = v - \tau x. \tag{4.24}$$

Consequently, the equilibrium sharing price is

$$p \equiv p(x) = v - \tau(1-x). \tag{4.25}$$

By the prices specified in Eqs. (4.24)-(4.25) the sharing market clears, and the supply and demand are equal to

$$S = D = \lambda\theta_H(1-\theta_H).$$

The remaining portion of the demand

$$D_r = \bar{D} - D = 2(1-\lambda)\theta_L,$$

is served by the firm's rental service.

Remark 4.7. By the set of prices specified in Eqs. (4.24)-(4.25), consumers employ a mixed strategy where they choose the firm or the sharing market with probabilities α and $1-\alpha$, where

$$\alpha = \frac{D_r}{D_r + D} = \frac{2\ell}{2\ell + (1-\theta_H)}.$$

Under a "high-end selling & rental" strategy, the optimality requires the firm to increase the purchase premium π up to the point that the (IC') constraint binds for the θ_H consumers. Substituting Eqs. (4.24)-(4.25) into La. 4.5 determines the optimal purchase premium

$$\pi(x) = \delta(v - 2\tau(1-x)(1-\theta_H)). \tag{4.26}$$

The following lemma compares the optimal purchase premiums with or without sharing, under the high-end selling strategy.

Lemma 4.6. *Assume that $\underline{x} > 0$, such that a region with natural sharing choke-off exists. Under the "high-end selling & rental" strategy, the purchase premium is strictly higher with sharing than without it.*

By La. 4.6, the firm is able to charge a higher purchase premium when the peer-to-peer market is active. Although the firm loses revenue from her rental service due to the competition with the sharing market, she increases her revenue from selling. In other words, the sharing market plays a dual-role for the firm. The sharing service substitutes one product (rental service), but complements the other (the purchase option). In presence of sharing, the high-end selling strategy yields the profit

$$\Pi_3(x) = \lambda\theta_H(\phi(x) + \pi(x) - c) + 2(1-\lambda)\theta_L\left(\phi(x) - \frac{c}{2}\right), \tag{4.27}$$

where $\phi(x)$ and $\pi(x)$ are determined in Eqs. (4.24) and (4.26). Compared to the no-sharing profit in Eq. (4.12), the firm benefits from an increase in the sales revenue, thanks to the sharing premium. Nevertheless, her revenue from the rental service strictly decreases. The resulting effect of the sharing market on the firm's profit is therefore ambiguous.

Mass Selling. Purchasing is the dominant strategy for all young agents, if

$$\bar{\theta}(r;x) \le \theta_L.$$

The sharing service is only used by the non-owners among the older generation. This includes all agents who happened to be in the high-need state for the first time in their latest stages of life. The demand for sharing is

$$D = \lambda(1-\theta_H)\theta_H + (1-\lambda)(1-\theta_L)\theta_L.$$

The sharing supply consists of all owners who bought and used the item in the first period, but do not need it at the second period. That is

$$S = \lambda\theta_H(1-\theta_H) + (1-\lambda)\theta_L(1-\theta_L).$$

Clearly, under the mass selling strategy $S = D$ regardless of the model parameters. The sharing market is sustainable at any viable sharing price

$$\tau(1-x) \le p \le \min\{v - \tau(1-x), \phi + \tau(2x-1)\}. \tag{4.28}$$

In most sharing platforms, the price is determined by the suppliers. Examples include AirBnB, BlaBlaCar, TURO, and Eloue, among others. When this is the case and the renters are price takers, it is in the suppliers best interest to raise the sharing price up to its upper bound in Eq. (4.4), i.e.,

$$p = p(\phi; x) = \min\{v - \tau(1-x), \phi + \tau(2x-1)\}. \tag{4.29}$$

Note that setting any rental price $\tau(2-3x) < \phi \le v - \tau(2-3x)$ is not a viable pricing strategy in equilibrium. In fact, the sharing suppliers can react by further cutting down the price such that they can attract all the demand. If the firm sets $\phi = \tau(2-3x)$, the sharing market is choked-off in an induced manner as discussed earlier. In this section, we concentrate on the case where the firm shuts down its own rental service (by artificially choosing $\phi \in (v-\tau(2-3x), \infty)$). She exclusively sells to all consumers at the nominal purchase price $r(x)$. By Eq. (4.29), the sharing market clears at

$$p(x) = v - \tau(1-x).$$

The monopolist increases the purchase price such the (IC') constraint binds for the low-type agents. By La. 4.5, that is

$$r \equiv r(x) = v - \tau x + \delta(v - 2\tau(1-x)(1-\theta_L)). \tag{4.30}$$

The following lemma compares the purchase price in a mass selling strategy with and without sharing.

Lemma 4.7. *Assume that $\underline{x} > 0$. Under the "mass selling" strategy,*

1. *the purchase price is strictly higher with sharing than without it.*
2. *the surcharge is strictly increasing in x.*

When the sharing market is operating, the firm asks for a surcharge on each unit sold. Interestingly, La. 4.7 implies that the surcharge is increasing in the sharing propensity $x \in \mathscr{X}$. As the consumers' tendency towards sharing grows, consumers are willing to make bigger payments, so that they can participate in the sharing activities, later. Note that under the mass-selling strategy with sharing, the rental service is inactive and the firm's only source of revenue is from selling. This yields her the profit

$$\Pi_4(x) = (\lambda\theta_H + (1-\lambda)\theta_L)(r(x) - c), \tag{4.31}$$

where $r(x)$ is determined in Eq. (4.30).

Recall that the peer-to-peer market is active only if the firm does not find it beneficial to artificially shut it down. Let

$$\underline{\Pi}(x) = \max\{\Pi_0(x), \Pi_1(x), \Pi_2(x)\}$$

be the firm's net gain from employing an induced choke-off strategy by setting $\phi = \tau(2-3x)$. Then the sharing market is active if

$$\min\{\Pi_3(x), \Pi_4(x)\} \geq \underline{\Pi}(x).$$

We define the sharing threshold $\bar{x} \in [\underline{x}, 2/3]$ as the sharing propensity above which, sharing is strictly preferred, i.e.,

$$\min\{\Pi_3(\bar{x}), \Pi_4(\bar{x})\} = \underline{\Pi}(\bar{x}). \tag{4.32}$$

The "high-end selling" threshold in presence of sharing is defined as

$$x_3 = \min\left\{1, \max\left\{\bar{x}, \frac{2\delta(1+\zeta(1-\theta_L)) + \zeta\hat{v}(1-\delta)}{2\delta(1+\zeta(1-\theta_L)) + \zeta}\right\}\right\}, \tag{4.33}$$

where $\zeta = \ell/\Delta\theta$. When the sharing propensity is $x \in [\bar{x}, x_3]$, high-end selling is the firm's optimal strategy. The firm offers both the rental service and the purchase option and targets different consumer segments with different menu-items. Above this threshold, the rental service is not functioning and the firm exclusively offers purchase options to all consumers. The following proposition formally characterizes the optimality of the two regimes.

Proposition 4.2. *If the sharing market is active, then*

1. *for all $x \in [\bar{x}, x_3]$, the firm's optimal strategy is "high-end selling & rental," where the optimal menu comprises of $(v - \tau x, 1)$ and $(v(1+\delta) - \tau x - 2\delta\tau(1-x)(1-\theta_H), 2)$.*
2. *For all $x \in [x_3, 1]$, the firm's optimal strategy is"mass selling," where the rental service is inactive and optimal purchase option is $(v(1+\delta) - \tau x - 2\delta\tau(1-x)(1-\theta_L), 2)$.*

By Prop. 4.2, in the advent of cultural transition, sharing begins by high-end selling. For intermediate sharing propensities, only high-type users are willing to purchase, and both the sharing market and the rental service coexist and serve their own costumers. As the sharing propensity passes the threshold x_3, sharing becomes mainstream. All consumers are willing to invest in purchasing such that they can benefit from the additional revenue in the course of their lives.

Lemma 4.8. *x_3 is non-increasing in δ, ℓ, and θ_L and non-decreasing in $\hat{v}$, λ, θ_H, and $\Delta\theta$.*

With sharing, $x_3 < 1$, if $\hat{v}(1-\delta) < 1$. Mass selling becomes optimal at least for some x, if consumers are patient enough to perceive the NPV of future sharing-related activities as significant. By La. 4.8, a decrease in δ or an increase in $\hat{v}$ make the mass selling region grow in size. When δ is small, consumers are not willing to pay high purchase premiums. Hence, the optimal strategy is to offer a mixed bundle of high-end selling & rental for a larger interval of sharing propensities. It is less likely that the firm finds it optimal to shut down the rental market because the revenue stream generated from this service does not depend on the consumers' discount factor δ. This is also the case where $\hat{v}$ is large. As the normalized consumption value increases, the revenue from the rental service increases and it is more likely that the firm prefers to keep this service functional.

The size of the mass selling interval $[x_3, 1]$ also increases, as the low-type's usage frequency θ_L or their likelihood-ratio of consumption ℓ increases. When the low-type agents have a higher

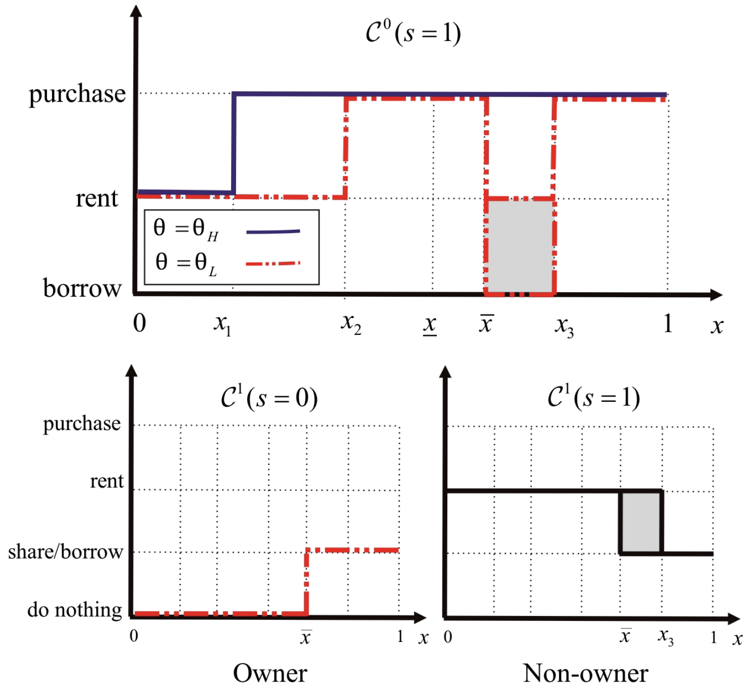

Figure 4.2: Agents' optimal actions in the early and late consumption phases.

willingness to pay, or their population increases, it is more likely for the firm to target them as purchasers at lower sharing propensities. On the other hand, when the high-types' usage frequency θ_H or their proportion in the population λ increases, the mass selling region shrinks. High-type consumers are more valuable in terms of revenue generation, and in a larger propensity interval, the firm designs the purchase option in a way that is specifically tailored to their specifications.

4.3.3 Market Structure

The firm's optimal selling vs. renting strategy as specified by Props 4.1-4.2 divides the $x-$space into two main regions, which establishes the market structure as a function of the sharing propensity. When the sharing propensity is small, the firm has monopolistic power and the sharing market is inoperative. As the sharing propensity grows, the firm and the P2P market engage in a competition, in which they offer options that are imperfect substitutes.

The optimal menu-design dictates the consumers' optimal actions. Note that the purchase decision is always taken by the young agents of each generation at $\mathscr{C}^0$ when $s^0 = 1$. The owners then take the sharing decision in their second stage of life $\mathscr{C}^1$, when $s^1 = 0$. The feasible market structures and the resulting actions of the agents are explained in what follows.

1. Firm's Monopoly. For all $x \in [0, \bar{x})$, sharing is not viable and the firm has monopolistic power. All owners in $\mathscr{C}^1$ who find themselves in the low need state do nothing with their idle

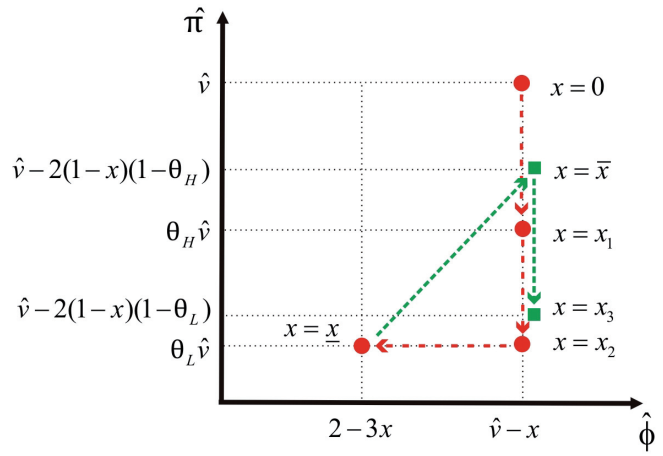

Figure 4.3: Optimal pricing strategy characterised by the $(\hat{\phi}, \hat{\pi})$.

capacity. The rental service is always active. The consumption menu is designed in a way that

1. for all $x \in [0, x_1)$, all non-owners of type $\theta \in \Theta$ rent in $\{\mathscr{C}^0, \mathscr{C}^1\}$.
2. for all $x \in [x_1, x_2)$, high-type (θ_H) non-owners purchase in $\mathscr{C}^0$ and rent in $\mathscr{C}^1$. Low-type (θ_L) non-owners rent in in $\{\mathscr{C}^0, \mathscr{C}^1\}$.
3. for all $x \in [x_2, \underline{x})$, all non-owners of type $\theta \in \Theta$ purchase in $\mathscr{C}^0$ and rent in $\mathscr{C}^1$.

2. (Imperfect) Competition Between the Firm and the Sharing Market. For all $x \in [\bar{x}, 1]$, the sharing market coexists and competes with the firm. All owners in $\mathscr{C}^1$ supply their unused items on the market. The available consumption offers are such that

1. for all $x \in [\bar{x}, x_3)$, renting, borrowing, and purchasing options are available on the market. High-type non-owners purchase in $\mathscr{C}^0$. High-type non-owners in $\mathscr{C}^1$ and Low-type non-owners in $\{\mathscr{C}^0, \mathscr{C}^1\}$ use a mixed strategy whether to rent or borrow.
2. for all $x \in [x_3, 1]$, the rental service is inoperative. All non-owners of type $\theta \in \Theta$ purchase in $\mathscr{C}^0$ and borrow in $\mathscr{C}^1$.

Fig. 4.2 shows the optimal actions of heterogeneous agents of type $\theta \in \Theta$ during their lifespan, for a case where all specified regions are non-degenerate. Note that for the older generation in $\mathscr{C}^1$, the type $\theta \in \Theta$ does not play a role in decision making. The end-of-horizon effect requires the agents to only consider their current need state $s^1 \in \{0, 1\}$ and ignore the future.

Depending on the market structure, the second-degree price discrimination is optimally performed to achieve the highest profit. Let $\hat{\phi} = \phi/\tau$ and $\hat{\pi} = \pi/\tau$ be the normalized rental price and the normalized purchase premium, respectively. Note that the pair $(\hat{\phi}, \hat{\pi})$ is sufficient to describe the optimal pricing strategy. Fig. 4.3 summarizes the results in Props. 4.1-4.2 and Las. 4.6-4.7 in the $(\hat{\phi}, \hat{\pi})$-state space. The red dots represent the optimal pricing strategy under the firm's monopoly, and the green squares represent the optimal second degree price discrimination when there is monopolist competition between the firm and the peer-to-peer market. The emergence of the peer-to-peer market creates non-monotonicity in the firm's pricing strategy. When there is no sharing, the normalized rental price is constant and designed to extract the consumers' maximum willingness to pay in all regimes, except for the induced sharing choke-off when $x \in [\underline{x}, \bar{x}]$. When the sharing market starts to operate, the rental price jumps back to its constant value.[7]

Note that the rental price and the purchase premium tend not to change at the same time, in any of the optimality regimes. The purchase premium changes while the rental price stays constant or vice versa. When there is no sharing, $\hat{\pi}$ jumps downward as the optimal regime changes. However, when the sharing market starts to operate, the firm benefits from an upward jump in the purchase premium, and the premium achieves the highest amount, at which selling is still feasible. As the sharing market matures and the optimal regime further changes at $x = x_3$, the purchase premium jumps downward again in a way that peer-to-peer transactions are embraced by all agents in the economy.

Optimal Consumption Bundling. A durable product can be viewed as a bundle of consumption opportunities (Jiang and Tian, 2016). In this regard, the firm's optimal portfolio design is also a product bundling strategy, where the firm's choice set comprises of *pure bundling*, *mixed bundling*, and *unbundling*.

Pure bundling requires the firm to offer an integral consumption bundle to all customers. This is equivalent to exclusively offering purchase options. The individual components of the bundle, i.e., single-use rental options are not separately available to the customers. By mixed bundling, the firm offers the integral bundle (here, the purchase option), and at the same time, customers have the option to separately select each component of the bundle (the per-period rental service). When the firm pursues an unbundling strategy, she decouples the single elements of the bundle and offers them only in the form of pure rental.

Fig. 4.4 depicts the firm's optimal consumption bundling strategy. As the economy's sharing propensity grows, the firm's strategy shifts from unbundling (pure rental), via mixed bundling (including the purchase option and the rental service in the menu) to pure bundling (exclusively selling). When unbundling, the firm is able to extract all consumers' surplus by charging them their highest willingness-to-pay for each unit of consumption and omitting the quantity discount. When sharing propensity is high, and the economy enjoys cultural shifts towards pooling and joint consumption, the firm finds pure bundling optimal. By exclusively offering purchase options,

[7] In equilibrium, when the rental market is inactive for all $x \in [x_3, 1]$, only the sum $\hat{r} = \hat{\phi} + \hat{\pi}$ matters for the consumers, and all $(\hat{\phi}, \hat{\pi})$ pairs that satisfy Eq. (4.30) are sustainable. The illustrated pair in this figure is chosen to better demonstrate the comparison between the two market structures.

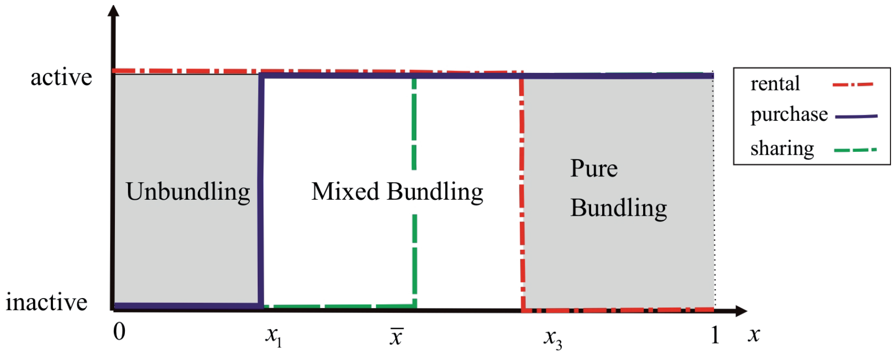

Figure 4.4: Optimal Consumption Bundling.

the firm encourages peer-to-peer transactions, while enjoying the revenue growth thanks to the increasing sharing premiums. When the customers are located between these two extreme conditions, mixed bundling is the optimal strategy for the firm. Mixed bundling can be optimal both when the sharing market is active or inactive.

4.4 Numerical Example

As observed in Las. 4.4 and 4.8 model parameters such as the discount rate, production cost, and likelihood ratio of consumption influence the propensity thresholds, and consequently the strategic propensity-regimes may grow, shrink, or completely vanish. In what follows, we use a numerical example to illustrate the results for a case where all six regions exist. Fig. 4.5 shows the firm's optimal profit when the discount factor is $\delta = 0.9$ and the production cost equals $c = 0.1$. The subjective probabilities of need for high and low types are $\theta_H = 0.5$ and $\theta_L = 0.4$, respectively. The firm believes that a given agent is of high type with probability $\lambda = 0.6$. The unit transportation cost is assumed to be equal to the consumption value $\upsilon = \tau = 0.6$, such that all agents find consumption appealing, in time of need. For all $x \leq \underline{x} = 0.5$, the peer-to-peer market naturally does not form. For an intermediate propensity region $x \in [0.5, 0.52]$, the firm artificially deactivates the sharing market by lowering the rental price. For higher sharing propensities, the peer-to-peer market is functioning. The optimal bundling strategy at each interval is shown in the figure.

Fig. 4.6 shows how the nominal prices ϕ and r change as the sharing propensity grows. As depicted in the left panel of the figure, the nominal rental price is strictly decreasing for all $x \in [0, \bar{x}]$, as the firm has to adjust her pricing to the growing cost of propensity mismatch. At the threshold $x_1 = 0.18$, the firm opens up the sales market exclusively for high-type customers. The nominal purchase price is seen in the right panel of Fig. 4.6. As the cost of propensity mismatch grows and the revenue from rental service further decreases, the firm adjusts its strategy at $x_2 = 0.41$ by opening the sales market to *all* consumers. If sharing was not possible such that $\underline{x} = \bar{x} = 1$, then mass selling was the optimal strategy for all $x \geq x_2$. As the sharing propensity passes this threshold, the firm is able to maintain the mixed bundling of mass selling & rental

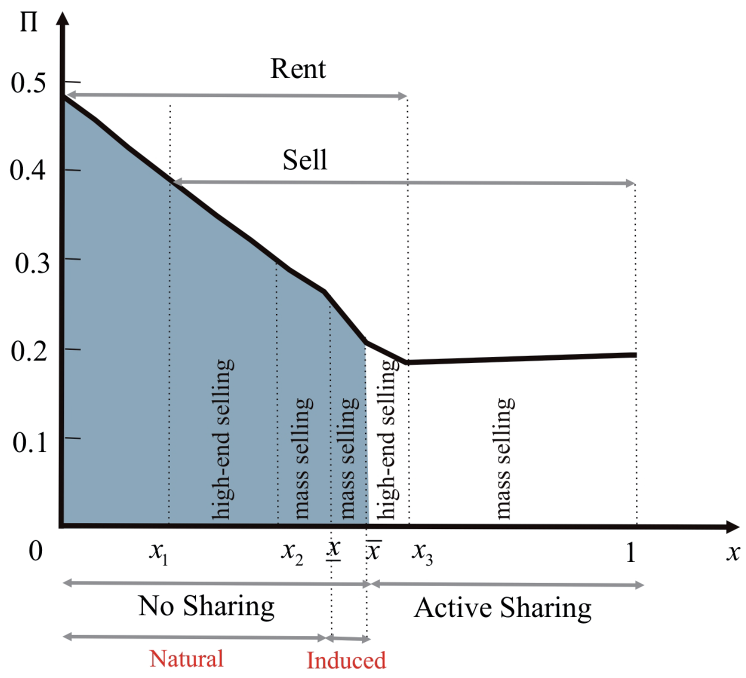

Figure 4.5: Firm's optimal stratgy and profit.

only by artificially reducing both the rental and purchase prices in order to deactivate the peer-to-peer market. However, this is optimal for the firm only when the sharing propensity lies in $x \in [0.5, 0.52]$. For $x \in [0.52, 0.6)$, the firm can mitigate the impact of the sharing market, using an upgraded purchase option that only suits the high-type users. This allows the firm to increase the rental price, too. The price increase in this region compensates for the shrinkage of the demand. The transitioning to mass selling happens once again at $x_3 = 0.6$ when the revenue from the rental service becomes negligible. Although the sharing price decreases compared to high-end selling, the increased demand for the purchase option compensates for the revenue loss from the rental service.

When the sharing propensity is low enough such that the peer-to-peer market does not function, the firm's profit constantly decreases in the sharing propensity. This is due to the fact that the mismatch cost is endured by the firm. As x increases, the monopolist is obligated to lower the prices in order not to lose her customers. In the absence of sharing, the firm might want to invest in a cultural transition towards trust in her established brand, such that the consumers mismatch-cost is perceived to be smaller. Equivalently, lowering the tax burden and/or the transportation cost on the consumers who use the firm's products and services can have the same effect. For example, the firm may want to offer additional services such as home-delivery and home pick-up for her rental service, in an attempt to reduce the consumers' transportation cost. However, when the consumers sharing propensity is high, the firm may in fact benefit from the presence of the peer-to-peer market. The firm's profit may be increasing in x. As shown in Fig. 4.5, in this

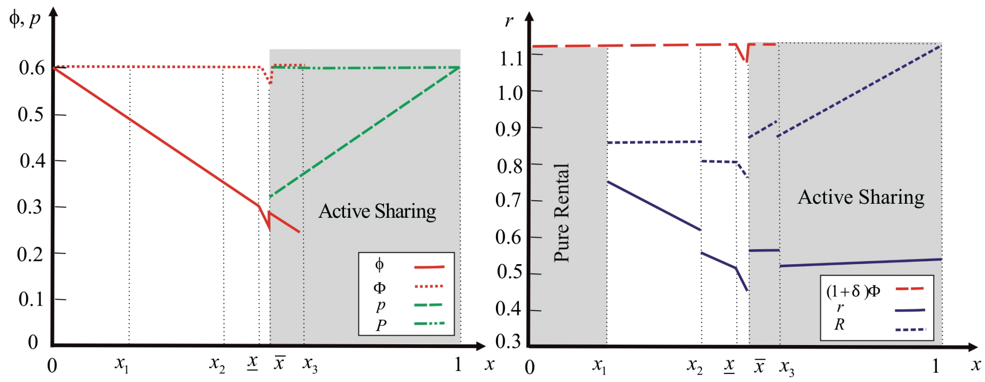

Figure 4.6: Nominal and effective Prices.

example the firm achieves her highest profit when $x = 0$. The profit constantly decreases for all $x \in [0, 0.6]$ and the total profit drop amounts to 61%. The profit then continuously increases and at $x = 1$, the firm's profit is 5% higher than its value at $x = 0.6$. The following lemma characterizes the changes in the firm's profit, as a function of the sharing propensity $x \in \mathscr{X}$.

Lemma 4.9. *i. For all $x \in [0, \bar{x}]$, the firm's profit is strictly decreasing in x.*

ii. For all $x \in [\bar{x}, x_3]$, the firm's profit is increasing in x, if

$$\delta \geq \frac{1+2\ell}{2(1-\theta_H)} \triangleq \delta_1.$$

iii. For all $x \in [x_3, 1]$, the firm's profit is increasing in x, if

$$\delta \geq \frac{1}{2(1-\theta_L)} \triangleq \delta_0.$$

By La. 4.9, if the consumers are patient enough to enjoy the benefits of sharing and joint consumption, the firm can increase her profits by increasing the purchase premium and extracting some of the consumers added surplus. This result is in-line with the previously found results by Razeghian and Weber (2016). By offering loans and financial services to the consumers, the firm can effectively increase their discount factor. By providing such services, the firm benefits from the cultural transition towards sharing and joint consumption. Clearly, $\delta_1 \geq \delta_0$. As the sharing propensity increases and the firm moves from mixed bundling to pure bundling, she can effectively downgrade her financial aids and yet benefit from the net profit increase as the population transitions further towards sharing.

Consumer Surplus, Transportation Market, and Gains from Trade

While the firm obtains the nominal prices ϕ or r from each unit provided, the consumers' perceived access/acquisition costs are in fact higher and include their transportation cost. This also holds for the sharing market. The nominal sharing price p is, in fact, smaller than the total cost paid by the borrowers. We define the *effective price* as the sum of the nominal price and the transportation cost, such that the effective sharing, rental, and purchase prices are

$$[\Phi, R, P] = [\phi, r, p] + \tau[x, x, 1-x]. \tag{4.34}$$

Fig. 4.6 shows the nominal and effective prices. When the sharing market is inactive, i.e., when $x \in [0, \bar{x}]$, the effective price is seen as either constant or decreasing by the consumers. Note that the consumers' valuation differs from their maximum willingness to pay. For example, for a one-time consumption, the valuation is v, while the willingness to pay is no more than $v - \tau x$. This implies that although the nominal prices decrease, consumers always face effective prices Φ and R which are equal to their valuation unless their sharing propensity implies an induced choke-off regime, i.e., $x \in (\underline{x}, \bar{x})$. However, when the sharing market is present, the effective purchase prices always grow in x. The underlying reason is the simultaneous growth of the sharing price p, which creates opportunities to compensate the surcharge. Meanwhile, the nominal purchase price may or may not increase.

Remark 4.8. In the presence of an operating sharing market, the effective purchase price grows at a faster speed than the nominal price, i.e.,

$$R'(x) = r'(x) + \tau \geq r'(x).$$

As the sharing propensity grows, consumers pay an increasing amount to get access to the purchase option. The existence of the peer-to-peer market per se allows the firm to pass a fraction of the transportation cost to the purchasers. This is in contrast to the case of no-sharing, where the burden of the transportation cost is entirely on the firm. In the case of active sharing, buyers are willing to accept this extra cost expecting to compensate the loss with the additional revenue from sharing transactions.

Note that second-degree price discrimination requires quantity discounts on the purchase option. That is, the consumers' spending on a purchase option must not exceed the cost of renting in all periods. Otherwise, no agent is willing to invest in purchasing. Since the consumers in total pay a sum equal to the effective prices, the quantity discount must satisfy

$$R < (1+\delta)\Phi. \tag{4.35}$$

That is, the effective purchase price must be strictly less than the total discounted effective rental price. Fig. 4.6 shows that the firm chooses the nominal price ϕ and r such that for all consumers purchasing is always cheaper than renting in both periods.

We now examine how the consumer surplus depends on the population's sharing propensity, the presence of a peer-to-peer market, and strategic decisions of the profit maximizing firm. At each period, the consumers' aggregate surplus is the sum of the benefits of both generations that live in that period. That is

$$\mathrm{CS}(x) = \mathrm{CS}^0(x) + \mathrm{CS}^1(x).$$

The surplus may stem either from self-consumption, or the additional revenue obtained from the sharing activities. Let $D^0_\phi \in [0,1]$ and $D^1_\phi \in [0,1]$ be the stationary demands for the rental service offered by the firm, in the early and late consumption phases, respectively. Similarly, let $D^0 \in [0,1]$ and $D^1 \in [0,1]$ be the stationary demands on the sharing market in $\mathscr{C}^0$ and $\mathscr{C}^1$, respectively. We also define $D_r \in [0,1]$ as the stationary demand for the purchase option. The demand for the purchase option consists of the demand coming from high and low type users, such that [8]

$$D_r = D_{rH} + D_{rL}.$$

Among the young generation, consumers either rent from the firm, borrow from the sharing market, purchase the item, or do nothing. Note that the consumers who do not engage in any transactions necessarily gain zero surplus. Hence, the benefits of the young agents in aggregate accounts to

$$\mathrm{CS}^0(x) = D^0(v - P(x)) + D^0_\phi(v - \Phi(x)) + D_r(v - R(x)). \tag{4.36}$$

The first term corresponds to the net benefit of the borrowers on the sharing market. The second and third terms describe the benefits of renters and buyers, respectively. The aggregate surplus of the older generation is

$$\begin{aligned}\mathrm{CS}^1(x) =& D^1(v - P(x)) + D^1_\phi(v - \Phi(x)) \\ &+ (D_{rH}\theta_H + D_{rL}\theta_L)v + (D_{rH}(1-\theta_H) + D_{rL}(1-\theta_L)(p(x) - \tau(1-x)).\end{aligned} \tag{4.37}$$

The first and second terms correspond to the benefits of the borrowers and renters, respectively. The third term describes the surplus of the owners who consume the product themselves. Note that for this consumer segment, the purchase price is already a sunk cost and they enjoy a positive surplus without incurring any cost. The fourth term corresponds to the owners who supply their unused capacity on the sharing market. Note that by Eq. (4.25), the sharing price in equilibrium is such that the users of the such service are left with zero surplus for all $x \in [\bar{x}, 1]$. Hence,

$$D^0(v - P(x)) = D^1(v - P(x)) = 0,$$

and the terms can be omitted from both Eqs. (4.36)-(4.37).

The total transportation cost $T(x)$ is the cost of propensity mismatch levied on the consumers of

[8] Note that the purchase option is only attractive for the young consumers. Hence the superscript is omitted for ease of notation.

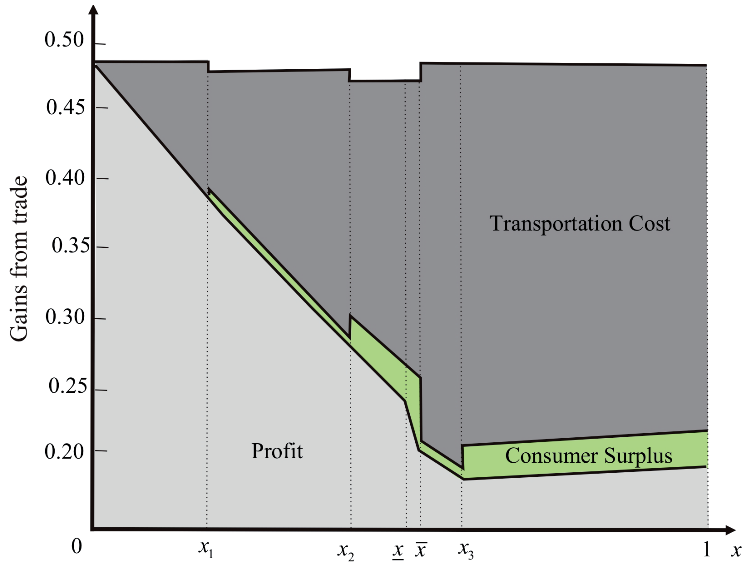

Figure 4.7: Distribution of gains from trade at different locations.

both markets. The aggregate cost is paid by the two generations that live in each period and is equal to

$$T(x) = T^0(x) + T^1(x).$$

$T^0(x)$ is paid by the younger agents and includes all travels either to the firm or the rental market. That is

$$T^0(x) = D^0\tau(1-x) + (D^0_\phi + D_r)\tau x. \tag{4.38}$$

For the older agents, $T^1(x)$ includes transportation costs to both the firm and the sharing market, such that

$$T^1(x) = [D^1 + D_{rH}(1-\theta_H)\theta_H + D_{rL}(1-\theta_L)\theta_L]\tau(1-x) + D^1_\phi\tau x. \tag{4.39}$$

Note that the cost of mismatch to the sharing market is paid both by borrowers and suppliers.

Fig. 4.7 shows the firm's profit, consumers surplus, and the transportation cost for the example used throughout the text. When the sharing propensity $x \in [0, x_1]$ is so low such that pure rental is the optimal strategy, consumers gain zero surplus. The firm absorbs all the consumers benefit by unbundling the consumption and charging the maximum willingness to pay per usage. This happens when the population finds their preferences closest to the firm or when the sharing services are heavily taxed. When $x \in [x_1, x_2]$, under the high-end selling strategy, the purchase option is designed so as to leave the young buyers with zero expected surplus. Nevertheless, for the owners among the older generation, the price paid in the first period is water under the bridge, and they gain a strictly positive surplus in their second stage of life. Consumer surplus increases, as the firm undertakes a mass selling strategy. In $x \in [x_2, \underline{x}]$, the optimal purchase option is designed to extract the surplus only from the low-type buyers, which allows the high-type buyers to enjoy a net positive gain from purchasing. This results in a positive jump in the consumer surplus at $x = x_2$. All owners of high and low types enjoy a positive net benefit in their late consumption phase if they find themselves in the high-need state. The transportation cost has its lowest value in this interval, since both types, enjoy the quantity discount and the number of travels to the firm is at its minimum.

Consumer surplus is the highest when the firm applies an induced choke-off strategy on the interval $x \in [\underline{x}, \bar{x}]$. The prices that disable the sharing market are smaller than the consumers' actual willingness to pay, which work in favor of the population. As the firm has to sacrifice some of her own gains to please the consumers, her profit rapidly decreases in x, and the optimality of the induced choke-off does not last for a large interval on sharing propensities.

When sharing market starts to operate for all $x \in [\bar{x}, x_3]$, all three services of renting, sharing, and purchasing coexist. At $x = \bar{x}$, the population experiences a large decrease in the aggregate surplus. The introduction of the sharing market allows the firm to increase the prices, and leaves the consumers with a smaller surplus. Yet, both suppliers and borrowers are still not close enough to the sharing market to enjoy a significant surplus. In the interval $x \in [\bar{x}, x_3]$ the transportation cost has its highest value, since all consumer segments, whether buyer, renter, supplier, or borrower incur relatively large transportation costs to access either the firm or the sharing market.

For all $x \in [x_3, 1]$, consumer surplus starts to increase. High-type agents gain a positive information rent as a result of the mass-selling strategy. Due to the presence of the peer-to-peer market, all owners (high or low type) gain a positive surplus either from their own usage or the additional income they gain by supplying their items on the sharing market. Suppliers in this region gain a significant surplus, since their transportation cost is relatively much smaller than the price they charge. The transportation cost decreases, as the number of travels to the firm decreases. Although the number of travels to the sharing market increases, the cost is negligible due to the consumers' proximity to this market.

The gains from trade in this economy is the sum of the consumer surplus, firm's profit, and the transportation cost, i.e.,

$$\begin{aligned} \mathrm{GT}(x) &\triangleq \Pi(x) + \mathrm{CS}(x) + T(x) \\ &= D_r(v - c + v \cdot \mathbb{1}_{\{x \geq \bar{x}\}}) + D_\phi \left(v - \frac{c}{2}\right) + (D_{rH}\theta_H + D_{rL}\theta_L)v. \end{aligned} \tag{4.40}$$

When the sharing market is not active, whether naturally or artificially, the gains from trade is piecewise. When the sharing market starts functioning, then gains from trade jumps upward and achieves its highest value. This is formalized in the following result.

Lemma 4.10. *1. Without sharing, gains from trade is piecewise constant.*

2. *With sharing, gains from trade is constant and it is equal to*

$$GT(x) = (\lambda\theta_H + (1-\lambda)\theta_L)\left(v - \frac{c}{2}\right).$$

3. *Gains from trade with sharing is higher than without sharing.*

As shown in Fig. 4.7, the gains from trade under each strategy are constant. When there is no sharing, the consumer surplus is also constant, demonstrating that the firm endures the total cost of mismatch. When the sharing market opens up, the society as a whole enjoys an increase in the gains from trade. The economy benefits from the sharing transactions and the social welfare increases as a result of cultural transitions towards sharing. Without sharing, the firm and the population have conflicts of interest whether or not to move towards sharing. However, in the presence of sharing, the firm is able to pass some of the burdens of the transportation cost to the consumers. Under certain conditions when the discount factor is significantly large, both the firm's profit and the consumer surplus increase in x and cultural transitioning towards sharing is in all players' interest.

4.5 Conclusion

The population's sharing propensity can play a major role in the strategic responses of the incumbents to emerging sharing markets. Firms can leverage from the society's positioning towards sharing and design their product portfolios to absorb the highest surplus from the consumers. This paper is the first to take into account the population's sharing propensity to address the firm's optimal second-degree price discrimination. In an economy with overlapping generations of consumers, the firm optimizes her menu design in the form of offering rental and/or purchase options. We use a Hotelling linear model to describe the sharing propensity of the consumers. Higher sharing propensity is equivalent to a smaller access cost to the sharing market and at the same time a larger cost to access the incumbent firm.

Sharing choke-off happens when the rental service is the cheaper substitute for the sharing service. When the sharing propensity is low, and the consumers show significant trust in the established brand, the rental service, and the sharing market engage in a Bertrand competition, in which they have asymmetric marginal supply costs. While the firm incurs only a fraction of the actual production cost to offer a rental service, the suppliers on the (potential) sharing market bear the transportation cost all the way to the sharing market. When accessing the sharing market is perceived to be very costly, the peer-to-peer market is naturally choked-off, and the firm attracts the entire demand.

Moreover, the firm always retains the option to artificially induce the sharing choke-off by offering overly low rental rates in a way that the peer-to-peer market cannot compete. However, employing this strategy may impose a significant profit loss for the firm. We show that for a range of intermediate sharing propensities, this strategy is optimal, but as the population moves further towards the sharing culture, the firm finds it optimal to join the flow and promote sharing.

Without sharing, when the consumers' mismatch cost is small, the firm only offers the rental service. As accessing the firm becomes more costly, the firm is compelled to offer quantity discounts by enabling the purchase market. The quantity discount is at first offered only to the high-types (high-end selling), and then includes the entire population (mass selling).

When the sharing propensity is high enough, both the incumbent firm and the sharing market engage in monopolistic competition. Consumers perceive the rental service and the sharing market as imperfect substitutes, as a result of their mismatch to the firm and/or to the sharing market. We show that for intermediate sharing propensities, both sharing and rental coexist (using a high-end selling strategy), while for larger propensities, the sharing market wins the total demand. When sharing is mainstream, mass-selling strategy becomes optimal, since the asset base of a sharing economy ultimately depends on the firm's output, so that a portion of the available surplus can be captured by the durable-goods monopolist.

Although the quantity of the firm's transactions decreases in total, the emergence of the peer-to-peer market allows the firm to increase both the purchase premium and the rental price, and potentially gain from sharing. We find that an increase in sharing propensity has an ambiguous effect on the firm's profit. While the sharing market decreases the revenues of the rental market, it causes an increase in the sales revenues. Sharing imperfectly substitutes one option (rental), and at the same time complements the other option (purchase option). The results indicate that the consumer's patience is a major factor in the upswing of the purchase premium, in the presence of sharing. Hence, the firm's profit increases in the sharing propensity if the discount factor is significant enough. By artificially increasing the discount factor, say by providing loans to the buyers, the firm always retains the option to increase her profits as the economy experiences cultural transitions towards sharing. The amount of financial aid decreases as the economy further embraces the sharing culture and peer-to-peer transactions become mainstream.

Cultural transitions from private ownership to collective consumption shapes the firm's portfolio design. As the sharing propensity grows, the firm's optimal strategy changes from unbundling (exclusively renting), via mixed bundling (renting and selling), to pure bundling (exclusively selling). The unbundling propensity-threshold increases in the production cost. As the cost of production increases, unbundling becomes more attractive for the firm, since it is more cost-effective not to sell and only rent out. On the other hand, as consumers become more patient, the unbundling threshold becomes smaller. The revenue from the purchase option makes it more appealing to offer both options at the same time. Mixed bundling happens when sharing and rental each has their own customers. The mixed-bundling threshold decreases in the low-type's consumption likelihood. When a large enough fraction of the aggregate consumption comes from low-types, it is more profitable to target them as purchasers and switch to mass selling. The mixed-bundling threshold is also increasing in the discount factor. Consumers increased patience has a positive effect on the sales price, hence on the firm's bottom-line.

The paper also elaborates on the power of regulatory interventions on the market structure. While the sharing propensity is intrinsic to the consumers, it can be manipulated by regulatory actions such as imposing taxes on consumption and changing the perceived cost of mismatch. We show that gains from trade increases when the sharing market is operating and it is constant regardless of the status of the economy. Imposing consumption taxes on the firm's services, or equivalently subsidizing sharing services brings the consumers closer to the sharing market. This, in fact, may benefit the society as a whole and result in an upward jump in the gains from trade. Moreover, if the consumers are patient enough, then not only the size of the pie increases but also both the firm and the customers benefit from transitioning towards sharing. Hence, regulatory intervention in the form of promoting the sharing economy can be beneficial to all players.

The model provided in this paper is a simplified version of a more complex problem and aims at providing insights on the role of the sharing propensity in shaping the portfolio of the firm. Several interesting extensions to this work could be used in future research. To limit the complexity of the problem, we have assumed that the entire population shares a common attitude towards sharing. A natural extension of the model is to include various consumer types in the economy with respect to the sharing propensity. When the attitudes of various groups are close, the results stay qualitatively the same. However, when consumer groups have very different tendencies towards sharing, arbitrage opportunities may arise, which is worth investigating.

Another interesting question that has been neglected in this analysis is the quality and features offered in each menu. In this model, the firm solely determines the price and the intertemporal consumption allowance, and consumers get the same per-period consumption value from renting or buying. However, the firm might be able to also leverage from adjusting the product range and quality. For example, while owners stick to the same product that they buy, the renters get the chance to choose from a portfolio of different products in each period.

Lastly, we consider a static Hotelling linear model, in which the positioning of the firm and the sharing market remain unaffected, and only the transportation cost may be subject to change. An interesting extension to this model is to study a dynamic Hotelling model. By investing in marketing activities, the firm can also change her positioning and strengthen the competition between herself and the sharing market by developing characteristics that are more similar to the ones of peer-to-peer markets. The resulting positioning of the firm and its effect on her profitability remain unexplored.

A Notations

Table A.1: Description of sharing companies mentioned in Chapter 1.

Startup	*About*	*Headquarter*
AirbnB	Short-term rental of lodging	San Francisco, CA
BlaBlaCar	Long-distance ridesharing	Paris, France
Couchsurfing	Homestay and social networking	San Francisco, CA
Dropbox	File sharing and storage	San Francisco, CA
Eloue	Equipment and every day goods rental	Paris, France
Peerby	Local household good borrowing	Amsterdam, Netherlands
Sharecash	Pay-per-download file sharing	New York, NY
Turo	Car-sharing marketplace	San Francisco, CA
Uber	Short-distance ridesharing	San Francisco, CA

Table A.2: Main notations of Chapter 2.

Symbol	*Description*	*Domain/Definition*
a	Agent's adjustment decision	$\mathscr{A} = \{0, 1\}$
c_{01}	Conversion cost from "sharing" to "keeping"	$\mathbb{R}_+$
c_{10}	Conversion cost from "keeping" to "sharing"	$\mathbb{R}_+$
$F(\cdot)$	Cumulative distribution function for types $\theta \in \Theta$	$F : \Theta \to [0, 1]$
$\bar{g}(\cdot)$	Agent's current-period expected net payoff	$\bar{g} : \mathscr{A} \times \mathscr{X} \times \mathbb{R}_+ \to \mathbb{R}$
n	Available supply of the shared item	$[0, 1]$
$\bar{n}$	Steady-state supply of the shared item	$[0, 1]$
$p(\cdot)$	Price of the shared item as a function of the available supply	$p : [0, 1] \to [0, \hat{\gamma}]$
$\hat{p}^0$	Critical (effective) sharing price	$[0, \hat{\gamma}]$
$\hat{p}^1$	Critical (effective) keeping price	$[0, \hat{\gamma}]$
$\hat{p}$	Effective transaction price	$\hat{p} = p \cdot q$
$\bar{p}$	Steady-state price	$[0, \hat{\gamma}]$

Table A.2 (continued) . . .

...Table A.2 (continued)

Symbol	*Description*	*Domain/Definition*
$\mathscr{P}(\theta)$	Agent θ's inaction region in the price space	$[\hat{p}^1(\theta), \hat{p}^0(\theta)]$
q	Transaction probability	$[0,1]$
r	Per-period interest (or discount) rate	$\mathbb{R}_+$
$\mathscr{R}$	Invariance region	$[\vartheta^0, \vartheta^1]$
s	Agent's need state	$\mathscr{S} = \{L, H\}$
t	Time	$\mathbb{N}$
$\bar{t}$	Type-dependent sharing-switch time	$\mathbb{N} \cup \{\infty\}$
t^μ	Maximum-diffusion time	$\mathbb{N}$
$u(\cdot)$	Utility as a function of the agent's need state and his sharing state	$u : \mathscr{S} \times \mathscr{X} \to \mathbb{R}$
u_0	Agent's (*dis*)utility from not having the item at disposal when needed	$\mathbb{R}_-$
u_1	Agent's utility from having the item at disposal when needed	$\mathbb{R}_+$
$V(\cdot\|\theta)$	Type-θ agent's value function	$V : \mathscr{X} \times \Theta^2 \to \mathbb{R}$
$\hat{V}(\cdot)$	Agent's transformed value function	$\hat{V} : \Theta^3 \to \mathbb{R}$
$\bar{V}^0(\theta\|\bar{\vartheta})$	Terminal payoff of a type-θ agent when sharing, at the steady-state $\bar{\vartheta}$	$\mathbb{R}$
$\bar{V}^1(\theta\|\bar{\vartheta})$	Terminal payoff of a type-θ agent when keeping, at the steady-state $\bar{\vartheta}$	$\mathbb{R}$
x	Sharing state	$\mathscr{X} = \{0, 1\}$
z	Excess demand	$[0,1]$
$\alpha(\cdot)$	Law of motion of the sharing threshold (system function)	$\alpha : \Theta \to \Theta$
$\alpha_0(\cdot)$	Law of motion of the sharing threshold below the invariance region	$\alpha_0 : [0, \vartheta^0] \to [0, \vartheta^0]$
$\alpha_1(\cdot)$	Law of motion of the sharing threshold above the invariance region	$\alpha_1 : [\vartheta^1, 1] \to [0, \vartheta^1]$
γ	Perceived quality by renters / demand-elasticity parameter	$\mathbb{R}_{++}$
$\hat{\gamma}$	Maximal sharing payoff / intermediated demand-elasticity parameter	$\mathbb{R}_{++}$
δ	Discount factor	$(0,1)$
Δ	Utility gain for use of shared item when needed, $u_1 - u_0$	$[\hat{\gamma}, \infty)$
ε	Price elasticity of demand for the shared item	$\mathbb{R}_+$
ϑ_t	Sharing threshold at time t	Θ
ϑ^0 / ϑ^1	Lower / upper invariance threshold	Θ
ϑ^2 / ϑ^μ	Rest / maximum-diffusion threshold	Θ
$\bar{\vartheta}$	Steady-state sharing threshold	Θ
θ	Agent's type (probability of being in high-need state)	Θ
Θ	Type space for θ	$[0,1]$

Table A.2 (continued) ...

...Table A.2 (continued)

Symbol	*Description*	*Domain/Definition*
κ	Per-period servicing cost	$\mathbb{R}_+$
λ	Compression factor for price-adjustment time scale	$(0,1]$
$\pi(\cdot\|\theta)$	Agent θ's optimal policy as a function of own sharing state and sharing threshold	$\pi : \mathscr{X} \times \Theta \to \mathscr{A}$
$\hat{\pi}(\cdot)$	Agent's transformed optimal policy as a function of type and sharing threshold	$\hat{\pi} : \Theta^2 \to \mathscr{A}$
ρ	Sharing intermediary's commission rate	$[0,1]$
$\xi(\cdot)$	Agent's sharing state as a function of type and sharing threshold	$\xi : \Theta^2 \to \mathscr{X}$
$\bar{\xi}(\cdot)$	Stationary sharing-state distribution	$\bar{\xi} : \Theta \to \mathscr{X}$
$\tau(\cdot)$	Time of convergence as a function of the initial condition	$\tau : \Theta \to \mathbb{N}$
$\varphi(\cdot)$	Mapping from the initial state of the economy to its limit	$\varphi : \Theta \to \Theta$

Table A.3: Main notations of Chapter 3.

Symbol	*Description*	*Domain/Definition*
c	Unit production cost	$[0,\bar{c}]$
$\hat{c}$ / $\check{c}$	Cost threshold for perfect durability / disposability (without sharing)	$[0,\bar{c}]$
$\underline{c}$	Cost threshold for existence of Nash Equilibrium (j = DC)	$[0,\bar{c}]$
c_0	Cost threshold for perfect durability versus full disposability (j = SP)	$[0,\bar{c}]$
$\mathscr{C}_0$ / $\mathscr{C}_1$	Early / late consumption phases	N/A
CS / $\widehat{\text{CS}}$	Consumer surplus (with / without sharing)	$\mathbb{R}$
D	Demand on the sharing market	$[0,2]$
i	Item availability	$\mathscr{I} = \{0,1\}$
j	Commitment regime	$\{\text{DC},\text{SMP},\text{SP}\}$
n	Total per-period sales (items in circulation before potential failure)	$[0,2]$
p	Sharing price (clearing price on the peer-to-peer market)	$[0,1]$
q	Product durability	$[0,1]$
Q	Transaction volume on the sharing market	$[0,1]$
r	Retail price	$\mathbb{R}_+$
$\underline{r}$ / $\bar{r}$	Lower / upper bound on the retail price in the liquidity condition (L')	$\mathbb{R}_+$
s	Agent's need state	$\mathscr{S} = \{L,H\}$
S	Supply on the sharing market	$[0,2]$
t	Time	$\mathbb{N}$
T_s	Agent's total expected payoff (with sharing)	$\mathbb{R}$
u_0 / u_1	Agent's (indirect) utility without / with the item	$\mathbb{R}$
U_s	Non-owner's payoff in $\mathscr{C}_1$ in need state s (with sharing)	$\mathbb{R}$
$\hat{U}_s$	Non-owner's payoff in $\mathscr{C}_1$ in need state s (without sharing)	$\mathbb{R}$
$\bar{U}$	Non-owner's *ex ante* expected payoff in $\mathscr{C}_1$ (with sharing)	$\mathbb{R}$
V_s	Owner's payoff in $\mathscr{C}_1$ (with sharing)	$\mathbb{R}$
$\bar{V}$	Owner's *ex ante* expected payoff in $\mathscr{C}_1$ (with sharing)	$\mathbb{R}$
$\hat{V}_s$	Owner's payoff in $\mathscr{C}_1$ (without sharing)	$\mathbb{R}$
W / $\hat{W}$	Per-period social welfare (with / without sharing)	$\mathbb{R}$
y	Agent's income	$\mathbb{R}_+$
δ	Discount factor	$(0,1]$
$\hat{\delta}$ / $\check{\delta}$	Discount-factor threshold for optimality of perfect durability / disposability	$[0,1]$
θ	Agent's likelihood of need	$\Theta = [0,1]$
θ_0	Purchase threshold in the early consumption phase (without sharing)	$[0,1]$
v	Agent's consumption value contingent on high need	$\mathscr{V} = [0,1]$
π	Sharing premium as a function of the durability	$\mathbb{R}_+$
Π / $\hat{\Pi}$	Firm's per-period profit (with / without sharing)	$\mathbb{R}$
Ω / $\hat{\Omega}$	Firm's per-period demand (with / without sharing)	$[0,2]$

Table A.4: Main notations of Chapter 4.

Symbol	*Description*	*Domain/Definition*
c	Unit production cost	$\mathbb{R}_+$
$\hat{c}$	Normalized value of unit production cost	$\mathbb{R}_+$
$\mathscr{C}_0$ / $\mathscr{C}_1$	Early / late consumption phases	N/A
CS	Consumer surplus	$\mathbb{R}$
D	Demand on the sharing market	$[0,2]$
GT	Gains from trade	$\mathbb{R}$
j	consumption phase	$\{0,1\}$
p	Sharing price (clearing price on the peer-to-peer market)	$[0,v]$
P	Effective sharing price	$[0,v]$
q	Quantity offered by the consumption bundle	$\{1,2\}$
r	purchase price	$[0,v]$
R	Effective purchase price	$[0,v]$
s	Agent's need state	$\mathscr{S} = \{L,H\}$
S	Supply on the sharing market	$[0,1]$
T	Total transportation cost	$\mathbb{R}$
$u(.)$	Agent's per-period utility	$\mathscr{S} \times \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}_+$
U_s	Non-owner's payoff in $\mathscr{C}_1$ in need state s	$\mathbb{R}$
v	Value of unit consumption	$\mathbb{R}_+$
$\hat{v}$	Normalized value of unit consumption	$\mathbb{R}_+$
$\bar{U}$	Non-owner's *ex ante* expected payoff in $\mathscr{C}_1$ (with sharing)	$\mathbb{R}$
V_s	Owner's payoff in $\mathscr{C}_1$	$\mathbb{R}$
$\bar{V}$	Owner's *ex ante* expected payoff in $\mathscr{C}_1$	$\mathbb{R}$
x	Sharing propensity	$\mathscr{X} = [0,1]$
y	Agent's income	$\mathbb{R}_+$
ℓ	Consumption likelihood ratio $((1-\lambda)\theta_L)/(\lambda\theta_H)$	$\mathbb{R}_+$
δ	Discount factor	$(0,1]$
$\Delta\theta$	Agent's heterogeneity measure $(\theta_H - \theta_L)$	$\{0,1\}$
θ	Agent's likelihood of need	$\Theta = \{\theta_L, \theta_H\}$
λ	Probability distribution of high-type agents	$[0,1]$
π	purchase premium	$\mathbb{R}_+$
Π	Firm's per-period profit	$\mathbb{R}$
ρ	price of the consumption bundle	$\{\phi, r\}$
τ	Propensity mismatch (transportation) cost per unit distance	$\mathbb{R}_+$
ϕ	Rental price	$[0,v]$
Φ	Effective rental price	$[0,v]$

B Proofs and Analytical Details

B.1 Analytical Details of Chapter 2

B.1.1 Proof of the Main Results

Proof of Lemma 2.1. By the one-shot deviation principle, to establish a subgame-perfect equilibrium, it is sufficient to check that no agent has an incentive to deviate from the equilibrium path in any single period (Fudenberg and Tirole 1991).[1] Accordingly, consider an agent of type θ at time $t > 0$. At that time, the agent prefers to keep the item in the next period, i.e., $\hat{\pi}(\theta, \vartheta_t) = 1$, if and only if

$$\begin{aligned} &\bar{g}(1, \xi(\theta, \vartheta_t), \hat{p}(\vartheta_t, \vartheta_{t-1})|\theta) + \delta \bar{g}(\hat{\pi}(\theta, \vartheta'), 1, \hat{p}(\vartheta', \vartheta_t)|\theta) + \delta^2 \hat{V}(\theta, \alpha(\vartheta'), \vartheta') \geq \\ &\quad \bar{g}(0, \xi(\theta, \vartheta_t), \hat{p}(\vartheta_t, \vartheta_{t-1})|\theta) + \delta \bar{g}(\hat{\pi}(\theta, \vartheta'), 0, \hat{p}(\vartheta', \vartheta_t)|\theta) + \delta^2 \hat{V}(\theta, \alpha(\vartheta'), \vartheta'). \end{aligned} \tag{B.1}$$

All other agent types follow the equilibrium threshold policy $\hat{\pi}$. Hence, the time-$(t+1)$ type threshold becomes

$$\begin{aligned} \vartheta' \quad = \quad &\inf\{\theta \in \Theta : \bar{g}(1, \xi(\theta, \vartheta_t), \hat{p}(\vartheta_t, \vartheta_{t-1})|\theta) + \delta \bar{g}(\hat{\pi}(\theta, \vartheta'), 1, \hat{p}(\vartheta', \vartheta_t)|\theta) \geq \\ &\quad \bar{g}(0, \xi(\theta, \vartheta_t), \hat{p}(\vartheta_t, \vartheta_{t-1})|\theta) + \delta \bar{g}(\hat{\pi}(\theta, \vartheta'), 0, \hat{p}(\vartheta', \vartheta_t)|\theta)\}, \end{aligned} \tag{B.2}$$

independent of agent θ's choice. The last condition defines the next-period type as an externality for all agents, and it provides a transversal equilibrium condition linking the individuals' payoff-

[1] The continuity-at-infinity condition (Fudenberg and Tirole 1991, Def. 4.1) for the application of the one-shot deviation principle in an infinite-horizon dynamic game is satisfied because the stage-game payoffs are uniformly bounded and future payoffs are discounted geometrically.

maximization problems in Eq. (2.12). Thus, $\vartheta' = \alpha(\vartheta_t)$ is such that

$$\bar{g}(1, \xi(\vartheta', \vartheta_t), \hat{p}(\vartheta_t, \vartheta_{t-1})|\vartheta') + \delta \bar{g}(\hat{\pi}(\vartheta', \vartheta'), 1, \hat{p}(\vartheta', \vartheta_t)|\vartheta') = \bar{g}(0, \xi(\vartheta', \vartheta_t), \hat{p}(\vartheta_t, \vartheta_{t-1})|\vartheta') + \delta \bar{g}(\hat{\pi}(\vartheta', \vartheta'), 0, \hat{p}(\vartheta', \vartheta_t))|\vartheta'). \tag{B.3}$$

To show that $\hat{\pi}(\vartheta', \vartheta') = 0$, suppose that, on the contrary, $\hat{\pi}(\vartheta', \vartheta') = 1$. Depending on whether the sharing market is about to stagnate, to expand, or to contract, we distinguish three cases.

1. If $\vartheta_t = \vartheta'$, the result is immediate. The threshold policy in Eq. (2.11) yields that $\hat{\pi}(\vartheta', \vartheta') = \hat{\pi}(\vartheta_t, \vartheta_t) = \mathbf{1}_{\{\vartheta_t > \vartheta'\}} = 0$.

2. $\vartheta_t < \vartheta'$. By Eq. (2.5), it is $\xi(\vartheta', \vartheta_t) = 1$, and by hypothesis $\hat{\pi}(\vartheta', \vartheta') = 1$. By Eq. (B.3) ϑ' is such that

$$\bar{g}(1,1,\hat{p}(\vartheta_t, \vartheta_{t-1})|\vartheta') + \delta \bar{g}(1,1,\hat{p}(\vartheta_t, \vartheta')|\vartheta') = \bar{g}(0,1,\hat{p}(\vartheta_t, \vartheta_{t-1})|\vartheta') + \delta \bar{g}(1,0,\hat{p}(\vartheta_t, \vartheta')|\vartheta'),$$

or equivalently,

$$\vartheta' u_1 + \delta \vartheta' u_1 = \vartheta' u_1 - c_{10} + \delta((1-\rho)\, p(\vartheta_t)\, q(\vartheta', \vartheta_t) + \vartheta' u_0 - c_{01}).$$

We therefore obtain that

$$\vartheta' = \frac{(1-\rho)\, p(\vartheta_t)\, q(\vartheta', \vartheta_t) - c_{01} - (1+r)c_{10}}{\Delta}. \tag{B.4}$$

The time-$(t+1)$ type threshold ϑ' can exceed ϑ_t, as long as $\vartheta_t \in [0, \omega)$, where ω solves a stationary version of Eq. (B.11), corresponding to a fixed-point problem with $q(\omega, \omega) = 1$, so

$$\omega = \min\left\{1, \frac{\hat{\gamma} - c_{01} - (1+r)c_{10}}{\hat{\gamma} + \Delta}\right\} \quad \left(= \frac{(1-\rho)\, p(\omega) - c_{01} - (1+r)c_{10}}{\Delta}\right).$$

Note that $\omega \leq \hat{\gamma}/(\hat{\gamma} + \Delta) \leq 1/2$, since $\hat{\gamma} \leq \Delta$ by Eq. (2.2). On the other hand, differentiation of ϑ' in Eq. (B.11) with respect to ϑ_t yields

$$\frac{\partial \vartheta'}{\partial \vartheta_t} = \frac{\hat{\gamma}(1 - 2\vartheta_t)}{2\Delta\vartheta' + c_{01} + (1+r)c_{10}}.$$

Thus, $\vartheta_t < \omega$ $(\leq 1/2)$, which in turn implies that $\partial\vartheta'/\partial\vartheta_t > 0$. As a result, for all $\vartheta_t \in [0, \omega)$, the next-period sharing threshold ϑ' is increasing in ϑ_t, and

$$\vartheta_t < \vartheta' < \vartheta'' < \omega \leq 1/2.$$

By Eq. (2.11) therefore $\hat{\pi}(\vartheta', \vartheta') = \mathbf{1}_{\{\vartheta' > \vartheta''\}} = 0$, in contradiction to our hypothesis.

3. $\vartheta_t > \vartheta'$. In this case, $q_t(\vartheta', \vartheta_t) = \min\{1, \vartheta_t/\vartheta'\} = 1$, and $\xi(\vartheta', \vartheta_t) = 1$. Thus, as before, by

Eq. (B.3) it is

$$\vartheta' = \frac{(1-\rho)\,p(\vartheta_t) + r c_{10}}{\Delta}. \tag{B.5}$$

Moreover, since by hypothesis $\hat{\pi}(\vartheta', \vartheta') = 1$, Eq. (2.11) implies that $\vartheta' > \vartheta''$, where $\vartheta' = \vartheta_{t+1}$ and $\vartheta'' = \vartheta_{t+2}$. Thus, as before, by Eq. (B.10) one obtains

$$\vartheta'' = \frac{(1-\rho)\,p(\vartheta') + r c_{10}}{\Delta}. \tag{B.6}$$

Since $\vartheta' > \vartheta''$, Eqs. (B.5)–(B.6) imply that $p(\vartheta_t) > p(\vartheta')$. As $p(\cdot)$ is decreasing, it is $\vartheta_t < \vartheta'$, which contradicts the initial assumption. Thus, necessarily $\hat{\pi}(\vartheta', \vartheta') = 0$.

The three cases taken together imply that $\hat{\pi}(\vartheta', \vartheta') = 0$, completing our proof. ■

Proof of Proposition 2.1. Depending on the direction of movement from ϑ_t to $\vartheta' = \vartheta_{t+1}$, three cases are examined separately.

Case 1: $\vartheta_t < \vartheta'$. By Eq. (2.5), the time-$(t+1)$ marginal type $\theta = \vartheta'$ is still out of the sharing market at time t, that is $\xi(\vartheta', \vartheta_t) = 1$. However, by La. 2.1, this agent type will join the market in the next time period: $\hat{\pi}(\vartheta', \vartheta') = 0$. As a result, Eq. (2.13) becomes

$$\bar{g}(1,1,\hat{p}(\vartheta_t,\vartheta_{t-1})|\vartheta') + \delta\bar{g}(0,1,\hat{p}(\vartheta_t,\vartheta')|\vartheta') = \bar{g}(0,1,\hat{p}(\vartheta_t,\vartheta_{t-1})|\vartheta') + \delta\bar{g}(0,0,\hat{p}(\vartheta_t,\vartheta')|\vartheta'),$$

or equivalently: $\vartheta' u_1 + \delta(\vartheta' u_1 - c_{10}) = \vartheta' u_1 - c_{10} + \delta((1-\rho)p(\vartheta_t)q(\vartheta', \vartheta_t) + \vartheta' u_0)$, which yields $\vartheta' = \alpha_0(\vartheta_t)$ as in Eq. (2.17). The marginal type ϑ' creates an action threshold, below which all agents are willing to share. As shown in the proof of La. 2.1, the action threshold is valid for $\vartheta_t \in [0, \vartheta^0)$, where the lower invariance threshold ϑ^0 is given in Eq. (2.14).

Case 2: $\vartheta_t > \vartheta'$. The sharing-state distribution in Eq. (2.5) implies that $\xi(\vartheta', \vartheta_t) = 0$ and by La. 2.1 it is $\hat{\pi}(\vartheta', \vartheta') = 0$. That is, the time-$(t+1)$ marginal type finds it optimal to exit the sharing market for one period, and rejoin afterwards. Furthermore, by Eqs. (2.8)–(2.9), the transaction probability equals 1, and $\hat{p}(\vartheta', \vartheta_t) = (1-\rho)p(\vartheta_t)$. Thus, Eq. (2.13) becomes

$$\bar{g}(1,0,\hat{p}(\vartheta_t,\vartheta_{t-1})|\vartheta') + \delta\bar{g}(0,1,\hat{p}(\vartheta',\vartheta_t)|\vartheta') = \bar{g}(0,0,\hat{p}(\vartheta_t,\vartheta_{t-1})|\vartheta') + \delta\bar{g}(0,0,\hat{p}(\vartheta',\vartheta_t)|\vartheta'),$$

or equivalently,

$$\hat{p}(\vartheta_t,\vartheta_{t-1}) + \vartheta' u_0 - c_{01} + \delta(\vartheta' u_1 - c_{10}) = \hat{p}(\vartheta_t,\vartheta_{t-1}) + \vartheta' u_0 + \delta((1-\rho)p(\vartheta_t) + \vartheta' u_0).$$

We therefore obtain that $\vartheta' = \alpha_1(\vartheta_t)$ as in Eq. (2.18). Analogous to Case 1, the action threshold is valid for $\vartheta_t \in (\vartheta^1, 1]$, where the upper invariance threshold ϑ^1 is specified in Eq. (2.15).

Case 3: $\vartheta_t \in [\vartheta^0, \vartheta^1]$. For type thresholds between the invariance thresholds, it is optimal for all agents to remain in their current sharing state, so $\vartheta' = \vartheta_t = \bar{\vartheta}$, where $\bar{\vartheta}$ implies a stationary sharing-state distribution $\bar{\xi}(\cdot, \bar{\vartheta})$ which stays in place for all times greater than t; see Section 2.3.3 for details.

Cases 1–3 together establish the law of motion in Eq. (2.16). ■

Proof of Proposition 2.2. Let $\vartheta_0 \in \Theta$. The fact that the agents' strategies $\hat{\pi}$ describe a subgame-perfect equilibrium of $\mathcal{G}(\vartheta_0)$ follows from the discussion in the main text, specifically because the agents' strategy profile implements value functions which together satisfy the system of Bellman equations (2.10). By application of the one-shot deviation principle in (B.1) we know that at time t, a type-θ agent's binary decision is to keep the item if and only if the following inequality holds:

$$\begin{aligned} &\bar{g}(1, \xi(\theta, \vartheta_t), \hat{p}(\vartheta_t, \vartheta_{t-1})|\theta) + \delta \bar{g}(\hat{\pi}(\theta, \vartheta'), 1, \hat{p}(\vartheta', \vartheta_t)|\theta) \geq \\ &\quad \bar{g}(0, \xi(\theta, \vartheta_t), \hat{p}(\vartheta_t, \vartheta_{t-1})|\theta) + \delta \bar{g}(\hat{\pi}(\theta, \vartheta'), 0, \hat{p}(\vartheta', \vartheta_t)|\theta). \end{aligned}$$

The entries in Table B.1 show the difference between the left-hand side and the right-hand side of the last inequality, which corresponds to the differential benefit of keeping ($a_t = 1$) over sharing ($a_t = 0$) for any given type $\theta \in \Theta$, conditional on the current sharing state and next period's optimal action.

	$\xi(\theta, \vartheta_t) = 0$	$\xi(\theta, \vartheta_t) = 1$
$\hat{\pi}(\theta, \vartheta') = 0$	$-c_{01} + \delta(\theta\Delta - \hat{p}_{t+1} - c_{10})$	$c_{10} + \delta(\theta\Delta - \hat{p}_{t+1} - c_{10})$
$\hat{\pi}(\theta, \vartheta') = 1$	$-c_{01} + \delta(\theta\Delta - \hat{p}_{t+1} + c_{01})$	$c_{10} + \delta(\theta\Delta - \hat{p}_{t+1} + c_{01})$

Table B.1: Payoff difference of keeping over sharing for an agent of type $\theta \in \Theta$, contingent on the current-period sharing state $\xi(\theta, \vartheta_t)$ and next period's equilibrium action $\hat{\pi}(\theta, \vartheta')$.

For all contingencies, the payoff difference is increasing in θ, so that a threshold strategy must necessarily be optimal, at any time t.[2] On the other hand, our equilibrium construction, beginning with the threshold policy in Eq. (2.11), leads to a unique strategy profile as subgame-perfect Nash equilibrium, so that the equilibrium is indeed unique. ■

Proof of Lemma 2.2. (i) We need to show that the limit of $\alpha(\cdot)$ vanishes when ϑ tends towards the lower end of the type space Θ, i.e., that $\lim_{\vartheta \to 0^+} \alpha_0(\vartheta) = 0$. For $\vartheta^0 = 0$ the result is immediate. Consider now the interesting case where $\vartheta^0 \in (0, 1)$. For $0 < \vartheta < \vartheta^0$, by Prop. 2.1 it is $\alpha(\vartheta) = \alpha_0(\vartheta) > 0$, and using the definition of the transaction probability in Eq. (2.8),

$$\alpha_0(\vartheta) = \frac{(1-\rho)\, p(\vartheta)(\vartheta/\alpha_0(\vartheta)) - r c_{10}}{\Delta}.$$

[2] If type θ prefers keeping over sharing, then all types $\hat{\theta} > \theta$ also prefer keeping over sharing.

Multiplying both sides by $\alpha_0(\vartheta) > 0$, and then taking the right-sided limit, for $\vartheta \to 0^+$, yields

$$0 \leq \lim_{\vartheta \to 0^+} \alpha_0^2(\vartheta)\Delta \quad = \quad \lim_{\vartheta \to 0^+} ((1-\rho)\, p(\vartheta)\vartheta - rc_{10}\alpha_0(\vartheta)) = -\lim_{\vartheta \to 0^+} rc_{10}\alpha_0(\vartheta) \leq 0,$$

which immediately implies that $\alpha_0(0) = 0$, by continuous completion.

(ii) If $\hat{\gamma} < rc_{10}$, then by Eq. (2.14) the lower invariance threshold vanishes, so for all $\vartheta \in (0, \vartheta^1]$ the sharing threshold lies in $\mathscr{R}$, and therefore $\alpha(\vartheta) \equiv \vartheta$. For all $\vartheta \in (\vartheta^1, 1]$, Prop. 2.1 yields $\alpha(\vartheta) = \alpha_1(\vartheta)$. Since $\alpha_1(\vartheta^1) = \vartheta^1$ and $\alpha_1'(\vartheta) < 0$, it follows that $\alpha_1(\vartheta) < \vartheta$ for all ϑ above $\mathscr{R}$. This implies the single-crossing property, as claimed. For $\hat{\gamma} > rc_{10}$, the lower invariance threshold $\vartheta^0 > 0$. By part (i), it is $\alpha_0(0) = 0$, and by La. 2.2 we know that $\alpha_0(\vartheta)$ has a positive slope for all $\vartheta \in (0, \vartheta^0)$. Thus, $\alpha_0(\vartheta) > \vartheta$ for $0 < \vartheta < \vartheta^0$. As for $\hat{\gamma} < rc_{10}$, $\alpha_1(\vartheta) < \vartheta$ for $\vartheta > \vartheta^1$. Hence, $\alpha(\vartheta) > \vartheta$ for $0 < \vartheta < \vartheta^0$ and $\alpha(\vartheta) \leq \vartheta$ for $\vartheta \geq \vartheta^0$, so the forward-difference of the sharing thresholds, $\alpha(\vartheta) - \vartheta$, has the (weak) single-crossing property for all $\vartheta \in (0, 1]$, as claimed.

(iii) By Prop. 2.1, $\alpha(\vartheta) = \alpha_0(\vartheta)$ for all ϑ below the lower invariance threshold. For $\hat{\gamma} \leq rc_{10}$, by Eq. (2.14) the lower invariance threshold ϑ^0 vanishes, so there is nothing to prove. For $\hat{\gamma} > rc_{10}$, we first consider the monotonicity of the system function. Note that $\vartheta^0 \leq \hat{\gamma}/(\hat{\gamma} + \Delta) \leq 1/2$. Differentiation of α_0 in Eq. (2.17) with respect to ϑ yields

$$\alpha_0'(\vartheta) = \frac{\hat{\gamma}(1-2\vartheta)}{2\Delta\alpha_0(\vartheta) + rc_{10}} > 0, \tag{B.7}$$

for all $\vartheta \in (0, \vartheta^0)$. Next we consider the concavity of the system function. Taking into account that $0 < \vartheta < \vartheta^0 \leq 1/2$, Eq. (B.7) yields

$$\alpha''(\vartheta) = -\frac{2\hat{\gamma}(2\Delta\alpha(\vartheta) + rc_{10}) + 2\Delta\hat{\gamma}(1-2\vartheta)\alpha'(\vartheta)}{(2\Delta\alpha(\vartheta) + rc_{10})^2} < 0,$$

for all $\vartheta \in (0, \vartheta^0)$, completing the proof. ■

Proof of Proposition 2.3. Let $\phi(\vartheta) \triangleq \alpha(\vartheta) - \vartheta$ for all $\vartheta \in [0, \vartheta^0]$ be the threshold increment, where we use the fact that by La. 2.2(i) the system function $\alpha(\cdot)$ is well-defined (by continuous completion) on the entire domain of ϕ. Then, $\phi(0) = \phi(\vartheta^0) = 0$, and by Rolle's theorem there exists a $\vartheta^\mu \in (0, \vartheta^0)$ such that $\phi'(\vartheta^\mu) = 0$. Since by La. 2.2 it is $\phi'' < 0$ on $(0, \vartheta^0)$, the maximum-diffusion threshold ϑ^μ is the unique maximizer of the threshold increment ϕ on $[0, \vartheta^0]$. ■

Proof of Proposition 2.4. (i),(ii): The claims follow directly from the definition of the invariance thresholds in Eqs. (2.14)–(2.15). (iii): Differentiating ϑ^0 in Eq. (2.14), with respect to γ and σ, respectively, yields

$$\frac{\partial\vartheta^0}{\partial\hat{\gamma}}\frac{\partial\hat{\gamma}}{\partial\gamma} = (1-\rho)\frac{\Delta + rc_{10}}{(\hat{\gamma}+\Delta)^2} > 0 \quad \text{and} \quad \frac{\partial\vartheta^0}{\partial\hat{\gamma}}\frac{\partial\hat{\gamma}}{\partial\rho} = -\gamma\frac{\Delta + rc_{10}}{(\hat{\gamma}+\Delta)^2} < 0,$$

as claimed. Similarly, differentiating ϑ^1 in Eq. (2.15), with respect to $\hat{\gamma}$ yields

$$\frac{\partial \vartheta^1}{\partial \hat{\gamma}} \frac{\partial \hat{\gamma}}{\partial \gamma} = (1-\rho) \frac{\Delta - c_{10} - (1+r)c_{01}}{(\hat{\gamma}+\Delta)^2} > 0 \quad \text{and} \quad \frac{\partial \vartheta^0}{\partial \hat{\gamma}} \frac{\partial \hat{\gamma}}{\partial \rho} = -\gamma \frac{\Delta - c_{10} - (1+r)c_{01}}{(\hat{\gamma}+\Delta)^2} < 0,$$

which is true for all $\vartheta^1 < 1$. ■

Proof of Lemma 2.3. As in the one-shot deviation principle applied to the general case in Eq. (B.1), the steady state is maintained in a (subgame-perfect) Nash equilibrium if no agent has an incentive to deviate (in any proper subgame of $\mathscr{G}(\vartheta_0)$). Given the optimality of a threshold-type strategy by Prop. 2.2, as well as optimality of sharing for the next-period sharing threshold by La. 2.1, $\bar{\vartheta}$ is stationary if

$$\delta \bar{V}^0(\theta|\bar{\vartheta}) \geq -c_{01} + \delta(\theta u_1 - c_{10}) + \delta^2 \bar{V}^0(\theta|\bar{\vartheta}), \quad \theta \in [0, \bar{\vartheta}],$$

and

$$\delta \bar{V}^0(\theta|\bar{\vartheta}) - c_{10} \leq \delta \bar{V}^1(\theta|\bar{\vartheta}), \quad \theta \in (\bar{\vartheta}, 1].$$

Combining the last two inequalities with the previous identities yields

$$\Big(\theta \in [0, \bar{\vartheta}] \;\Rightarrow\; (1+r)c_{01} + c_{10} \geq \theta\Delta - \bar{p}\Big) \quad \text{and} \quad \Big(\theta \in (\bar{\vartheta}, 1] \;\Rightarrow\; -rc_{10} \leq \theta\Delta - \bar{p}\Big).$$

Hence, for $\theta = \bar{\vartheta}$ one obtains

$$(\bar{p} - rc_{10})/\Delta \leq \bar{\vartheta} \leq (\bar{p} + (1+r)c_{01} + c_{10})/\Delta.$$

This completes the proof. ■

Proof of Proposition 2.5. (i) If $\vartheta_0 = 0$, then by La. 2.2(i) the sequence of sharing thresholds is stationary, and $\vartheta_t \equiv 0$. (ii) If $0 < \vartheta_0 \leq \vartheta^0$, then by La. 2.2 and Corollary 2.1, $(\alpha(\vartheta_t) - \vartheta_t)_{t=t^\mu}^\infty$ is a monotonically decreasing sequence, bounded from below by 0. Thus, by the monotone convergence theorem (see, e.g., Rudin 1976, p. 55)

$$\lim_{t\to\infty} (\alpha(\vartheta_t) - \vartheta_t) = \big(\alpha(\bar{\vartheta}) - \bar{\vartheta}\big)\Big|_{\bar{\vartheta}=\vartheta^0} = 0,$$

so that by continuity of $\alpha(\cdot)$, it is $\lim_{t\to\infty} \alpha(\vartheta_t) = \vartheta^0$. (iii) If $\vartheta^0 \leq \vartheta_0 \leq \vartheta^1$, then by Prop. 2.1, ϑ_0 lies in the invariance region $\mathscr{R}$, which implies the result. (iv) If $\vartheta^1 \leq \vartheta_0 \leq \vartheta^2$, then by Prop. 2.1, $\vartheta^0 \leq \alpha(\vartheta_0) = \alpha_1(\vartheta_0) \leq \vartheta^1$, so that $\alpha(\vartheta_0) \in \mathscr{R}$, which yields the claim. (v) If $\vartheta^2 \leq \vartheta_0 \leq 1$, then by Prop. 2.1, $\vartheta^0 \geq \alpha(\vartheta_0) = \vartheta_1$, and for $t \geq 1$ onwards the threshold ϑ_t lies in the interval $[\alpha_1(\vartheta_0), \vartheta^0]$. As in part (ii), we obtain that $\lim_{t\to\infty} \vartheta_t = \vartheta^0$. ■

Proof of Lemma 2.4. Let $\vartheta \in (0, \vartheta^0)$. By Prop. 2.1 and Eq. (2.8), we can rewrite the effective transaction price in Eq. (2.9) in the form

$$\hat{p}(\vartheta, \alpha(\vartheta)) = \hat{\gamma}(1-\vartheta)\vartheta / \alpha_0(\vartheta).$$

Differentiating the effective price with respect to the sharing threshold yields

$$\hat{p}'(\vartheta) = \frac{\hat{\gamma}(1-2\vartheta)}{\alpha_0^2(\vartheta)} \left(\alpha_0(\vartheta) - \frac{\hat{\gamma}\vartheta(1-\vartheta)}{2\Delta\alpha_0(\vartheta) + rc_{10}} \right). \tag{B.8}$$

Taking into account that $\vartheta^0 \leq 1/2$, to show that the effective price is increasing in ϑ, i.e., $\hat{p}'(\vartheta) > 0$, it is sufficient that the bracketed expression in Eq. (B.8) is positive, i.e.,

$$\alpha_0(\vartheta)(2\Delta\alpha_0(\vartheta) + rc_{10}) > \hat{\gamma}\vartheta(1-\vartheta). \tag{B.9}$$

By definition of $\alpha_0(\vartheta)$ in Prop. 2.1, for all $\vartheta \in (0, \vartheta^0)$ we obtain

$$\alpha_0(\vartheta)\,(\Delta\alpha_0(\vartheta) + rc_{10}) = \hat{\gamma}(1-\vartheta)\vartheta.$$

By substituting the last equation, inequality (B.9) is equivalent to

$$\Delta\alpha_0^2(\vartheta) > 0;$$

the latter holds for all $\vartheta \in (0, \vartheta^0)$, which completes our proof. ■

Proof of Proposition 2.6. By Prop. 2.5, for any nonzero initial condition the economy attains its steady state in finite time if and only if $\vartheta_0 \in [\vartheta^0, \vartheta^2]$. By Eq. (2.14), $\vartheta_0 < \vartheta^0$ if $rc_{10} < \hat{\gamma} - \vartheta_0(\hat{\gamma}+\Delta)$. Since $r = \delta/(1-\delta)$, rearranging the terms yields

$$c_{10} < \frac{\delta(\hat{\gamma}+\Delta)}{1-\delta} \left(\frac{\hat{\gamma}}{\hat{\gamma}+\Delta} - \vartheta_0 \right),$$

as claimed. Further, by Eq. (2.19), $\vartheta_0 > \vartheta^2$ if and only if

$$c_{10}(\hat{\gamma} + \Delta/\delta) + c_{01}(\hat{\gamma}+\Delta) < \delta(\hat{\gamma}+\Delta) \left(\vartheta_0 - \frac{\hat{\gamma}}{\hat{\gamma}+\Delta} \right),$$

which, once written in matrix form, yields the result immediately. ■

Proof of Lemma 2.5. By Eq. (2.14), $0 \leq \vartheta^0 \leq \hat{\gamma}/(\hat{\gamma}+\Delta) \leq 1/2$, and by Remark 2.3 it is $\vartheta^0 \leq \vartheta^1 \leq 1$. Since $\alpha_1(\vartheta)$ is decreasing in ϑ, it follows that $\vartheta^1 = \alpha_1^{-1}(\vartheta^1) \leq \alpha_1^{-1}(\vartheta^0) = \vartheta^2$, as claimed. We now prove the three remaining claims. (i) By Eq. (2.14), $\vartheta^0 > 0$ if $\hat{\gamma} > rc_{10}$. Since $r = \delta/(1-\delta)$, the result follows immediately. (ii) Using the definition of ϑ^1 in Eq. (2.15) and rearranging the terms yields the desired result. (iii) If $\hat{\gamma} > rc_{10}$, since $\alpha_1(\vartheta)$ is a decreasing function, for $\vartheta^2 < 1$ we require $\alpha_1(1) < \vartheta^0$, i.e.,

$$\frac{c_{10} + (1+r)c_{01}}{\Delta} \le \frac{\hat{\gamma} - rc_{10}}{\hat{\gamma} + \Delta}.$$

Rearranging the terms yields $c_{10}(1 + \frac{\hat{\gamma}}{\Delta}) + c_{01}(1 + \delta\frac{\hat{\gamma}}{\Delta}) < \delta\hat{\gamma}$. If $\hat{\gamma} > rc_{10}$, the lower invariance threshold vanishes and $(c_{10} + (1+r)c_{01})/\Delta \ge 0$ implies that $\vartheta^2 = 1$. ■

Proof of Lemma 2.6. Let $y \in \mathbb{R}$ be fixed, and let $\Phi \subset \mathbb{R}$ be a nonempty, convex, compact set (i.e., a closed interval). Consider the mapping $H : \Phi \to \mathbb{R}$ with $H(\varphi) \triangleq (p(\varphi) + y)/\Delta$ for all $\varphi \in \Phi$. Then $H(\cdot)$ is continuous as a composition of continuous functions. By a similar argument it is also differentiable, and

$$H'(\varphi) = \frac{p'(\varphi)}{\Delta} \le 0,$$

for all $\varphi \in \mathbb{R}$, since the inverse demand function $p(\cdot)$ is downward-sloping by assumption. Let $\Phi = [0, 1 + y/\Delta]$, and let $h(x) \triangleq H(x) - x$. Since $h(1 + y/\Delta) < 0 < h(0)$, by the intermediate value theorem there exists a solution $\varphi = \varphi(y, \Delta)$ (in the set Φ) such that $h(\varphi) = 0$, or equivalently

$$\varphi = H(\varphi) = \frac{p(\varphi) + y}{\Delta}.$$

The uniqueness of $\vartheta(y, \Delta)$ follows from the fact that the left-hand side of the last equation is strictly increasing whereas its right-hand side is (weakly) decreasing. ■

Proof of Lemma 2.7. Similar to the procedure undertaken in the proof of La. 2.1, we use the one-shot deviation principle to establish a subgame-perfect equilibrium, when the economy is about to (i) stagnate; (ii) expand; or (iii) contract. Accordingly we distinguish the three cases (i)–(iii). The proofs of cases (i) and (iii) are analogous to the ones in La. 2.1, since the arguments only depend on the basic non-Giffen-good property, i.e., that the demand curve is downward-sloping. For case (ii), we now check that in an expanding sharing economy, no agent has an incentive to deviate from the equilibrium path in any single period, and $\vartheta' = \alpha(\vartheta_t)$ is such that

$$\begin{aligned}\bar{g}(1, \xi(\vartheta', \vartheta_t), \hat{p}(\vartheta_t, \vartheta_{t-1})|\vartheta') + \delta\bar{g}(\hat{\pi}(\vartheta', \vartheta'), 1, \hat{p}(\vartheta', \vartheta_t)|\vartheta') = \\ \bar{g}(0, \xi(\vartheta', \vartheta_t), \hat{p}(\vartheta_t, \vartheta_{t-1})|\vartheta') + \delta\bar{g}(\hat{\pi}(\vartheta', \vartheta'), 0, \hat{p}(\vartheta', \vartheta_t))|\vartheta').\end{aligned} \quad \text{(B.10)}$$

We establish this by contradiction. Suppose that $\hat{\pi}(\vartheta', \vartheta') = 1$. By Eq. (2.5), it is $\xi(\vartheta', \vartheta_t) = 1$, so that by Eq. (B.3):

$$\vartheta' = \frac{p(\vartheta_t)q(\vartheta', \vartheta_t) - c_{01} - (1+r)c_{10}}{\Delta}. \quad \text{(B.11)}$$

The time-$(t+1)$ type threshold ϑ' can exceed ϑ_t, as long as $\vartheta_t \in [0,\omega)$, where ω solves a stationary version of Eq. (B.11), corresponding to a fixed-point problem with $q(\omega,\omega) = 1$, so

$$\omega = \max\left\{0, \frac{p(\omega) - c_{01} - (1+r)c_{10}}{\Delta}\right\} \le \vartheta^0. \tag{B.12}$$

By La. 2.6, there exists a unique $\omega \in \Theta$ that satisfies Eq. (B.12). Differentiating ϑ' in Eq. (B.11) with respect to ϑ_t and evaluating at $\vartheta_t = \omega$ yields, together with the demand-elasticity condition (A),

$$\left.\frac{\partial\vartheta'}{\partial\vartheta_t}\right|_{\vartheta_t=\omega} = \left.\frac{p'(\vartheta_t)\vartheta_t + p(\vartheta_t)}{2\Delta\vartheta' + c_{01} + (1+r)c_{10}}\right|_{\vartheta_t=\omega} = \frac{\omega(\Delta + p'(\omega)) + c_{01} + (1+r)c_{10}}{2\Delta\vartheta' + c_{01} + (1+r)c_{10}} \ge 0,$$

since $\varepsilon(\omega) \ge 1$ means that $p'(\omega)\omega + p(\omega) \ge 0$. Hence, for all $\vartheta_t \in [0,\omega)$, the next-period sharing threshold ϑ' is increasing in ϑ_t, and

$$\vartheta_t < \vartheta' < \vartheta'' < \omega.$$

The strict inequalities are a consequence of the uniqueness of the fixed point ω. Thus, by virtue of Eq. (2.11) it is $\pi(\vartheta',\vartheta') = \mathbf{1}_{\{\vartheta' > \vartheta''\}} = 0$, in direct contradiction to our hypothesis, so that the claim holds for case (ii). ■

Proof of Lemma 2.8. The result is obtained in a manner analogous to the proof of Prop. 2.3. We restrict attention to establishing the convexity of $\alpha_0(\vartheta)$ as claimed by La. 2.2. The demand-elasticity condition (A) implies that

$$\alpha_0'(\vartheta) = \frac{p'(\vartheta)\vartheta + p(\vartheta)}{2\Delta\alpha_0(\vartheta) + rc_{10}} \ge 0, \quad \vartheta \in (0,\vartheta^0). \tag{B.13}$$

Eq. (B.13) yields the second derivative,

$$\alpha_0''(\vartheta) = \frac{(p''(\vartheta)\vartheta + 2p'(\vartheta))(2\Delta\alpha_0(\vartheta) + rc_{10}) - 2\Delta\alpha_0'(\vartheta)(p'(\vartheta)\vartheta + p(\vartheta))}{(2\Delta\alpha_0(\vartheta) + rc_{10})^2}, \quad \vartheta \in (0,\vartheta^0).$$

To obtain the convexity of the system function for all $\vartheta \in (0,\vartheta^0)$, it is enough to show that $\hat{p}''(\vartheta)\vartheta \le -2\hat{p}'(\vartheta))$, which can be accomplished by considering each demand class separately:

- *constant-elasticity demand:* $p''(\vartheta)\vartheta = \left(1 + \frac{1}{\eta}\right)\frac{\gamma}{\eta}\vartheta^{-1/\eta} \le 2\frac{\gamma}{\eta}\vartheta^{-1/\eta} = -2p'(\vartheta)$, for $\eta \ge 1$ and $\gamma > 0$;
- *semi-logarithmic demand:* $p''(\vartheta)\vartheta = \frac{\gamma_1}{\vartheta} \le 2\frac{\gamma_1}{\vartheta} = -2p'(\vartheta)$, for $\gamma_0 \ge \gamma_1 > 0$;
- *quasi-affine demand:* $p''(\vartheta)\vartheta = \left(1 - \frac{1}{\eta}\right)\frac{\gamma_1}{\eta}\vartheta^{1/\eta} \le 2\frac{\gamma_1}{\eta}\vartheta^{1/\eta} = -2p'(\vartheta)$, for $\eta \ge (\gamma_1/\gamma_0)/(1 - (\gamma_1/\gamma_0))$ and $\gamma_0 > \gamma_1 > 0$.

Thus, the mapping $\alpha_0(\cdot)$ is strictly concave in each case, which establishes our claim. ■

Proof of Lemma 2.9. Suppose that the utilities u_0, u_1 and the quality γ are measured over the normalized adjustment-interval length $dt = 1$, as in the main text. Assume also that the demand-elasticity parameter $\gamma > 0$ is given. If prices can adjust after the reduced time $dt' = \lambda dt < dt$, where $\lambda \in (0,1)$, the per-period utilities adjust accordingly to λu_1 and λu_0, while the consumers perceive the quality $\gamma' = \lambda\gamma$. Furthermore, the per-period interest rate for the compressed time scale becomes

$$r' = (1+r)^{\lambda} - 1. \tag{B.14}$$

As in Prop. 2.1, the system function for the compressed time scale can be written in the implicit form

$$\hat{\alpha}_0(\vartheta) = \frac{(1-\rho)\lambda p(\vartheta)\vartheta/\hat{\alpha}_0(\vartheta) - r'c_{10}}{\lambda\Delta} = \frac{(1-\rho)p(\vartheta)\vartheta/\hat{\alpha}_0(\vartheta) - rc_{10}}{\Delta} + \left(r - \frac{(1+r)^{\lambda}-1}{\lambda}\right)\frac{c_{10}}{\Delta}. \tag{B.15}$$

The first term on the right-hand side of Eq. (B.15) is identical to the right-hand side in the implicit definition of the original (not time-compressed) system function $\alpha_0(\cdot)$ in Prop. 2.1. Using elementary methods, one can show that the second term is nonnegative, decreasing in λ, and increasing in r. Moreover, using implicit differentiation it becomes clear that the value of the time-compressed system function $\hat{\alpha}_0(\vartheta)$ is increasing in the value of this second term, so that

$$\alpha_0(\vartheta) < \hat{\alpha}_0(\vartheta) \quad \text{and} \quad \max\left\{\frac{\partial\hat{\alpha}_0(\vartheta)}{\partial\lambda}, \frac{\partial\hat{\alpha}_0(\vartheta)}{\partial r}\right\} < 0,$$

for all $\vartheta \in (0, \vartheta^0)$. Even in the limit, for $\lambda \to 0^+$, the difference between the compressed and uncompressed system function stays positive (for $r > 0$):

$$\lim_{\lambda\to 0^+}[\hat{\alpha}_0(\vartheta) - \alpha_0(\vartheta)] = \frac{(1-\rho)p(\vartheta)\vartheta}{\Delta}\left(\frac{1}{\hat{\alpha}_0(\vartheta)} - \frac{1}{\alpha_0(\vartheta)}\right) + (r - \ln(1+r))\frac{c_{10}}{\Delta} > 0.$$

Because the first term on the right-hand side is negative, the second term is an upper bound for the deviation between the compressed and uncompressed system functions, i.e.,

$$0 < \hat{\alpha}_0(\vartheta) - \alpha_0(\vartheta) < (r - \ln(1+r))\frac{c_{10}}{\Delta},$$

for any time-compression factor $\lambda \in (0,1)$. Similarly, the time-compressed lower invariance threshold $\hat{\vartheta}^0$ can be computed using Eq. (2.14),

$$\hat{\vartheta}^0 = \frac{\lambda\hat{\gamma} - r'c_{10}}{\lambda\hat{\gamma} + \lambda\Delta} = \vartheta^0 + \left(r - \frac{(1+r)^{\lambda}-1}{\lambda}\right)\frac{c_{10}}{\hat{\gamma}+\Delta} > \vartheta^0,$$

for all $\lambda \in (0,1)$. Using the same limit-argument as before, we can therefore conclude that

$$0 < \hat{\vartheta}^0 - \vartheta^0 < (r - \ln(1+r))\frac{c_{10}}{\hat{\gamma}+\Delta},$$

for all $\lambda \in (0,1)$. This completes the proof. ■

Proof of Lemma 2.10. To establish the claim, it is sufficient to show that Las. 2.1 and 2.2 hold for a generic affine demand function of the form $p(\vartheta) = \gamma_0 - \gamma_1\vartheta$, where $\gamma_0 = (1-\rho)\gamma - \kappa$ and $\gamma_1 = (1-\rho)\gamma$. By replacing ω with

$$\hat{\omega} = \min\left\{1, \frac{\gamma_0 - c_{01} - (1+r)c_{10}}{\gamma_1 + \Delta}\right\},$$

La. 2.1 obtains, since $\hat{\omega} \leq \gamma_0/(\gamma_1 + \Delta) \leq 1/2$, and

$$\frac{\partial\vartheta'}{\partial\vartheta_t} = \frac{\gamma_0 - 2\gamma_1\vartheta_t}{2\Delta\vartheta' + c_{01} + (1+r)c_{10}} > 0,$$

for all $\vartheta_t \in [0, \hat{\omega})$. The rest of the proof of La. 2.1 remains essentially unchanged. Furthermore, parts (i) and (ii) of La. 2.2 can be proved as before, with the condition $\hat{\gamma} < rc_{10}$ replaced by the condition $\gamma_0 < rc_{10}$ in part (ii) of the proof. We can therefore restrict attention to showing that $\alpha''(\vartheta) < 0 < \alpha'(\vartheta)$, as claimed in part (iii) of La. 2.2. Since $\vartheta^0 \leq \gamma_0/(\gamma_1 + \Delta)$,

$$\alpha_0'(\vartheta) = \frac{\gamma_0 - 2\gamma_1\vartheta}{2\Delta\alpha_0(\vartheta) + rc_{10}} > 0, \tag{B.16}$$

for all $\vartheta \in [0, \vartheta^0)$. We now establish the convexity of the system function. Differentiating Eq. (B.16) yields

$$\alpha_0''(\vartheta) = -\frac{2\gamma_1(2\Delta\alpha_0(\vartheta) + rc_{10}) + 2\Delta(\gamma_0 - 2\gamma_1\vartheta)\alpha_0'(\vartheta)}{(2\Delta\alpha_0(\vartheta) + rc_{10})^2} < 0,$$

for all $\vartheta \in [0, \vartheta^0)$, as claimed. The generalized versions of La. 2.1 and La. 2.2 together imply La. 2.10. ■

B.1.2 Auxiliary Results

Lemma B.1.1. *The time-t price elasticity of demand ε_t is (weakly) decreasing in $\hat{\gamma}$ on the equilibrium path.*

Proof. Consider the interesting case where $\vartheta_0 > 0$. By footnote 10 and Eq. (2.6), the price elasticity of demand at time t is $\varepsilon_t = (1-n_t)/n_t = (1-\vartheta_t)/\vartheta_t$, provided $\vartheta_t > 0$. Differentiating ε_t with respect to the demand-elasticity parameter $\hat{\gamma}$ yields

$$\frac{\partial\varepsilon_t}{\partial\hat{\gamma}} = -\frac{1}{\vartheta_t^2}\left(\frac{\partial\vartheta_t}{\partial\vartheta_{t-1}} \cdot \frac{\partial\vartheta_{t-1}}{\partial\vartheta_{t-2}} \cdot \ldots \cdot \frac{\partial\vartheta_1}{\partial\hat{\gamma}}\right). \tag{B.17}$$

If $\vartheta_0 \in \mathscr{R}$, by Prop. 2.5 the sharing economy becomes stationary, independent of the demand-elasticity parameter $\hat{\gamma}$, i.e., $\partial\varepsilon_t/\partial\hat{\gamma} = 0$. For all $\vartheta_0 \in (0, \vartheta^0)$, on the equilibrium path given by Prop. 2.1, it is

$$\frac{\partial\vartheta_1}{\partial\hat{\gamma}} = \frac{\partial}{\partial\hat{\gamma}}\left(\frac{p(\vartheta_0)\cdot\vartheta_1/\vartheta_0 - rc_{10}}{\Delta}\right) = \frac{(1-\vartheta_0)\cdot\vartheta_1/\vartheta_0}{\Delta + \hat{\gamma}(1-\vartheta_0)} > 0, \tag{B.18}$$

which, together with La. 2.2, implies that $\partial\varepsilon_t/\partial\hat{\gamma} < 0$. Similarly, for all $\vartheta_0 \in (\vartheta^1, 1)$ we obtain

$$\frac{\partial\vartheta_1}{\partial\hat{\gamma}} = \frac{1-\vartheta_0}{\Delta} > 0. \tag{B.19}$$

If $\vartheta_0 \in (\vartheta^1, \vartheta^2]$, then $\partial\vartheta_t/\partial\vartheta_{t-1} = 1$ for all $t \geq 1$. Hence, $\partial\varepsilon_t/\partial\hat{\gamma} = \partial\vartheta_1/\partial\hat{\gamma} < 0$. On the other hand, for $\vartheta_0 \in (\vartheta^2, 1]$, La. 2.2 implies that $\partial\vartheta_t/\partial\vartheta_{t-1} > 0$, and the right-hand side of Eq. (B.17) becomes negative, thus completing the proof. ∎

B.2 Analytical Details of Chapter 3

B.2.1 Auxiliary Results

Lemma B.2.1. *Let $\varkappa \triangleq f/g$, where $f, g : \mathscr{X} \to \mathbb{R}_{++}$ are differentiable functions defined on the closed interval $\mathscr{X} \subset \mathbb{R}$ such that $f(x_0) = g(x_0)$ for some $x_0 \in \mathscr{X}$. If furthermore $f' \geq g'$ on $\mathscr{X}$, then $\varkappa \geq 1$ on $\mathscr{X} \cap [x_0, \infty)$.*

Proof. Since $f' \geq g'$ on $\mathscr{X}$, it is $f(x) \geq g(x) \geq f(x_0) = g(x_0) > 0$ for all $x \in \mathscr{X}$ with $x \geq x_0$, which implies that necessarily $\varkappa \geq 1$ on $\mathscr{X} \cap [x_0, \infty)$. ∎

Lemma B.2.2. *Let $\varkappa \triangleq f/g$, where $f, g : \mathscr{X} \to \mathbb{R}_{+}$ are differentiable functions defined on the closed interval $\mathscr{X} \subset \mathbb{R}$, with $g > 0$, such that $f(x_0) \geq g(x_0)$ for some $x_0 \in \mathscr{X}$. If furthermore $0 \geq f' \geq g'$ on $\mathscr{X}$, then $\varkappa' \geq 0$ on $\mathscr{X} \cap [x_0, \infty)$.*

Proof. Since $f' \geq g'$ on $\mathscr{X}$, it is $f(x) \geq g(x) \geq f(x_0) \geq g(x_0) > 0$ for all $x \in \mathscr{X}$ with $x \geq x_0$. Thus, $0 \geq f'(x)g(x) \geq g'(x)g(x) \geq g'(x)f(x)$ for all $x \in \mathscr{X} \cap [x_0, \infty)$. But this implies that $\varkappa'(x) = (f'(x)g(x) - g'(x)f(x))/g^2(x) \geq 0$ for all $x \in \mathscr{X} \cap [x_0, \infty)$. ∎

Lemma B.2.3. *For all $q \in [0,1]$: $\rho'(q) \leq 0 \leq \rho''(q)$.*

Proof. Let $\hat{\rho}(q) \triangleq [1 - \ln(1+\delta q)/(\delta q)]/(\delta q)$. Then, $\rho'(q) = \hat{\rho}'(q) - 1/3$. To prove that $\rho'(q) < 0$, it is sufficient to show that $\hat{\rho}(q)$ is decreasing in q. Differentiating $\hat{\rho}(q)$ yields

$$\hat{\rho}'(q) = -\frac{1}{\delta^2 q^3}\left(\delta q + \frac{\delta q}{1+\delta q} - 2\ln(1+\delta q)\right) = -\frac{f(q)}{\delta^2 q^3},$$

where we have set $f(q) \triangleq \delta q + (\delta q)/(1+\delta q) - 2\ln(1+\delta q)$. Note that

$$f'(q) = \delta + \frac{\delta}{(1+\delta q)^2} - \frac{2\delta}{(1+\delta q)} = \delta\frac{(1+\delta q)^2 + 1 - 2\delta(1+\delta q)}{(1+\delta q)^2} = \delta\frac{(\delta q)^2}{(1+\delta q)^2} \geq 0, \quad q \in [0,1].$$

This inequality, together with the initial value $f(0) = 0$, implies that $f(q) \geq 0$ on $[0,1]$, so that $\rho'(q) \leq 0$ for all $q \in [0,1]$, as claimed. The convexity of ρ is established by first introducing

$h(q) \triangleq \delta^2 q^3$, so

$$\rho''(q) = \hat{\rho}''(q) = -\frac{f'(q)h(q) - h'(q)f(q)}{h(q)^2}, \quad q \in (0,1].$$

Furthermore, $f'(q)h(q) - h'(q)f(q) \leq 0$ if and only if

$$\frac{(\delta q)^3/3}{(1+\delta q)^2} \leq \delta q + \frac{\delta q}{1+\delta q} - 2\ln(1+\delta q).$$

By setting $g(q) \triangleq 3(\delta^3 q^3)/(1+\delta q)^2$ the preceding inequality is equivalent to $f(q)/g(q) \geq 1$ for all $q \in (0,1]$. Note that $f(0) = g(0) = 0$, and

$$f'(q) = \delta \frac{(\delta q)^2}{(1+\delta q)^2} \geq \frac{\delta}{3} \frac{(3+\delta q)(\delta q)^2}{(1+\delta q)^3} = g'(q), \quad q \in (0,1].$$

By La. B.2.1. (including a continuous completion at $q = 0^+$), it follows that $f/g \geq 1$ a.e. on the interval $[0,1]$, which implies the monotonicity of ρ' and thus the convexity of ρ on $[0,1]$, as claimed. ■

B.2.2 Proof of the Main Results

Proof of Proposition 3.1. For $r \in [0,1]$, the no-sharing demand by consumers in the early consumption phase is [3]

$$\hat{\Omega}_0(r) = \frac{1-r}{2} + \int_{r/(1+\delta q)}^{r} \left(\int_{(r-v)/(\delta q v)}^{1} \theta \, d\theta \right) dv = \frac{1-r}{2} + \frac{r}{2\delta q} \int_0^1 \frac{1-\xi^2}{((1/(\delta q)) + \xi)^2} \, d\xi,$$

so that

$$\hat{\Omega}_0(r) = \frac{1}{2} - \frac{r}{\delta q}\left(1 - \frac{\ln(1+\delta q)}{\delta q}\right), \quad r \in [0,1]. \tag{B.20}$$

For $r \in [1, 1+\delta q]$, one obtains—by similar calculations—that

$$\begin{aligned}\hat{\Omega}_0(r) &= \int_{r/(1+\hat{\delta})}^{1} \left(\int_{(r-v)/(\hat{\delta} v)}^{1} \theta \, d\theta \right) dv \\ &= \frac{1}{2}\left[\left(1 - \frac{1}{\hat{\delta}^2}\right)\left(1 - \frac{r}{1+\delta}\right) + \frac{r^2}{\hat{\delta}^2}\left(1 - \frac{1+\delta}{r}\right)\right] - \frac{r}{\hat{\delta}^2} \ln\left(\frac{r}{1+\hat{\delta}}\right).\end{aligned} \tag{B.21}$$

Lastly, for $r \geq 1+\delta q$ it is $\hat{\Omega}_0(r) = 0$. Combining Eqs. (B.20) and (B.21) yields Eq. (3.6). On the other hand, the ownership demand by consumers in the late consumption phase is

$$\hat{\Omega}_1(q,r) = (1-r)\left(\int_0^1 (1-\theta)\theta \, d\theta + (1-q)\int_0^1 \theta^2 \, d\theta\right) = (1-r)\left(\frac{1}{2} - \frac{q}{3}\right), \quad r \in [0,1],$$

[3] The inner integral evaluates to $\int_{(r-v)/(\delta q v)}^{1} \theta \, d\theta = \frac{1}{2} - \frac{((r/v)-1)^2}{2(\delta q)^2}$.

and $\hat{\Omega}_1(q,r) = 0$ for $r \geq 1$, corresponding to Eq. (3.7). The aggregate demand for ownership is $\hat{\Omega} = \hat{\Omega}_0 + \hat{\Omega}_1$. ■

Proof of Corollary 3.1. It is straightforward to show that the partial derivative of Ω_1 with respect to q is nonpositive,

$$\frac{\partial \hat{\Omega}_1}{\partial q} = -\max\left\{0, \frac{1-r}{3}\right\} \leq 0.$$

We now prove that the sales in the early consumption phase is increasing in q. Note that by Prop. 3.1, for all $r \in [0,1]$, the demand for ownership by the young generation (in $\mathscr{C}_0$) is

$$\hat{\Omega}_0(q,r) = \frac{1}{2} - r\hat{\rho}(q),$$

where $\hat{\rho}(q) = \left(1 - \ln(1+\delta q)/(\delta q)\right)/(\delta q)$. Partially differentiating $\hat{\Omega}_0$ with respect to q yields

$$\frac{\partial \hat{\Omega}_0}{\partial q} = \frac{\partial \hat{\rho}}{\partial q} \leq 0,$$

thus implying the result by virtue of La. B.2.3. ■

Proof of Lemma 3.1. Differentiating $\hat{r}$ in Eq. (3.9) with respect to δ and taking into account La. B.2.3, one obtains

$$\frac{\partial \hat{r}}{\partial \delta} = -\left(1 - \frac{q}{3}\right)\frac{\rho'(q)}{2\rho^2(q)} \geq 0, \quad \delta, q \in (0,1).$$

Similarly, differentiating $\hat{r}$ with respect to q yields

$$\hat{r}'(q) = -\frac{1}{2\rho^2}\left(\left(1-\frac{q}{3}\right)\rho'(q) + \frac{\rho}{3}\right) = \frac{\partial \hat{r}}{\partial \delta} - \frac{1}{6\rho} < \frac{\partial \hat{r}}{\partial \delta}.$$

Using the abbreviation $f(q) \triangleq 1 - q/3$, it is $f(0) = \rho(0) = 1$, and

$$\hat{r}'(q) = \frac{1}{2}\frac{d}{dq}\left(\frac{k(q)}{\rho(q)}\right).$$

Note that $f(q)$ and $\rho(q)$ are positive for all $q \in [0,1]$. Furthermore,

$$f' = \frac{1}{3} \geq \hat{\rho}' - \frac{1}{3} = \rho',$$

where $\hat{\rho}(q) = [1 - \ln(1+\delta q)/(\delta q)]/(\delta q)$ and $\hat{\rho}'(q) \leq 0$ by La. B.2.3. Thus, applying La. B.2.2 to the fraction $\varkappa \triangleq f/\rho$ yields that $\varkappa$ is nondecreasing, whence, by construction, $\hat{r}' \geq 0$, which completes our proof. ■

Proof of Proposition 3.2. It is $\rho'(0) = -(1+\delta)/3$. Thus, differentiating the profit function in

Eq. (3.10) yields $d\hat{\Pi}(q,\hat{r}(q))/dq\big|_{q=0} \geq 0$, if and only if

$$c \geq \frac{1-\delta}{1+\delta} \triangleq \check{c}. \tag{B.22}$$

(i) Let $c \in [0,\check{c}]$, so that the no-sharing profit is has nonpositive slope at $q=0$. Note that $\hat{\Pi}'(q)$ is unimodal with the local maximum always preceding the local minimum (if it exists in the domain). As $\hat{\Pi}'(q) \leq 0$ for all $c \in [0,\check{c}]$, in order to find the global maximum it is enough to consider the profit at the boundary, i.e., where $q \in \{0,1\}$. Using l'Hôpital's rule it is $\lim_{q\to 0^+} \rho(q) = 1$, so that the profit at zero durability becomes

$$\lim_{q\to 0^+} \hat{\Pi}(\hat{r}(q),q) = \frac{(1-c)^2}{4}.$$

On the other hand, perfect durability generates a profit equal to

$$\hat{\Pi}(\hat{r}(1),1) = \frac{(2-3c\rho(1))^2}{36\,\rho(1)}.$$

Comparing the profit at the boundary points yields that $\hat{\Pi}(1,\hat{r}(1)) \leq \hat{\Pi}(0,\hat{r}(0))$ if and only if

$$c < \frac{3\sqrt{\rho(1)}-2}{3\sqrt{\rho(1)}-3\rho(1)} \leq \check{c},$$

which is satisfied for all $c \in [0,\check{c}]$. This completes the proof of part (i).

(ii) For any $c \geq \check{c}$, the corner solution $\hat{q}=0$ is suboptimal. The other corner solution, $\hat{q}=1$, is also suboptimal, as long as

$$\hat{\Pi}(1,\hat{r}(1)) < \hat{\Pi}(\hat{q},\hat{r}(\hat{q})),$$

where $\hat{q}$ is the interior maximizer implicitly defined in Eq. (3.12) of Cor. 3.3. Substituting Eq. (3.12) in Eq. (3.10) and rearranging the terms provides us with the global optimality of the interior maximizer as long as

$$0 \leq c \leq \frac{2-\sqrt{8\rho(1)\rho(\hat{q})/\rho'(\hat{q})}}{3\rho(1)} \triangleq \hat{c} \in (\check{c},\bar{c}), \tag{B.23}$$

as claimed.

(iii) As already established in our proof of (ii), for all $c \in (\hat{c},\bar{c}]$ one obtains $\hat{\Pi}(\hat{q},\hat{r}(\hat{q})) < \hat{\Pi}(1,\hat{r}(1))$, so $\hat{q}=1$.

Parts (i)–(iii) together establish the result. ■

Analytical Details for Footnote 8: see the proof of Prop. 3.2(i). ■

Proof of Corollary 3.2. Part (i) follows from solving the inequality in Eq. (B.22) in the proof of Prop. 3.2 for the discount factor: $\delta \geq (1-c)/(1+c) \triangleq \check{\delta}$. Parts (ii) and (iii) revolve around the discount-factor threshold $\hat{\delta} \in (\check{\delta}, 1]$ which can be obtained, for a given c, by (numerically) solving the second inequality in Eq. (B.23). ■

Proof of Corollary 3.3. An interior maximizer $\hat{q} \in (0,1)$ of the firm's profit $\hat{\Pi}(q, \hat{r}(q))$ in Eq. (3.10) satisfies the Fermat condition

$$\frac{d\hat{\Pi}(q,\hat{r}(q))}{dq} = \frac{2(1-c\rho(q)-q/3)(-c\rho'(q)-1/3)\rho(q)-(1-c\rho(q)-q/3)^2\rho'(q)}{2\rho^2(q)} = 0,$$

which—provided a positive optimal profit—is equivalent to Eq. (3.12). ■

Proof of Proposition 3.3. We first consider the dependence of the optimal durability $\hat{q}$ on the production cost c. To establish the monotone dependence, it is sufficient to show that the firm's objective function in Eq. (3.10) has increasing differences in (q,c). The corresponding cross-partial derivative is

$$\frac{\partial^2\hat{\Pi}(q,\hat{r}(q))}{\partial q \partial c} = c\rho'(q) + \frac{1}{3} \geq c\rho'(0) + \frac{1}{3} = \frac{1-(1+\delta)c}{3}.$$

The last expression on the right-hand side is positive if $c > \check{c}$, which by Cor. 3.2 is a necessary and sufficient condition for the optimal durability to be nonzero. Since $1/(1+\delta) > \check{c}$ (see Eq. (B.22)), this guarantees that $\hat{q}$ is (weakly) increasing in c as claimed. — Next, we consider the monotonicity of $\hat{q}$ in δ, restricting attention to an interior solution ($0 < \hat{q} < 1$) which satisfies

$$\left(1 + c\rho(\hat{q}) - \frac{q}{3}\right)(-\rho'(\hat{q})) = \frac{2\rho(\hat{q})}{3}, \tag{B.24}$$

as indicated in Cor. 3.3. Differentiating implicitly with respect to δ yields

$$\frac{\rho'(\hat{q})}{3} = \left[\left(\frac{2}{3} - c\right)\rho'(\hat{q}) + \left(1 + c\rho(\hat{q}) - \frac{q}{3}\right)\rho''(\hat{q})\right]\frac{\partial\hat{q}}{\partial\delta},$$

so that $\partial\hat{q}/\partial\delta \geq 0$ if and only if

$$\left(1 + c\rho(\hat{q}) - \frac{\hat{q}}{3}\right)\rho''(\hat{q}) \leq \left(\frac{2}{3} - c\right)\left(-\rho'(\hat{q})\right).$$

Using again the optimality condition (B.24), the preceding inequality is equivalent to

$$\left.\frac{\rho\rho''}{(\rho')^2}\right|_{q=\hat{q}} \leq 1 - \frac{3c}{2},$$

which holds for all $(\delta, c) \in (0,1) \times (0, \bar{c})$. Note also that the transition from zero durability to intermediate durability takes place at a lower cost threshold $\check{c}$ as δ goes up. The same holds for the cost threshold $\hat{c}$, which is increasing in δ. ■

Proof of Lemma 3.2. Given the optimal profit $\hat{\Pi}^* = \hat{\Pi}(\hat{q}, \hat{r}(\hat{q}))$ in Eq. (3.10), for intermediate durability levels the claims obtain immediately by using the envelope theorem:

$$\frac{\partial \hat{\Pi}^*}{\partial c} = -\left(1 - c\rho(\hat{q}) - \frac{\hat{q}}{3}\right) < 0 < -\left(c + \hat{\Pi}^*\right) \frac{\hat{q}\hat{\rho}'(\hat{q})}{\delta\rho(\hat{q})} = \frac{\partial \hat{\Pi}^*}{\partial \delta},$$

for all $(\delta, c) \in (0,1) \times (0, \bar{c})$. In the boundary cases where the optimal durability is either 0 or 1, the conclusion of the envelope theorem continues to hold, given that $\partial\hat{q}/\partial(\delta, c) = 0$. ■

Proof of Proposition 3.4. For any given $r \in (\underline{r}(q), \bar{r}(q)]$, the equilibrium sharing price solves the market-clearing condition by equating the supply and demand obtained in Eqs. (3.22)–(3.23). That is,

$$\frac{q}{2}\left(\frac{1}{3} + \left(\frac{2}{3} + \delta q\right) p - r\right) = \frac{1-p}{2}\left(1 - \frac{2q}{3}\right),$$

which results in the best market response

$$p(q,r) = \frac{1 - q + qr}{1 + \delta q^2}, \tag{B.25}$$

as claimed. The sharing price from Eq. (B.25) in the sharing demand of Eq. (3.23) yields the transaction volume:

$$Q(q,r) = D(q, p(q,r)) = \frac{q}{2}\left(1 - \frac{2q}{3}\right)\left(\frac{1 + \delta q - r}{1 + \delta q^2}\right).$$

Substituting $\bar{r}(q) = 1 + \delta q$ in the preceding equation completes the proof. ■

Proof of Lemma 3.3. (i) Partially differentiating the clearing price in Prop. 3.4 with respect to q yields

$$\frac{\partial p}{\partial q} = \frac{1(1-r)(1+\delta q^2) - 2\delta q(rq + 1 - q)}{(1+\delta q^2)^2} \leq 0.$$

Similarly, partial differentiation of the sharing price with respect to r yields

$$\frac{\partial p}{\partial r} = \frac{q}{1 + \delta q^2} \geq 0.$$

(ii) For the transaction volume in Eq. (3.26), partial differentiation with respect to r yields

$$\frac{\partial Q}{\partial r} = -\frac{q}{2(1+\delta q^2)}\left(1 - \frac{2q}{3}\right) \leq 0.$$

With respect to q, partial differentiation yields

$$\frac{\partial Q}{\partial q} = \left(1 - \frac{4q}{3}\right)\left(\frac{1 + \delta q - r}{1 + \delta q^2}\right) + \delta q\left(1 - \frac{2q}{3}\right)\left(\frac{1 - \delta q^2 - 2q(1-r)}{(1+\delta q^2)^2}\right).$$

One can verify that for all $\delta \in [0,1]$, there exists $\bar{q} \in [0,1]$, below which $\partial Q/\partial q > 0$, and above which $\partial Q/\partial q < 0$. This completes the proof. ■

Proof of Lemma 3.4. The result follows directly from Eq. (3.21) and the liquidity condition (L'). ■

Proof of Lemma 3.5. Differentiating $r(q) - \hat{r}(q)$ with respect to q yields

$$\frac{d}{dq}[r(q) - \hat{r}(q)] = \frac{1}{2}\frac{d}{dq}\left[1 + \delta q - \frac{1 - \frac{q}{3}}{\rho(q)}\right] = \frac{1}{2}\left(\delta + \frac{1}{3\rho(q)} + \left(1 - \frac{q}{3}\right)\frac{\rho'(q)}{\rho^2(q)}\right).$$

Let $\phi(q) \triangleq 1/(3\rho(q)) + (1 - q/3)(\rho'(q)/\rho^2(q))$. It is easy to show that $\phi(q)$ is positive as $q \to 0^+$. That is,

$$\lim_{q\to 0^+}\left(\delta + \frac{1}{3\rho} + \left(1 - \frac{q}{3}\right)\frac{\rho'(q)}{\rho^2(q)}\right) = \delta + \frac{1}{3} - \left(\frac{\delta}{3} + \frac{1}{3}\right) > 0.$$

We complete the proof by showing that $\phi'(q) > 0$. Differentiating with respect to q, together with La. B.2.3, yields

$$\phi'(q) = -\frac{\rho'(q)}{3\rho^2} + \rho''(q)\left(1 - \frac{q}{3}\right) - \frac{\rho'(q)}{3} > 0,$$

as claimed. ■

Proof of Proposition 3.5. Provided the firm's profit is positive, the derivative $d\Pi_{\text{SP}}(q, r(q))/dq$ is positive (i.e., warrants increasing the durability level) if and only if $(1 + \delta q - c)(1 + \delta q^2)\delta - (1 + \delta q - c)^2\delta q > 0$. But the last inequality is automatically satisfied for all $q \in (0,1)$, which therefore implies that with sharing the optimal level of durability is $q^* = 1$. ■

Proof of Lemma 3.6. Given that $p < r$, the firm's profit is $\Pi(q, r; p) = (1 - r + \delta pq)(r - c)/2$. The first-order necessary optimality condition with respect to the purchase price, combined with the nonnegativity of the profit function, yields the monopolist's best response,

$$r(q;p) = \max\left\{c, 1 + \delta pq, \frac{1 + c + \delta pq}{2}\right\};$$

equivalently, for all $p \in [0, (1+c)/(2 - \delta q))$:

$$r(q;p) = \begin{cases} c, & \text{if } p \in [0, c/(1+\delta q)), \\ p(1+\delta q), & \text{if } p \in [c/(1+\delta q), (1+c)/(2+\delta q)], \\ (1 + c + \delta pq)/2, & \text{if } p \in ((1+c)/(2+\delta q), (1+c)/(2-\delta q)). \end{cases} \tag{B.26}$$

Note that $p \geq (1+c)/(2 - \delta q)$ results in $p > r(q;p)$, which violates the liquidity condition (L').

The firm is able to deactivate the sharing market and its best response is to charge the optimal price $\hat{r}(q)$, as obtained in Eq. (3.9). – We now examine the optimal durability for a given $p \in [0,1]$. for all $c < (1+c)/(2-\delta q))$ such that the $p < r$. The first-order necessary optimality condition with respect to q yields

$$\frac{\partial \Pi(q,r;p)}{\partial q} = \delta p(r-c) > 0,$$

which results in the optimality of the corner solution $q(p) = 1$, as claimed. For all $c \geq (1+c)/(2-\delta q)$, the liquidity condition (L') is not satisfied and the sharing market is not active. Hence, the results in Prop. 3.2 are applicable. Plugging the optimal $q(p)$ into Eq. (B.26) completes the proof. ∎

Proof of Proposition 3.6. The simultaneous-move stage-game Nash equilibrium in each time period is obtained at the intersection of the best-response functions indicated by Eqs. (3.25) and (3.35)–(3.36), respectively. That is,

$$p^*_{\text{SMP}} = \frac{r^*_{\text{SMP}}}{1+\delta}, \tag{B.27}$$

and

$$r^*_{\text{SMP}} = (1 + c + p^*_{\text{SMP}})/2. \tag{B.28}$$

Solving the system of Eqs. (B.27)–(B.28) yields the desired results. Note that for the equilibrium sharing price p^*_{SMP}, the liquidity condition (L') is satisfied regardless of the value of c. This completes the proof. ∎

Proof of Lemma 3.7. The proof is similar to the proof of La. 3.6, for a fixed $q \in [0,1]$. ∎

Proof of Proposition 3.7. The Nash equilibrium is obtained as the intersection of the two best-response functions in Eqs. (3.25) and (3.37). That is,

$$p^*(q) = \frac{2-q+qc}{2+\delta q^2},$$

and

$$r^*(q) = \frac{1}{2}\left(1 + c + \delta q\left(\frac{2-q+qc}{2+\delta q^2}\right)\right).$$

Note that the liquidity condition (L') is satisfied if

$$c > \frac{1-q-\delta q}{1-q+\delta q^2} \triangleq \underline{c}(q),$$

as claimed. ∎

Proof of Proposition 3.8. Note first that $\Pi_{\text{DC}}(q) = (\Phi(q))^2/2$ with $\Phi(q) \triangleq (1-c+\delta q)/(2+\delta q^2)$ for $q \in [0,1]$. A maximizer of $\Phi(q)$ also maximizes $\Pi_{\text{DC}}(q)$. The first-order necessary optimality condition to the optimal durability problem solves $\max_{q\in[0,1]} \Pi_{\text{DC}}(q)$ is $(2+\delta q^2)-2q(1-c+\delta q) = 0$, with unique nonnegative solution

$$q^*_{\text{DC}} = \frac{\sqrt{(1-c)^2+2\delta}-(1-c)}{\delta} \in [0,1], \quad c \in [0,\delta/2].$$

Perfect durability is optimal ($q^*_{\text{DC}} = 1$) if and only if $\delta+(1-c) \geq \sqrt{(1-c)^2+2\delta}$, which is equivalent to $c \geq \delta/2$. By Prop. 3.7, the firm's equilibrium retail price is $r^*_{\text{DC}} = r_{\text{DC}}(q^*_{\text{DC}})$, which completes our proof. ■

Proof of Proposition 3.9. By Eq. 3.8 and Corollary 3.2, for all $c \in [0,\check{c}]$, the optimal profit is $\hat{\Pi}^*(c) = (1-c)^2/4$. On the other hand, by Eq. 3.34 the optimal profit in the presence of an active sharing market in the Stackelberg regime is

$$\Pi^*(c) = \frac{(1+\delta-c)^2}{8(1+\delta)}.$$

Note that at $\hat{\Pi}^*(0) \leq \Pi^*(0)$, and both functions are strictly decreasing in c. Hence there exists at most one intersection c_0, below which for all $0 \leq c \leq c_0$, the monopolist prefers no sharing, i.e., $\hat{\Pi}^*(c) \leq \Pi^*(c)$ for all $c \in [0,c_0]$, where

$$c_0 \triangleq 1 - \frac{\delta}{\sqrt{2(1+\delta)}-1}.$$

One can verify that $c_0 \leq \check{c}$ as follows:

$$1-\frac{\delta}{\sqrt{2(1+\delta)}-1} \leq \frac{1-\delta}{1+\delta} \Leftrightarrow \frac{1}{\sqrt{2(1+\delta)}-1} \geq \frac{2}{1+\delta} \Leftrightarrow 9+\delta^2+6\delta \geq 8+8\delta \Leftrightarrow (1-\delta)^2 \geq 0,$$

which completes the proof. ■

B.3 Analytical Details of Chapter 4

Proof of Lemma 4.1. Substituting Eqs.(4.6)-(4.7) in the (IC) constraint and rearranging the terms yields $\delta\theta(\phi+\tau x) \geq \pi$, as claimed. ■

Proof of Lemma 4.2. By Eq. (4.8), the sharing market is naturally choked off for all $x \leq \underline{x}$. In the absence of sharing, the (IR) constraint requires that the firm increases the rental price, such that the constraint binds, i.e.,

$$\phi(x) = v - \tau x.$$

For all $x \geq \underline{x}$, the rental service preferred if Eq. (4.1) is satisfied. Substituting the equilibrium sharing price in Eq. (4.5) into Eg. (4.1) and rearranging the terms yields

$$\phi(x) \leq \tau(2-3x). \tag{B.29}$$

As, it is never optimal for the firm to charge a rental price that is strictly less than $\tau(2-3x)$, Eq. (B.29) holds with equality in equilibrium. Furthermore, $\phi \geq 0$ requires that $x \leq 2/3$. This completes the proof. ■

Proof of Lemma 4.3. If the monopolist decides to shut down the rental service, her strategy set shrinks to selling to young high-type agents, selling to young agents of low and high-type, and selling to both generations. We show that all three strategies are dominated. By targeting the young consumers, the (IC) constraint simplifies to

$$v - r - \tau x + \delta\theta v \geq 0.$$

If the (IC) constraint binds only for the high-types, the purchase price is $r(x) = (1+\delta\theta_H)v - \tau x$, at this price, the firm's optimal profit is

$$\lambda\theta_H[(1+\delta\theta_H)v - \tau x - c]_+ \leq \Pi_1(x), \tag{B.30}$$

where $\Pi_1(x)$ is defined in Eq. (4.12). If the (IC) constraint binds for the low-type agents such that all consumers in need prefer to purchase, i.e, if $r(x) = (1+\delta\theta_L)v - \tau x$, the firm gains the profit

$$(\lambda\theta_H + (1-\lambda)\theta_L)\,[(1+\delta\theta_L)v - \tau x - c]_+ \leq \Pi_2. \tag{B.31}$$

where $\Pi_2(x)$ is defined in Eq. (4.12). If the monopolist chooses to sell to both generations, the purchase price drops to

$$r(x) = v - \tau x.$$

The firm's profit would be

$$(\lambda\theta_H + (1-\lambda)\theta_L + \lambda\theta_H(1-\theta_H) + (1-\lambda)\theta_L(1-\theta_L))\,[v - \tau x - c]_+ \leq \Pi_2. \tag{B.32}$$

This concludes the proof. ■

Proof of Proposition 4.1. If the sharing market is naturally chocked off, by La. 4.2, the sharing propensity is less than $1 - v/(\tau 2)$ and the rental price is $r(x) = v - \tau x$. Define $\underline{x} \in \{1 - v/(\tau 2), 1\}$, where $\underline{x} = 1$, describes the artificial case, where the sharing market does not exist at all. The firm's profit from a pure rental strategy outweighs her profit from a applying a high-end selling & rental strategy if

$$\Pi_0(x) \geq \Pi_1(x),$$

where $\Pi_0(x)$ and $\Pi_1(x)$ are specified in Eqs. (4.10)-(4.12). It follows that, when the pure rental strategy dominates high-end selling & rental in a natural choke-off if

$$x \geq \left\{ \frac{(1-\theta_H)c}{2\tau\theta_H} + \frac{v(1-\delta)}{\tau}, \underline{x} \right\} \equiv x_1.$$

Similarly, for all $x \leq \underline{x}$, where the sharing market is naturally inactive, the firm's profit from a high-end selling & rental strategy is greater than mass selling & rental if

$$\Pi_1(x) \geq \Pi_2(x),$$

where $\Pi_1(x)$ and $\Pi_2(x)$ are specified in Eqs. (4.10)-(4.12), and $r(x) = v - \tau x$. It follows that high-end selling & rental is feasible and yields a higher profit if

$$x \geq \left\{ \frac{\delta v \lambda \theta_H(\theta_H - \theta_L)}{\tau(1-\lambda)\theta_L^2} + \frac{(1-\theta_L)c}{2\tau\theta_L} + \frac{v(1-\delta)}{\tau}, \underline{x} \right\} \equiv x_2.$$

Furthermore, it is easy to show that

$$\frac{(1-\theta_H)c}{2\tau\theta_H} \leq \frac{(1-\theta_L)c}{2\tau\theta_L} + \frac{\delta v \lambda \theta_H(\theta_H - \theta_L)}{\tau(1-\lambda)\theta_L^2},$$

since the first term on the RHS is strictly is strictly greater than the one on the LHS, and the second term on the RHS is non-negative. It follows that $x_1 \leq x_2$. Hence, for all $x \in [0, x_1]$,

$$\Pi_0(x) \geq \Pi_1(x) \geq \Pi_2(x),$$

which completes the proof of part (i). For all $x \in [x_1, x_2]$,

$$\Pi_1(x) \geq \max\{\Pi_0(x), \Pi_2(x)\}.$$

This completes the proof of part (ii). For all $x \in [x_2, \underline{x}]$,

$$\Pi_2(x) \geq \Pi_1(x) \geq \Pi_0(x),$$

which concludes the proof of part (iii). By La. 4.2, the rental price in the induced choke-off strategy is $\tau(2-3x)$, which is strictly smaller than $v - \tau x$, for all $x \leq 1 - v/(2\tau)$. Hence for all $x \leq \underline{x}$, induced choke-off strategy is suboptimal. Applying La. 4.2 again, induced choke-off strategy leads to a negative payoff for all $x > 2/3$, which indicates that it is not the optimal strategy in that region. This strategy may be optimal only on the interval $x \in [\underline{x}, 2/3]$. This concludes the proof of part (iv). ∎

Proof of Lemma 4.4. (i) By Eqs. (4.8) and (4.17), the inequality $x_1 \leq \underline{x}$ holds, if

$$\hat{c}\frac{1-\theta_H}{2\theta_H} + \hat{v}(1-\delta) \leq 1 - \frac{\hat{v}}{2},$$

or equivalently if

$$\hat{v} \le \frac{\theta_H(2+\hat{c})-\hat{c}}{\theta_H(3-2\delta)} \triangleq \hat{v}_0 \quad (= v_0\tau).$$

If $\hat{v} \ge \hat{v}_0$, then by definition $x_1 = 1 - v/2\tau$, which is decreasing in v. We now, examine the case where $\hat{v} < \hat{v}_0$. Differentiating x_1 with respect to v yields

$$\frac{\partial x_1}{\partial v} = \frac{1-\delta}{\tau} \ge 0,$$

indicating that the threshold x_1 is locally increasing in v. Hence, the maximum is achieved at $v = \hat{v}_0\tau$, as claimed. Comparative statics with respect to other parameters only requires analysis for the case where $\hat{v} < \hat{v}_0$. For all $\hat{v} \ge \hat{v}_0$, by definition $x_1 = \underline{x}$ and it is unaffected by the changes in parameters other than v and τ. Differentiating x_1 with respect to c yields

$$\frac{\partial x_1}{\partial c} = \frac{1-\theta_H}{2\theta_H\tau} \ge 0,$$

as claimed. Differentiating x_1 with respect to δ yields

$$\frac{\partial x_1}{\partial \delta} = -\hat{v} \le 0.$$

And finally, differentiating x_1 with respect to θ_H yields

$$\frac{\partial x_1}{\partial \theta_H} = -\frac{\hat{c}}{2{\theta_H}^2} \le 0,$$

completing the proof of part (i).
(ii) By Eqs. (4.8) and (4.17), the inequality $x_2 \le \underline{x}$ holds if

$$\hat{c}\left(\frac{1-\theta_L}{2\theta_L}\right) + \hat{v}\left(1-\delta+\frac{\delta\Delta\theta}{\ell\theta_L}\right) \le 1-\frac{\hat{v}}{2}.$$

Rearranging the terms yields

$$\hat{v} \le \frac{\theta_L(2+\hat{c})-\hat{c}}{3\theta_L-\delta(2\theta_L-\Delta\theta)} \triangleq \hat{v}_1 \quad (= v_1\tau).$$

If $\hat{v} \ge \hat{v}_1$, then $x_2 = \underline{x} = 1 - v/2\tau$, which is decreasing in v. If $\hat{v} < \hat{v}_1$, the derivative of x_1 with respect to $\hat{v}$ is

$$\frac{\partial x_2}{\partial v} = \frac{1-\delta}{\tau} + \frac{\tau\delta\Delta\theta}{\ell\theta_L} \ge 0.$$

Hence, the maximum is achieved at $v = v_1$, as claimed. Similar to the reasoning provided in part (i), comparative statics with respect to other variables only entails focusing on the case where $\hat{v} < \hat{v}_1$. Differentiating x_2 with respect to c yields

$$\frac{\partial x_2}{\hat{c}} = \frac{1-\theta_L}{2\theta_L\tau} \ge 0.$$

Differentiating x_2 with respect to θ_L yields

$$\frac{\partial x_2}{\partial \theta_L} = -\frac{\hat{c}}{2{\theta_H}^2} \leq 0.$$

Differentiating x_2 with respect to $\Delta\theta$ yields

$$\frac{\partial x_2}{\partial \Delta\theta} = \hat{v}\frac{\delta}{\theta_L \ell \theta_L} ge0.$$

Differentiating x_2 with respect to ℓ yields

$$\frac{\partial x_2}{\partial \ell} = -\hat{v}\frac{\delta\Delta\theta}{\theta_L \ell^2} \leq 0.$$

Differentiating x_2 with respect to λ yields

$$\frac{\partial x_2}{\partial \lambda} = \frac{\partial x_2}{\partial \ell}\frac{\partial \ell}{\partial \lambda} = \hat{v}\frac{\delta\Delta\theta}{\ell^2\lambda^2\theta_H} \geq 0.$$

Differentiating x_2 with respect to θ_H yields

$$\frac{dx_2}{d\theta_H} = \frac{\partial x_2}{\partial \ell}\frac{\partial \ell}{\partial \theta_H} + \frac{\partial x_2}{\partial \theta_H} = \frac{\partial x_2}{\partial \ell}\frac{\partial \ell}{\partial \lambda} = \hat{v}\left(\frac{\delta}{\ell\theta_L} + \frac{(1-\lambda)\delta\Delta\theta}{\lambda\ell^2{\theta_H}^2}\right) \geq 0.$$

Differentiating x_2 with respect to δ yields

$$\frac{\partial x_2}{\partial \delta} = \hat{v}\left(-1 + \frac{\Delta\theta}{\ell\theta_L}\right),$$

which is greater than zero, if $\ell \leq \Delta\theta/\theta_L$. This completes the proof of part (ii). ■

Proof of Lemma 4.5. Substituting Eqs. (4.21)-(4.22) into the (IC') and rearranging yields

$$2\delta\theta\delta\tau(1-x) \geq r - p(1+\delta) - \tau(1-2x) + \delta\tau(1-x),$$

which implies the results, as claimed. ■

Proof of Lemma 4.6. By Eq. (4.8), the inequality $\underline{x} > 0$ holds if

$$v \geq 2\tau. \tag{B.33}$$

If $x \leq \underline{x}$, then by La. 4.2, the rental price is $\phi = v - \tau x$. Comparing the optimal purchase premium in Eq (4.11) with the one characterized in Eq. (4.26) and substituting the optimal rental price requires

$$\delta(v - 2\tau(1-x)(1-\theta_H)) \overset{!}{\geq} \delta\theta_H v.$$

Rearranging the terms yields

$$\delta(1-\theta_H)(v-2\tau(1-x)) \overset{!}{\geq} 0,$$

which holds by assumption (B.33). If $x > \underline{x}$, then by La. 4.2, the sharing is deactivated only by means of applying the induced choke-off strategy, and the rental price is $\phi = \tau(2-3x)$. The purchase premium in Eq (4.26) is greater than the purchase premium in Eq. (4.26) if

$$\delta(v-2\tau(1-x)(1-\theta_H)) \overset{!}{\geq} 2\delta\theta_H\tau(1-x),$$

Rearranging the terms yields

$$\delta(v-2\tau(1-x)) \overset{!}{\geq} 0,$$

which holds for all parameter values, given that $v > 2\tau$. This completes the proof. ■

Proof of Lemma 4.7. By assumption, $\underline{x} = 1 - v/(2\tau) > 0$. By La. 4.2, if $x \leq \underline{x}$, the rental price is $\phi = v - \tau x$. The optimal $r(x)$ under the mass selling strategy with sharing in Eq (4.30) is greater than the purchase price $r(x) = \phi(x) + \pi(x)$, as determined by Prop. 4.1. That is

$$v(1+\delta) - \tau x - 2\delta\tau(1-x)(1-\theta_L) \overset{!}{\geq} v - \tau x + \delta\theta_L v.$$

Rearranging the terms yields

$$\delta(1-\theta_L)(v-2\tau(1-x)) \overset{!}{\geq} 0,$$

which holds by assumption for all $x \in \mathscr{X}$. If $x > \underline{x}$, by La. 4.2 determines the rental price is $\phi = \tau(2-3x)$. The optimal purchase price under the mass selling strategy with sharing is greater than without it if

$$v(1+\delta) - \tau x - 2\delta\tau(1-x)(1-\theta_L) \overset{!}{\geq} \tau(2-3x) + 2\delta\theta_H(1-x).$$

Rearranging the terms yields

$$v(1+\delta) \overset{!}{\geq} 2\tau(1-x)(1+\delta),$$

which is true by assumption for all $x \in \mathscr{X}$. This completes the proof. ■

Proof of Proposition 4.2. By Eq. (4.32), sharing is feasible for all $x \in [\bar{x}, 1]$. In presence of sharing, high-end selling is more profitable than mass selling if $\Pi_3(x) \geq \Pi_4(x)$. Using Eqs. (4.27) and (4.31), and rearranging yields

$$(1-\lambda)\theta_L(-v(1-\delta) + \tau x - 2\delta\tau(1-x)(1-\theta_L)) \leq \lambda\theta_H(2\delta\tau(1-x)(\theta_H-\theta_L)).$$

Solving for $x \in [\bar{x}, 1]$ yields

$$x \geq \min\left\{1, \max\left\{\bar{x}, \frac{2\lambda\theta_H\delta\tau(\theta_H - \theta_L) + 2(1-\lambda)\theta_L\delta\tau(1-\theta_L) + (1-\lambda)\theta_L v(1-\delta)}{2\lambda\theta_H\delta\tau(\theta_H - \theta_L) + 2(1-\lambda)\theta_L\delta\tau(1-\theta_L) + (1-\lambda)\theta_L\tau}\right\}\right\}.$$

Dividing the nominator and denominator by $\lambda\theta_H\tau(\theta_H - \theta_L)$ and using the definition $\ell = ((1-\lambda)\theta_L)/(\lambda\theta_H)$, $\Delta\theta = \theta_H - \theta_L$, and $\hat{v} = v/\tau$ returns the result as claimed. ■

Proof of Lemma 4.8. By Eq. (4.33), $x_3 < 1$ if $\hat{v}(1-\delta) < 1$. If this is the case, then differentiating x_3 with respect to ζ yields

$$\frac{\partial x_3}{\partial \zeta} = \frac{2\delta(\hat{v}(1-\delta) - 1)}{(2\delta(1+\zeta(1-\theta_L)) + \zeta^2)} < 0. \tag{B.34}$$

By definition

$$\zeta = \frac{\ell}{\Delta\theta} = \frac{(1-\lambda)\theta_L}{\lambda\theta_H\Delta\theta},$$

is decreasing in λ, θ_H, and $\Delta\theta$ and increasing in θ_L. Hence by Eq. (B.34), x_3 is increasing in λ, θ_H, and $\Delta\theta$. Differentiating x_3 with respect to θ_L yields

$$\frac{dx_3}{d\theta_L} = \frac{\partial x_3}{\partial \zeta}\frac{\partial \zeta}{\partial \theta_L} + \frac{\partial x_3}{\partial \theta_L} = \frac{2\delta(\hat{v}(1-\delta) - 1)}{(2\delta(1+\zeta(1-\theta_L)) + \zeta)^2}\left(\frac{1-\lambda}{\lambda(\Delta\theta^2)} + \zeta^2\right) < 0.$$

Differentiating x_3 with respect to $\hat{v}$ yields

$$\frac{\partial x_3}{\partial \delta} = \frac{\zeta(1-\delta)}{(2\delta(1+\zeta(1-\theta_L)) + \zeta^2)} > 0.$$

Differentiating x_3 with respect to δ yields

$$\frac{\partial x_3}{\partial \delta} = \frac{2\zeta(1+\zeta(1-\theta_L))(1-\hat{v}) - \zeta^2\hat{v}}{(2\delta(1+\zeta(1-\theta_L)) + \zeta^2)} < 0,$$

given that $\hat{v} = v/\tau > 1$. This completes the proof. ■

Proof of Lemma 4.9. (i) Let $D_r \geq 0$ and $D_\phi \geq 0$ be the demand for the purchase and rental service without sharing, such that

$$\Pi(x) = D_r(r(x) - c) + D_\phi\left(\phi(x) - \frac{c}{2}\right),$$

as described by Eqs. (4.10), (4.12), and (B.31). Differentiating the profit function with respect to x yields

$$\frac{\partial \Pi(x)}{\partial x} = D_r\frac{\partial r(x)}{\partial x} + D_\phi\frac{\partial \phi(x)}{\partial x} \leq 0,$$

since by Eq. (4.13) and Eq. (4.16), the rental price and the purchase premium are such that

$$\frac{\partial\phi(x)}{\partial x} \in \{-\tau, -3\tau\} \leq 0,$$

and

$$\frac{\partial\pi(x)}{\partial x} \in \{0, -2\delta\theta_L\tau, -2\delta\theta_H\tau\} \leq 0.$$

Note that

$$\frac{\partial r(x)}{\partial x} = \frac{\partial\pi(x)}{\partial x} + \frac{\partial\phi(x)}{\partial x} < 0.$$

Since, there is no jump in the profit function, $\Pi(x)$ is continuously decreasing in x.

(ii) For parts (ii)-(iii) of this proof, we assume that the model parameters are non-degenerate, i.e. $\tau, (1-\lambda), \lambda, \theta_H, \theta_L$ are strictly positive and $\theta_L \leq \theta_H < 1$.

For all $x \in [\bar{x}, x_3]$, the profit is given by Eq. (4.27). The profit function is increasing in x if $\Pi_4'(x) \geq 0$. This requires

$$\Pi_3'(x) = \lambda\theta_H\tau(2\delta(1-\theta_H) - 1) - 2(1-\lambda)\theta_L\tau \geq 0.$$

Rearranging the terms and using the definition of $\ell = (1-\lambda)\theta_L/(\lambda\theta_H)$ yields

$$\delta \geq \frac{\ell}{(1-\theta_H)} + \frac{1}{2(1-\theta_H)},$$

as claimed.

(iii) For all $x \in [x_3, 1]$, the profit is increasing, if the derivative of the profit function in Eq. (4.31) is positive. That is

$$\Pi_4'(x) = \tau(\lambda\theta_H + (1-\lambda)\theta_L)(2\delta(1-\theta_L) - 1) \geq 0.$$

The profit function is increasing if

$$\delta \geq \frac{1}{2(1-\theta_L)},$$

concluding the proof. ■

Proof of Lemma 4.10. (i) By Props. 4.1-4.2, the sharing market is inactive when $x \leq \bar{x}$. Hence, the gains from trade equals

$$\mathrm{GT}(x) = D_\phi\left(v - \frac{c}{2}\right) + D_r\,(v-c) + (D_{rH}\theta_H + D_{rL}\theta_L)\,v.$$

For all $x \in [0, x_1]$, by Prop. 4.1 the demand for rental is $D_\phi^0 = D_\phi^1 = \lambda\theta_H + (1-\lambda)\theta_L$. Hence, the gains from trade is

$$\mathrm{GT}_0 = \lambda\theta_H(2+2\ell)\left(v-\frac{c}{2}\right). \tag{B.35}$$

For all $x \in (x_1, x_2]$, the demand for rental is $D^0_\phi = (1-\lambda)\theta_L$ and $D^1_\phi = \lambda\theta_H(1-\theta_H)+(1-\lambda)\theta_L$. The demand for purchase comes only from the young high-type agents and equals $D_r = D_{rH} = \lambda\theta_H$. Hence, the gains from trade is

$$\mathrm{GT}_1 = \lambda\theta_H\Big((2\ell+1-\theta_H)\left(v-\frac{c}{2}\right)+(v-c)+\theta_H\Big). \tag{B.36}$$

For all $x \in (x_2, \bar{x}]$, he demand for rental in the second period is $D^1_\phi = \lambda\theta_H(1-\theta_H)+(1-\lambda)\theta_L(1-\theta_L)$. The demand for purchase comes from both high and low types agents in $\mathscr{C}^0$ and equals $D_r = D_{\rho H} = \lambda\theta_H + (1-\lambda)\theta_L$. Hence, the gains from trade is

$$\mathrm{GT}_2 = \lambda\theta_H\Big((\ell(1-\theta_L)+1-\theta_H)\left(v-\frac{c}{2}\right)+(\ell+1)(v-c)+(\theta_H+\ell\theta_L)v\Big). \tag{B.37}$$

(ii) By Props. 4.1-4.2, the sharing market is inactive when $x > \bar{x}$. Hence, the gains from trade equals

$$\mathrm{GT}(x) = D_\phi\left(v-\frac{c}{2}\right)+D_r\,(v-c)+(D_{rH}+D_{rL})\,v.$$

For all $x \in (\bar{x}, x_3]$, by Prop. 4.2 high-end selling & rental is the optimal strategy. The demand for rental is $D_\phi = D^0_\phi + D^1_\phi = 2(1-\lambda)\theta_L$. The demand for purchase comes from young high-type consumers and is equal to $D_r = \lambda\theta_H$. Gains from trade is equal to

$$\mathrm{GT}_3 = 2(1-\lambda)\theta_L\left(v-\frac{c}{2}\right)+\lambda\theta_H\,(2v-c)\,.$$

For all $x \in (x_3, 1]$, by Prop. 4.2 mass selling is the optimal strategy. The demand for rental is $D_\phi = D^0_\phi + D^1_\phi = 0$. The demand for purchase is equal to $D_r = \lambda\theta_H + (1-\lambda)\theta_L$. Gains from trade is equal to

$$\mathrm{GT}_4 = (\lambda\theta_H+(1-\lambda)\theta_L)\,(2v-c)\,.$$

Clearly, $\mathrm{GT}_3 = \mathrm{GT}_4$, as claimed.

(iii) It is enough to show that the gains from trade with sharing is larger than the maximum gains from trade without sharing, i.e.,

$$\mathrm{GT}_3 - \max\{\mathrm{GT}_0, \mathrm{GT}_1, \mathrm{GT}_2\} = D_{rH}\theta_H(1-\theta_H)+D_{rL}\theta_L(1-\theta_L) > 0,$$

concluding the proof. ∎

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