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Electromagnetic forces in the time domain

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Abstract: We look beyond the standard time-average approach and investigate optical forces in the time domain. The formalism is developed for both the Abraham and Minkowski momenta, which appear to converge in the time domain. We unveil an extremely rich – and by far unexplored – physics associated with the dynamics of the optical forces, which can even attain negative values over short time intervals or produce low frequency dynamics that can excite mechanical oscillations in macroscopic objects under polychromatic illumination. The magnitude of this beating force is tightly linked to the average one. Implications of this work for transient optomechanics are discussed.

1. Introduction

Electromagnetic forces produced by light acting on physical objects have received significant research attention ever since the observation of comet tail bending by Kepler in the 17th century [1]. The resulting effect stems from momentum exchange between photons and physical objects. The human eye perceives this force as being static and is described as optical "pressure" [2,3], "pulling" [4–8] or "trapping" [9–23], all terms associated with a static behavior. This perception is illustrated in Fig. 1(a) where a rectangular rod attached to the wall is illuminated at frequency ω_0 and consequently experiences constant optical pressure.

However, the electromagnetic wave theory predicts that the momentum carried by an electromagnetic wave includes not only a static momentum but also a part that oscillates in time. This has been discussed in classical works [24–29].

The presence of the oscillating component of the optical force can be easily understood by analyzing the instantaneous value of the Poynting vector for a monochromatic plane wave at the frequency ω_0 [30]:

$$\begin{aligned}\mathbf{S}(\mathbf{r},t) &= \mathrm{Re}\left(\mathbf{E}(\mathbf{r},\omega_0)e^{-i\omega_0 t}\right)\times\mathrm{Re}\left(\mathbf{H}(\mathbf{r},\omega_0)e^{-i\omega_0 t}\right) = \\ &\frac{1}{2}\,\mathrm{Re}\left(\mathbf{E}(\mathbf{r},\omega_0)\times\mathbf{H}^*(\mathbf{r},\omega_0) + \mathbf{E}(\mathbf{r},\omega_0)\times\mathbf{H}(\mathbf{r},\omega_0)e^{-2i\omega_0 t}\right).\end{aligned} \tag{1}$$

The first term in Eq. (1) is responsible for the static force, while the second accounts for a harmonic force. Hence, the rod experiences a periodically varying force at the frequency $2\omega_0$ that induces mechanical oscillations in the rod, Fig. 1(b).

At first sight, such oscillations are not easy to observe. Indeed, if the excitation frequency is high (GHz and higher), those oscillations are too far from the mechanical resonance of the oscillator (typically in the kHz-MHz range) and their amplitude becomes negligible. To overcome this issue, we will show that two waves with slightly different frequencies ω_0 and $\omega_1 = \omega_0 + \Omega$ can be used ($\Omega << \omega_0$). Together, they produce a beating effect that induces mechanical oscillations at the beating frequency Ω. Such an idea was experimentally realized in a cavity via the dynamical backaction effect [31,32], which essentially appears through the beating of the pump wave with the Stokes or anti-Stokes components induced by the mechanical mode of the cavity. This can

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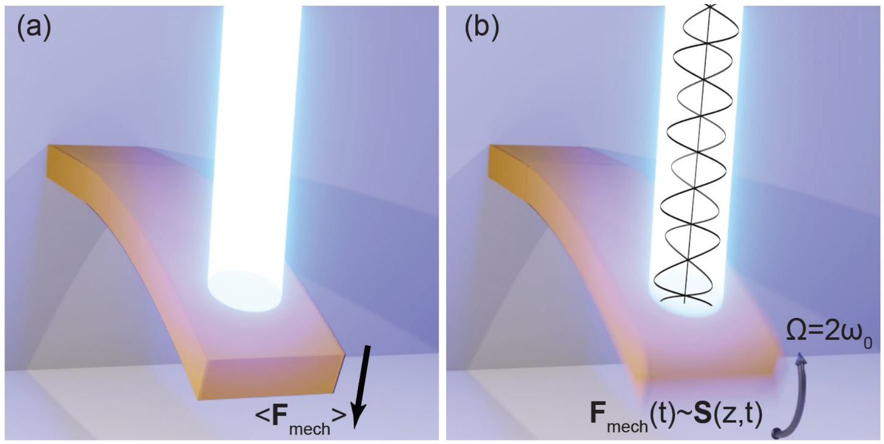

Fig. 1. (a) An electromagnetic wave impinges on a rod making it bend. Classical considerations suggest that only the constant component $<\mathbf{F}_{\mathrm{mech}}>$ of the force is present. (b) A more thorough consideration of the electromagnetic interaction between the wave and the rod reveals that the magnitude of the wave transfer momentum changes within the period of the wave. Consequently, the rod experiences an oscillating electromagnetic force $\mathbf{F}_{\mathrm{mech}}(t)$.

lead to either enhancement of the mechanical oscillations [33–36] or rather efficient oscillation quenching [37].

Alternatively, an optical wave can be modulated with the mechanical frequency Ω, such that a signal of the form $A\cos(\Omega t)\cos(\omega_0 t)$ can effectively drive the mechanical oscillator, leading to the experimental observation of mechanical oscillations even at the macroscopic scale [38–40].

Optical forces in the time domain are also very important for trapping experiments where optical pulses are used as incident illumination [24,41–53]. Besides, some very challenging ideas of using complex-valued frequency excitation have emerged recently to control the optical force in the time-domain [54,55].

The optical forces were studied in great details in experiments on radiation pressure exerted on muons, protons, atoms, molecules and larger particles with single frequency light [56] and multi-frequency illuminations [57]. The two-[38,58–62] and four-wave [63] illuminations showed that an increased optical force can be achieved, thus overcoming the limit on the optical force imposed by the spontaneous emission rate factor [57]. It is interesting to note that concepts related to complex-frequency illumination or forces appearing under transient illumination discussed above are of multi-scale nature and can be realized in a very broad range of frequencies, including acoustics [64–66].

On looking through the literature, we notice that several previous studies undertook the analysis of the optical force in the time domain, but at a certain point always shifted their focus to the time average force [67–69] or, alternatively, use numerical approaches to find the force in the time domain [44,70–75]. To the best of our knowledge, only a few publications conducted analytical studies of the optical force evolution. Very recent paper employs the signal theory to derive the imaginary part of the Maxwell stress tensor, which is responsible for the oscillating optical force and torque [76]. The optical force is studied under two-wave excitation acting on a half-space [40] and on cylinders [77], and a systematic analytical study of the time evolution of the optical force has not yet been reported.

In this paper, we address this shortcoming and provide a comprehensive theoretical study that establishes the foundations for the dynamics of the optical forces in the time domain. We discuss first in Sec. 2. the general formalism that can be used to find the optical forces in the time

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domain. Next, in Sec. 3. we apply this approach and describe how the simplest possible form of excitation – a plane wave – triggers the optical force in the time domain. To make the analysis as clear as possible and as illustrative as we can, we start by examining the dynamics of the optical force acting on a thin film under normal incidence. Next, in Sec. 4, we study the dynamics of the beating force and derive a simple formula for arbitrary objects. Finally, we analyze the derived formula and discuss possible applications of that beating effect.

2. Optical force in the time domain: general theory

The momentum conservation law is one of the fundamental laws that govern the mechanical motion of physical objects. Newton's second law states that the first time-derivative of the mechanical momentum $\mathbf{G}_{\mathrm{mech}}(t)$ carried by the system is equal to the mechanical force $\mathbf{F}_{\mathrm{mech}}(t)$ acting on it [78],

$$\mathbf{F}_{\mathrm{mech}}(t) = d\mathbf{G}_{\mathrm{mech}}(t)/dt. \tag{2}$$

The mechanical momentum of an object that is subjected to electromagnetic fields can be calculated by evaluating the total momentum $\mathbf{G}_{\mathrm{tot}}(t)$ stored within the object minus the momentum stored in the electromagnetic waves $\mathbf{G}_{\mathrm{e.m.}}(t)$ travelling inside the object [27]:

$$\mathbf{G}_{\mathrm{mech}}(t) = \mathbf{G}_{\mathrm{tot}}(t) - \mathbf{G}_{\mathrm{e.m.}}(t). \tag{3}$$

The time derivative of the mechanical momentum can thus be written as

$$d\mathbf{G}_{\mathrm{mech}}(t)/dt = d\mathbf{G}_{\mathrm{tot}}(t)/dt - d\mathbf{G}_{\mathrm{e.m.}}(t)/dt. \tag{4}$$

Maxwell's equations provide explicit relations for the time derivatives of the total and electromagnetic momenta in Eq. (4) based on the electric and magnetic fields $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{H}(\mathbf{r}, t)$. Let us assume that the object under study is nonmagnetic, has a homogeneous dielectric permittivity ε and is placed in vacuum. We further assume that ε has negligible dispersion over the frequency range considered. In this case, the time derivative of the total momentum can be rewritten in terms of the surface integral as [27,79,80],

$$d\mathbf{G}_{\mathrm{tot}}(t)/dt = \int_{\Sigma} \overleftrightarrow{\mathbf{T}}(\mathbf{r}, t) \cdot d\boldsymbol{\sigma}, \tag{5}$$

where Σ is the total surface of the object and $d\boldsymbol{\sigma} = d\sigma\mathbf{n}$ is an elementary outward-directed surface element. Maxwell's stress tensor $\overleftrightarrow{\mathbf{T}}(\mathbf{r}, t)$ is defined by [27]

$$\begin{aligned}
&\overleftrightarrow{\mathbf{T}}(\mathbf{r}, t) \cdot d\boldsymbol{\sigma} = (\varepsilon_0 \operatorname{Re}(\mathbf{E}(\mathbf{r}, t))(\operatorname{Re}(\mathbf{E}(\mathbf{r}, t)) \cdot \mathbf{n}) + \\
&\mu_0 \operatorname{Re}(\mathbf{H}(\mathbf{r}, t))(\operatorname{Re}(\mathbf{H}(\mathbf{r}, t)) \cdot \mathbf{n}) \\
&- \varepsilon_0 \frac{1}{2}(\operatorname{Re}(\mathbf{E}(\mathbf{r}, t)) \cdot \operatorname{Re}(\mathbf{E}(\mathbf{r}, t)))\mathbf{n} \\
&- \mu_0 \frac{1}{2}(\operatorname{Re}(\mathbf{H}(\mathbf{r}, t)) \cdot \operatorname{Re}(\mathbf{H}(\mathbf{r}, t)))\mathbf{n})d\sigma,
\end{aligned} \tag{6}$$

where ε_0 and μ_0 are the vacuum permittivity and permeability and we use the SI unit system throughout the manuscript. The time-derivative of the electromagnetic momentum in Eq. (4) can also be calculated. However, at this point we must make a very important remark about the form of $d\mathbf{G}_{\mathrm{e.m.}}(t)/dt$. In fact there exist two slightly different forms of the electromagnetic momentum.

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The first one is based on the Minkowski formalism [25,28],

$$d\mathbf{G}^{\mathrm{M}}_{\mathrm{e.m.}}(t)/dt = \frac{1}{c^2}\int_V \frac{\partial(\mathrm{Re}\,(\varepsilon\mathbf{E}(\mathbf{r},t))\times \mathrm{Re}\,(\mathbf{H}(\mathbf{r},t)))}{\partial t}dV, \tag{7}$$

and the second one on the Abraham form of the momentum,

$$d\mathbf{G}^{\mathrm{A}}_{\mathrm{e.m.}}(t)/dt = \frac{1}{c^2}\int_V \frac{\partial(\mathrm{Re}\,(\mathbf{E}(\mathbf{r},t))\times \mathrm{Re}\,(\mathbf{H}(\mathbf{r},t)))}{\partial t}dV. \tag{8}$$

(We note in passing that other formalisms exist beyond that of Abraham or Minkowski [28,73].) Here, the integration is performed over the volume V of the system and c is the speed of light in vacuum. Both forms of the electromagnetic momentum, Eqs. (7)–(8), are in fact accepted in the scientific literature [28]. The ambiguity in the definition stems from a century long debate about whether the momentum of the photon inside a medium is $p^M = n\hbar\omega_0/c$ or $p^A = \hbar\omega_0/nc$ (here n is the refractive index of the medium, $n^2 = \varepsilon$) [28]. Significant efforts have been invested to prove one expression against the other [24–27,29,74,81–90]; however, it is now well accepted that both are valid and neither is more correct than the other [28]. To circumvent this issue we decided to adopt both versions of momentum and test our theory for both cases. Henceforth, we will be adding the "M" superscript for solutions obtained within the framework of the Minkowski's approach, while "A" will stand for the Abraham's one.

3. Optical force in the time domain under monochromatic illumination

3.1. Average and oscillating components

Let us consider harmonic electromagnetic fields with a frequency ω_0. In this case, one can find the average mechanical force $\langle\mathbf{F}_{\mathrm{mech}}(t)\rangle_T$ (averaging is performed over a period $T = 2\pi/\omega_0$ of the wave) that can be calculated by evaluating the Maxwell stress tensor,

$$\begin{aligned}\langle\mathbf{F}_{\mathrm{mech}}(t)\rangle_T &= \langle d\mathbf{G}_{\mathrm{tot}}(t)/dt\rangle_T = \int_\Sigma \left\langle \overleftrightarrow{\mathbf{T}}(\mathbf{r},t)\right\rangle_T\cdot\mathbf{n}d\sigma = \\ &\frac{1}{2}\int_\Sigma \mathrm{Re}(\varepsilon_0\mathbf{E}(\mathbf{r},t)(\mathbf{E}^*(\mathbf{r},t)\cdot\mathbf{n}) + \mu_0\mathbf{H}(\mathbf{r},t)(\mathbf{H}^*(\mathbf{r},t)\cdot\mathbf{n}) \\ &-\varepsilon_0\frac{1}{2}(\mathbf{E}(\mathbf{r},t)\cdot\mathbf{E}^*(\mathbf{r},t))\mathbf{n} - \mu_0\frac{1}{2}(\mathbf{H}(\mathbf{r},t)\cdot\mathbf{H}^*(\mathbf{r},t))\mathbf{n})d\sigma.\end{aligned} \tag{9}$$

It is interesting to note that the average mechanical force depends only on the average of the total momentum and not on the electromagnetic part since Eqs. (7)–(8) contain the time derivative $\partial/\partial t$ which period-average is $\langle\partial\mathbf{f}(\mathbf{r},t)/\partial t\rangle_T = \mathbf{f}(\mathbf{r},T) - \mathbf{f}(\mathbf{r},0) = 0$ for time-harmonic fields $\mathbf{f}(\mathbf{r},T)$. Therefore, $\left\langle d\mathbf{G}^{\mathrm{M}}_{\mathrm{e.m.}}(t)/dt\right\rangle_T = \left\langle d\mathbf{G}^{\mathrm{A}}_{\mathrm{e.m.}}(t)/dt\right\rangle_T = 0$. For a monochromatic wave, Eqs. (7)–(8) can be simplified further [91]:

$$d\mathbf{G}^{\mathrm{M}}_{\mathrm{e.m.}}(t)/dt = \frac{\omega_0}{c^2}\int_V \mathrm{Im}(\varepsilon\mathbf{E}(\mathbf{r},t)\times\mathbf{H}(\mathbf{r},t))dV \tag{10}$$

and

$$d\mathbf{G}^{\mathrm{A}}_{\mathrm{e.m.}}(t)/dt = \frac{\omega_0}{c^2}\int_V \mathrm{Im}(\mathbf{E}(\mathbf{r},t)\times\mathbf{H}(\mathbf{r},t))dV. \tag{11}$$

Let us introduce the new variable $d\mathbf{G}_{\mathrm{tot.osc.}}(t)/dt$ as

$$\begin{aligned}d\mathbf{G}_{\mathrm{tot.osc.}}(t)/dt &= d\mathbf{G}_{\mathrm{tot}}(t)/dt - \langle d\mathbf{G}_{\mathrm{tot}}(t)/dt\rangle_T \\ &= \int_\Sigma \left(\overleftrightarrow{\mathbf{T}}(\mathbf{r},t) - \left\langle\overleftrightarrow{\mathbf{T}}(\mathbf{r},t)\right\rangle_T\right)\cdot\mathbf{n}d\sigma,\end{aligned} \tag{12}$$

which represents the oscillating part of the total momentum. The mechanical force in the time domain can then be calculated in the Minkowski and Abraham forms as

$$\mathbf{F}^{\mathrm{M}}_{\mathrm{mech}}(t) = \langle \mathbf{F}_{\mathrm{mech}}(t)\rangle_T + d\mathbf{G}_{\mathrm{tot.osc.}}(t)/dt - d\mathbf{G}^{\mathrm{M}}_{\mathrm{e.m.}}(t)/dt, \tag{13}$$

$$\mathbf{F}^{\mathrm{A}}_{\mathrm{mech}}(t) = \langle \mathbf{F}_{\mathrm{mech}}(t)\rangle_T + d\mathbf{G}_{\mathrm{tot.osc.}}(t)/dt - d\mathbf{G}^{\mathrm{A}}_{\mathrm{e.m.}}(t)/dt. \tag{14}$$

In the following, we find the analytical expressions for the average and oscillating forces for both forms.

3.2. Optical force acting on a film under normal incidence

In this section, we analyze the average and oscillating components of the mechanical optical force for a plane wave impinging at normal incidence on a film of thickness d, as illustrated in Fig. 2. The frequency of the wave is ω_0, which corresponds to the wavelength $\lambda_0 = 2\pi c/\omega_0$.

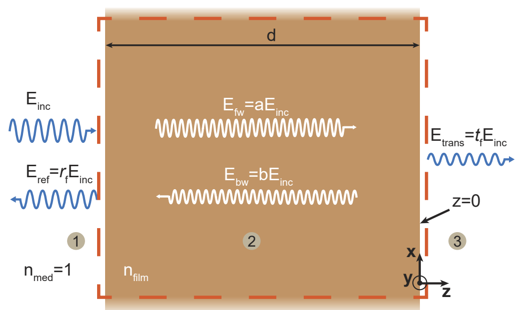

Fig. 2. Plane wave impinging at normal incidence on a film with thickness d and refractive index n_{film}, in vacuum.

Finding the evolution of the optical force using Eqs. (13)–(14) requires the knowledge of the electric and magnetic fields at every point inside and on the surface of the volume of the film. To find the force, we seek for the electric and magnetic fields in the bulk as well as on the boundaries of the film analytically by matching the boundary conditions at the interfaces of the film, which is assumed to be infinite in the xy-plane, see Fig. 2. We present here the reflection (r_f) and transmission (t_f) coefficients.

$$r_f = \frac{-0.5i\sin(\delta)\Delta_-}{\cos(\delta) - 0.5i\Delta_+\sin(\delta)}, \tag{15}$$

$$t_f = \frac{1}{\cos(\delta) - 0.5i\Delta_+\sin(\delta)}, \tag{16}$$

which relate incident $\mathbf{E}_{\mathrm{inc}}$, reflected $\mathbf{E}_{\mathrm{ref}}$ and transmitted $\mathbf{E}_{\mathrm{trans}}$ fields. We have introduced the variables $\Delta_\pm = n^{-1}_{\mathrm{film}} \pm n_{\mathrm{film}}$ and the normalized thickness of the film $\delta = 2\pi n_{\mathrm{film}} d/\lambda_0$. The explicit expression of the electric field as well as the amplitudes of the waves propagating inside the film in the forward, respectively backward (subscripts "a", respectively "b") directions can be found in Sec. A of the Supplement 1. The resulting fields are then integrated analytically using Eqs. (13)–(14). It follows that the total mechanical force can be written in a relatively simple form. The average force component (normalized to the intensity of the incident wave) can be expressed through the transmission and reflection coefficients:

$$\langle \mathbf{F}_{\mathrm{mech}}(t)\rangle_T = \frac{\Sigma_0}{c}(1 + r_f r_f^* - t_f t_f^*)\mathbf{z}, \tag{17}$$

where $\mathbf{z}$ is the unit vector in the z-direction and Σ_0 represents the front/back surface elements of the film, Fig. 2. Here $\Sigma = 2\Sigma_0 + \Sigma_{\mathrm{side}}$. For normal incidence, the integration over the side Σ_{side}

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always vanishes and henceforth is not considered. Note that due to the introduced normalization, the mechanical force in Eq. (17) has units $m \cdot s$; also, for normal incidence we do not need to distinguish s- and p-polarizations. The oscillating part of the total momentum becomes

$$\begin{aligned} d\mathbf{G}_{\text{tot.osc.}}(t)/dt = \frac{\Sigma_0}{c} \operatorname{Re}(e^{-2i\omega_0 t}(1 + r_f^2 - t_f^2))\mathbf{z} \\ = \frac{\Sigma_0}{c} \operatorname{Im}(e^{-2i\omega_0 t}\Delta_+ \sin(\delta) t_f)\mathbf{z}. \end{aligned} \tag{18}$$

Whilst the oscillating component of the electromagnetic momentum is

$$d\mathbf{G}^{\text{M}}_{\text{e.m.}}(t)/dt = \frac{\Sigma_0}{c} \operatorname{Im}(2e^{-2i\omega_0 t} n_{\text{film}} \sin(\delta) t_f)\mathbf{z}, \tag{19}$$

and

$$d\mathbf{G}^{\text{A}}_{\text{e.m.}}(t)/dt = \frac{\Sigma_0}{c} \operatorname{Im}\left(\frac{2e^{-2i\omega_0 t} \sin(\delta) t_f}{n_{\text{film}}}\right)\mathbf{z}. \tag{20}$$

In the classical literature the Minkowski form is usually considered as the total momentum density (field+matter) [28], while the Abraham form is the momentum density of the field. It is interesting to note that from Eqs. (19)–(20) it follows that the difference between the two $\mathbf{G}^{\text{M}}_{\text{e.m.}}(t)/dt - \mathbf{G}^{\text{A}}_{\text{e.m.}}(t)/dt$ is proportional to $n_{\text{film}} - 1/n_{\text{film}}$, the parameter that also enters Eq. (91) in Ref. [28] and defines the momentum density of a medium. In that context, in the calculation of the mechanical force, we believe that the Abraham form is more suitable for our numerical simulations, rather than the Minkowski form that takes into account the field and the medium momentum densities. Nonetheless, for completeness, we proceed with both forms.

It follows that the total oscillating mechanical force can be written as

$$\begin{aligned} \mathbf{F}^{\text{M}}_{\text{mech.osc.}}(t) = d\mathbf{G}_{\text{tot.osc.}}(t)/dt - d\mathbf{G}^{\text{M}}_{\text{e.m.}}(t)/dt = \\ \frac{\Sigma_0}{c} \operatorname{Im}(e^{-2i\omega_0 t}\Delta_- \sin(\delta) t_f)\mathbf{z} = \frac{2\Sigma_0}{c} \operatorname{Re}(e^{-2i\omega_0 t} r_f)\mathbf{z} \end{aligned} \tag{21}$$

and

$$\begin{aligned} \mathbf{F}^{\text{A}}_{\text{mech.osc.}}(t) = d\mathbf{G}_{\text{tot.osc.}}(t)/dt - d\mathbf{G}^{\text{A}}_{\text{e.m.}}(t)/dt = \\ -\frac{\Sigma_0}{c} \operatorname{Im}(e^{-2i\omega_0 t}\Delta_- \sin(\delta) t_f)\mathbf{z} = -\frac{2\Sigma_0}{c} \operatorname{Re}(e^{-2i\omega_0 t} r_f)\mathbf{z}. \end{aligned} \tag{22}$$

We note that both expressions for the force differ only by the sign. This happens because, as follows from Eqs. (18)–(20), the different forms of momentum constituting the oscillating force are related by $d\mathbf{G}_{\text{tot.osc.}}(t)/dt = (\mathbf{G}^{\text{M}}_{\text{e.m.}}(t)/dt + \mathbf{G}^{\text{A}}_{\text{e.m.}}(t)/dt)/2$. The total optical force in the time domain takes the form

$$\mathbf{F}^{\text{M}}_{\text{mech}}(t) = \frac{\Sigma_0}{c}(1 + r_f r_f^* - t_f t_f^* + 2\operatorname{Re}(e^{-2i\omega_0 t} r_f))\mathbf{z}, \tag{23}$$

$$\mathbf{F}^{\text{A}}_{\text{mech}}(t) = \frac{\Sigma_0}{c}(1 + r_f r_f^* - t_f t_f^* - 2\operatorname{Re}(e^{-2i\omega_0 t} r_f))\mathbf{z}. \tag{24}$$

As mentioned, the two forms differ by the sign of the dynamic part and will lead to forces that are out-of-phase. It is also very interesting to note that Eqs. (23)–(24) predict that within the cycle of the wave, for a semi-transparent film ($\left|t_f\right| \neq 0$), the instantaneous magnitude of the optical force acting on the film can be negative. This will be shown in Sec. 3.4.

3.3. Model validation

To validate this model, we first consider the rather trivial case of a film with refractive index $n_{\mathrm{film}} = 1$ (vacuum), Fig. 3(a). In this case, the reflection and transmission coefficients are $\left|r_f\right| = 0$ and $\left|t_f\right| = 1$. Consequently, we expect a vanishing mechanical force ($\mathbf{F}_{\mathrm{mech}}(t) = \mathbf{F}^{\mathrm{M}}_{\mathrm{mech}}(t) = \mathbf{F}^{\mathrm{A}}_{\mathrm{mech}}(t) = 0$ in Eqs. (23)–(24)). Hence, no momentum is transferred to the mechanical force, and the total momentum of the system is stored in the propagating electromagnetic wave ($d\mathbf{G}_{\mathrm{tot.osc.}}(t)/dt = d\mathbf{G}_{\mathrm{e.m.}}(t)/dt$), as can be seen from the dynamics of the force components presented in Fig. 3(b).

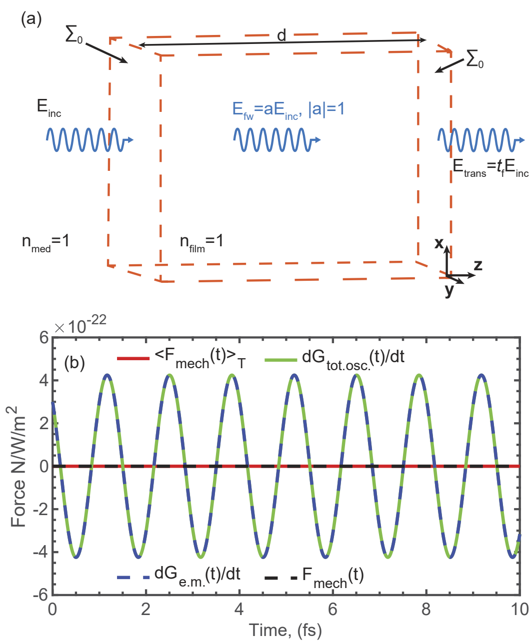

Fig. 3. (a) Plane wave impinging at normal incidence on a film with thickness d and refractive index $n_{\mathrm{film}} = 1$, in vacuum. (b) Dynamics of the optical force normalized to the incident intensity, acting on a $\Sigma_0 = 300 \times 300\ \mathrm{nm}^2$ section of an infinitely large air film (thickness $d = 100$ nm) placed in air, illuminated by a monochromatic plane wave $\lambda_0 = 800$ nm. The average force $\langle \mathbf{F}_{\mathrm{mech}}(t) \rangle_T$ acting on the film is zero (red line), while the oscillating part of the total momentum (green) equals the electromagnetic component (dashed blue). The mechanical force acting on the film is also zero (dashed black).

The oscillations amplitude of the electromagnetic momentum part can be found from Eqs. (19)–(20) as $2\Sigma_0 \left|\sin(\delta)\right|/c$ since $n_{\mathrm{film}} = 1$. Note that this amplitude vanishes for $\delta = q\pi$, $q \in \mathbb{N}$, corresponding to ($d = q\lambda_0/2$). This is related to the fact that the Poynting vector in the film, Eq. (1), is a periodic function with a period equal to $\lambda_0/2$ when $n_{\mathrm{film}} = 1$. Consequently, the integration in Eqs. (10)–(11) over a volume with thickness $d = q\lambda_0/2$ vanishes. In summary, this trivial situation emphasizes the importance of the electromagnetic momentum part, Eqs. (19)–(20), in the calculations of the mechanical force. Indeed, taking into account only the total oscillating component, Eq. (18), we would get a nonzero mechanical force in the considered case, which is unphysical.

3.4. Force acting on a lossless film in air

Let us now consider a 100 nm thick fused silica film, with a refractive index $n_{\mathrm{film}} \approx 1.5$ without dispersion or losses (the refractive index data are taken from Ref. [92]). As a matter of fact,

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dispersive materials bring about additional complications for the definition of the electromagnetic momentum [90]. The reflection coefficient as a function of wavelength for such a film is calculated by enforcing the boundary conditions, see Sec. A of the Supplement 1.

At $\lambda_0 = 300$ nm we observe a dip in the reflection, which is associated with the Fabry-Perot destructive interference condition for reflection ($\lambda_0 = 2dn_{\text{film}}$). For wavelengths above 600 nm the reflection coefficient is close to $\left|r_f\right| \approx 0.3$. For non-vanishing reflection, the electromagnetic momentum can now couple into the mechanical force. The dynamics of the total oscillating, as well as Minkowski and Abraham electromagnetic momenta for λ_0=800 nm are presented in Fig. 4(b). The total oscillating momentum is clearly not equal to either the Minkowski or the Abraham forms of the electromagnetic momentum. Hence, the mechanical force calculated for both formalisms is nonzero and oscillates in time around the average, Fig. 4(c). From Eqs. (23)–(24) it follows that for a lossless film, the minimum of the mechanical force can be found as $2\frac{\Sigma_0}{c}|r_f|(|r_f| - 1)\mathbf{z}$. From the that it directly follows that the minimum of the optical force is always less than zero since $|r_f| \leq 1$. This result is actually not surprising. One can derive even from the simple consideration of Eq. (1) using the explicit expression of the electric field, Eq. (S.1), that the Poynting vector of the incident field interfering with the reflected field reaches negative values, which can be the origin of the appearance of a negative force. In Ref. [54] a negative optical force acting on a thin film was observed for illumination with a decaying monochromatic wave with complex frequency. It was shown that for short periods of time the instantaneous optical force can attain negative values. In our case we show that the instantaneous optical force can attain negative values even for simple harmonic illumination.

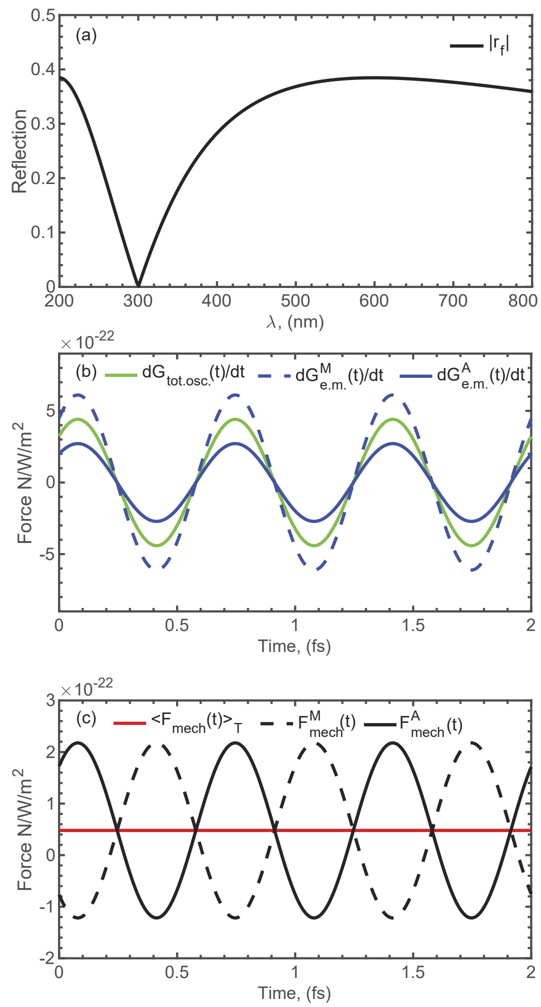

Fig. 4. (a) Reflection coefficient for a 100 nm thick fused silica film under normal incidence. (b) Dynamics of the total oscillating momentum, as well as the Minkowski and Abraham forms of the electromagnetic momentum. (c) Average mechanical force as well as dynamics of the mechanical forces obtained within the Minkowski and Abraham formalisms.

4. Optical force acting on an object under two-wave illumination

In the previous sections we developed the fundamental analysis of the optical force in the time domain under monochromatic illumination. We observed that the optical force was oscillating at $2\omega_0$ - a frequency that is too high for the mechanical oscillator to develop a response. In this section, we aim at decreasing this frequency down to the one accessible for mechanical oscillators. To this end, let us consider an object illuminated by two monochromatic plane waves with frequencies ω_0 and $\omega_0 + \Omega$, see the sketch in Fig. 5(a). Here ω_0 is an optical frequency and $\Omega \ll \omega_0$. In the sketch, we also introduce an angle θ between the two beams for visualization purposes. Hereafter, we consider the case $\theta = 0$, for which the analytical expression becomes very simple.

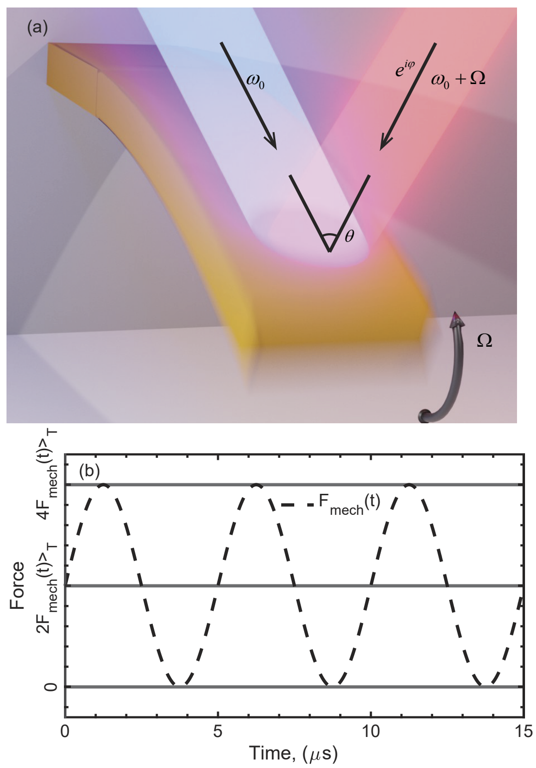

Fig. 5. (a) A rod is illuminated by two plane waves with different frequencies ω_0 and $\omega_0 + \Omega$. Here $\Omega \ll \omega_0$ and the two beams have different colors for illustration purposes. The wave with frequency $\omega_0 + \Omega$ has additional phase shift φ with respect to the wave ω_0. The angle between two waves is θ. (b) Dynamics of the beating component of the optical force in the time domain (dashed black line) acting on the rod at the frequency $\Omega/2\pi = 200$ kHz for $\theta = 0$ according to Eq. (25). The average and maximum/minimum values are highlighted with grey horizontal lines.

Let us assume that the illumination conditions (fields amplitude, direction of propagation) are equal for both waves. Let us further assume that the wave $\omega_0 + \Omega$ has a phase shift φ with respect to the wave ω_0. In the Supplement 1 we carefully analyse the Abraham and Minkowski forms of the force, Eqs. (13)–(14), in the considered case. Our developments show that the total oscillating component $d\mathbf{G}_{\text{tot.osc.}}(t)/dt$ is proportional to twice the average force $\langle \mathbf{F}_{\text{mech}}(t)\rangle_T$ created by each of the waves. It is also shown that the electromagnetic momentum of both forms $d\mathbf{G}^{\text{M}}_{\text{e.m.}}(t)/dt$ and $d\mathbf{G}^{\text{A}}_{\text{e.m.}}(t)/dt$ are by factor of Ω/ω_0<<1 smaller

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than the same for a single-frequency illumination. The latter is not surprising given that the calculation of the electromagnetic momentum requires the calculation of the time derivative of $\mathrm{Re}\,(\mathbf{E}(\mathbf{r},t)) \times \mathrm{Re}\,(\mathbf{H}(\mathbf{r},t))$ in Eqs. (7)–(8). For a monochromatic wave, ω_0 appears as a prefactor after differentiation, Eqs. (10)–(11). For two waves, the beating frequency Ω appears instead of ω_0. Consequently, the electromagnetic momentum contribution can be neglected and the Minkowski and Abraham force are equal to each other $\mathbf{F}^{\mathrm{M}}_{\mathrm{mech}}(t) = \mathbf{F}^{\mathrm{A}}_{\mathrm{mech}}(t) = \mathbf{F}_{\mathrm{mech}}(t)$. Hence, the analytical form of the force takes the form

$$\mathbf{F}_{\mathrm{mech}}(t) = 2\langle \mathbf{F}_{\mathrm{mech}}(t)\rangle_T (1 + \cos(\Omega t - \varphi)). \tag{25}$$

Note that Eq. (25) is applicable for objects of arbitrary shapes and normal incidence is not required, as can be noticed from the derivations given in Sec. B in Supplement 1. Besides, Eq. (25) contains only the part associated with the low-frequency Fourier component Ω of the optical force acting in the time-domain. The parts associated with $2\omega_0$, $(2\omega_0 + \Omega)$ and $2\,(\omega_0 + \Omega)$ take more complicated forms and can be derived following the procedure described in Supplement 1. However, it is the 2Ω component that is usually measured in optomechanics [36,93] and therefore our results could be applied to the classical interpretation of experiments on semitransparent membranes [94–97]. The derived Eq. (25) provides a simple form to calculate the oscillating force in the time-domain for semi-transparent objects. In that context, the results of the average force calculations performed previously [9,15,16,21,22,72,98–107] can now be directly applied to calculate the amplitude of the oscillating force acting on arbitrary objects in the time domain.

Let us analyze the implications of Eq. (25). It predicts that for a quarter of the period of the beating frequency $2\pi/(4\Omega)$, the instantaneous mechanical force overcomes the average force and can reach the maximum value of $4\langle \mathbf{F}_{\mathrm{mech}}(t)\rangle_{T=2\pi/\omega_0}$. In Fig. 5(b) the dynamics of the mechanical force in the time domain is presented for the beating frequency $\Omega/2\pi = 200$ kHz. This frequency corresponds to the typical resonance frequency of an AFM cantilever that is sometimes used in cavity cooling experiments [108]. From this figure and from Eq. (25) we note that the optical force cannot attain negative values in case of two frequency illuminations. Also, the optical force can now overcome the average force by a factor of two for durations in the order of microseconds. This time is sufficient to displace the nanorod from its equilibrium position and possibly even break it. Note that a similar idea of increasing the peak force acting on nanoparticles has been proposed for pulsed illumination, since in that case the different frequency components of the pulse are closely packed, thus increasing the peak optical force [52]. Pulsed illumination was used in the past to rapture the adhesion forces that hold a particle on a surface [42,44].

5. Conclusion

The analytical formalism presented in this work revealed the evolution of the optical force in the time-domain under monochromatic and two-wave illuminations. For the monochromatic case, we highlighted the different elements required to calculate the optical force in the time domain with both the Abraham and Minkowski approaches. Quite interestingly, the two approaches give optical forces with the same magnitude that only differ by a phase shift of π. The possibility of an instantaneous negative optical force acting on a thin film was also discovered. When a physical system is illuminated with two waves of slightly different frequencies, a relation between the magnitude of the amplitude of the beating force and the magnitude of the average force was established.

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Supplemental document. See Supplement 1 for supporting content.

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