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# Multipolar origin of electromagnetic transverse force resulting from TE/TM waves interference

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## ABSTRACT

In this paper, we aim at unveiling the underlying physical mechanism for transversal optical forces, appearing due to the simultaneous illumination of a spherical object with two plane waves possessing different polarizations. The appearance of such a transversal force is quite counterintuitive since it seems to contradict the law of momentum conservation. We consider the cases of perfect electric conductor (PEC) and silver spheres illuminated by two orthogonally polarized plane waves propagating obliquely with respect to each other. Interestingly, the Poynting vector in these cases acquires a nonzero component transverse to the plane of propagation. Since the momentum transfer is related to the energy transfer, or equivalently, to non-negligible Poynting vector pointed in a particular direction, an arbitrary object placed in such external field is expected to experience a transversal force. To cast light upon this peculiar effect, we use a surface integral equation method and, along with the Maxwell stress tensor formalism, find the optical force acting on various spheres. We observe this effect for PEC spheres of different sizes and find that they are indeed subject to such transversal force. We find an explanation for this phenomenon via interference effects between selected multipoles excited in the structure. With recently developed methods, we expand the optical force into contributing pairs of selected multipoles and show that, depending on the phase between each multipole pair, the sign and direction of the force can be controlled. We also compare the results for silver and PEC spheres and find that the transversal force magnitude in silver has higher values for more limited range of sphere radii, as compared to PEC.

**Keywords:** Optical force, PEC, surface integral equation, Maxwell stress tensor

## 1. INTRODUCTION

The abilities to precisely manipulate, propel, pull and rotate nanoparticles at the nanoscale have attracted tremendous interest recently.[1-13] Likewise, propelling macroscopic particles can lead to very peculiar forces that can find applications in the concept of a solar engine with adjustable thrust.[14-16]

The momentum conservation law stands as a convenient tool to explain these forces and usually the direction of the force is co-directed with the photons' direction of propagation. Quite interestingly, this is the case only while the system conserves the symmetry with respect to the incident light propagation direction. As soon as the symmetry is broken, the scattered light can be asymmetrically redirected, leading to a net force.[17] A force transversal to the photons stream direction can appear in systems with broken symmetry of the scattering objects such as Janus particles[18] or parity-time symmetric objects.[19]

Alternatively, such a force can appear for an illumination with broken symmetry.[20-23] In this case, the system can "see" the upcoming wavefront as being asymmetrical and, consequently, develop an asymmetric response. One interesting study case for such a force appears when a sphere is illuminated with two plane waves possessing different polarizations and propagating at an angle with respect to each other. In this situation, an optical force, that is simultaneously orthogonal to the propagation direction of the first and the second waves, can be observed. The emergence of this force has been recently explained by an effective interplay between electric and magnetic multipoles inside a scattering object.[20] This interplay creates interesting oscillations of the force acting on a PEC sphere for different particles' radii.[20] These oscillations have nodes and maxima/minima peak values that are defined by the polarizability function of the sphere.

In this paper, we aim at extending the analysis of the transversal force acting on PEC spheres by comparing it with the one acting on more realistic metals. We show that in the case of silver spheres, the presence of a weak magnetic optical

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response leads to a transversal optical force, which direction is more sensitive to the radius of the particle, thus enabling a possibility to sort nanoparticles with higher precision.

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## 2. RESULTS

Let us consider a metallic sphere and two orthogonally propagating TE- and TM-polarized waves with their **k**-vectors aligned along the $\mathbf{x}$- and $\mathbf{y}$-axes, respectively, as shown in Figure 1. The propagation of these two waves leads to an intuitive diagonal force $\mathbf{F}_{diag}$ aligned along the $\mathbf{x}+\mathbf{y}$ direction. As was shown in Ref.[20], an additional optical force along the $\mathbf{z}$-axis also appears in this case due to asymmetric scattering from the sphere with respect to the $xOy$ plane.

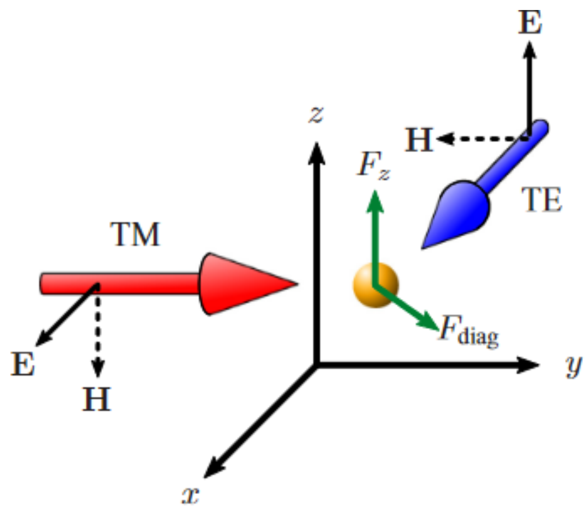


Figure 1. Two plane waves with electric and magnetic fields $\mathbf{E}$ and $\mathbf{H}$ propagating at $90°$ with respect to each other and possessing TE- and TM-polarizations hit a spherical particle causing a diagonal force in the xOz-plane and a transversal $F_z$ force along the **z**-axis.

The link between the Poynting vector, asymmetric scattering and optical force becomes more apparent upon considering the dipolar approximation and studying the scattered fields far away from the scatterer. In this case, the optical force acting on an object can be rewritten in terms of the integral of the difference between the total time-averaged Poynting vector $\langle \mathbf{S} \rangle$ and the time-averaged Poynting vector of the incident field $\langle \mathbf{S}^{(i)} \rangle$:[24]

$$\langle \mathbf{F} \rangle = -\frac{\varepsilon^2}{c} \int_{S_0} \left( \langle \mathbf{S} \rangle - \langle \mathbf{S}^{(i)} \rangle \right) d\sigma. \quad (1)$$

Here, $c$ is the speed of light in vacuum and $\varepsilon$ is the dielectric permittivity of the object. Henceforth, we assume vacuum as a background medium. Note that the integration in this case should be performed in the far-field over a surface $S_0$ that completely encompasses the illuminated object. From Eq. (1) it becomes clear that asymmetric scattering immediately leads to the optical force. One can also go beyond the dipolar approximation and consider Maxwell's stress tensor for the force calculations.[25] The transversal $\mathbf{F}_z$ component of the optical force calculated with Maxwell's stress tensor and a surface integral method[26] for a perfect electric conductor sphere (PEC) illuminated by two plane waves having a wavelength of 500 nm with different sphere radii is shown in Figure 2 (blue curve). As can be seen, the direction and the magnitude of the force strongly depends on the radius of the sphere.

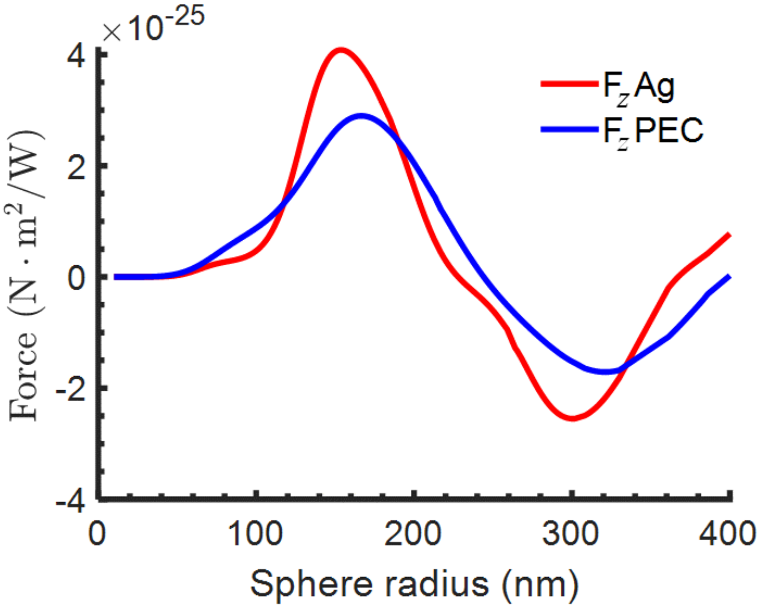


Figure 2. Transversal optical force $\mathbf{F}_z$ normalized to the illumination power, acting on PEC (blue) and silver (red) spheres having different radii. The wavelength is 500 nm.

In general (beyond the dipolar approximation), the optical force can be described through the interaction between incident and scattered fields and also the interaction between scattered fields themselves.[27] The first interaction type plays a very important role in optical trapping.[6,7,12,13,28-31] Interestingly, it was shown that it does not give a contribution to the transversal force for the illumination considered in the current article.[20] On the contrary, the scattered – scattered light interaction is the one that plays an important role. In order to describe the force dependency presented in Figure 2, a multipolar theory can be applied.[27,32] As was shown by Chen et al.[32] and Achouri et al.[20], in this case the optical force can be fully described by the interaction of the vector spherical harmonic pairs effectively excited in such a sphere. The contributions giving rise to the force along the **z**-direction for illumination presented in Figure 1 can be described as

$$\begin{aligned}\langle\mathbf{F}\rangle_z = &-\frac{k^4}{12\pi\varepsilon_0 c_0}\mathrm{Re}\left[p_x m_y^* - p_y m_x^*\right]\\ &-\frac{k^5}{40\pi\varepsilon_0}\mathrm{Im}\left[Q_{zx}^e p_x^* + Q_{zy}^e p_y^* + Q_{zz}^e p_z^*\right]\\ &-\frac{k^5}{40\pi\varepsilon_0 c_0^2}\mathrm{Im}\left[Q_{zx}^m m_x^* + Q_{zy}^m m_y^* + Q_{zz}^m m_z^*\right]\\ &-\frac{k^6}{240\pi\varepsilon_0 c_0}\mathrm{Re}\begin{bmatrix} Q_{xx}^e Q_{xy}^{m*} - Q_{yy}^e Q_{yx}^{m*} + Q_{yx}^e Q_{yy}^{m*} + \\ -Q_{xy}^e Q_{xx}^{m*} + Q_{zx}^e Q_{zy}^{m*} - Q_{zy}^e Q_{zx}^{m*}\end{bmatrix}.\end{aligned} \tag{2}$$

Here, $p_i$ and $m_i$ stand for the electric and magnetic multipoles excited in the sphere. And $Q_{ij}^e$ and $Q_{ij}^m$ are the magnitudes of electric and magnetic quadrupoles.[33] For each radius in Figure 2, we present a multipolar decomposition of the far field to estimate the magnitudes of the multipoles.[34] This allows us to find the individual contributions for each multipolar pair in Eq. (2). We present the result of such a decomposition for PEC in Figure 3(a). As can be seen, the electric dipole-magnetic dipole contributions $(\mathrm{PM})$ are the first to appear for the smallest sphere radii. As the radius increases, the electric dipole-electric quadrupole contribution $(\mathrm{PQ}_e)$ starts playing a more important role. For spheres larger than 200 nm in radius, the magnetic dipole-magnetic quadrupole contribution $(\mathrm{MQ}_m)$ becomes significant. Additionally, we see a small contribution from the interaction between the electric and magnetic quadrupoles $Q_{zx}^e$ and $Q_{zy}^m$. We also plot in Figure 3 the sum of the four abovementioned contributions as a solid grey line, as well as the total force found by the full-wave simulations as a solid black line. It becomes evident that these four contributions are simultaneously strong. This is because for a PEC sphere, in the quasistatic approximation, the dipolar electric $\alpha_e$ and magnetic $\alpha_m$ polarizabilities show simultaneous cubic dependence on the radius of the sphere as $\alpha_e \approx \varepsilon_0 4\pi r_s^3$ and $\alpha_m \approx -\mu_0 4\pi r_s^3$.[25] Interestingly, this is not the case for metallic spheres, where a magnetic response is effectively suppressed.[35] Consequently, replacing the PEC material by a real metal metal is expected to reduce the force contributions containing magnetic multipoles, thus effectively reducing the extent of a strong transversal force to a narrower radii range .

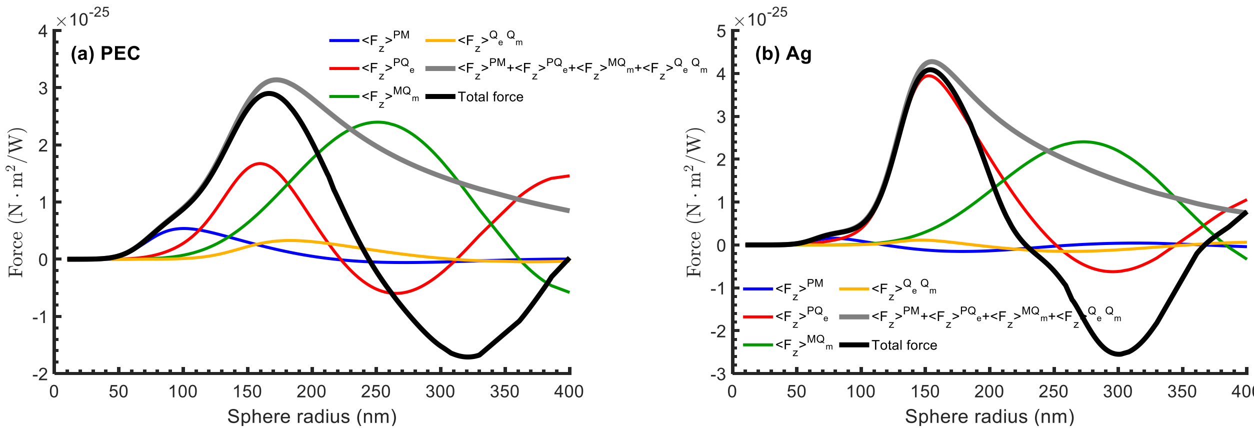


Figure 3. (a) and (b) Averaged transversal optical force $\langle \mathbf{F}_z \rangle$ acting on PEC and Silver spheres, respectively, for different radii and their multipolar decomposition given by Eq. (2).

We confirm that by analysing the transversal force acting on Ag sphere. The refractive index of Ag is modelled with the experimental data fit obtained from Johnson and Christy.[36] The tranversal optical force acting on Ag is plotted in Figure 1, where the peak of the optical force as a function of radius for silver spheres (red curve, for $r_s = 154$ nm) is more narrow than that for PEC (blue curve). This is explained by analysing Figure 3(b), where multipolar contribution pairs are presented. It is seen that the relative contributions of pairs containing magnetic contributions are now significantly reduced, which explains the observed effect for Ag.

## 3. CONLUSIONS

We have provided an analysis of the transversal force caused by crossed TE/TM plane wave illumination acting on realistic silver spheres and compared it with that observed for a PEC model. In both cases, the emergence of the force is attributed to the interplay between the radiation scattered by different multipoles. In the case of Ag spheres, thanks to the dominant electric multipole response, the effect of transversal force appears to be limited to a narrower range of sphere radii. This effect could be used for example for sorting particle sizes with light.


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