MR3767234 65-02 35L65 65M06 65M08 65M12 65M60 65M70 76Mxx
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★**Numerical methods for conservation laws.**
From analysis to algorithms.
Computational Science & Engineering, 18.
Society for Industrial and Applied Mathematics (*SIAM*), *Philadelphia*, *PA*, 2018.
xvi+570 *pp. ISBN* 978-1-611975-09-3

This book is dedicated to the construction and practical implementation of numerical approximations of hyperbolic conservation laws. One of the main features of such nonlinear partial differential equations is the finite time formation of discontinuities in their solutions, which gives rise to many (now well-documented) difficulties in the design of efficient numerical schemes. Though the book covers more introductory material, such as the theory of monotone schemes, its main topic is the construction and analysis of high order numerical methods together with their practical implementation. Throughout the book, the author illustrates the design of numerical schemes by providing MATLAB codes that correspond to the implementation of the proposed schemes on the most celebrated hyperbolic equations, among which the reader will find the transport equation, Burgers' equation, the Euler system of gas dynamics and the Maxwell system of electromagnetics. The provided routines correspond to simulations in either one or two space dimensions.

Probably the most striking feature of the book is the presentation of a wide variety of numerical techniques that have been developed for the numerical simulation of hyperbolic wave propagation phenomena. The material covered in the book includes monotone finite difference schemes, flux limited methods, (weighted) essentially nonoscillatory schemes, spectral methods and some of the most recent discontinuous Galerkin (DG) techniques. The book thus deals with major families of numerical schemes that have been designed in this field. Equipped with the MATLAB routines provided along the way, the reader will be eventually immersed in some of the up-to-date computational challenges in the theory of hyperbolic conservation laws. All chapters display detailed examples and an extensive bibliography which gives the reader the opportunity to focus on a particular class of methods. Though the variety of techniques and problems considered in this book makes its reading quite demanding, this new reference will definitely be of interest to students and researchers willing to learn high order discretization techniques for hyperbolic conservation laws.

Let us now go into more detail about the contents of the book. A general introduction covers some of the most fundamental concepts associated with hyperbolic conservation laws: characteristic curves, finite speed of propagation and finite time formation of discontinuities in the nonlinear case. All these notions are presented with documented examples and numerical illustrations. The book is then organized into three main parts. Part I (chapters 2 and 3) gathers the main theoretical background on hyperbolic conservation laws. Part II (chapters 4 through 7) deals with monotone numerical schemes. Part III (chapters 8 through 13) deals with high order numerical methods.

More specifically, chapter 2 gathers some material on the theory of scalar conservation laws, including integration along the characteristics, finite time blowup, weak solutions, admissibility criteria, L^1 contraction and vanishing viscosity. Chapter 3 deals with the harder case of systems of conservation laws and includes material on the Riemann problem, entropy conditions for shocks and symmetrization issues. Though some of the notions are only sketched, with appropriate more advanced references for the interested reader, these two introductory chapters gather all the main notions that will guide the design and analysis of numerical methods throughout the book.

Chapter 4 initiates the study of numerical methods and presents the fundamental notions of conservation at the discrete level as well as monotone and total variation diminishing schemes. The celebrated first-order restriction for monotone schemes is also presented. Chapter 5 covers the theory of finite difference schemes for linear and nonlinear problems. In the linear setting, stability is dealt with in ℓ^2 by means of Fourier analysis, while in the nonlinear case, the author presents a general convergence result for monotone discretizations. Numerical simulations are presented based on the most common examples, such as the Lax-Friedrichs, Lax-Wendroff or Roe schemes. Chapter 6 deals with finite volume methods and primarily Godunov's method. Several approximate Riemann solvers are presented. The author also introduces the class of central schemes. Chapter 7 explains how the one-dimensional theory allows for the design of numerical schemes on cartesian grids in any space dimension. Numerical simulations are presented for several two-dimensional problems.

Chapter 8 is the starting point of the study of high order methods. These are first motivated on linear equations, and specific attention is paid to the phase error analysis. The analysis then switches to total variation and/or entropy stability. The author explains at length the issue of oscillations in the neighborhood of discontinuities for high order methods, and the use of filtering techniques. Chapter 9 is devoted to time integration issues, namely the so-called strong stability preserving Runge-Kutta and multistep integrators, be they explicit or implicit. Numerous examples are given and order barriers are discussed. The author then focuses on spatial discretization issues. Chapter 10 deals with a first class of high order methods, namely flux limited and slope limited schemes. The many possible limiters as well as total variation stability are extensively discussed. Central schemes are then further discussed, complementing the introductory material of Chapter 6.

Chapter 11 deals with ENO and WENO schemes by first constructing essentially nonoscillatory interpolation polynomials. The author introduces ENO schemes, discusses stability issues and provides various numerical simulations to illustrate their efficiency. WENO methods are then introduced together with some possible extensions. Chapter 11 also contains some material on well-balanced schemes (when the underlying partial differential equation incorporates a source term) and some boundary extrapolation procedures known as inverse Lax-Wendroff methods. Chapter 12 introduces discontinuous Galerkin methods. After a general introduction and a motivating discussion, the methods are presented and implementation issues are discussed. The analysis then focuses on stability and convergence of the DG method in the scalar case. For nonsmooth problems, filtering techniques as well as nonlinear dissipation are introduced to enhance stability and diminish oscillations in the vicinity of discontinuities. Alternative strategies based on slope limiters are also presented, closing the loop with the slope limited methods of Chapter 10 and the WENO methods of Chapter 11. Connections with even further families of numerical schemes are made and numerical implementations are provided. Chapter 13 is devoted to spectral methods. After recalling classical convergence results on Fourier series, the author introduces several spectral methods and discusses their stability and convergence properties for either the transport or Burgers' equation. For nonlinear problems, filtering techniques or more advanced tools based for instance on Padé approximants may be applied in order to enhance stability.

Jean-François Coulombel

MR3829062 65-01

Conte, S. D. [Conte, Samuel Daniel];
de Boor, Carl [de Boor, Carl R.] (1-WI-CS; Madison, WI)

★**Elementary numerical analysis.**
An algorithmic approach.
Updated with MATLAB.
Reprint of the third (1980) edition.
For the 1965 edition see [MR0202267].
Classics in Applied Mathematics, 78.

Society for Industrial and Applied Mathematics (*SIAM*), *Philadelphia*, *PA*, 2018.
xxiv+456 *pp.* *ISBN* 978-1-611975-19-2

Publisher's description: "This book provides a thorough and careful introduction to the theory and practice of scientific computing at an elementary, yet rigorous, level, from theory via examples and algorithms to computer programs. The intended audience is upper-division undergraduates in engineering, mathematics, and the sciences, including computer science. The book has served well as a text book. The original FORTRAN programs have been rewritten in MATLAB and now appear in a new appendix and online, offering a modernized version of this classic reference for basic numerical algorithms."

MR3829063 00A71 37-01 49-01 70-01 76-01 91Bxx

Wan, Frederic Y. M. (1-CA3-NDM; Irvine, CA)

★**Mathematical models and their analysis.**

Reprint of the 1989 original.
Classics in Applied Mathematics, 79.
Society for Industrial and Applied Mathematics (*SIAM*), *Philadelphia*, *PA*, 2018.
xx+402 *pp. ISBN* 978-1-611975-26-0

Publisher's description: "A great deal can be learned through modeling and mathematical analysis about real-life phenomena, even before numerical simulations are used to accurately portray the specific configuration of a situation. Scientific computing also becomes more effective and efficient if it is preceded by some preliminary analysis. These important advantages of mathematical modeling are demonstrated by models of historical importance in an easily understandable way.

"The organization of *Mathematical Models and Their Analysis* groups models by the issues that need to be addressed about the phenomena. The new approach shows how mathematics effective for one modeled phenomenon can be used to analyze another unrelated problem. For instance, the mathematics of differential equations useful in understanding the classical physics of planetary models, fluid motion, and heat conduction is also applicable to the seemingly unrelated phenomena of traffic flow and congestion, offshore sovereignty, and regulation of overfishing and deforestation. The formulation and in-depth analysis of these and other models on modern social issues, such as the management of exhaustible and renewable resources in response to consumption demands and economic growth, are of increasing concern to students and researchers of our time.

"The modeling of current social issues typically starts with a simple but meaningful model that may not capture all the important elements of the phenomenon. Predictions extracted from such a model may be informative but not compatible with all known observations; so the model may require improvements. The cycle of model formulation, analysis, interpretation, and assessment is made explicit for the modeler to repeat until a model is validated by consistency with all known facts.

"This book is recommended for advanced undergraduates, early graduate students, or anyone with a strong background in calculus and ordinary differential equations, for use in mathematical modeling and introduction to applied mathematics courses."

{For the 1989 original see [F. Y. M. Wan, *Mathematical models and their analysis*, Harper & Row Ser. Appl. Math., Harper & Row, New York, 1989].}