ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Prof. Marco PICASSO
Mathematics Institute of Computational Science and Engineering - MATHICSE

SEMINAR OF NUMERICAL ANALYSIS

- **WEDNESDAY 18 MAI 2016 - ROOM CM 09 - 16h15**

Prof. Andrea BONITO (Texas A & M University, Tamu, USA)
will present a seminar entitled:

"Numerical approximations of fractional operators: a journey with the Balakrishnan formula"

Abstract:

The mathematical theory and numerical analysis of non-local operators has been the topic of intensive research in recent years. As prototypes, we focus on fractional powers of elliptic operators and advocate the so-called Balakrishnan formula to guide the design and analysis of numerical algorithms.
The case of elliptic operators on bounded domains is considered first. It turns out that the Balakrishnan formula provides an adequate expression for the targeted solution in terms of an improper integral involving the solution of auxiliary, parameter dependent, elliptic problems. The improper integral is approximated by an exponentially convergent SINC quadrature method. At each quadrature point, a standard finite element method is used to approximate the auxiliary problems. They are independent of each other and therefore suited for parallel implementations.
One class of applications come from replacing Brownian motion diffusion by diffusion coming from symmetric alpha-stable Levy processes, i.e., the Laplace operator is replaced by a fractional Laplacian defined on an unbounded domain. It is conceptually feasible to study the Galerkin approximation but such a direct approach is questionable as the exact computation of the resulting stiffness matrix entries does not seem to be possible (at least in two or more spatial dimensions). The Balakrishnan formula cannot be used to represent the inverse of the operator either but is instrumental in deriving a variational formulation. As in the bounded case, we invoke a SINC quadrature formula coupled with finite element discretizations on parameter dependent truncated domains. We develop a non-conforming method by approximating the action of the stiffness matrix on a vector (sometimes referred to as a matrix free approach).

This is joint work with Wenyu Lei and Joseph E. Pasciak

Lausanne, 12 April 2016/MP/cr

The MATHICSE seminars are announced at http://mathicse.epfl.ch/seminars