Low Complexity Iterative Multimedia Resource Allocation based on Game Theoretic Approach

Eunji Kim, Hyunggon Park
Multimedia Communications and Networking Laboratory
Ewha Womans University, Seoul, Korea
doojiq@ewhain.net; hyunggon.park@ewha.ac.kr

Pascal Frossard
Signal Processing Laboratory (LTS4)
Ecole Polytechnique Fédérale de Lausanne (EPFL), Switzerland
pascal.frossard@epfl.ch

Abstract— **Efficient resource management strategies are important for multiuser multimedia applications, as they are often serviced over resource-constrained and shared network infrastructure. Moreover, an acceptable level of quality e.g., Quality of Service (QoS) should be guaranteed. In this paper, we consider a game-theoretic resource management strategy, where the bargaining solutions are deployed in the resource allocation. We are in particular interested in the Nash Bargaining Solution (NBS) that can allocate resources in a fair and optimal way, while explicitly considering the achieved utility. Finding the NBS, however, is a challenging task due to its potentially high computational complexity, especially when a large number of users and the large amount of resources are available. In order to overcome the problem, we propose an iterative approach that requires significantly lower computational complexity compared to the conventional approach. The proposed approach decomposes the bargaining problem into sub-bargaining problems, where a sub-bargaining problem considers smaller feasible set and computes the corresponding sub-NBS. This step is iteratively repeated for successive sub-bargaining problems until the NBS is obtained. We show that the proposed sub-NBS approaches the NBS with a small error while significantly reducing the complexity required to find the NBS.**

Index Terms - **Nash bargaining solution (NBS), gradual bargaining, strategic step-by-step negotiation, resource allocation**

I. Introduction

The bargaining strategies and solutions based on game theory [1], [2] have been proposed to solve multimedia resource allocation problems [3]. Several bargaining solutions such as the Nash bargaining solution (NBS) and the Kalai-Smorodinsky bargaining solution (KSBS) have been exploited to allocate resource efficiently ensuring the necessary QoS over resource-constrained network environment. However, it is challenging to compute the bargaining solutions due to their computationally-intensive task. This becomes critical when the amount of resources and the number of supporting users increase, as it leads to exponential computation complexity for finding the bargaining solutions.

In order to overcome this problem, we propose to decompose the feasible utility set into smaller sub-feasible utility sets, and then find the NBS by iteratively applying the NBS approach in each sub-feasible utility sets. This enables us to compute the NBS with significant less computation complexity compared to a conventional approach, where users bargain over the whole feasible utility set to find the NBS.

The iterative sub-NBS algorithm is inspired by the strategic step-by-step negotiation [4]. While the negotiation process in [4] considers the exogenous bargaining set, we consider mutually exclusive and discrete sub-bargaining sets. Thus, the proposed algorithm can be directly deployed in practice. It is shown in [5] that a solution to gradual bargaining problems is compatible with a solution to one-shot bargaining problems. They demonstrate that the gradual NBS corresponds to a path of agreements that satisfies a certain differential equation. The monotone path discussed in [6] is interpreted as the differential approach in the context of a gradual bargaining problem. These analytical solutions to the gradual bargaining problems show that decomposing the feasibility utility set into sub-feasible utility sets can also lead to the bargaining solutions. Note that it is shown that the step-by-step negotiation leads to the KSBS [7]. The key property of the KSBS, i.e., constant relative payoffs, can be simply achieved by the movements through the bargaining in sub-feasible utility sets. Consequently, the KSBS of the multi-stage bargaining game is the same as the KSBS of the bargaining over the whole set in one stage. Unlike the KSBS, the NBS cannot simply be computed based on the decomposition of the feasible utility set because of its characteristics of non-linearity.

The proposed algorithm has the following advantages. Compared to a conventional approach to compute the NBS, where it generally relies on the exhaustive search, the iterative sub-NBS algorithm requires significantly lower complexity. This is a direct benefit by considering smaller feasible utility sets that include the NBS. The idea is similar to the *pruning* in a decision tree, i.e., removing the potentially unnecessary feasible utility points, thereby leading to smaller search space. Hence, the performance speed is significantly improved. While the computation complexity required for the iterative sub-NBS decreases, the performance degradation measured by the accuracy is negligible, as shown in our experiment results. Therefore, it can be concluded that the iterative sub-NBS can achieve the almost same performance while requiring significantly small computation complexity. We also study the tradeoff between the performance degradation and the complexity reduction. Thus, an optimal number of divisions can also be determined.

This work was supported in part by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (2011-0001546 and 2011-0005184) and in part by the 2nd Strategic Korean-Swiss Cooperative Program in Science and Technology.

This paper is organized as follows. In Section II, the problem formulation and the key ideas of the proposed iterative sub-NBS algorithm are discussed. In Section III, we quantitatively evaluate the iterative sub-NBS approach. Simulation results are presented in Section IV and the conclusions with future research work are drawn in Section V.

II. Problem Formulation and Sub-NBS Approach

A. Problem Formulation

The problem of resource allocation for multiuser can be formulated as follows. We assume that there are n multimedia users and individual user i has its own utility function defined by utility function $U_i(\cdot)$. In our case, the utility function is defined such that it represents the achieved multimedia quality as in [3]. The utility that user i achieves is denoted by $U_i(x_i)$ given the allocated resource x_i. All possible utility pairs that n users jointly achieve form a feasible utility set S, which is assumed to be convex, nonempty, and closed [2], [3], [8]. The disagreement point d represents a set of minimum utilities of users. In order to highlight the performance of the proposed algorithm, we also assume that there is no packet loss in the network, so that the whole network resources can be allocated to the users. More details on the problem formulation can be found in our prior work [3].

The NBS is conventionally found based on the exhaustive search, i.e., all feasible utility pairs in S are examined. However, the exhaustive search becomes significantly inefficient when the feasible utility set becomes large. Hence, in order to reduce the search complexity, we decompose the feasible utility set into m smaller sub-feasible utility sets $S_1', S_2', \ldots, S_m'$. Then, the NBS in each sub-feasible utility set, which is referred to as sub-NBS in this paper, can be computed successively. More details of the proposed iterative sub-NBS algorithm are discussed next. A conceptual illustration of the proposed sub-NBS approach for two-user case is shown in Fig 1.

B. Iterative Sub-NBS Algorithm

The feasible utility set S, which is the set of all possible pairs of payoffs and is given by

$$S = \left\{ (U_1(x_1), \ldots, U_n(x_n)) \in \mathbb{R}_+^n \,\middle|\, \sum_{k=1}^{n} x_k \leq R \right\}$$

where R is the available resource and x_i is the resource allocated to user i. $\mathbb{R}_+^n$ denotes a set of non-negative real number. The ith sub-feasible utility set S_i' can be similarly defined as

$$S_i' = \left\{ (U_1(x_1), \ldots, U_n(x_n)) \in \mathbb{R}_+^n \,\middle|\, R_{i-1} \leq \sum_{k=1}^{n} x_k \leq R_i \right\}$$

where R_0 is determined by the disagreement point d and $R_m = R$. The iterative sub-NBS algorithm begins by considering the first sub-feasible utility set S_1' with disagreement point d, and the first sub-NBS is determined as $\mathbf{U}_1^* \triangleq NBS(S_1', d) = (U_1(x_1^*), \ldots, U_n(x_n^*))$. The sub-NBS, $\mathbf{U}_1^*$, is

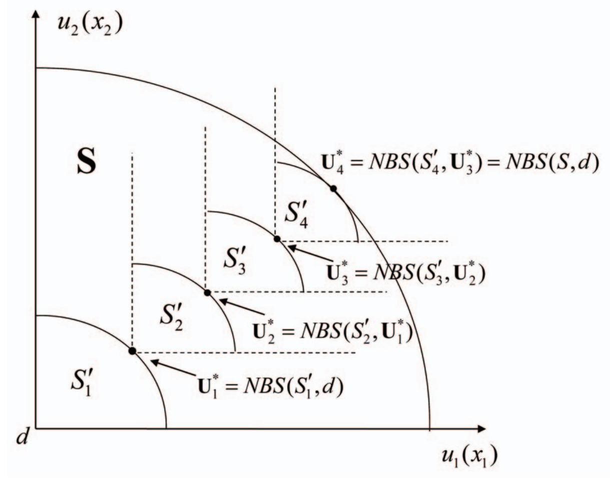

Fig. 1. An illustrative example of the iterative sub-NBS processes for two-user case. The entire feasible utility set is decomposed into four non-overlapped subsets S_i', and u_i^* represents the corresponding sub-NBS.

considered as the disagreement for the next sub-NBS with the next sub-feasible utility set S_2'. Correspondingly, $\mathbf{U}_2^* = NBS(S_1', \mathbf{U}_1^*)$ is computed. In general, the ith sub-NBS is computed by considering ith sub-feasible utility set S_i' and the $(i-1)$th sub-NBS $\mathbf{U}_{i-1}^*$ as the disagreement point, i.e., $\mathbf{U}_i^* = NBS(S_i', \mathbf{U}_{i-1}^*)$. The process continues iteratively until the last sub-NBS is obtained, i.e., $\mathbf{U}_m^*$.

Note that a solution to gradual bargaining problems, which can be considered as a continuous version of the proposed approach, is converged to the solution to one-shot bargaining problems [5]. It was also demonstrated that the gradual NBS corresponds to a path of agreements that satisfies a differential equation. The monotone path discussed in [6] is interpreted as the differential approach in the context of a gradual bargaining problem. Hence, the proposed iterative sub-NBS approach can also converge to the NBS with a small error that may occur in dividing sub-feasible utility sets. This is quantitatively evaluated in the next section.

III. Evaluation of Iterative Sub-NBS Algorithm

In this section, we quantitatively evaluate the proposed iterative sub-NBS approach in terms of required complexity and the corresponding performance.

A. Complexity of Iterative Sub-NBS Algorithm

We first study the computation complexity required for the iterative sub-NBS approach. In this evaluation, we assume that the computation complexity is measured by the number of operations needed to find NBS. As mentioned, conventional approach to find NBS requires exhaustive search for all possible pairs over the entire feasible utility set in general. In the proposed iterative sub-NBS algorithm, however, the feasible utility set is decomposed into m smaller sub-feasible utility sets, where they are not overlapped. Note that as the number of sub-feasible utility sets increases, the number of iterations for computing NBS also increases. The results for

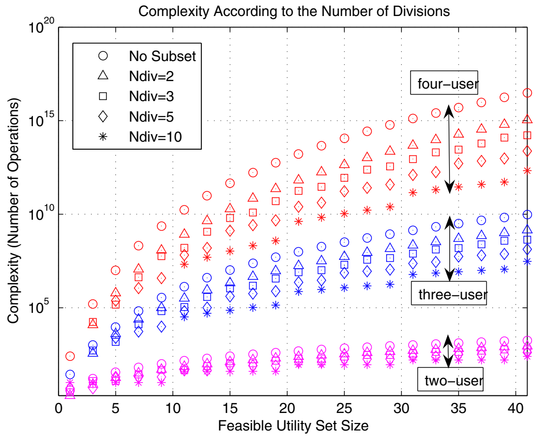

Fig. 2. The computational complexity required for the conventional approach and the iterative sub-NBS approach to find the NBS.

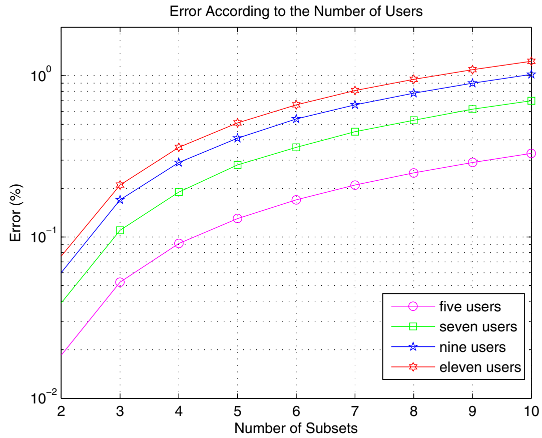

Fig. 3. Errors occurred in different number of subsets and participating users.

complexity required to find the NBS based on the iterative sub-NBS algorithm is shown in Fig. 2. In this illustration, we consider the cases of two, three, and four users in the bargaining games given a set of available resources. Different numbers of sub-feasible utility sets (denoted by N_{div}) are considered for the iterative sub-NBS algorithm.

It is clearly observed that as N_{div} (i.e., the number of decomposed sub-feasible sets) increases, the total complexity required to find the NBS is reduced. This implies that the computational complexity can be significantly lowered by dividing the whole feasible utility set into small sub-feasible utility sets. In addition, it is also observed that the complexity increases as the number of participating users increases. This is because as the dimension of the feasible utility set becomes enlarged. However, the complexity required by the iterative sub-NBS algorithm is always significantly lower than that by the conventional approach for NBS.

B. *Performance of Iterative Sub-NBS Algorithm*

We next evaluate how the iterative sub-NBS algorithm can accurately find the NBS. The accuracy is measured by the errors incurred between the NBSs computed by the conventional approach with the exhaustive search and the proposed iterative sub-NBS approach.

Simulation results for different numbers of participating users and subsets are shown in Fig. 3. It is observed that smaller errors occurred as the number of subsets decreases (or equivalently, the sizes of the sub-feasible utility sets are enlarged). This is because of the structure of the iterative sub-NBS algorithm, where the sub-NBS in a stage is used as the disagreement point in the next sub-NBS. Thus, if there are errors occurred in the early stages of the sub-NBS, they may be propagated to its successive sub-NBS computations. This is also consistent with the intuition, where if there is only one iteration, i.e., the largest sub-feasible utility set (or a single sub-NBS), the iterative sub-NBS is the same as the one-shot NBS, which implies no error between two approaches.

TABLE I
MODEL PARAMETERS FOR VIDEO SEQUENCES. (VIDEO TYPE, TEMPORAL LEVEL(TL), FRAME RATE)

Video Sequence	μ	D_0	R_0
Foreman (CIF, TL=4, 30Hz)	5232400	0	0
Coastguard (CIF, TL=4, 30Hz)	6329700	4.3	0
Mobile (CIF, TL=4, 30Hz)	38230000	1	44040

From the results above, we can conclude that there is a tradeoff between operational complexity and performance for the iterative sub-NBS approach. Therefore, the number of iteration or the size of the decomposed feasible utility set can be determined appropriately by considering the tradeoff.

IV. SIMULATION RESULTS

In this section, the performance of the iterative sub-NBS approach is evaluated in a network where multimedia users are sharing the network resources.

A. *System Setup*

In order to be consistent with the referenced simulation results in [3], we assume that the available resources (bandwidth) are allocated to the users without loss. Moreover, in order to explicitly consider the video quality, the distortion-rate (DR) model used in [3] is considered, which is given by

$$D = \frac{\mu}{(R - R_0)} + D_0, R \geq R_0, D_0 \geq 0, \mu > 0, \quad (1)$$

where D is the distortion of the sequence, and R is the video sequence rate. Parameter values of the DR model such as μ, R_0, and D_0 for different video sequences are given in Table I, which are obtained in [3]. The utility function is accordingly defined as

$$U_i(x_i) \triangleq \frac{c}{D_i} = \frac{c(x_i - R_{0i})}{D_{0i}(x_i - R_{0i}) + \mu_i} \quad (2)$$

where c is a nonnegative constant (often it is set by $c = 255^2$) and subscript i denotes user i. The resource allocations are determined based on the approach used in [3] and the iterative sub-NBS approach.

TABLE II

ALLOCATED RATES BY THE GENERALIZED NBS OBTAINED IN CONVENTIONAL METHOD AND FINAL SUB-NBS OBTAINED FROM ITERATIVE APPROACH
USER 1: FOREMAN, USER 2: COASTGUARD, USER 3: MOBILE (CIF, TL=4, 30HZ)

N_{div}=5	NBS (conventional approach)			Sub-NBS (iterative approach)			
R_{MAX} (Mbps)	x_1^* (Kbps)	x_2^* (Kbps)	x_3^* (Kbps)	$\hat{x}_1^*$ (Kbps)	$\hat{x}_2^*$ (Kbps)	$\hat{x}_3^*$ (Kbps)	Error (%)
0.3	89.4 (29.1%)	84.6 (27.5%)	133.2 (43.4%)	90.0 (28.6%)	90.0 (28.6%)	134.6 (42.8%)	2.70
0.5	161.1 (31.5%)	146.5 (28.6%)	204.4 (39.9%)	160.0 (30.5%)	160.0 (30.5%)	204.3 (39.0%)	3.32
1.0	346.7 (33.8%)	289.7 (28.3%)	387.6 (37.9%)	340.0 (32.4%)	326.0 (31.1%)	382.6 (36.5%)	5.25
1.5	539.7 (35.1%)	419.9 (27.4%)	576.4 (37.5%)	520.0 (33.1%)	491.0 (31.2%)	561.9 (35.7%)	7.70
2.0	738.7 (36.1%)	540.3 (26.4%)	769.0 (37.5%)	701.0 (33.4%)	652.0 (31.1%)	744.2 (35.5%)	9.67

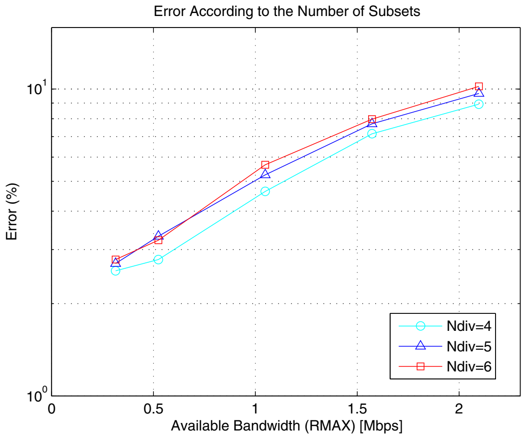

Fig. 4. Errors for different number of sub-feasible utility sets.

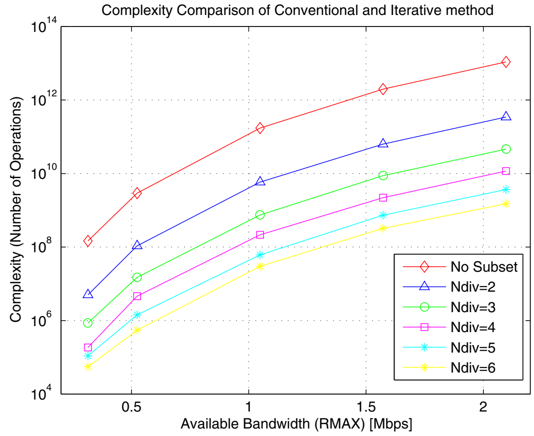

Fig. 5. Complexity required for the conventional NBS and proposed iterative sub-NBS approaches.

B. Performance Evaluation of Iterative sub-NBS Approach

We assume that there are three multimedia users trying to transmit video sequences (*Foreman*, *Coastguard*, and *Mobile*) at CIF resolution 30Hz, respectively. The resources are allocated based on the conventional approach in [3] and the iterative sub-NBS approach. The allocated bitrates are shown in Table II. In this case, $N_{div} = 5$ (i.e., five sub-feasible utility sets) are used for the iterative sub-NBS approach. Fig. 4 also shows errors occurred for different numbers of sub-feasible utility sets. The results shown in Fig. 4 are consistent with the evaluation results in Section III-B, where the more sub-feasible utility sets, the larger errors may occur. However, all the errors are still less than 10% compared to the NBS computed by the conventional approach.

Fig. 5 shows the complexity required for the conventional approach (No Subset case) and for the iterative sub-NBS approach. This result is also consistent with the discussion in Section III-A, where the complexity requirement for finding NBS increases as more resources are considered and decreases as more sub-feasible utility sets are considered.

These results confirm that the proposed iterative sub-NBS can compute the NBS with a small error while significantly reducing the computation complexity.

V. CONCLUSION AND FUTURE WORK

In this paper, we propose an efficient algorithm of computing NBS with significantly low complexity. The key idea is to decompose the feasible utility set into smaller sub-feasible utility sets and the sub-NBS from early stages becomes the disagreement point in the next sub-NBS stage. We quantitatively show that the computation complexity can be significantly lowered compared to the conventional approach while achieving the NBS with a smaller error. In the future work, analytical analysis for the performance and the complexity can be studied. Moreover, the optimal number of sub-feasible utility sets can be determined by considering the tradeoff between the complexity and performance.

REFERENCES

[1] J. Nash, "The bargaining problem," *Econometrica*, vol. 18, pp. 155–162, 1950.

[2] K. Binmore, *Fun and Games: A Text on Game Theory*. Lexington, MA: D.C. Health, 1992.

[3] H. Park and M. van der Schaar, "Bargaining strategies for networked multimedia resource management," *IEEE Trans. Signal Processing*, vol. 55, no. 7, pp. 3496–3511, July 2007.

[4] R. John and M. G. Raith, "Strategic step-by-step negotiations," *Journal of Economics*, vol. 70, no. 2, pp. 127–154, Mar. 1999.

[5] Z. W. Barry O' Neill, Dov samet and E. Winter, "Bargaining with an agenda," *Games and Economic Behavior*, vol. 48, pp. 139–153, 2004.

[6] G. Rocheteau and C. Waller, "Bargaining and the value of money," *Working papers of the Federal Reserve Bank of Cleveland*, vol. 05-01, Mar. 2005.

[7] E. Kalai, "Proportional solutions to bargaining situations: Interpersonal utility comparisons," *Econometrica*, vol. 45, no. 7, pp. 1623–1630, Oct. 1977.

[8] M. J. Osborne and A. Rubinstein, *A Course in Game Theory*. The MIT Press, 1994.