# Super-resolution from unregistered omnidirectional images

Zafer Arıcan and Pascal Frossard
*Ecole Polytechnique Fédérale de Lausanne (EPFL)*
*Signal Processing Laboratory - LTS4*
*Lausanne - 1015, Switzerland*
{zafer.arican,pascal.frossard}@epfl.ch

## Abstract

*This paper addresses the problem of super-resolution from low resolution spherical images that are not perfectly registered. Such a problem is typically encountered in omnidirectional vision scenarios with reduced resolution sensors in imperfect settings. Several spherical images with arbitrary rotations in the SO(3) rotation group are used for the reconstruction of higher resolution images. We first describe the impact of the registration error on the Spherical Fourier Transform coefficients. Then, we formulate the joint registration and reconstruction problem as a least squares norm minimization problem in the transform domain. Experimental results show that the proposed scheme leads to effective approximations of the high resolution images, even with large registration errors. The quality of the reconstructed images also increases rapidly with the number of low resolution images, which demonstrates the benefits of the proposed solution in super-resolution schemes.*

## 1. Introduction

Image super-resolution typically describes the reconstruction of high-resolution images from multiple low-resolution images that are typically produced by imperfect vision sensors. Super-resolution has been an active field of research, and efficient solutions have been proposed when planar images are perfectly registered[9, 3]. Similar approaches have been proposed for omnidirectional images [6, 7], where the true geometry of these particular images is however left unexploited. In omnivision, the images created catadioptric cameras can alternatively be mapped onto the unit sphere [4] in order to provide an efficient representation of the geometry of the scene in the spherical images. Super-resolution with spherical images has however not been widely studied, except in the case of spherical microphone arrays [10].

Most super-resolution algorithms work with images that are perfectly registered. This is however often an unrealistic assumption, as registration can generally not be computed exactly, especially with very low resolution images, and imperfect sensor settings. Recent methods have investigated the joint problem of registration of low resolution images, and super-resolution reconstruction. Subspace methods and a projection theorem are used in [12] for estimating the registration parameters, followed by reconstruction. However, the complex structure of the projection matrix limits the development of fast iterative algorithms. Other approaches have been proposed using respectively alternating minimization, or structured nonlinear total least-squares norm with Gauss-Newton method in the pixel domain [8, 5]. The former solution may require many iterations for convergence. The latter offers a relatively fast solution, but the computation of matrices in the pixel domain is not trivial. Moreover, no solution can be easily adapted to spherical images.

In this work, we propose a method that jointly estimates the registration errors and reconstructs high resolution images from low resolution spherical images with arbitrary rotations in the SO(3) rotation group. We perform the super-resolution reconstruction with help of the Spherical Fourier Transform (SFT) computed from non-uniformly sampled data on the sphere. We further represent the registration problem in the SFT domain, which permits to formulate a least-squares norm minimization problem in the tranform domain. The solution of this problem provides effective approximation of spherical images even with relatively large registration errors for the low resolution images. Experimental results with images of different resolution demonstrate the validity of the proposed solution for super-resolution in omnivision applications.

## 2. Superresolution on the sphere

We consider several low resolution omnidirectional images that are mapped on the sphere in order to preserve the geometry of the scene. Formally, we denote by$X(\theta, \phi)$ a square-integrable continuous signal lying on 2-sphere, $S^2$ where $\theta$ is the longitude angle in the range $[0, \pi]$ and $\phi$ is the colatitude angle defined in $[-\pi, \pi)$ that forms an equiangular grid. We assume that we have $Q$ such signals that represent $L \times L$ low resolution images with different orientations in SO(3). Let $g_k = g_{ZYZ}(\alpha_k, \beta_k, \gamma_k)$ denote a non-commutative rotation operator in the rotation group $SO(3)$. It describes the orientation of $k^{th}$ spherical image, so that the point $\nu(\theta, \phi)$ corresponds to $g_k \nu$ in the $k^{th}$ image. When points on the low-resolution images are registered on the high resolution sphere using the rotation operator $g_k$, it produces an interlaced sampling scheme, illustrated in Figure 1.

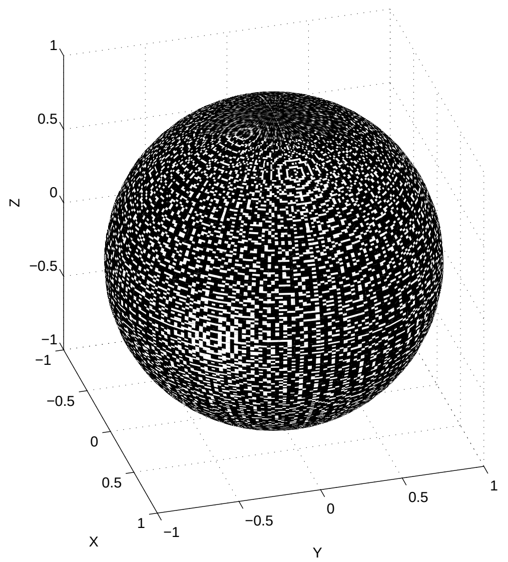


Figure 1: Non-uniform sampling grid formed by registration of low resolution images with different orientations.

The super-resolution problem becomes equivalent to the problem of reconstruction with non-uniformly sampled data on the sphere [11]. The high resolution image can be reconstructed with the help of the Spherical Fourier Transform (SFT). The spherical image $X(\theta, \phi)$ can be decomposed into a series of spherical harmonics using discrete SFT as:

$$X(\theta, \phi) = \sum_{l \in \mathbb{N}} \sum_{|m| \leq l} \hat{x}(l, m) Y_l^m(\theta, \phi), \qquad (1)$$

where $Y_l^m(\theta, \phi)$ is the spherical harmonic of degree $l$, order $m$ and $\hat{x}(l, m)$ is the corresponding Fourier coefficient. When $X(\theta, \phi)$ is bandlimited to $B$, it can be perfectly reconstructed from uniformly sampled data on a $2B \times 2B$ equiangular grid [2]. However, we do not have a uniformly sampled set of data after registration, but rather a set of intensity values $X(\nu)$ that can be written as

$$X(\nu) = \sum_{l=0}^{N-1} \sum_{|m| \leq l} a(l, m) Y_l^m(\nu). \qquad (2)$$

where $a(l, m) \approx \hat{x}(l, m)$ are now the Fourier coefficients that have to be estimated. If $V^l(\nu)$ is a $(2l+1)$-tuble column vector in the form

$$\mathbf{V}^l(\nu) = \left[ \begin{array}{ccc} Y_l^{-l}(\nu)^{\mathrm{T}} \cdots Y_l^0(\nu)^{\mathrm{T}} \cdots Y_l^l(\nu)^{\mathrm{T}} \end{array} \right]^{\mathrm{T}}, \quad (3)$$

and $\mathbf{V}$ is $B^2$-tuble vector formed by concatenation of $\mathbf{V}^l$ for $l = 0 \ldots (B-1)$, we can equivalently write the following linear system of equations:

$$\mathbf{M} \cdot \mathbf{a} = \mathbf{z} \qquad (4)$$

where

$$\begin{aligned} \mathbf{M} &= \{\mathbf{V}(\nu)\}_{QL^2 \times B^2} \\ \mathbf{a} &= \{a(l, m)\}_{B^2 \times 1} \\ \mathbf{z} &= \{X(\nu)\}_{QL^2 \times 1} \end{aligned} \qquad (5)$$

The solution of this linear system for the coefficients $a(l, m) \approx \hat{x}(l, m)$ finally permits to reconstruct the high-resolution image by inverse Spherical Fourier Transform.

## 3. Super-resolution with unregistered images

We consider now the problem where the low-resolution images are not perfectly registered, which represents a likely scenario in practice. We first render explicit the registration parameters in the linear systems described in the previous section, by exploiting the relation between the SO(3) rotation group and the SFT. In particular, the spherical harmonic $Y_n^l(g\nu)$ for a rotated sampling point $\nu$ by $g$ can be related to $Y_m^l(\nu)$ by

$$Y_l^n(g\nu) = \sum_{m=-l}^{l} U_{mn}^l(g) Y_l^n(\nu) \qquad (6)$$

where $U_{mn}^l(g)$ is the elements of Wigner-D matrix [1]. We can therefore separate the sampling from the registration in the data matrix $\mathbf{M}$ that is formed by concatanation of $V(\nu)$ for all registered sample points as

$$\mathbf{M} = \left[ \begin{array}{c} \mathbf{V}_1^{\mathrm{T}} \mathbf{V}_2^{\mathrm{T}} \cdots \mathbf{V}_Q^{\mathrm{T}} \end{array} \right]^{\mathrm{T}}, \qquad (7)$$

where $\mathbf{V}_k$ represents the spherical harmonics for the sample points on $k^{th}$ image. When we choose the first low resolution image as reference, we can further write

$$\mathbf{M} = \underbrace{\begin{bmatrix} \mathbf{V}_1 & & & \\ & \mathbf{V}_1 & & \\ & & \ddots & \\ & & & \mathbf{V}_1 \end{bmatrix}}_{\mathbf{K}} \underbrace{\begin{bmatrix} \mathbf{I}^{\mathrm{T}} \\ \mathbf{R}_2^{\mathrm{T}} \\ \vdots \\ \mathbf{R}_{Q-1}^{\mathrm{T}} \\ \mathbf{R}_Q^{\mathrm{T}} \end{bmatrix}}_{\mathbf{E}}, \tag{8}$$

where $\mathbf{R}_q^{\mathrm{T}}$ describes rotations of the low resolution images. The linear system of equations in Eq. (4) therefore takes the form

$$\mathbf{KE}\,\mathbf{a} = \mathbf{z}, \tag{9}$$

where $\mathbf{K}$ characterizes the sampling and $\mathbf{E}$ models the registration.

When the registration is not perfect (i.e., $g_k$ is not accurate), the data matrix $\mathbf{M} = \mathbf{KE}$ induces erroneous Fourier coefficients in the solution of the above system of equations. When the rotation angles that are embedded in $\mathbf{E}$ are unknown, the system becomes non-linear. We propose to formulate the reconstruction of the high resolution image as a nonlinear least-squares norm optimization problem [5]. We propose to work in the Fourier domain, where the computational complexity can be reduced by exploiting the relation between the rotation on the sphere and the spherical harmonics.

We denote the unknown vector of rotation angles by $\mathbf{b}$, and it represents a column vector formed by concatenation of $\{\alpha_k, \beta_k, \gamma_k\}$. The joint registration and reconstruction problem becomes

$$\{\hat{\mathbf{a}}, \hat{\mathbf{b}}\} = \operatorname*{argmin}_{\mathbf{a},\mathbf{b}} ||\mathbf{z} - \mathbf{KE}(\mathbf{b})\mathbf{a}||_2\,, \tag{10}$$

where $\mathbf{E}(\mathbf{b})$ models the registration when the rotations are given by $\mathbf{b}$. When the first image is chosen as a reference, the minimization problem can be rewritten as

$$\min \left\| \begin{array}{c} \mathbf{r}_1(\mathbf{a}) \\ \mathbf{r}(\mathbf{a},\mathbf{b}) \end{array} \right\|_2 \tag{11}$$

where $\mathbf{r}_1 = \mathbf{z}_1 - \mathbf{V_1}\mathbf{a}$ and $\mathbf{r} = \mathbf{z} - \tilde{\mathbf{K}}\tilde{\mathbf{E}}(\mathbf{b})\mathbf{a}$ where $\tilde{\mathbf{K}}$ and $\tilde{\mathbf{E}}$ represents the reduced versions of the matrices $\mathbf{K}$ and $\mathbf{E}$ where the parts corresponding to the first image has been removed.

If $\Delta\mathbf{a}$ denotes a small change in $\mathbf{a}$, and $\Delta\mathbf{b}$ a small change in $\mathbf{b}$, we then have the following approximation:

$$\mathbf{r}(\mathbf{a}+\Delta\mathbf{a}, \mathbf{b}+\Delta\mathbf{b}) \approx \mathbf{r}(\mathbf{a},\mathbf{b}) - \mathbf{J}(\mathbf{a},\mathbf{b})\Delta\mathbf{b} - \tilde{\mathbf{K}}\tilde{\mathbf{E}}(\mathbf{b})\Delta\mathbf{a} \tag{12}$$

where $\mathbf{J}(\mathbf{a},\mathbf{b})$ is the jacobian of $\tilde{\mathbf{K}}\tilde{\mathbf{E}}(\mathbf{b})\mathbf{a}$ with respect to $\mathbf{b}$. With such a linearization, the minimization problem of Eq. (11) takes the form

$$\begin{pmatrix} \mathbf{0} & \mathbf{V_1} \\ \mathbf{J}(\mathbf{a},\mathbf{b}) & \tilde{\mathbf{K}}\tilde{\mathbf{E}}(\mathbf{b}) \\ \mathbf{L} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \Delta\mathbf{b} \\ \Delta\mathbf{a} \end{pmatrix} = \begin{pmatrix} \mathbf{r}_1(\mathbf{a}) \\ \mathbf{r}(\mathbf{a},\mathbf{b}) \\ \mathbf{0} \end{pmatrix}, \tag{13}$$

where the additional parameter $\mathbf{L} = \sqrt{c}\mathbf{I}$ is a regularization term that increases the stability when $\mathbf{J}(\mathbf{a},\mathbf{b})$ is close to zero. The minimization problem becomes equivalent to an iterative algorithm that estimates $\mathbf{a}$,$\mathbf{b}$. Due to the structure of the matrix in Eq. (13) the problem is actually a Levenberg-Marquardt minimization algorithm, that can be solved efficiently by Levenberg-Marquardt method.

## 4. Experiments & Results

We analyze now the performance of the proposed algorithm with synthetic spherical images of realistic looking room scene. We first reconstruct a $128 \times 128$ image from 80 low resolution images of $16 \times 16$ pixels. The rotation angles for the low-resolution images are randomly selected and a random registration error of maximum $5$ degrees is further introduced for each rotation angle. Figure 2 shows the reconstruction of the high resolution image, while Figure 3 proposes a zoom on a highly textured region in the image. We see that the registration errors highly affect the reconstructed image, where the PSNR is 24.30 dB. The proposed method that jointly performs registration and reconstruction is able to correct these registration errors, and provides an effective approximation with a PSNR of 44.28 dB. Finally, we analyze the effect of number of low-resolution images on the convergence of the proposed algorithm for different registration errors. We use $16 \times 16$ low-resolution images to reconstruct a $64 \times 64$ high-resolution image. We randomly generate registration errors of maximum $5$, $10$ and $15$ degrees with a uniform distribution with zero mean. The Table 1 shows the PSNR values for the reconstructed images. We can see that, for small registration errors, an accurate high-resolution image can be generated from a small number of low resolution images. The number of images required for a good approximation augments with the registration errors, as expected.

## 5. Conclusions

We have proposed an algorithm that simultaneously estimates the registration parameter of low resolution spherical images with arbitrary rotations in SO(3) and

Figure 2: Groundtruth high-resolution image (top-left), one low-resolution image (top-right), reconstructed image with registration errors (bottom-left), reconstructed image with the proposed algorithm (bottom-right).

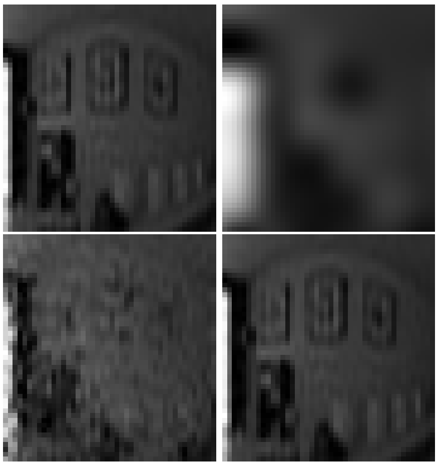

Figure 3: Zoom images: groundtruth high-resolution image (top-left), one low-resolution image (top-right), reconstructed image with registration errors (bottom-left), reconstructed image with the proposed algorithm (bottom-right).

reconstructs a high resolution image. The algorithm is based on a nonlinear least-squares norm optimization problem in the Fourier domain, which is solved by a Levenberg-Marquardt method. The effective reconstruction performance in experiments with synthetic omnidirectional images demonstrates the benefits of the proposed solution for super-resolution problems in omnivision applications with imperfect settings.

Table 1: PSNR of the reconstructed images from $N$ low resolution images with registration errors $Err$.

| | $Err = 5^o$ | $Err = 10^o$ | $Err = 15^o$ |
|---|---|---|---|
| N=5 | 16.48 | 16.08 | 16.15 |
| N=10 | 43.33 | 18.74 | 15.97 |
| N=15 | 44.02 | 41.66 | 16.75 |
| N=20 | 44.49 | 41.49 | 25.56 |
| N=25 | 42.49 | 37.57 | 38.56 |
| N=30 | 47.27 | 39.41 | 43.06 |

## References


[1] G. Chirikjian and A. Kyatkin. *Engineering Applications of Noncommutative Harmonic Analysis: With Emphasis on Rotation and Motion Groups.* CRC Press, 2000.

[2] J. Driscoll and D. Healy Jr. Computing Fourier transforms and convolutions on the 2-sphere. *Advances in Applied Mathematics*, 15(2):202–250, 1994.

[3] S. Farsiu, D. Robinson, M. Elad, and P. Milanfar. Advances and challenges in super-resolution. *International Journal of Imaging Systems and Technology*, 14(2):47–57, 2004.

[4] C. Geyer and K. Daniilidis. Catadioptric Projective Geometry. *International Journal of Computer Vision*, 45(3):223–243, 2001.

[5] Y. He, K. Yap, L. Chen, and L. Chau. A Nonlinear Least Square Technique for Simultaneous Image Registration and Super-Resolution. *Image Processing, IEEE Transactions on*, 16(11):2830–2841, 2007.

[6] H. Kawasaki, K. Ikeuchi, and M. Sakauchi. Super-resolution Omnidirectional Camera Images Using Spatio-temporal Analysis. *Electronics and Communications in Japan Part 3 Fundamental Electronic Science*, 89(6):47, 2006.

[7] H. Nagahara, Y. Yagi, and M. Yachida. Super-resolution from an omnidirectional image sequence. *Industrial Electronics Society, 2000. IECON 2000. 26th Annual Confjerence of the IEEE*, 4, 2000.

[8] M. Ng, J. Koo, and N. Bose. Constrained Total Least-Squares Computations for High-Resolution Image Reconstruction with Multisensors. *sensors*, 50:35–42, 2002.

[9] S. Park, M. Park, and M. Kang. Super-resolution image reconstruction: a technical overview. *Signal Processing Magazine, IEEE*, 20(3):21–36, 2003.

[10] B. Rafaely and M. Park. Super-resolution spherical microphone arrays. *Electrical and Electronics Engineers in Israel, 2004. Proceedings. 2004 23rd IEEE Convention of*, pages 424–427, 2004.

[11] I. Tosic and P. Frossard. FST-based Reconstruction of 3D-models from Non-Uniformly Sampled Datasets on the Sphere. *Picture Coding Symposium*, 2006.

[12] P. Vandewalle, L. Sbaiz, J. Vandewalle, and M. Vetterli. Super-resolution from unregistered and totally aliased signals using subspace methods. *IEEE Trans. Signal Processing*, 55:3687–3703, 2007.