L_1 REGULARIZED SUPER-RESOLUTION FROM UNREGISTERED OMNIDIRECTIONAL IMAGES

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ABSTRACT

In this paper, we address the problem of super-resolution from multiple low-resolution omnidirectional images with inexact registration. Such a problem is typically encountered in omnidirectional vision scenarios with reduced resolution sensors in imperfect settings. Several spherical images with arbitrary rotations in the SO(3) rotation group are used for the reconstruction of higher resolution images. We propose an l_1 regularized total least squares norm minimization method for joint registration and reconstruction with better stabilization and denoising. Experimental results show that regularization offers a quality improvement of up to 1dB. In addition, it reduces the number of low resolution images that are necessary to reconstruct a high resolution image at a target quality.

***Index Terms*—** Super-resolution, omnidirectional images, spherical Fourier Transform, l_1 regularization

1. INTRODUCTION

Image super-resolution typically describes the reconstruction of high-resolution images from multiple low-resolution images that are produced by imperfect vision sensors. Super-resolution is however an ill-defined inverse problem leading often to unstable systems, especially in the case where images are not perfectly registered. Regularization has been proved to be useful to increase the stability of such systems. For example, Tikhonov and total variation (TV) algorithms are two common regularization methods using l_2 and l_1 norms respectively in order to improve the performance and compensate for some small registration errors.

In this work, we address the problem of super-resolution of omnidirectional images mapped on the unit sphere (e.g., as captured by catadioptric systems [1]). We propose an l_1 regularized least-squares method that jointly estimates the registration errors and reconstructs high resolution images from low resolution spherical images with arbitrary rotations in the SO(3) rotation group. We represent registration and sampling problem with the help of the Spherical Fourier Transform (SFT), which permits to formulate a least-squares norm minimization with l_1 norm regularization in the transform domain. The formulation of the problem in the transform domain simplifies the handling of registration errors, and further it permits the use of methodologies developped for denoising. The solution of our optimization problem provides effective approximation of spherical images even with relatively large registration errors on the low resolution images. Experimental results with images of different resolutions demonstrate the validity of the proposed solution and the benefit of the regularization term for super-resolution in omnivision applications.

Super-resolution has been an active field of research, and efficient solutions have been proposed when planar images are perfectly registered [2, 3]. Similar approaches have been proposed for registered omnidirectional images [4, 5], where the true geometry of these particular images is however left unexploited. Some recent works have further addressed the joint problem of registration of low resolution images and super-resolution reconstruction of images from perspective cameras. For example, researchers have proposed techniques based on subspace methods and projection theorem [6], alternating minimization [7], or structured nonlinear total least-squares norm [8] with a total variation regularization step in the pixel domain. Unfortunately, none of these solutions can be easily adapted to omnidirectional images that present a very specific geometry. Finally, it should be noted that l_1 regularization techniques have received quite some attention in emerging fields like compressed sensing. While it nicely extends to super-resolution, the l_1 regularized least-squares problem generally does not consider inexact system matrices, which happen for unregistered images.

2. SUPER-RESOLUTION ON THE SPHERE

Image super-resolution is an inverse problem which is the reconstruction of an high-resolution image, $\mathbf{X}$ from multiple low-resolution input images, $\mathbf{z}$. Typically, the low-resolution image formation is modeled as

$$\mathbf{z} = \mathbf{DTX} + \mathbf{n_z} \tag{1}$$

where $\mathbf{n_z}$ is additive noise, $\mathbf{D}$ and $\mathbf{T}$ are downsampling and transformation operators, respectively.

In this work, we consider several low resolution omnidirectional images that are mapped on the sphere in order to preserve the geometry of the scene. Formally, we denote by $X(\theta, \phi)$ a square-integrable continuous signal lying on 2-sphere, S^2 where θ is the longitude angle in the range $[0, \pi]$ and ϕ is the colatitude angle defined in $[-\pi, \pi)$ that forms an equiangular grid. We assume that we have Q such signals that represent $L \times L$ low resolution images with different orientations in SO(3). Let $g_k = g_{ZYZ}(\alpha_k, \beta_k, \gamma_k)$ denote a non-commutative rotation operator in the rotation group $SO(3)$. It describes the orientation of k^{th} spherical image, so that the point $\nu(\theta, \phi)$ corresponds to $g_k \nu$ in the k^{th} image. When points on the low-resolution images are registered on the high resolution sphere using the rotation operator g_k, it produces an interlaced sampling scheme, illustrated in Figure 1.

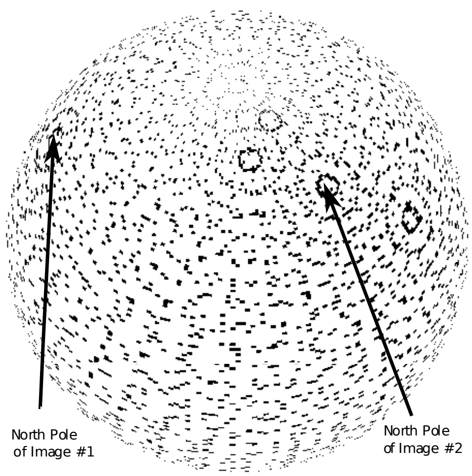

Fig. 1: Non-uniform sampling grid formed by registration of low resolution images with different orientations. Polar regions of low res. images can be observed on the grid.

Note that neither downsampling nor transformation can be directly performed on the image pixels by linear operators on spherical images. We therefore move to the transform domain and use spherical Fourier transform (SFT) to model both sampling and rotations in transform domain. The spherical image, $X(\theta, \phi)$, can be decomposed into a series of spherical harmonics using discrete SFT as:

$$X(\theta, \phi) = \sum_{l \in \mathbb{N}} \sum_{|m| \leq l} \hat{x}(l, m) Y_l^m(\theta, \phi), \tag{2}$$

where $Y_l^m(\theta, \phi)$ is the spherical harmonic of degree l, order m and $\hat{x}(l, m)$ is the corresponding Fourier coefficient. When $X(\theta, \phi)$ is bandlimited to B, it can be perfectly reconstructed from uniformly sampled data on a $2B \times 2B$ equiangular grid [9]. However, we do not have a uniformly sampled set of data after registration, but rather a set of intensity values $X(\nu)$ that can be written as

$$X(\nu) = \sum_{l=0}^{N-1} \sum_{|m| \leq l} a(l, m) Y_l^m(\nu). \tag{3}$$

where $a(l, m) \approx \hat{x}(l, m)$ are now the Fourier coefficients that have to be estimated. If $V^l(\nu)$ is a $(2l+1)$-tuple column vector in the form

$$\mathbf{V}^l(\nu) = \left[\ Y_l^{-l}(\nu)^{\mathtt{T}} \cdots Y_l^0(\nu)^{\mathtt{T}} \cdots Y_l^l(\nu)^{\mathtt{T}}\ \right]^{\mathtt{T}}, \tag{4}$$

and $\mathbf{V}$ is B^2-tuple vector formed by concatenation of $\mathbf{V}^l$ for $l = 0 \ldots (B-1)$, we can equivalently write the following linear system of equations:

$$\mathbf{M} \cdot \mathbf{a} = \mathbf{z} \tag{5}$$

where

$$\begin{aligned} \mathbf{M} &= \{\mathbf{V}(\nu)\}_{QL^2 \times B^2} \\ \mathbf{a} &= \{a(l, m)\}_{B^2 \times 1} \\ \mathbf{z} &= \{X(\nu)\}_{QL^2 \times 1} \end{aligned} \tag{6}$$

The solution of this linear system for the coefficients $a(l, m) \approx \hat{x}(l, m)$ finally permits to reconstruct the high-resolution image by inverse Spherical Fourier Transform.

Interestingly enough, rotations in SO(3) can also be explicitly represented in Spherical Fourier domain. In particular, the spherical harmonic $Y_l^n(g\nu)$ for a rotated sampling point ν by g can be related to $Y_l^m(\nu)$ by

$$Y_l^n(g\nu) = \sum_{m=-l}^{l} U_{mn}^l(g) Y_l^n(\nu) \tag{7}$$

where $U_{mn}^l(g)$ is the elements of Wigner-D matrix [10]. Note that the data matrix, $\mathbf{M}$, is now a function of the rotation values. If we select the first images as reference, and form a vector $\mathbf{b}$ by concatenation of the rotation angles of each image, $\mathbf{M}$ can be written as

$$\mathbf{M}(\mathbf{b}) = \begin{bmatrix} \mathbf{W_1} \\ \mathbf{W}(\mathbf{b}) \end{bmatrix} \tag{8}$$

where $\mathbf{W_1}$ is the set of spherical harmonics for the first image while $\mathbf{W}$ is the set of spherical harmonics of the other images as a function of the rotation vector $\mathbf{b}$.

3. L_1 REGULARIZED SUPER-RESOLUTION WITH UNREGISTERED IMAGES

We consider now the problem where the low-resolution images are not perfectly registered. In other words, the rotation vector $\mathbf{b}$ is not exact and needs to be estimated together with the transform coefficient vector $\mathbf{a}$, which makes the overall system nonlinear. In [11], we proposed a least-squares based method to jointly estimate the rotation parameters and the

Fourier coefficients. We propose here to exploit the sparsity of the Fourier coefficients in order to achieve a better denoising performance on both intensity and rotation values. We therefore add an l_1 regularization term to the least squares cost function, and the optimization problem becomes

$$\underset{\mathbf{a},\mathbf{b}}{\text{argmin}} \left[\|\mathbf{M}(\mathbf{b})\mathbf{a} - \mathbf{z}\|_2^2 + \lambda \|\mathbf{a}\|_1 \right]. \tag{9}$$

As the regularization term is not differentiable, the Newton-based methods cannot be applied directly. As proposed in [12], the minimization problem is converted into

$$\begin{aligned} & min \left[\|\mathbf{M}(\mathbf{b})\mathbf{a} - \mathbf{z}\|_2^2 + \lambda \sum_i u_i \right] \\ & s.t. \quad -u_i < a_i < u_i \end{aligned} \tag{10}$$

where $u_i > 0$. The inequality constraint is added to the cost function via a logarithmic barrier function to form

$$min \left[\|\mathbf{M}(\mathbf{b})\mathbf{a} - \mathbf{z}\|_2^2 + \lambda \sum_i u_i + t\phi(\mathbf{u}, \mathbf{a}) \right]. \tag{11}$$

The barrier function $\phi(.,.)$ for complex variables is selected as

$$\phi(\mathbf{u}, \mathbf{a}) = -\sum_i log(u_i^2 - Re(a_i)^2 - Im(a_i)^2). \tag{12}$$

The new cost function is now differentiable. For derivation, however, the other elements of the system should be real-valued too. By using the method presented in [12] we convert the complex valued matrices and vectors into real-valued ones and denote them with $\tilde{.}$ for the rest of the paper. After the conversion and the linearization of the square norm part of the cost function, the search direction for the minimization is found by solving

$$\mathbf{H} \begin{bmatrix} \Delta\mathbf{b} \\ \Delta\tilde{\mathbf{x}} \\ \Delta\mathbf{u} \end{bmatrix} = -\mathbf{g} \tag{13}$$

where $\mathbf{H}$ is the Hessian matrix and $\mathbf{g}$ is the gradient.

Note that the calculation of Hessian for registration parameters is computationally expensive. Thus, the components of the Hessian matrix corresponding to the registration parameters are approximated by first-order derivatives. The approximated Hessian matrix is

$$\mathbf{H} \approx \begin{bmatrix} \tilde{\mathbf{J}}^T\tilde{\mathbf{J}} + L\mathbf{I} & \tilde{\mathbf{J}}^T\tilde{\mathbf{W}} & \mathbf{0} \\ \tilde{\mathbf{W}}^T\tilde{\mathbf{J}} & \mathbf{D_1} & \mathbf{D_2} \\ \mathbf{0} & \mathbf{D_3} & \mathbf{D_4} \end{bmatrix}. \tag{14}$$

where $\tilde{\mathbf{J}}$ is the Jacobian of the cost function with respect to $\tilde{\mathbf{b}}$ and $L\mathbf{I}$ is the regularization term proposed in [11] for registration parameters. The term

$$\begin{bmatrix} \mathbf{D_1} & \mathbf{D_2} \\ \mathbf{D_3} & \mathbf{D_4} \end{bmatrix} \tag{15}$$

is the Hessian with respect to the $\tilde{\mathbf{x}}$ and $\tilde{\mathbf{u}}$.

At each iteration, the $\mathbf{b}$, $\mathbf{x}$ and $\mathbf{u}$ are updated by solving the system for the step direction and incrementing them by $\mathbf{\Delta b}$, $\mathbf{\Delta x}$, and $\mathbf{\Delta u}$ until maximum number of iterations are reached or increments are less than a threshold.

4. EXPERIMENTAL RESULTS

We have tested the proposed algorithm on multiple 16×16 low-resolution spherical images to reconstruct a 64×64 image. We use synthetic images in the experiments, since they provide groundtruth information for performance evaluation, as well as control on the registration parameters. The parameters of the optimization problem have been fixed $\lambda = t = 10$.

The first experiment compares the proposed algorithm with a typical l_2 regularization [12] and a super-resolution algorithm without regularization constraint [11]. Figure 2 shows the results for a registration error of 10 degrees. The methods are applied for 5 different registration errors picked from a uniform distribution with 10 degrees maximum. Both l_1 and l_2 regularization provide improved image quality but l_1 has better improvement and provided up to $1dB$ quality improvement. In addition, we can observe that, for the quality obtained around the saturation point, one could remove 20 images compared to a scheme without regularization [11].

Figure 3 illustrates the performance of the algorithm for different values of the maximum error on the rotation parameter. It shows that the proposed method is able to correct errors up to 15 degrees on the registration parameters, when the number of low resolution images is sufficient.

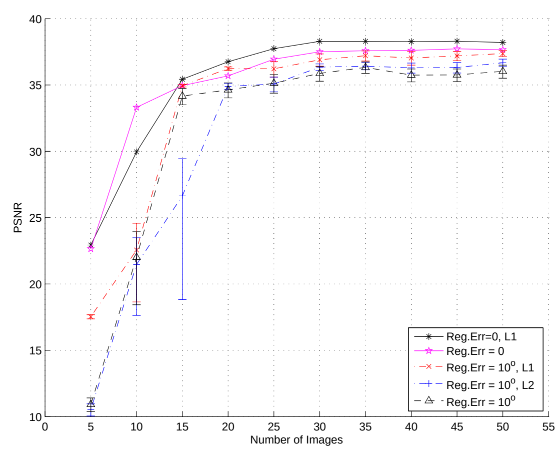

Fig. 2: Comparison of the effect of different regularization terms.

We have also tested the robustness of the proposed method to noise in the low resolution images. We have applied white gaussian noise such that the SNR is $30dB$ for the low-resolution images. As shown in Figure 4, the proposed method is quite robust to noise. It is able to correct reg-

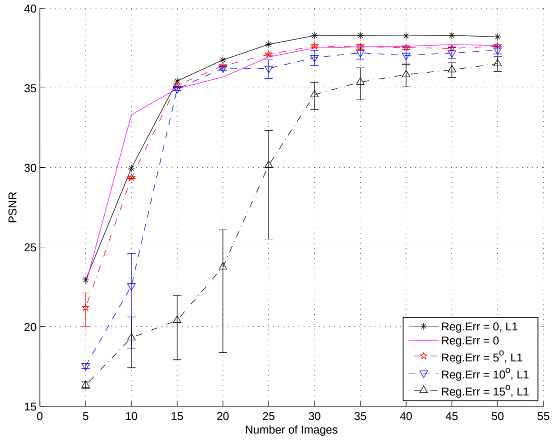

Fig. 3: Performance of the regularized super-resolution method for different registration errors.

istration errors as well as to provide a good reconstruction quality.[1]

Computation of **J** and **W** are the most computationally demanding parts of the algorithm, however, spherical harmonics have recursion property and this can be exploited for faster computations. S2kit [13] software kit provides faster computation of the harmonics exploiting this property and FFT. In addition, iterative methods to solve the system can be used to reduce the memory load.

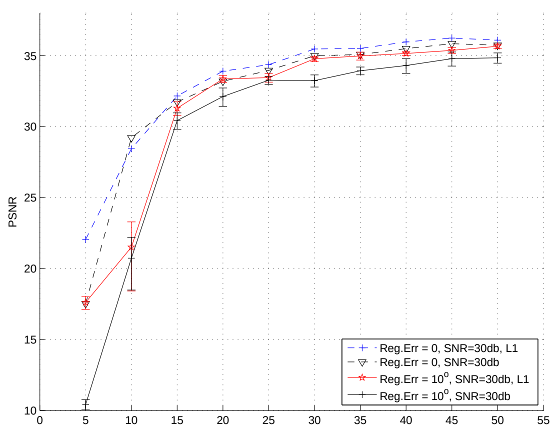

Fig. 4: Performance of the proposed method for noisy low-resolution images.

5. CONCLUSION

We proposed a method to solve l_1 regularized least-squares problem with inexact system matrices that represents the super-resolution from multiple low-resolution omnidirectional images with arbitrary rotations in SO(3) rotation group with only coarse approximation of the rotation parameters. The experimental results show that the proposed method efficiently reconstructs the high-resolution image while correcting the registration errors due to the regularization constraint.

[1]The low-resolution images and generated high resolution images are available at http://lts4www.epfl.ch/˜arican/research.php.

6. REFERENCES

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