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NETWORK CODING OF RATELESS VIDEO IN STREAMING OVERLAYS

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Abstract

We investigate the problem of collaborative video streaming in overlay networks. We exploit path and source diversity, as well as basic processing capabilities of network nodes in order to increase the overall throughput and improve the video quality at the clients. We consider an architecture where several streaming servers simultaneously deliver video information to a set of clients. The servers apply Raptor coding on the video packets for error resiliency, and the forwarding peer nodes selectively combine the Raptor coded video packets in order to increase the packet diversity. We analyze the performance of selective network coding and we describe its application in video streaming systems. We further define the source and channel rate allocation in such a collaborative streaming system by estimating the expected symbol diversity at clients. The optimization is defined as *minimax* and it is solved by a low-cost bisection based method. The experimental evaluation demonstrates that our system typically outperforms Raptor video streaming systems that do not use network coding, or systems that perform decoding and encoding in the network nodes. Finally, our solution presents a low complexity and only requires small buffers in the network coding nodes, which are certainly important advantages for deployment in practical streaming systems.

Index Terms

Network coding, Raptor codes, overlay networks, p2p streaming, optimization, rate allocation.

I. INTRODUCTION

The last decade has witnessed the rapid development of novel overlay network architectures such as peer-to-peer or wireless mesh networks that offer source and path diversity to data delivery applications. Network diversity permits to augment the quality of service for media streaming applications, with increased throughput and improved resilience to failures. Streaming applications can typically benefit from the network diversity by designing multipath or multisource streaming strategies. Such algorithms however generally rely appropriate source coding, routing mechanisms or coordination of the peers in the system in order to prevent packet duplicates that cause waste of resources. The need for coordination can however be avoided if the overlay network nodes are able to perform simple processing or coding operations on the packet streams. Hence, network coding [1] can prove to be particularly useful in overlay streaming applications, where it could help to increase the network throughput and data persistence.

In this paper, we propose a network coding algorithm for video streaming in lossy overlay networks. We consider a framework where a compressed video sequence is sent from possibly multiple servers to multiple clients via several network peers which are able to perform network coding operations by combination of packets. This framework is illustrated in Fig. 1. The network links are typically characterized by the available bandwidth, and the error probability. The nodes are organized in a mesh network, which is generally more appropriate than tree-based overlays for streaming applications with stringent timing constraints due to self-organization and resiliency to node failure. The network coding peers could be virtually arranged into successive coding stages, depending on the hop-count distance to the servers. Efficient solutions for streaming on overlay networks have to prevent important duplication for effective resource usage. Network coding techniques brings interesting benefit in such a context [2], since it permits to take advantage of the network diversity for increased performance. As decoding and encoding in the network nodes might induce important latency in the delivery, the coded packets are simply combined together in the nodes, and forwarded to the children peers.

We first encode the source packets at the server in order to combat losses in the network and to provide enough packet diversity for effective delivery. The packets are encoded with Raptor codes [3], which offer interesting rateless properties and low complexity. When several servers are used, the encoding process forms disjoint packet sets at each server, in order to provide additional robustness to failures and losses. After Raptor encoding, the packets are sent towards the clients via overlay nodes. The overlay nodes implement network coding by combining Raptor symbols, where the coding strategy depends on the set of received packets and on the output bandwidth. Selective combination of packets in the node permits to increase the symbol diversity in the overlay network and proves to be particularly efficient to combat losses that could be particularly important in wireless environments. Furthermore, selective network coding in the nodes only necessitates the use of small buffers and hence induces small latency. This is a clear advantage compared to schemes that perform decoding and re-encoding in the network

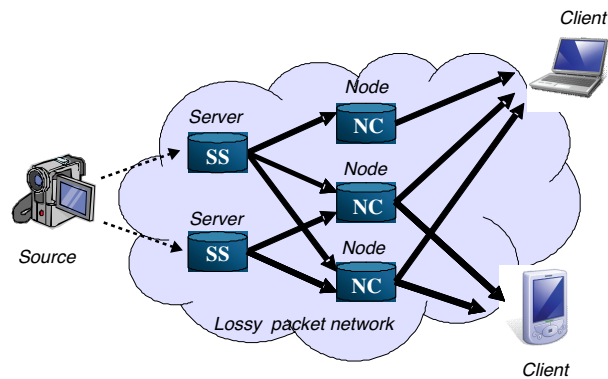


Fig. 1. Illustration of a system for video streaming on overlay networks. Multiple streaming servers (SS) send information to video clients on a lossy packet network via network coding (NC) peers. Each network segment is typically characterized by an available bandwidth and a packet loss probability.

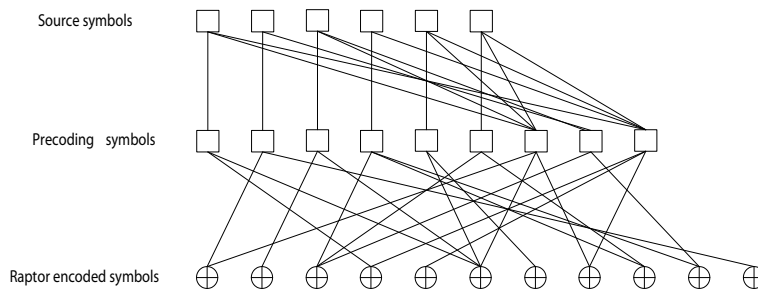


Fig. 2. Encoding of non-systematic Raptor codes. The source symbols are first pre-coded and successively LT encoded.

nodes (e.g., [4]). We finally address the rate allocation problem and determine the optimal source and channel rates under given network statistics. The optimization problem is formulated as a *minmax* problem, and the algorithm seeks for the optimal trade-off between source distortion, and resiliency to loss such that the source rate is maximized for the less reliable clients. We evaluate the performance of our system in various random network topologies, and we show that it typically outperforms Raptor coding systems that do not utilize peers for network coding.

The paper is arranged as follows. In Section II, we give an overview of Raptor codes and describes the selective network coding algorithm applied to Raptor streams. In Section III, we present the application of the network coding algorithms to compressed video sequences. We propose a source and channel rate allocation algorithm that minimizes the maximal distortion at the clients. Simulation results are proposed in IV for a variety of scenarios that demonstrate the benefits of the proposed solution. The related work is finally discussed in Section V.

II. NETWORK CODING OF RAPTOR STREAMS

A. Overview of Raptor codes

Our collaborative streaming system proposes to use Raptor codes [3] at the sources and network coding in the overlay nodes in order to combat packet erasures and augment the network throughput. Maximum distance separable (MDS) codes such as Reed-Solomon codes (RS) are often used in packet erasure networks, but their application is generally limited to small coding blocks due to their decoding complexity that typically grows quadratically with the block size. On the other hand, the family of Fountain codes and specifically Raptor codes has received a lot of interest recently to overcome these limitations, since they provide advanced performance with reduced complexity. In addition, they have an implicit coding structure that permits to generate encoded packets on the fly in a flexible way. They typically combine packets to generate encoded symbols, in a similar way to the randomized network coding systems [5] that perform simple linear operations with packets. We present below an overview of Raptor codes, and their application in our network coding framework.

Raptor codes [3] are based on Luby Transform (LT) codes [6] and designed to achieve linear encoding-decoding time complexity. They typically concatenate weakened LT codes with outer codes (pre-coding step), as represented in Fig. 2. In the LT coding stage, the source symbols are XOR-ed for generating the encoded symbols, and the implicit structure of the corresponding Tanner graph is passed to the decoder as a decoding key along with the encoded symbols. This key, also called the encoded symbol ID (ESI) represents the seed of a pseudo-random generator that determines both the degree of the encoded symbols as well as the source symbols to be combined. Raptor codes perform close to perfect codes as they are able to recover

the source symbols from any set of encoded symbols that is slightly larger than the set of encoded symbols. Hence, they have a very small overhead that further tends to zero when the number of encoded symbols in a block increases. This rateless property is particularly attractive for streaming applications with strict timing constraints. Raptor codes have been proposed for peer-to-peer multimedia delivery [4] and wireless broadcast systems [7]. The Raptor codes have also been recently adopted in the 3GPP standard [8].

We describe now in more details the 3GPP implementation that will be used in this paper. In this implementation, the pre-coding step consists of regular LDPC codes and high density Half codes. This encoding stage enables low complexity encoding and decoding, as it permits the use of LT codes with sparse parity check matrices. In more details, the encoding procedure grows as follow. Let \mathbf{C} denote a source vector of K input (source) symbols with $\mathbf{C} = [C_1^T C_2^T \dots C_K^T]$. If we represent as \mathbf{C}_P the redundant pre-coded symbols, the coding constraints among source and redundant symbols are given by

$$\mathbf{C}_P = \mathbf{G}_P \cdot \mathbf{C},$$

where \mathbf{G}_P is the generator matrix of the pre-coder. The LT encoder then takes as input the pre-coding symbols $\mathbf{F}_{1:L} = [\mathbf{C}^T \mathbf{C}_P^T]^T$ and generates the Raptor symbols, where $L = K + S + H$ is the number of pre-coded symbols, and S and H are respectively the number of LDPC and Half codes constraints. If \mathbf{G}_{LT} is the generator matrix of non-systematic LT encoder, then it holds that

$$E_{1:N} = [E_1^T E_2^T \dots E_N^T]^T = \mathbf{G}_{LT}(1, 2, \dots, N) \cdot \mathbf{F},$$

where N is the number of Raptor encoded symbols. The generator matrix of the LT coder \mathbf{G}_{LT} is defined as

$$\mathbf{G}_{LT} = [\mathbf{\Gamma}_1^T \mathbf{\Gamma}_2^T \dots \mathbf{\Gamma}_n^T],$$

where $\mathbf{\Gamma}_i^T$ with $i = 1, \dots, n$ is a row vector with ones at the positions corresponding to the indices of the pre-coded symbols that are combined in the generation of encoded symbols. Therefore, each encoded symbol E_i , with $i = 1, \dots, n$, can be written as

$$E_i = \sum_{j \in \Psi_i} \bigoplus F_j,$$

where Ψ_i is the set of pre-coding symbols indices that are combined and $\sum \bigoplus$ is the XOR operator. The Ψ_i is determined by a pseudo-random generator whose seed is controlled by the ESI information.

On the receiver side, Raptor decoding first constructs the generator matrix \mathbf{A} with the encoding constraints given by the ESI's of the received symbols. The first rows of \mathbf{A} contains the pre-coding conditions given by the generator matrices \mathbf{G}_{LDPC} and \mathbf{G}_{Half} for both the LDPC and the Half encoders, respectively. The last rows of the matrix \mathbf{A} represent the operations of the LT encoder given by \mathbf{G}_{LT} . The matrix \mathbf{A} can be written as

$$\mathbf{A} = \begin{bmatrix} \mathbf{G}_{LDPC} & I_S & O_{S \times H} \\ \mathbf{G}_{Half} & I_H & \\ \mathbf{G}_{LT} & & \end{bmatrix},$$

where I_S and I_H are square unit matrices of size S and H , respectively, and $O_{S \times H}$ is a zero matrix of size $S \times H$. It is interesting to note that the matrices \mathbf{G}_{LDPC} and \mathbf{G}_{Half} are not transmitted to the decoder as they can be constructed based only on the number K of input symbols. The matrix \mathbf{G}_{LT} can be constructed based on the ESI information.

Whenever symbols have been erased during transmission, the decoder modifies the original generator matrix and forms a new matrix \mathbf{G}'_{LT} by pruning the rows in \mathbf{G}_{LT} that correspond to the symbols that have been lost. This results into a modified decoder matrix \mathbf{A}_d that is given by

$$\mathbf{A}_d = \begin{bmatrix} \mathbf{G}_{LDPC} & I_S & O_{S \times H} \\ \mathbf{G}_{Half} & I_H & \\ \mathbf{G}'_{LT} & & \end{bmatrix},$$

If \mathbf{A}_d has full rank, then the probability for decoding the original source symbols stays high even in the case of symbol erasures. The decoder has to solve the following linear system of equations :

$$\mathbf{A}_d \cdot \mathbf{C} = \mathbf{D}',$$

\mathbf{D}' is a vector composed of both a zero vector of length $S + H$ and the vector of received symbols. The above linear system is typically solved by Gauss-Jordan elimination or by fast belief propagation algorithms for some specific codes.

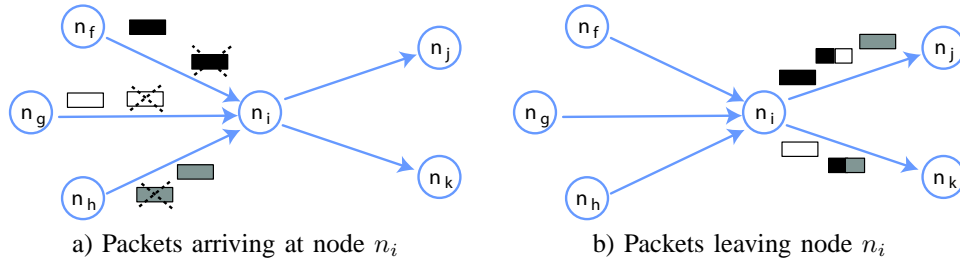


Fig. 3. Selective network coding in a network peer. a) Three packets are erased from the incoming streams. b) Successfully received packets are forwarded to the children peers, along with network coded packets that replace the lost packets and permit to fully use the available output bandwidth.

B. Selective network coding

We propose to implement network coding in the nodes of the overlay network in order to augment the performance of the streaming system. The peers selectively perform combinations of packets that have been encoded with non-systematic Raptor codes [8] by the servers in order to compensate for packet loss in the network (see Fig. 3).

We model the overlay network as a directed acyclic graph $\mathcal{G} = \{\mathcal{N}, \mathcal{E}\}$, where the set of vertices \mathcal{N} corresponds to the nodes n_i in the overlay network, and the set of edges \mathcal{E} represents the link between the nodes. Each link ϵ_{ij} between the nodes n_i and n_j is characterized by a loss probability π_{ij} and an available bandwidth b_{ij} , given in packets per time unit. We call *parents* and *children* of the node n_i the peers that transmit information to, or respectively receive packets from n_i . We denote by \mathcal{P}_i the set of parents nodes for node n_i , and by \mathcal{C}_i the set of its children nodes. We further denote by t_i^j the number of packets sent by the node n_i to its children n_j in each time unit. Similarly, r_i^h is the number of packets received by the node n_i from its parent n_h in each time unit. The network constraints are finally given as

$$t_i^j \leq b_{ij}$$

and

$$r_i^j = (1 - \pi_{ij}) t_i^j$$

Network coding is applied selectively in overlay nodes, and new packets are generated only if packets have been lost, as long as the outgoing links can accommodate new packets. In more details, a network node n_i gathers r_i encoded symbols from its parents nodes, where $r_i = \sum_{j|n_j \in \mathcal{P}_i} r_i^j$ represents the sum of the packets received from all the parent nodes. If the overall outgoing bandwidth b_i with $b_i = \sum_{j|n_j \in \mathcal{C}_i} b_{ij}$ is smaller than the number of received packets r_i , then the node n_i forwards to its children the received packets in the order of their arrival and distribute them randomly on the outgoing paths. It further drops the packets that cannot be forwarded due to bandwidth limitations. If, on the other hand, the outgoing bandwidth b_i is larger than the number of received packets ($b_i > r_i$), the node implements selective network coding in order to fully use the outgoing links. Selective network coding combines incoming packets and forwards the resulting packets to the children nodes such that the total number of transmitted packets $t_i = \sum_{j|n_j \in \mathcal{C}_i} t_i^j$ is equal to the outgoing bandwidth, i.e., $t_i = b_i$.

The coding strategy is a crucial part of the collaborative streaming algorithm. It is important for the efficiency of the overall system that the symbol diversity stays as high as possible, such that the probability of decoding failure is low on the client side. The selective network coding has therefore to choose properly the symbols to be combined together, in order to maximize the symbol diversity. The network coding node first forms a modified generating matrix \mathbf{A}_{nc} based on the ESI information that is present in the received packets. This matrix typically contains some rows of the initial generating matrix \mathbf{A} that correspond to the original symbols that have been correctly received. It also contains rows that describe the network coding operations in the parent nodes. Network coded packets are generated by the combination of a random pair of received symbols that respect the following conditions:

- *Rule 1:* the selected symbols correspond to some rows of \mathbf{A}_{nc} that are orthogonal to all the rows in the original matrix \mathbf{A} that correspond to lost symbols
- *Rule 2:* the selected symbols correspond to some rows of \mathbf{A}_{nc} that are orthogonal to each other.

These rules permit to maintain a high symbol diversity. They further permit to prevent decoding problems in the Gauss-Jordan elimination process at the receiver. Some bad symbol combinations could indeed mislead the decoder, where the matrix \mathbf{A}_{nc} stays full rank, but the solution differs from the correct one. In particular, the first condition ensures that the new symbols are independent from the lost symbols. The second condition prevents the combination of non-orthogonal symbols that can result in erroneous codewords. Once a symbol is used in network coding, it is marked in order to force the network coding node to use other symbols for encoding. Once the network coding node cannot find valid pairs of symbols for encoding, all the symbols markers are set to zero. All the symbols can be used again for encoding. The encoding proceeds until the outgoing bandwidth can be fully utilized. In practice, the constraint imposed by *Rule 1* is often relaxed, as it is difficult to track the missing symbols in distributed settings. Note that in our system, the nodes only combine pairs of packets, in order to maintain

the sparsity of the modified generator matrix \mathbf{A}_{nc} even in multihop networks. It is possible to combine more than two packets in the network coding process, but it decreases the probability to find symbols that satisfy the two conditions above.

Obviously, the re-encoding strategy does not increase the rank of the matrix \mathbf{A} , since the symbols generated by network coding are combinations of received symbols. The redundancy therefore augments with the number of coding stages. The performance of the coding system decreases with the number of network coding nodes augments, since some information conveyed by symbols generated by symbol combination is also available in other encoded symbols. Symbol erasures and re-encoding render the generating matrix \mathbf{A} less sparse, and network coding can only be applied a limited number of times. This phenomenon could however be mitigated by taking advantage of the path diversity in the overlay network. In particular, the rank of \mathbf{A} decreases more gracefully when multi-path transmission is combined with efficient packet splitting policy. The peers in our system therefore implement a routing policy that splits the packets to be transmitted to children nodes into different subsets, which are sent on different paths. In general, the packets that have been combined together and the resulting network coded packet are sent on different outgoing links. If the network topology does not permit it, the encoded packet is sent on the same outgoing channel as one of the two packets used in the combination. This simple splitting policy tries to prevent a rapid increase of redundancy in children peers.

Finally, it should be noted that the choice of a non-systematic version of the Raptor codes is vital in the proposed scheme. It provides high symbol diversity that is crucial in network coding. Even if they directly provide some original symbols among the received symbols, systematic Raptor codes are not appropriate in our network coding scheme since they lead to increased probability of receiving duplicate symbols. Systems that use systematic codes are thus more fragile to erasures and the decoding probability drops after a few network coding stages as the symbol diversity decreases extremely fast.

C. Performance analysis

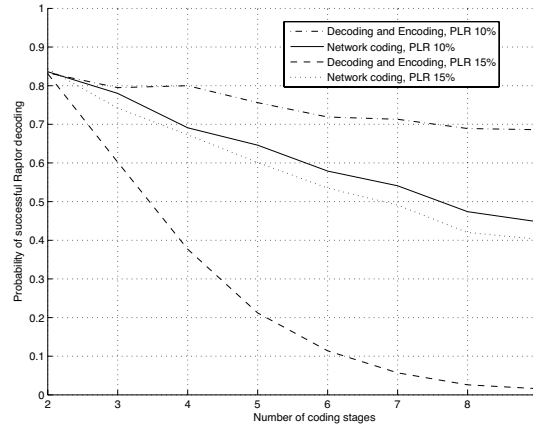


Fig. 4. Probability of decoding vs number of coding stages, in a regular topology with 3 nodes per stage.

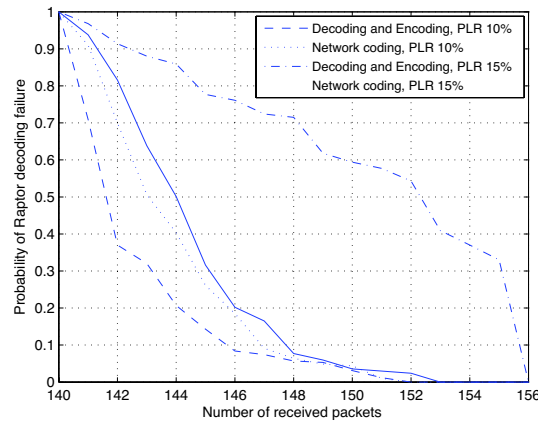


Fig. 5. Probability of decoding failure vs number of symbols in a regular topology with 5 coding stages and 3 nodes per stage.

We now analyze the performance of the proposed network coding algorithm in a few sample scenarios in regular topologies. For the clarity of the analysis, we consider here that the network nodes are grouped into *coding stages* depending on the

hop-distance to the server nodes. We consider here regular topologies with 3 peers per coding stage, which are connected to all the peers in the neighbouring coding stages. We assume that each link has a capacity of 300 kbps, and that the packet loss probability is the same on each segment. Two servers encode 158 packets of 512 bytes from 140 source packets with different ESIs. We compare the performance of the network coding algorithm to an algorithm that performs Raptor decoding and re-coding at each of the network peers. Whenever Raptor decoding is not successful in a peer, a random packet replication is performed so that the available bandwidth is fully used in both schemes.

We look at the evolution of the performance of the Raptor network coding algorithm with respect to the number of coding stages between the encoders and the decoder for different packet loss probabilities, as illustrated in Fig. 4. We first observe that the performance of both schemes decreases with the number of coding stages, as expected. The decoding/recoding strategy performs better for low to medium loss rates (up to 10% in our sample scenario). In this case, it is indeed better to try to completely recover from losses at each coding stage and then to generate new Raptor symbols. However, when the loss rate gets high (e.g., 15%), Raptor decoding in the nodes cannot be performed correctly, and network coding becomes more beneficial as it permits to maintain a higher packet diversity. These observations are confirmed in Fig. 5. It shows that the probability of decoding failure in the network coding algorithm decreases faster when the number of received packets increases. This is mostly due to the high number of packet duplicates in the decoding/recoding strategy when the loss rate is high. Note that at high loss rates, the decoding/recoding strategy tends to behave as poorly as a random packet replication strategy.

Decoding and recoding is probably not a viable option in large networks, since it introduces important complexity penalties and large delays. The computational complexity requirements are also more important than in the network coding scheme. From [9], the number of XORs operations in a (N, K) Raptor code is given by

$$\mathcal{C}_{enc} = K \cdot \left(10 + 4.5 \cdot \frac{K}{N} \right).$$

At the same time, Raptor decoding requires \mathcal{C}_{dec} XORs operations, with

$$\mathcal{C}_{dec} = 10 + 4.5 \cdot \frac{\min(N - K, K)}{K}.$$

In every node, decoding and re-coding necessitates a total number of XOR operations that can be written as

$$\mathcal{C}_{total} = N \cdot \mathcal{C}_{enc} + K \cdot \mathcal{C}_{dec} = 10 \cdot K \cdot (N + 1) + 4.5 \cdot (K^2 + \min(N - K, K)).$$

For large values of N and K the \mathcal{C}_{total} is approximated by

$$\mathcal{C}_{total} = 10 \cdot K \cdot N + 4.5 \cdot K^2.$$

On the other hand, nodes perform packet combinations only in case of loss in the network coding scheme. Therefore, the number of XORs operations is directly driven by the packet loss rate p . It can be written as

$$\mathcal{C}_{NC} = p \cdot N,$$

and tends to zero when the loss probability gets small. In practice, \mathcal{C}_{NC} is slightly larger than $p N$, since the second network coding conditions has to be satisfied. This might necessitate a few additional combinations or tests. In any case, the complexity of the network coding solution is dramatically smaller than the complexity of decoding and recoding strategies, even if we assume that Raptor decoding is performed only in case of packet loss. In the case of an independent loss process, we have

$$\mathcal{C}_{total} = (1 - (1 - p)^K) (10 \cdot K \cdot N + 4.5 \cdot K^2),$$

We finally investigate the latency issues related to the selective network coding solution. We simulate a regular and homogeneous network topology with links of 400 kbps and a packet loss rate of 5%, and we send 270 packets of 512 bytes each from two servers. We compute the time required for the delivery of enough packets to enable the decoding of all source packets, in the case where each coding stage has 3 network coding nodes. We compare the proposed scheme to a solution where nodes simply replicate randomly chosen packets in case of loss. We also provide comparisons with a randomized network coding solution (RNC) [10] where all the packets buffered at a node are combined before forwarding to next hop nodes. Note that the header in this case needs to be adapted in order to keep track of the coding operations. Typically, we need a header of 270 bytes for 270 packets, if network coding is performed in a Galois field $GF(2^8)$. We omit the comparisons to decoding and recoding strategies in the nodes, where the delay becomes rapidly very large as each node has to perform full decoding prior to re-coding.

The Fig. 6 illustrates the delivery time as a function of the number of coding stages, when the buffer size in the node is set to 32 packets. We can see that selective network coding performs close to the packet replication solution. The randomized scheme experiences larger latencies as each node forwards packets when its buffer is full. It is interesting to note that the selective network coding scheme does not present any significant latency increase as the network size grows. Recall that network coded packets are generated only when loss happens. Since the packet loss rate is small and the buffer size sufficiently large, the probability that the network coding peer is able to find orthogonal packets in the buffer is kept high. Finally, it can be pointed out that the buffer size in the peers does not influence the delivery time as long as it is bigger than a few packets (typically 16 packets in our simulations); large buffers only offer a larger set of solutions for packet combinations.

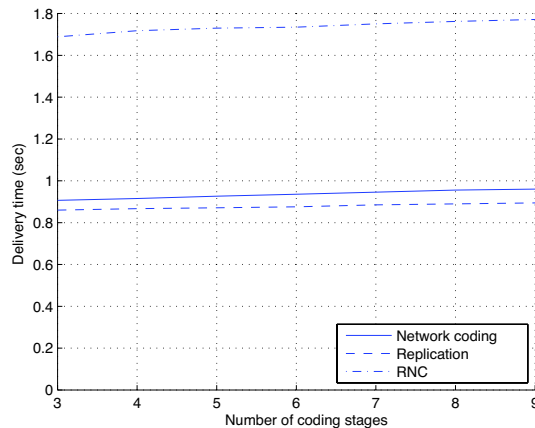


Fig. 6. Delay necessary for receiving enough packets to decode 270 source packets with respect to the number of coding stages (3 nodes per coding stages).

III. NETWORK CODING OF RAPTOR VIDEO

A. Overview

Network coding offers a promising alternative for media streaming systems, as it is able to improve the error robustness and to increase the network throughput. The selective network coding solution proposed above is particularly interesting in this context, as it is highly adaptive, low complexity and induces only very small delays.

The extension of network coding to video streaming applications however requires a proper analysis of the source and channel coding rates, so that the overall performance is maximized. Under timing, complexity and bandwidth constraints, a joint source and channel rate allocation strategy is necessary in order to find the proper trade-off between distortion due to compression of the video sequence and distortion due to channel loss. If the coding rate or the number of source symbols increases, the source distortion decreases. The redundancy created by the Raptor encoders or the network nodes generally tend to reduce the loss probability, or equivalently to increase the decoding probability for the video clients. When the overall network resources are constrained, one typically has to find the right trade-off between the source rate and the redundancy in the network, so that the end-to-end quality is maximized.

We first compute the decoding failure probability, as a function of the number of source symbols when the nodes implement selective network coding. Then we propose an optimization algorithm that determines the coding rate for the Raptor encoders for a given network topology, such that the distortion is minimized for the client that experiences the lowest decoding probability. This maximizes the probability that all clients are able to decode the video stream. We finally describe the video streaming system built on selective network coding of Raptor encoded video.

B. Decoding failure probability

The decoding failure probability p_R at a client denotes the fact that the decoder cannot recover the source symbols when Raptor decoding fails. This probability depends on the network topology and on the number of source symbols K used by the Raptor encoders. We propose to estimate the decoding failure probability based on the redundancy of the modified generator matrix built by the clients. Each client m forms an equation system with the received symbols and typically solves it by Gauss-Jordan elimination. The decoding probability is directly linked to the rank of the matrix. The redundancy of the system is however increased by the successive network coding operations in the network nodes that can generate symbol replicates. Therefore, the rank of the system at decoder might not increase as fast as the number of packets that are received. The linear system of equations at the client might therefore not be full rank anymore and thus not solvable. Based on the probability of unsuccessful decoding developed in [11], we write the probability of unsuccessful decoding at client m in the case of non-systematic 3GPP Raptor codes [8], as

$$p_R(r_m, K) = \begin{cases} 1, & r_m < K \\ 0.85 \cdot 0.567^{r_m - K - \delta_m}, & r_m \geq K \end{cases} \quad (1)$$

where the penalty term δ_m has been added to capture the effect of network coding. This penalty term represents the number of packets that are purely redundant and thus useless at decoder.

We estimate an upper-bound on the penalty term δ_m , which eventually leads to an upper-bound on the unsuccessful decoding probability. We consider each node independently. We estimate the number of redundant packets transmitted by a network coding node, which finally reach the client m . Recall that, per construction of the proposed selective coding algorithm, the number of transmitted packets in a node is equal to the outgoing bandwidth, i.e., $t_i = b_i$. Under the assumption that network

peers can always find orthogonal symbols in their buffer to perform selective network coding, the number c_i of network coded packets generated at node n_i can be computed as

$$c_i = \begin{cases} 0, & \text{if } t_i \leq r_i \\ t_i - r_i, & \text{if } t_i > r_i \end{cases}. \quad (2)$$

It is equal to the number of packets that are necessary to compensate for the difference between the outgoing bandwidth b_i and the incoming rate r_i at node n_i . We further compute the probability p_i^m for a packet sent by the node n_i to reach the client node m , where both nodes are connected by a set of paths \mathcal{W}_i^m . This probability is computed by considering all the paths $w \in \mathcal{W}$, as

$$p_i^m = \sum_{w \in \mathcal{W}_i^m} \left\{ \prod_{\epsilon_{jk} \in w} \rho_{jk} \cdot (1 - \pi_{jk}) \right\}$$

where ρ_{jk} is the percentage of the packets sent by node n_j that are forwarded towards the children node n_k on the network link ϵ_{jk} with a packet loss probability π_{jk} . The percentage of packets sent over the link ϵ_{jk} is driven by the respective bandwidth on this link. ρ_{jk} is equal to

$$\rho_{jk} = \begin{cases} \frac{t_j^k}{r_j^k}, & \text{if } t_j < r_j \\ \frac{t_j^k}{t_j}, & \text{otherwise} \end{cases}. \quad (3)$$

We can then estimate the expected number of packet triples τ_m received by each client m , where a packet triple represents a network coded packet along with the two packets used in the generation of this packet. When a triple is present at the decoder, one of the packets is completely redundant, and it does not participate to decreasing the probability of decoding failure. In this case, the rank of the system does not increase proportionally to the number of received packets, and the unsuccessful decoding probability of Eq. (1) is penalized by incrementing δ_m . The expected number of packet triples received by the client m that include a network packet generated at node n_i is given by

$$\tau_i^m = \alpha_i c_i (p_i^m)^3 \quad (4)$$

The parameter α_i captures the influence of the participation of packets to multiple network coding operations in a node. Our selective network coding policy indeed authorizes multiple packet combinations if the outgoing bandwidth at node n_i is very large compared to the incoming rate, i.e., if $2c_i > r_i$. In this case, the redundancy at decoder increases faster. This acceleration is however upper-bounded by the number of combinations for each packet. We therefore have:

$$\alpha_i = \begin{cases} \frac{2 \cdot (t_i - r_i)}{r_i}, & \text{if } 2 \cdot (t_i - r_i) \geq r_i \\ 1, & \text{otherwise} \end{cases} \quad (5)$$

Finally, we can estimate an upper-bound on the penalty term δ_m . When a set of servers \mathcal{S} is available for stream delivery, the penalty term for a client $m \in \mathcal{M}$ is given by:

$$\delta_m = \sum_{s \in \mathcal{S}} \sum_{w \in \mathcal{W}_s^m} \sum_{i \in w} \tau_i^m$$

This clearly represents an upper-bound on the packet redundancy at client m , as we have considered all the network coding nodes independently. The actual redundancy can only be smaller than the value captured by δ_m .

C. Source rate selection

When the network nodes process packets distributedly by selective network coding as described in Section II-B, the optimization of the decoding quality becomes equivalent to the proper selection of the source rate. The source rate or equivalently the number of source symbols per timeframe is determined such that the distortion is minimized for the client that experiences the lowest decoding probability. This maximizes the probability that all clients are able to decode the video stream.

Formally, the problem of optimal allocation of the source and Raptor coding rate in the streaming servers is cast as a *minmax* problem that minimizes the maximal distortion among all clients $m \in \mathcal{M}$. Let us denote by D_s and D_c the source and channel distortion, respectively. The source distortion represents the distortion due to source encoding only. Therefore, the source distortion is directly driven by the number of source symbols K in the Raptor code. The channel distortion describes the degradation due to packet loss and happens when the Raptor decoding fails. It includes the effect of error concealment when available and represents the distortion of the video displayed at the decoder. Finally, we denote by $p_R(r_m, K)$ the probability of failure of the Raptor decoding process when the clients m receives r_m symbols. The optimization problem can finally be written as follows.

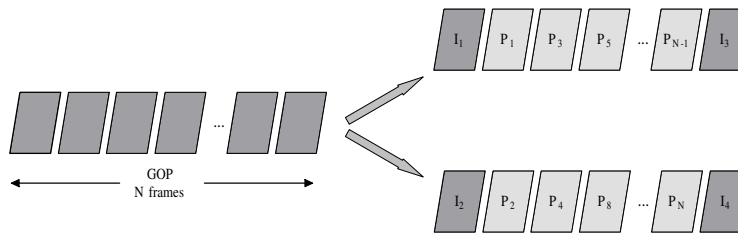


Fig. 7. GOP structure of the MSVC encoder [12].

Optimization problem: **RA**

$$K^* = \arg \min_K \left\{ \max_{m \in \mathcal{M}} \left\{ (1 - p_R(r_m, K)) \cdot D_s(K) + p_R(r_m, K) \cdot D_c(K) \right\} \right\}$$

subject to $t_i^j \leq b_{ij}$.

The condition expresses the link capacity constraints, as the number of transmitted packets on a given network segment cannot be larger than the available bandwidth on this segment. Other constraints are given by the construction of the selective network coding algorithm. In particular, the network coding algorithm transmits new packets when loss happens or when the outgoing bandwidth is large, as explained in Section II-B.

We compute for each client its $p_R(r_m, K)$ and then we apply a bisection search algorithm in order to solve the RA optimization problem. The source distortion $D_s(K)$ decreases when K increases. On the other hand, the decoding failure probability $p_R(r_m, K)$ also increases when K gets larger. The optimization problem is therefore convex with K . The proof of convexity is presented in the Appendix. Initially, the lower bound for the source rate is set to zero, which means that only channel symbols are transmitted. The upper bound is equal to the receiving rate of each client. Bisection search then converges very fast to the optimal source rate per client K_m . The overall optimal source rate is calculated as $K^* = \min_{m \in \mathcal{M}} K_m$.

D. Video streaming system

We consider a system where the video sequence is possibly available at multiple servers. These may offer different subsets of Raptor packets. Since we use a non-systematic Raptor code, the efficiency of error concealment at the decoder is very limited in case of decoding failures. We therefore propose to improve the robustness to error with a simple Multiple Description Coding (MDC) scheme [12] that provides a sufficient high number of source packets. The video sequence is initially split into two independent encoding threads, as illustrated in Fig. 7. The odd P frames use the first I frame as reference frame, while the even P frames refer to the second I frame. The MDC coder enhances system robustness because the simultaneous loss of both descriptions is quite improbable. Whenever the decoder fails to retrieve the source information from one description, the other description is used for error concealment with simple frame replication. Note however that our scheme is independent on the source coding scheme and it can be used with any video coder with advanced concealment properties like the scalable extension of the H.264/AVC standard.

We then segment each subsequence into independent Group of Pictures (GOP). For transmission of CIF sequences coded at 30 *fps*, we typically select a GOP size of 30 frames for each description. It represents a good compromise between latency constraints and number of source symbols, which has to be large for Raptor and network coding efficiency. Each GOP is then compressed with a source rate corresponding to the optimal value of K^* symbols. The source symbols typically corresponds to fixed-length chunks of the compressed video stream. The size of these chunks is a trade-off between coding efficiency that increases with the number of symbols, and the overhead due to the header that is appended to each symbol by the transmission protocols.

The source symbols are then encoded with a non-systematic Raptor coder. The total number of Raptor symbols is driven by the transmission bandwidth availability. Each symbol is then fed into a transmission packet, which is augmented with the RTP/UDP/IP protocol headers for transmission. As proposed in the 3GPP implementation of the Raptor codes [8], each packet as a size of 512 bytes. We also append ESI information to each packet as a header of 2 bytes that convey the information about the Raptor encoding structure. The Raptor symbols are sent in packets to the overlay nodes, which implement selective network coding as described in Section II-B. The video clients finally decode the network coded symbols and eventually the Raptor symbols to re-assemble the video streams. The video streams are finally decoded. Error concealment can finally be activated to mask the effect of losses or decoding failures.

IV. SIMULATION RESULTS

A. Simulation setup

We analyze in this section the performance of the proposed streaming system in various scenarios. The video sequences are encoding with a MSVC variant of the H.264 AVC encoding (i.e., JM 12.2 [13]), which divides the original video sequence into two sub-bitstreams of odd and respectively even frames. The GOP size is set to 60 frames while the frame rate is equal to 30 *fps*. The Raptor encoder is non-systematic version of 3GPP Raptor codes [8], with a Raptor symbol size of 512 bytes. Each of the Raptor symbols forms a network packet.

Simulations are run on several network topologies. Regular overlay network topologies consist in several coding stages, which group the nodes that are at the same hop distance from the server. Although, regular networks are not very good models of real peer-to-peer networks, they can give useful insights about the efficiency the streaming algorithms. The peer nodes in regular networks have constant node degree, so that all peers in the same coding stages have the same number of ancestors, respectively descendants. All network paths connecting servers and clients have an identical hop distance. An overlay network with T sources, P coding stages with Q_p nodes in the p^{th} coding stage, and R receivers is denoted as a $T \rightarrow Q_1 \rightarrow \dots \rightarrow Q_P \rightarrow R$ network topology. In addition to regular topologies, we also build irregular network overlays for our simulations. The irregular graphs are generated from regular ones, where some links are randomly pruned or shifted. A link is shifted if the destination is changed to a node in one of the subsequent coding stage. Shifting does not affect the out-degree of a peer. After shifting and pruning operations we examine whether the resulting network graph is directed acyclic. Only directed acyclic graphs are allowed as network coding faces problems in network with cycles. The links are pruned and shifted with a uniform probability distribution. We however constrain all the peers to have at least two incoming and two outgoing links, as path diversity is critical for illustrating the performance of network coding. Network coding obviously does not perform nicely if there is no path diversity in the network.

We simulate packet loss on the network links with a Gilbert-Elliott (GE) model, which is a two-state Markov chain, where the 'good' and 'bad' state represent respectively the correct reception, or the loss of a packet. The transition probabilities between the two states drive the packet loss ratio and the average error burst length. In our model, the average burst length is equal to nine packets. We compare the performance of the proposed system with a strategy that decodes and re-encodes packets in the network nodes and with end-to-end error protection. All results reported are averages of 150 simulations.

B. Baseline distributed streaming

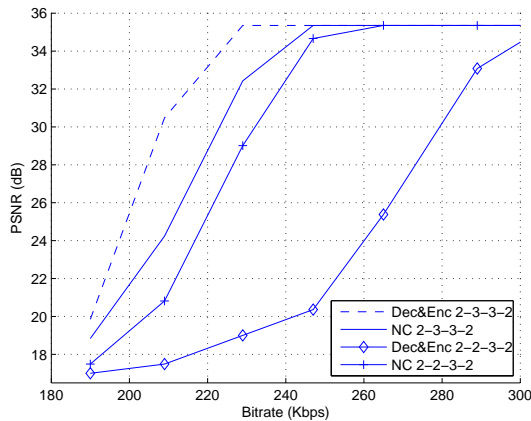


Fig. 8. PSNR comparison for transmission of the “Foreman” CIF sequence coded at 512 Kbps with respect to average link capacity for 2 – 3 – 3 – 2 and 2 – 2 – 3 – 2 overlay network topologies.

We first study the performance of the proposed streaming system in a totally distributed scenario, where the rate allocation is chosen without knowledge about the network topology. We compare the selective network coding algorithm with a strategy that performs Raptor decoding and re-coding in the network nodes. This baseline scheme replicates randomly the received packets, whenever Raptor decoding fails or when the number of encoded packets is not sufficient to completely fill in the outgoing bandwidth. When the outgoing bandwidth is insufficient, both schemes randomly select a subset of packets to be transmitted. It is worth noting that both schemes do not assume any reconciliation among network nodes.

We first analyze the behavior of the network coding strategy as a function of the average link bandwidth, \bar{b} . The average packet loss ratio is set to 5%, while the link bandwidth is randomly selected in $[\bar{b} - 40, \bar{b} + 40]$ kbps. The “Foreman” CIF sequence is encoded at 512 Kbps. We compare in Fig. 8 the performance of the selective network coding and the baseline schemes for two regular topologies, respectively of the form 2 – 3 – 3 – 2 and 2 – 2 – 3 – 2. It can be seen that the decoding

and encoding approach performs better than the Raptor network coding approach for the 2-3-3-2 topology, which provides increased path diversity. The performance gap is quite significant for low average link bandwidth. The performance difference then decreases sharply as the average link capacity increases, and both methods perform similarly when the bandwidth is high. The improved performance of the decoding and encoding approach is due to the replication policy implemented in the intermediate nodes. It ensures successful decoding at the second hop nodes and at the receivers. On the contrary, the Raptor network coding scheme cannot maintain a sufficiently high packet diversity for low bandwidth links. It hardly succeeds in exploiting path diversity to compensate for the decay of the packet diversity, which reduces the probability that the end-nodes receive a set of packets that corresponds to full rank generator matrix.

The influence of the link capacity is even more apparent when we evaluate the coding scheme in the 2-2-3-2 topology, as shown in Fig. 8. Since the first hop has only two nodes, the packet replication policy can not assist decoding and encoding scheme because the set of packets that arrive at the second hop does not correspond to a linear equation system with full rank. For this topology, the network coding system even performs best, since it becomes less sensitive to the above issues. However, it should be noted that the system needs slightly more bandwidth to guarantee error free video decoding, when compared to the 2-3-3-2 topology.

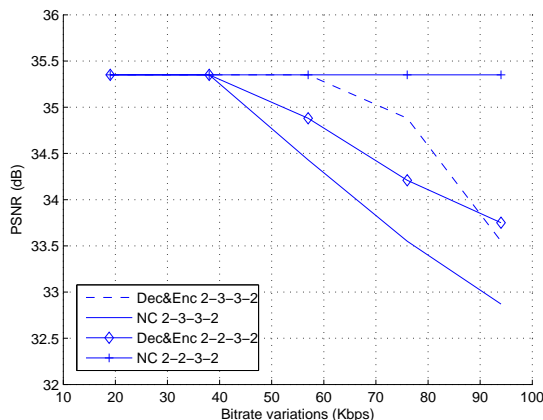


Fig. 9. PSNR comparison for transmission of the “Foreman” CIF sequence coded at 512 Kbps with respect to the variation at the average link capacity for 2-3-3-2 and 2-2-3-2 overlay network topologies.

We also compare the performance of both schemes when the link bandwidth is heterogeneous. The average link capacity \bar{b} is set to 250 and 300 kbps for the 2-3-3-2 and the 2-2-3-2 topologies, respectively. The actual link bandwidths are then randomly selected as $\bar{b} + \sigma$, with $\sigma \in [20, 100]$ kbps. The results in Fig. 9 confirm the previous analysis, as the decoding and re-encoding strategy performs best for the regular topology in presence of large bandwidth variations. For the irregular topology, the performance are again better for the network coding algorithm, as the baseline scheme fails to transmit enough different packets due to the reduced number of peers.

While the baseline decoding/recoding solution generally performs well for distributed streaming, its computational complexity and latency are serious limitations for its deployment in practical systems. On the other hand, the selective network coding strategy generally performs close to the baseline solution in network with high path diversity, and performs better when the diversity reduces. It has low complexity and requires only small buffers in the network nodes. It provides a viable solution for low cost networks and we focus on this algorithm in the rest of this section.

C. Selective NC with optimized rate allocation

We have shown above that the selective network coding strategy represents an interesting solution for distributed streaming. We evaluate now the benefits of optimized rate allocation in order to take full advantage of the proposed algorithm. We first analyze the solution provided by the rate allocation algorithm in Section III-C, and then we compare the performance of the optimized algorithm with optimized end-to-end FEC protection. In this latter scheme, the peer nodes only forward packets. They replicate packets to fill up the outgoing links and provide additional robustness. The optimal source rate K in this case is found by solving the optimization problem of Eq. (6) with exhaustive search, where we assume that the servers are aware of the channel statistics and that the peer nodes do not perform network coding.

In order to solve the rate allocation problem, we consider a source rate-distortion model of the form $D_s = x \cdot r^y$ where r is the encoding rate or the number of source symbols. The parameters x and y are determined by training the model for a large number of points acquired from different encodings. We adopt this model in our system because of its simplicity and accuracy in modeling the average distortion of compressed video sequence [14].

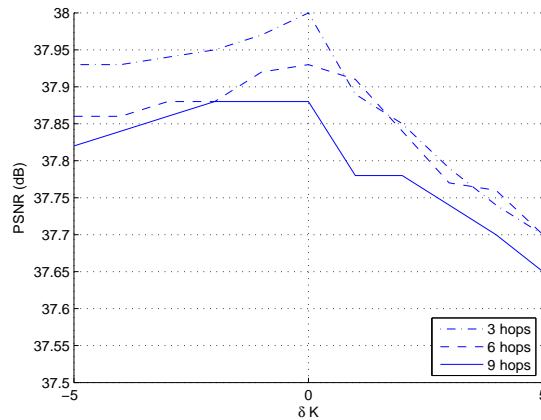


Fig. 10. Examination of the optimal rate allocation strategy. Transmission of “Foreman” CIF sequence was considered for regular network topologies. δK is the searching area around the solution determined by the optimization algorithm.

We evaluate the solution obtained by the rate allocation algorithm in Sec. III-C. In particular, we examine the quality of the received video for different values of K selected around the K^* value computed by the rate allocation algorithm. We consider the transmission of the “Foreman” CIF sequence for regular networks with three nodes per coding stage ($3 - 3 - \dots - 3$) and different number of coding stages. The loss link rate and the link capacity have been respectively set to 5% and 400 kbps. The PSNR evaluation is presented in Fig. 10 for different source rate values. The horizontal axis denotes the difference δK (in packets) between the selected source rate allocation and the one computed by the proposed optimization algorithm. We can see that the optimization algorithm is able to find the optimal rate allocation in all the topologies under test, which confirms the validity of the proposed algorithm. Note that this does not provide a proof of optimality, but rather shows that the rate allocation algorithm provides good performance despite the conservative assumptions necessary for solving the RA problem.

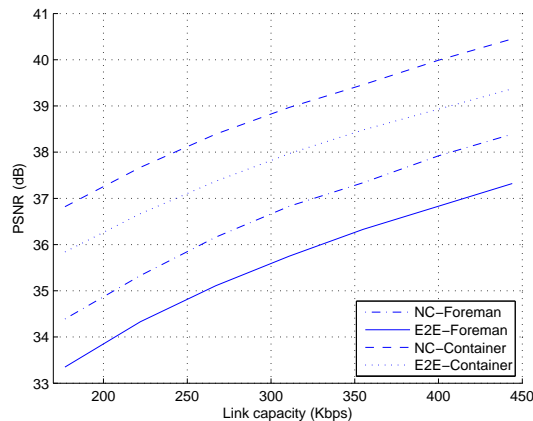


Fig. 11. PSNR performance evaluation for the proposed Raptor network coding (NC) and an end-to-end Raptor coding systems (E2E), for regular topologies with six coding stages and 3 node per stage. The PSNR is measured as a function of the link bandwidth \bar{b} ($\pi = 5\%$).

We now compare the selective network coding scheme with optimized rate allocation to optimal end-to-end error protection with packet replication in the nodes. We first study the performance of both schemes as a function of the link bandwidth \bar{b} . The packet loss ratio is set to 5% while the link bandwidth \bar{b} varies between 170 kbps and 450 kbps. We consider a regular topology with six encoding stages between the servers and the decoders and three nodes per coding stage. We examine the performance of both schemes for the transmission of two CIF sequences “Foreman” and “Container”. Note that these very different sequences lead to very different performances of the error concealment scheme. Typically, MSVC decoding and error concealment performs relatively poorly for the high motion “Foreman”, while it gives excellent results for “Container” that has quite a static background. The streaming results for “Foreman” and “Container” sequences are presented in Fig. 11. We can see that the selective network coding algorithm perform better than the end-to-end solution, by roughly 1 dB. We also observe that the performance gap remains unaltered as the link capacity increases. This is due to fact that the schemes are not very sensitive to bandwidth variations, but rather to the overall packet loss rate.

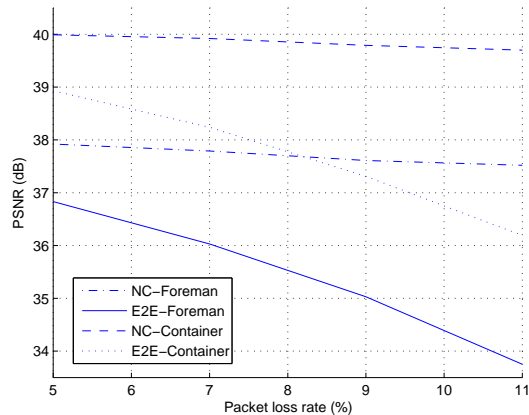


Fig. 12. PSNR performance evaluation for the proposed Raptor network coding (NC) and an end-to-end Raptor coding systems (E2E), for regular topologies with six coding stages and 3 node per stage. The PSNR is measured as a function of the packet loss rate π ($\bar{b} = 400$ kbps).

We also investigate the influence of the packet loss rate for the same network topology. We vary the link loss rate between 5% and 11%, while the link capacity is fixed to 400 kbps. Fig. 12 present the results for streaming of “Foreman” and “Container” sequences. We can see that network coding performs better by more than 1dB when the loss rate is low. When channel conditions deteriorate the performance of the network coding schemes degrades smoothly, while the end-to-end scheme collapses. For 11% packet loss ratio the performance of the proposed scheme for “Foreman” is close to 3.5 dB, while for “Container” the performance gaps grows up to 4 dB. This additional gain for the “Container” is due to the better concealment performance of the MSVC scheme. This impressive performance difference shows the resiliency of the network coding schemes for channels with heavy losses; in these conditions the replication policy can not assist the end-to-end scheme as the optimized source rate is selected very low.

D. Influence of the network topology

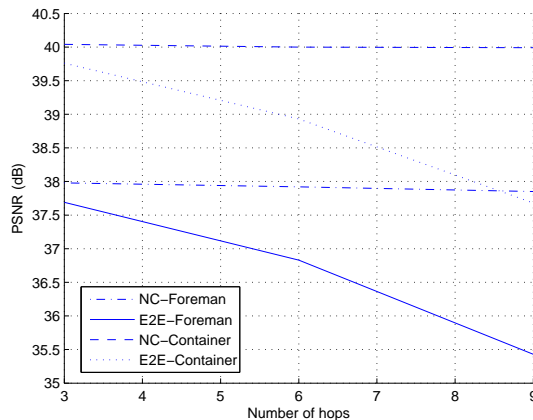


Fig. 13. PSNR performance evaluation for the proposed Raptor network coding (NC) and an end-to-end Raptor coding systems (E2E) for link capacity of $\bar{b} = 400$ kbps and loss rate of $\pi = 5\%$. The quality is measured for different path lengths.

We study now the influence of the network topology on the performance of the network coding algorithm. We compare both streaming algorithms with respect to the hop distance between the encoder and the decoder for regular topologies of three nodes per coding stage. The link capacity is set to 400 kbps and the packet loss ratio to 5%. As above, we present results for streaming “Foreman” and “Container” CIF sequences in Fig. 13. The results show that for small size topologies the end-to-end scheme performs close to the Raptor network coding. However, as the number of hops increases, the advantages of the network coding become more apparent and the performance difference widens. In general, the performance degrades when the path length increases, as the diversity of symbols becomes smaller in the last coding stages. Still, the performance of the network coding scheme remains high, while the end-to-end scheme performs worse. This difference is attributed to the high loss rate

that reduces significantly the optimized source rate for the end-to-end scheme. At the same time, the optimized source rate for the Raptor network coding is not significantly affected by the loss rate value due to its ability to properly exploit path diversity.

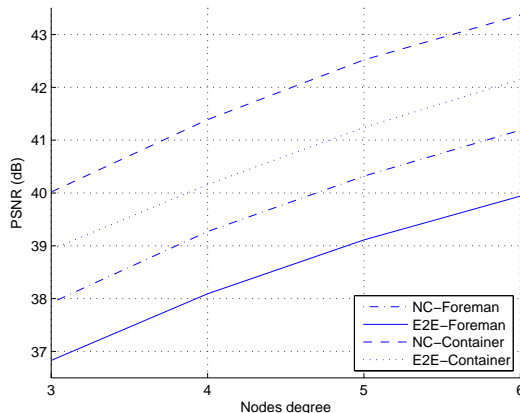


Fig. 14. PSNR performance evaluation for the proposed Raptor network coding (NC) and an end-to-end Raptor coding systems (E2E) for link capacity of $\bar{b} = 400$ kbps and loss rate of $\pi = 5\%$. The quality is measured for different node degrees.

We also examine the performance of both schemes considering regular network topologies with six coding stages. The number of nodes per coding stage is in the range $[3, 6]$. Again the transmission of “Foreman” and “Container” CIF sequences is considered and the results are presented in Fig. 14. It can be seen that the network coding outperforms the end-to-end scheme by approximately 1 dB. The performance generally improves with the node degree as the path diversity becomes more important. For high nodes degree, the sequence “Container” benefits from the increased number of nodes per coding stages and the performance gain reaches 1.5 dB due to improved error concealment compared to the “Foreman” sequence. The advanced performance of the proposed streaming solution is due to the efficiency of both the network coding scheme and the channel rate allocation algorithm that permit the use of higher code rate in the Raptor encoder. Furthermore, as the number of nodes per hop increases, the symbol diversity in the system becomes also higher, which is beneficial for our network coding algorithm.

Finally, we study the effect of the network regularity on the streaming performance. We consider transmission of “Foreman” CIF sequence over irregular networks that are generated from regular topologies by link pruning and shifting. We run simulations over different random topologies and provide average performance results for different degrees of irregularity in the topology. We avoid pruning the links that connect the last coding stages to the clients, since this would mislead the system towards very low source rate values. We consider network topologies with six coding stages and three nodes per coding stage, where we set the link pruning and shifting probabilities to 10% for the irregular topologies. We show in Figs. 15 (a) and (b) the PSNR performance as a function of the link bandwidth \bar{b} (with $\pi = 5\%$) and the loss probability π (with $\bar{b} = 400$ kbps). It can be seen that the network coding is clear that the network coding scheme is resilient to the irregularity of the network topology, and that the performance only degrades by 0.2 dB compared to regular topologies at high loss rate. The proposed method is resilient to network variations as the scheme takes advantage of both path and symbol diversity.

V. RELATED WORK

Due to the advent of novel network architectures with path and source diversity, several works have addressed the problem of exploiting the network diversity for increased streaming performance. For example, a system for data overlay networks called CoolStreaming [15] has recently been designed. The system exploits path diversity and systems scalability. Specifically, the peer nodes periodically inform a set of neighboring nodes for their data availability and retrieve missing data from them. Each node maintains two lists: one with the neighboring nodes and another with the unavailable data that neighboring nodes possess. However, the periodic data exchange increases significantly the delays in the system. Avalanche [16] has been developed for large scale content distribution on peer-to-peer networks. It envisions the deployment of huge overlay networks that allow fast downloading. This collaborative system architecture improves significantly file download times over BitTorrent [17].

In parallel, network coding [1] has received a lot of attention from the research community in the past few years as it permits to effectively exploit the network diversity for increased throughput or robustness to losses. It is a promising method to improve network throughput utilization and better approach max-flow min-cut bound. In the network coding systems, the nodes do not only forward packets but perform operations with the received packets for exploiting network diversity and improving data persistence. The network coding schemes in the literature can be classified in two major categories, namely the systems based on algebraic and channel coding. An algebraic perspective on network coding has first been provided

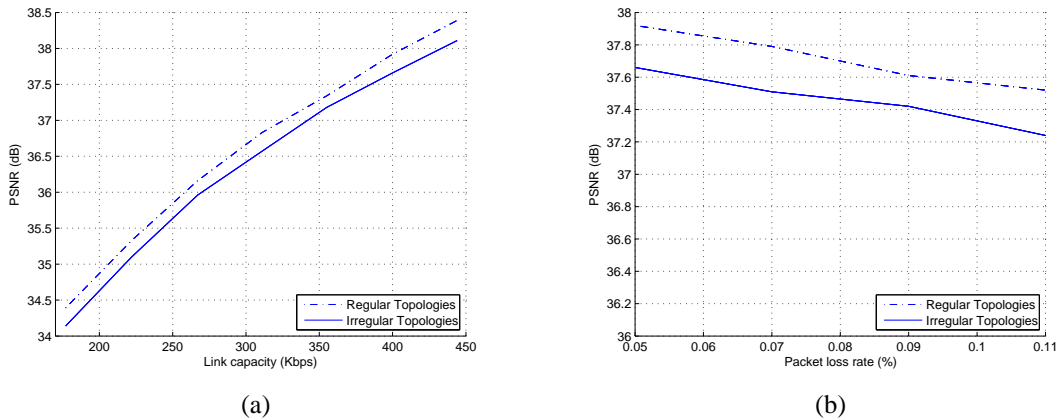


Fig. 15. PSNR performance for transmission of the “Foreman” CIF video sequence on regular and irregular topologies where link shifting and pruning probabilities are set to 10%. The topologies have 6 coding stages and three nodes per coding stage. Performance are given as a function of (a) the link capacity \bar{b} ($\pi = 5\%$) and (b) the loss probability π ($\bar{b} = 400\text{kbps}$).

in [18], which shows that maximal robustness to non-ergodic link failures can be achieved. Linear network codes [19] have been considered for multi-hop networks, where they achieve the max-flow capacity bound with controllable complexity in the network nodes. Recently, the benefits of network coding has been thoroughly examined in wireless sensor networks with dynamically changing topologies [20]. Alternatively, channel codes have also been proposed for network coding. For example, a network coding system based on LDPC codes has been proposed in [21] for transmission over wireless relay networks. Joint network-channel coding (JNCC) has been presented for transmission over relay channels [22], based on Turbo codes.

A few works have addressed lately the application of network coding principles to streaming applications. Reed-Solomon codes or LT codes have been implemented in network-embedded FEC nodes [23] and in network peers [24], respectively, in order to enhance the robustness to transmission errors. In both cases, the packets are decoded and re-encoded in the network nodes, before transmission towards the streaming client. Both schemes show that the network throughput can be significantly improved with network coding. However, decoding and re-coding operations in network nodes augments the latency of the streaming system. To reduce the end-to-end delays of [24] in [25] the relay and encode approach is proposed. The peers forward the received packets to their descendant peers upon their reception. Packet duplicates are eliminated since each packet is forwarded once. As soon as a peer is able to decode the LT encoded packets, the peer first performs LT encoding and then it generates a fresh set of packets. Practical network coding for streaming applications [26] generally rely on randomized linear network coding [10] that permits the deployment of distributed solutions with low complexity. In addition, streams are generally split into generations of symbols, in order to deal with the timing issues in streaming scenarios [26]. In [26], prioritized network coding is achieved by employing a MD-FEC scheme [27]. Motivated by this system, several unequal error protection schemes based on RNC have been presented [28]–[30].

Network coding has also been proposed for multicasting in overlay or peer-to-peer networks [31]–[34], where it takes advantage of path diversity. The benefit of network coding with peer-to-peer networks has been evaluated in [35], which shows that network coding is very useful in peer-to-peer networks, since it provides granularity, resiliency to network dynamics and leads to better bandwidth exploitation. A new streaming algorithm called R^2 [36] incorporates random network coding along with a random pushing algorithm to make feasible live peer-to-peer streaming and smoothening latency problems. Recently, a network coding system that benefits from both network coding and Raptor codes was presented in [37]. This scheme was very robust to erroneous channel estimations, especially at high loss rates and limited network diversity.

A method similar in many aspects with [37] was presented in [38] where network coding was used for video streaming over wireless mesh networks. It takes into account the significance of each video packet and intelligently selects network codes for combining packets that can be decoded by several peers. To select the most important packet, the scheme ignore the packets that has been already transmitted. This scheme employs the rate-distortion optimization framework proposed in RaDiO [39] and incorporate it into the design of the network coding algorithm such that the expected distortion is minimized. In [40], a network coding system was used for WLAN-like Access Point or WiMAX-like broadcast stations. This method employs an optimized scheduling algorithm based on the Markov Decision Process to maximize the multimedia transmission in both broadcast and unicast settings. A practical wireless multiparty video conferencing with network coding has been presented in [41]. A special protocol is devised for achieving real-time video scheduling of network coded video streams. LT codes [6] has been proposed for peer-to-peer multimedia delivery [4] eliminating the need for coordination among peer nodes and packet scheduling. This approach ensures very low but not negligible probability of multiple reception of the same packet. Although the scheme is efficient it has high computational complexity as it performs LT decoding/encoding in every peer node. This prohibits its deployment into real time systems with strict timing constraints and limited computational resources.

In this paper, we have proposed to take benefit from the advantage of both the Raptor codes and selective network coding. The Raptor codes provides a low complexity solution for rateless coding that provides high symbols diversity. Selective network coding efficiently exploits the path diversity with low complexity and small buffering requirements. It provides an effective and practical solutions for distributed streaming that is robust to packet loss and network irregularity.

VI. CONCLUSIONS

We have proposed in this paper a video network coding algorithm based on Raptor codes at the server and selective packet combinations in the network nodes. Raptor encoding permits to achieve a high symbol diversity in the network, and selective network coding effectively exploits path diversity in the network. The proposed solution has a small complexity and low delay requirements compared to state-of-the-art methods. We have also proposed an optimized rate allocation solution, such that the system minimizes the average distortion at the least reliable client. Compared to optimized end-to-end FEC protection, the selective network coding schemes shows improved quality performance and increased robustness to packet loss and network irregularity. Overall, the low complexity nature and the effective behavior of the proposed algorithm in regular as well as irregular network topologies position it as an interesting solution for distributed streaming in practical overlay network scenarios.

APPENDIX

A. Convexity of the expected distortion

We show here that the objective function given in the Eq. (6) is a function that is convex with the number of source packets, K . The expected distortion function is given by

$$E_d(K) = (1 - \bar{p}(K)) \cdot D_s(K) + \bar{p}(K) \cdot D_c(K) = (D_c(K) - D_s(K)) \cdot \bar{p}(K) + D_s(K) \quad (6)$$

where we have removed the dependency in the number of received packets r_m . The source distortion can be modeled as $D_s(K) = a \cdot (p_s \cdot K)^b$ where p_s denotes the packet size, while the concealed distortion according to [14] can be approximated by an affine function $D_c(K) = \mu \cdot p_s K + \nu$.

For examining the convexity of $E_d(K)$ convexity, we have to show that its second derivative is positive, i.e., $E_d(k)'' > 0$. We have

$$E_d(K)'' = D_c(K)'' \cdot \bar{p}(K) + (1 - \bar{p}(K)) \cdot D_s(K)'' + 2 \cdot (D_c(K)' - D_s(K)') \cdot \bar{p}(K)' + (D_c(K) - D_s(K)) \cdot \bar{p}(K)'' \quad (7)$$

It can be easily shown that the first term of Eq. (7) is equal to zero, while the second and the fourth terms are positive as $\bar{p}(K)$ is a decreasing convex function. In general, the third term is not convex as $(D_c(K)' - D_s(K)')$ can take both positive and negative values. However, we can prove that for typical values of K $(D_c(K)' - D_s(K)')$ is less than zero and since $\bar{p}(k)' < 0$, the third term is positive.

Let us now show that $(D_c(K)' - D_s(K)') < 0$ for small values of K . We have to investigate when $(D_c(K)' - D_s(K)')$ shifts from negative to positive values. For equality, it holds

$$a \cdot b (p_s \cdot K)^{b-1} = \mu \Rightarrow K = \frac{1}{p_s} \cdot \sqrt[b-1]{\frac{\mu}{a \cdot b}} \quad (8)$$

We examine the typical parameters values in practice. For a Foreman QCIF sequence encoded by the employed MDC H.264/AVC based coder the distortion model parameters are $b = 0.98$, $a = 8493000$ and $\mu = -0.0026$ while $p_s = 4096$ bits. This means that difference of the derivatives becomes positive for $K < 1$. As K denotes the number of source symbols, it is always positive, which means that the last term of the second derivative is also positive. Hence, the expected distortion is convex with K in practice.

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