Graph Signal Processing for Dynamic Geometry

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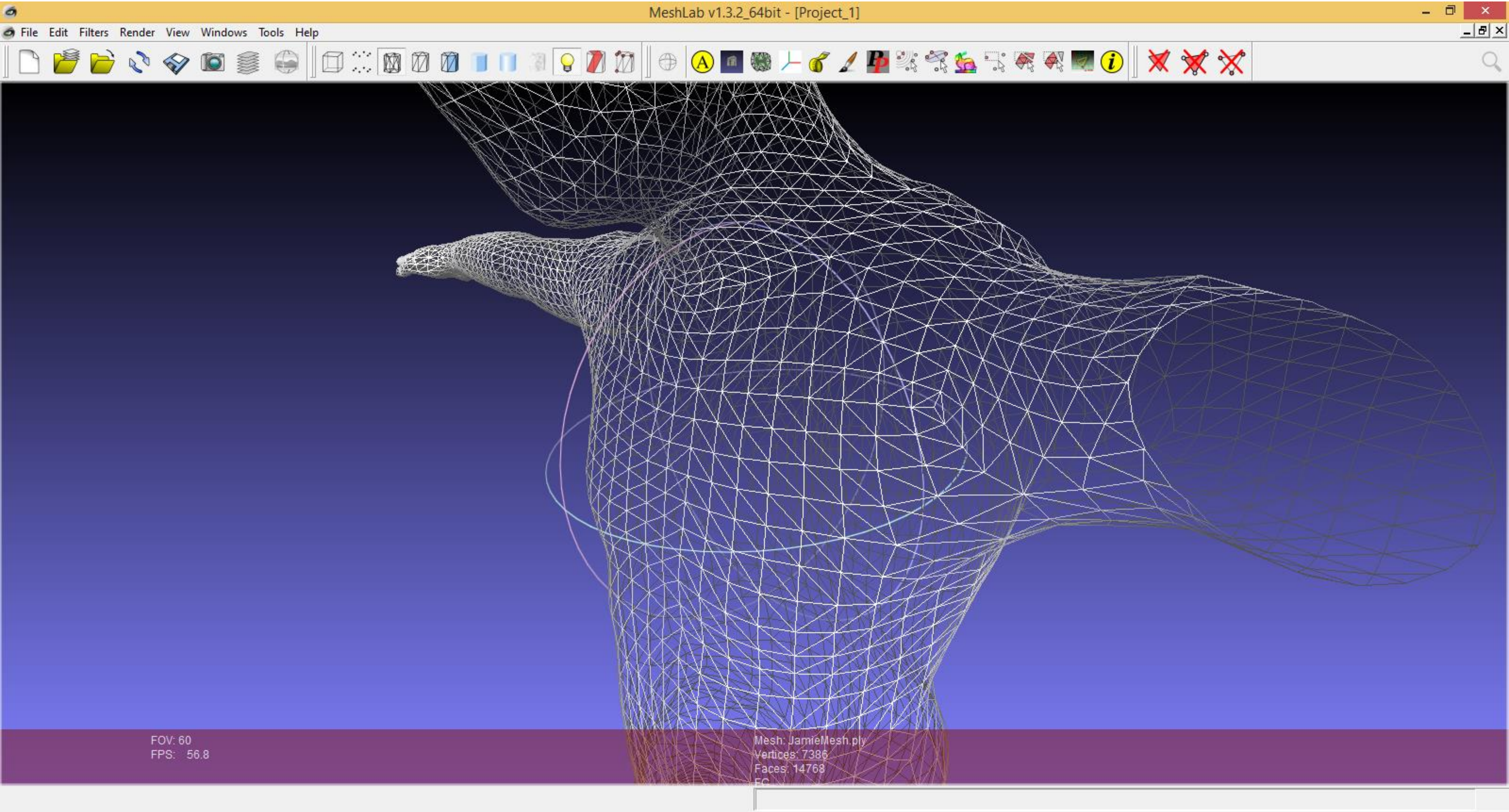
WORKSHOP ON GRAPH SIGNAL PROCESSING FOR IMAGING
OCTOBER 31, 2014

Outline

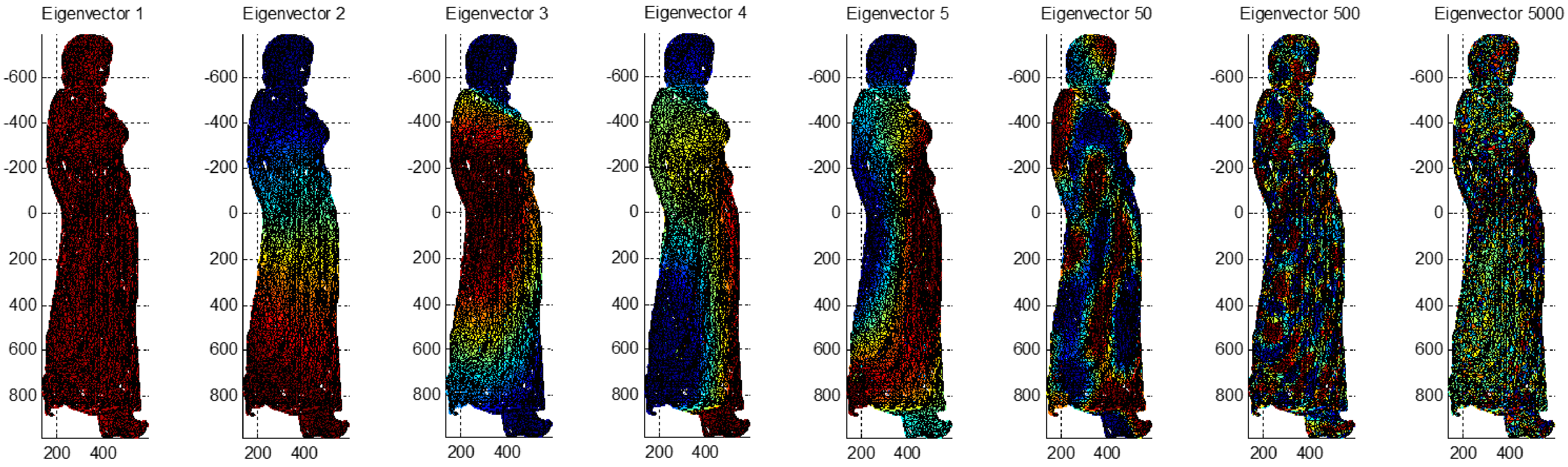
Graph Signal Processing for Static Geometry

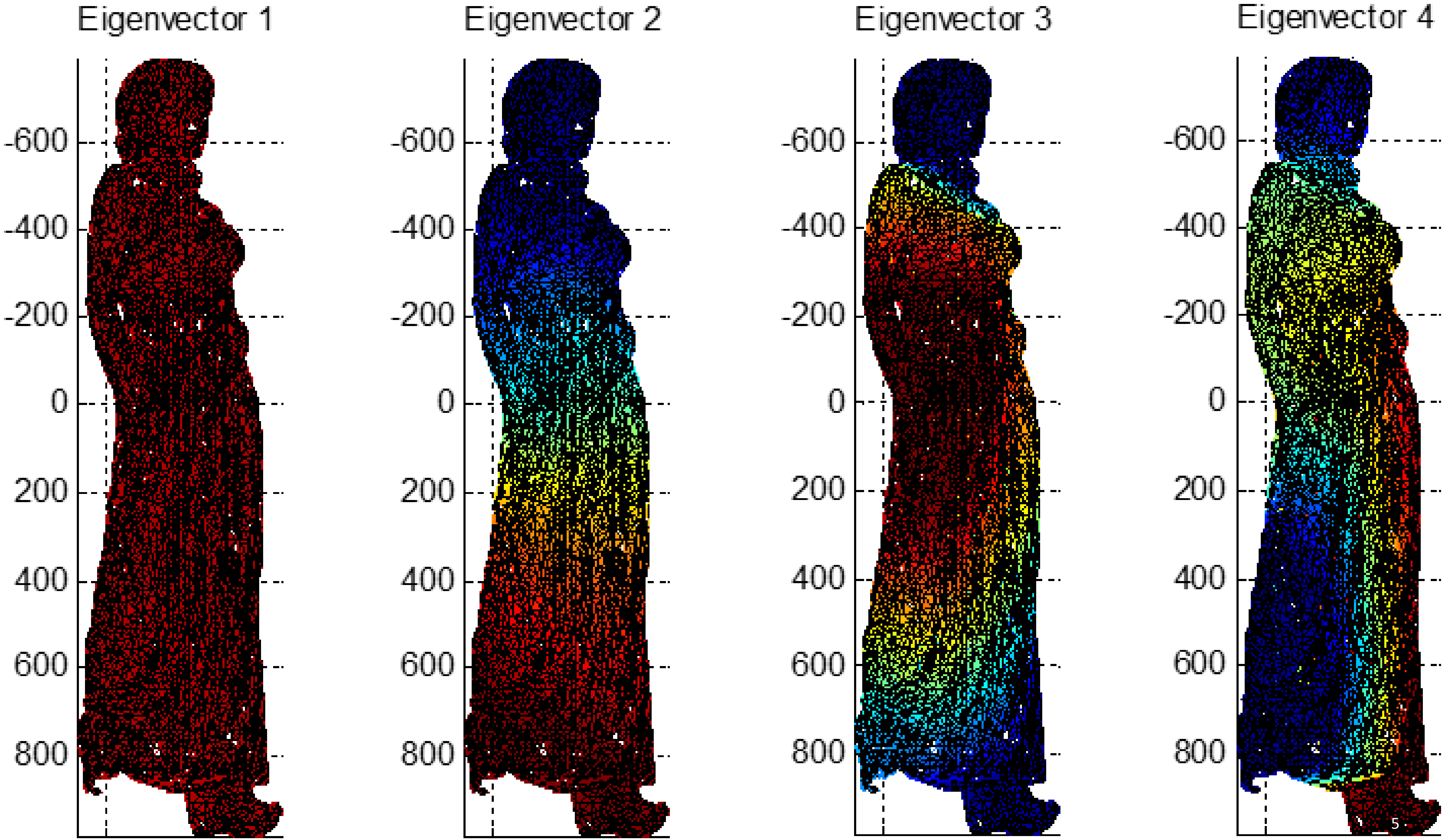
Graph Signal Processing for Dynamic Geometry (topology consistent over time)

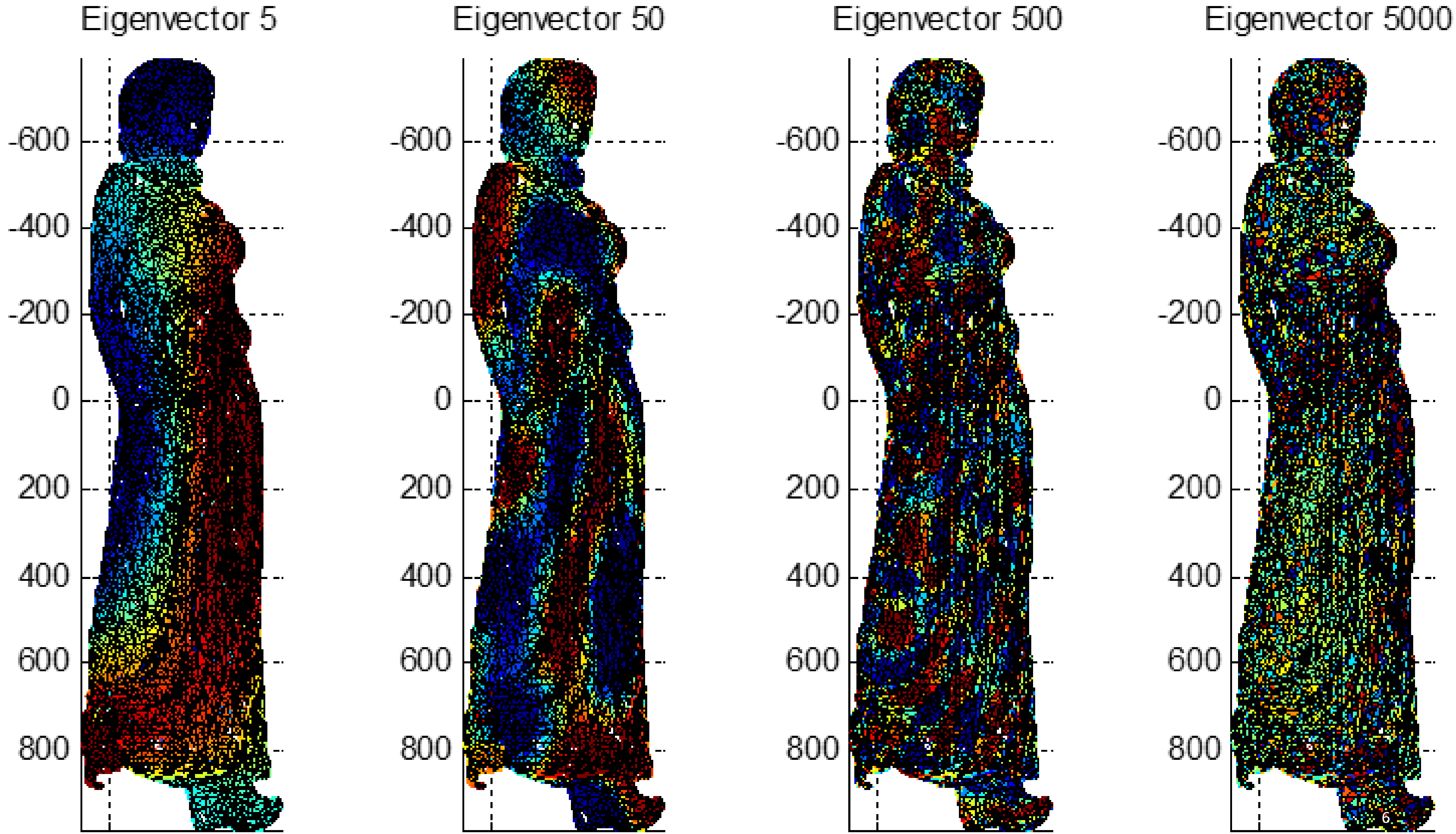
Graph Signal Processing for Dynamic Geometry (topology not consistent over time)


MeshLab v1.3.2_64bit - [Project_1]
File Edit Filters Render View Windows Tools Help
FOV: 60
FPS: 56.8
Mesh: JamieMesh.ply
Vertices: 7386
Faces: 14768

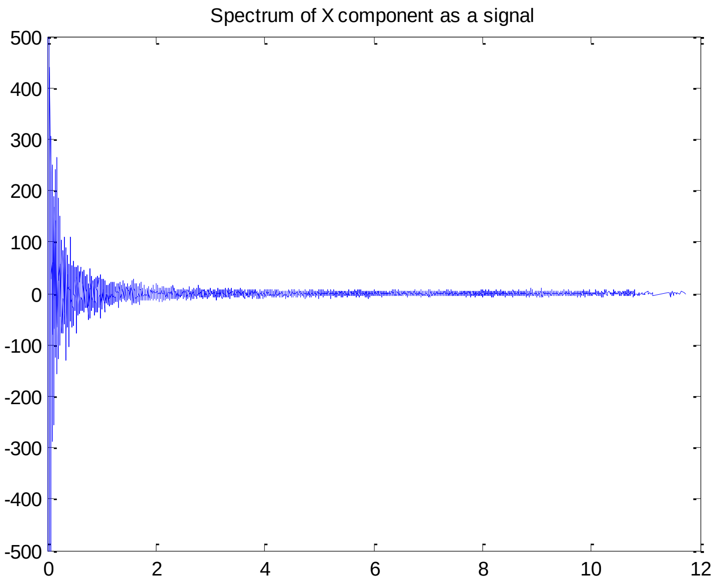
Eigenvectors of a Mesh

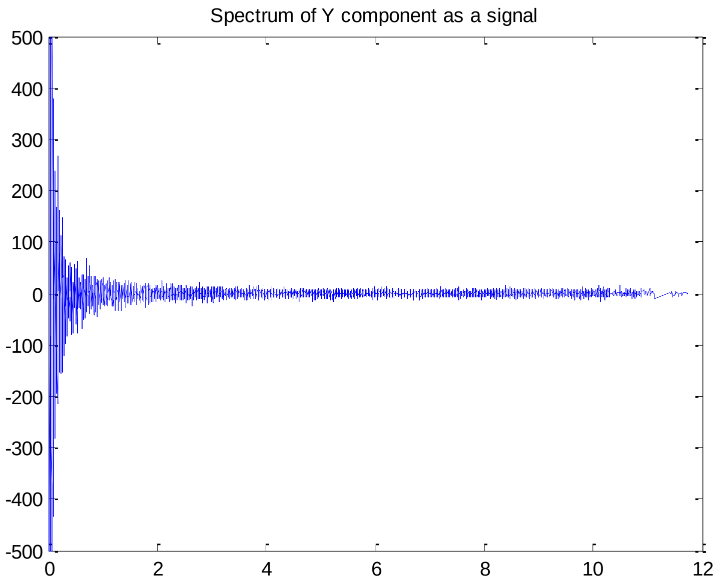


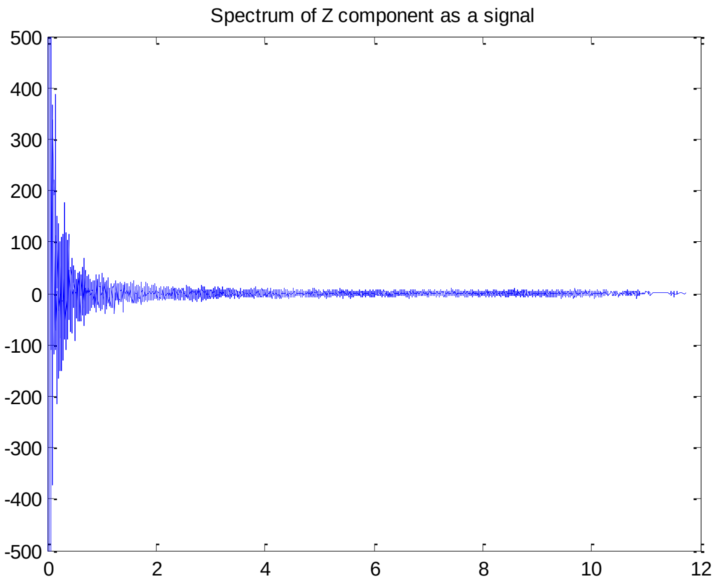

Eigenvector 1
Eigenvector 2
Eigenvector 3
Eigenvector 4
-600
-400
-200
0
200
400
600
800


Eigenvector 5
Eigenvector 50
Eigenvector 500
Eigenvector 5000
-600
-400
-200
0
200
400
600
800

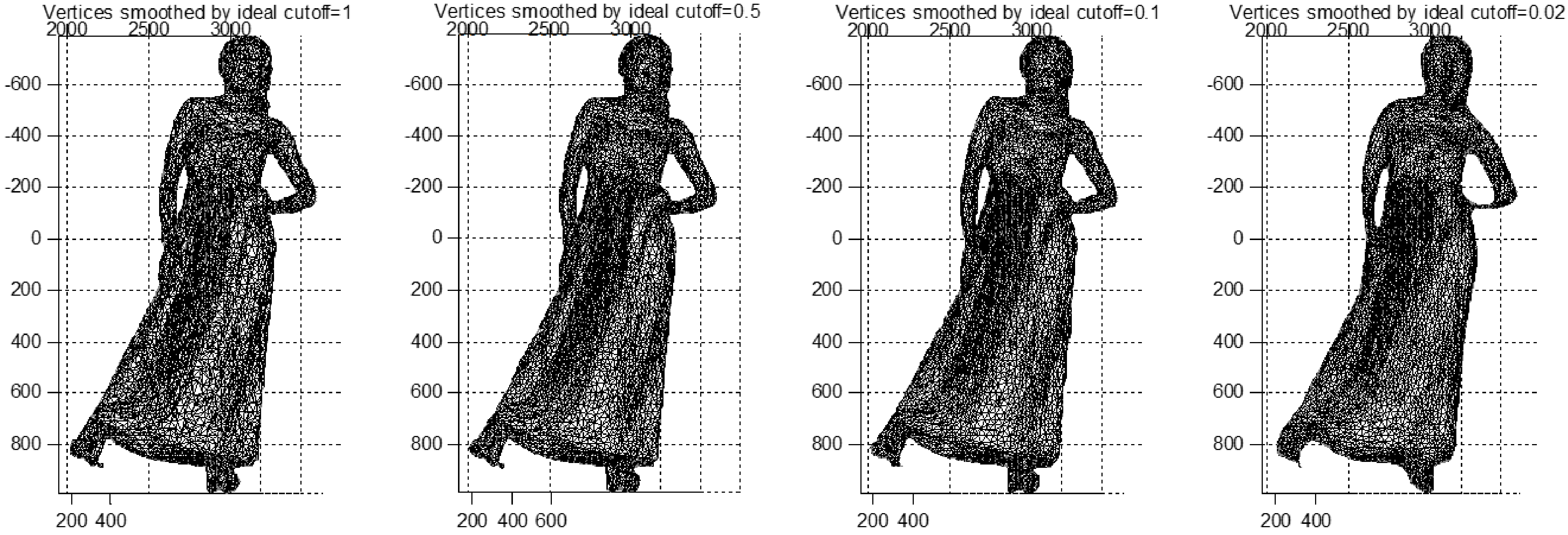
Spectrum of Geometry as a Signal



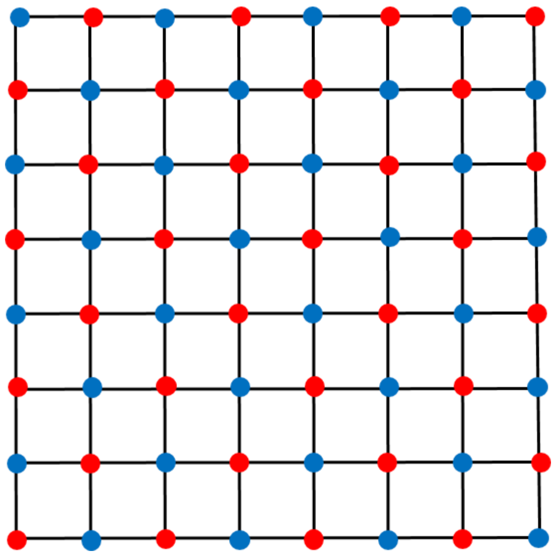




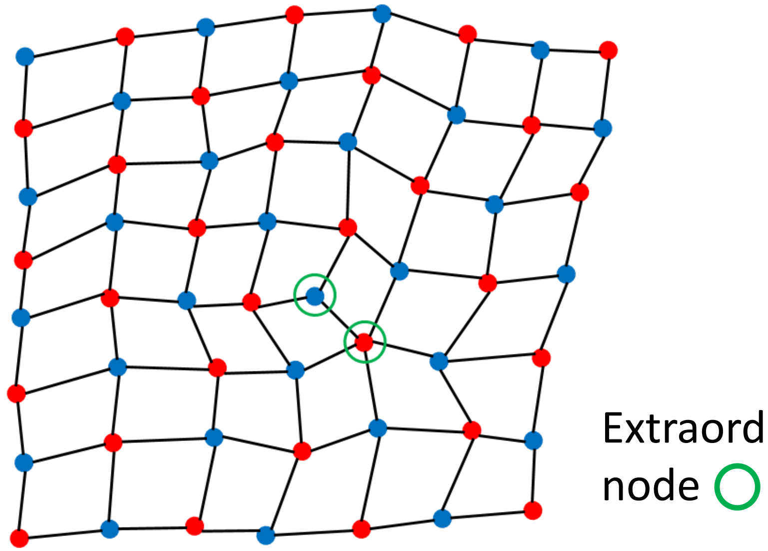
Smoothing by Ideal Low-Pass Filtering



Dealing with Complexity

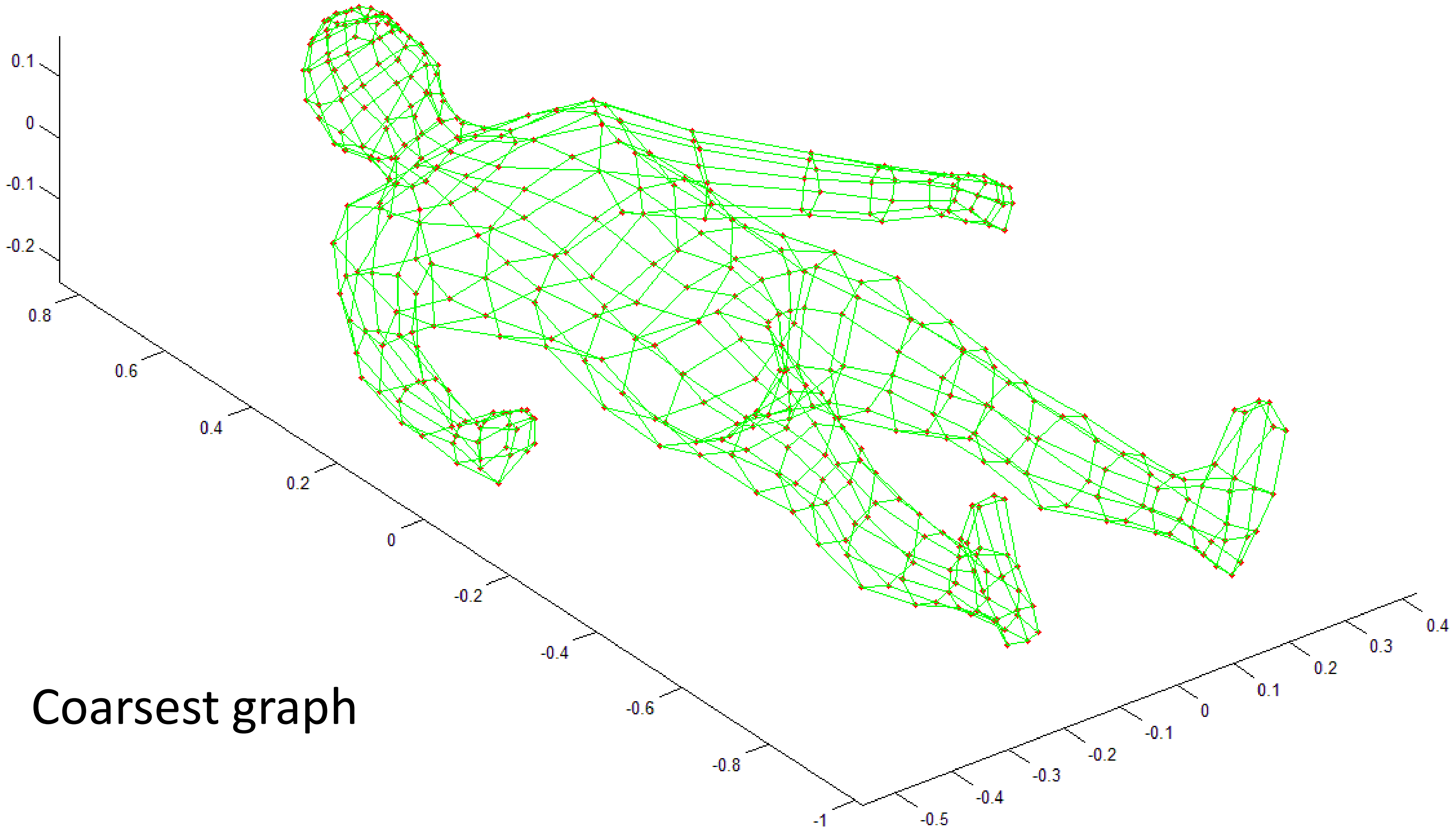


Traditional images:
process in blocks
or wavelets



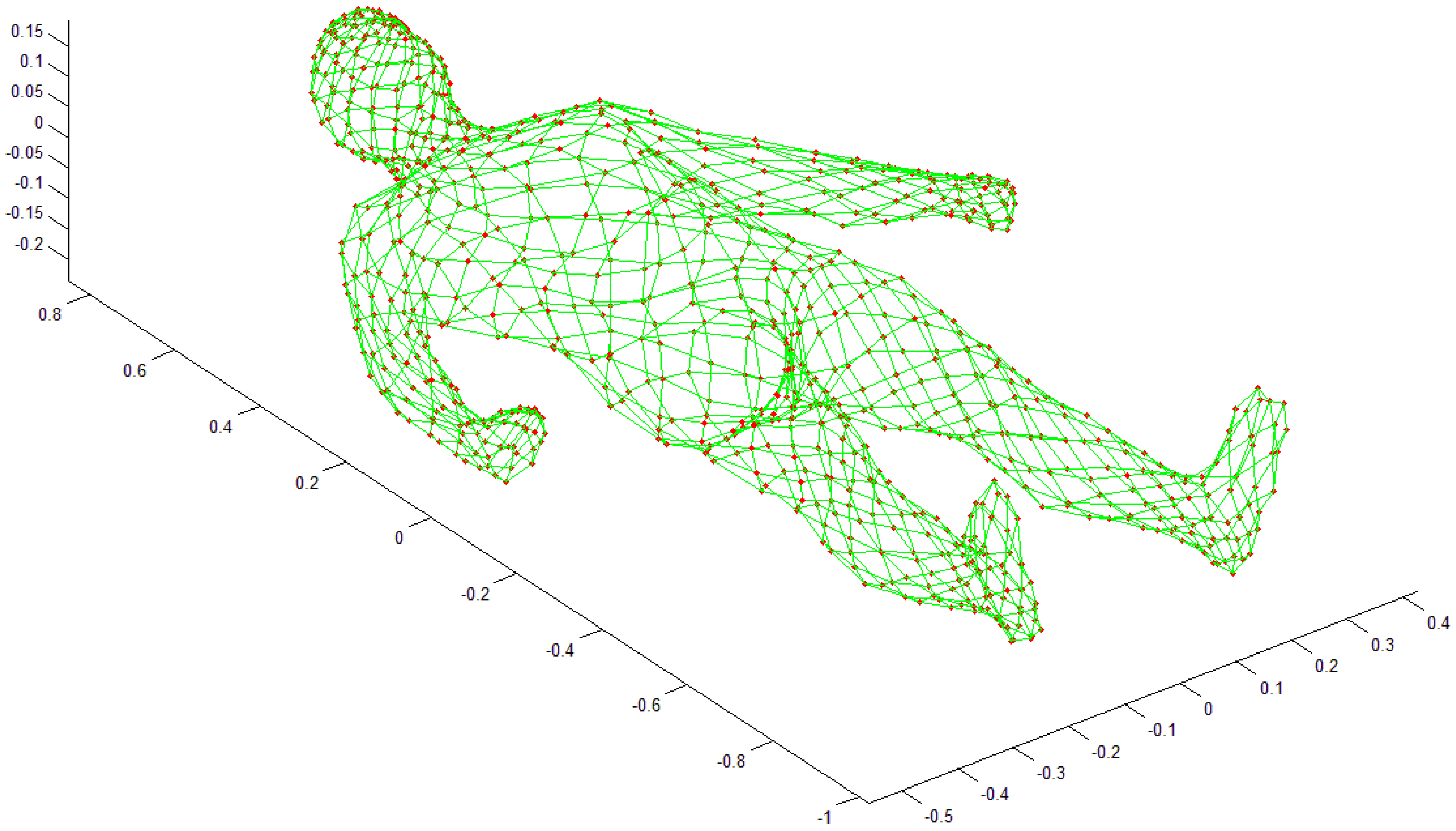
Arbitrary meshes:
not so easy

Multiresolution Graph

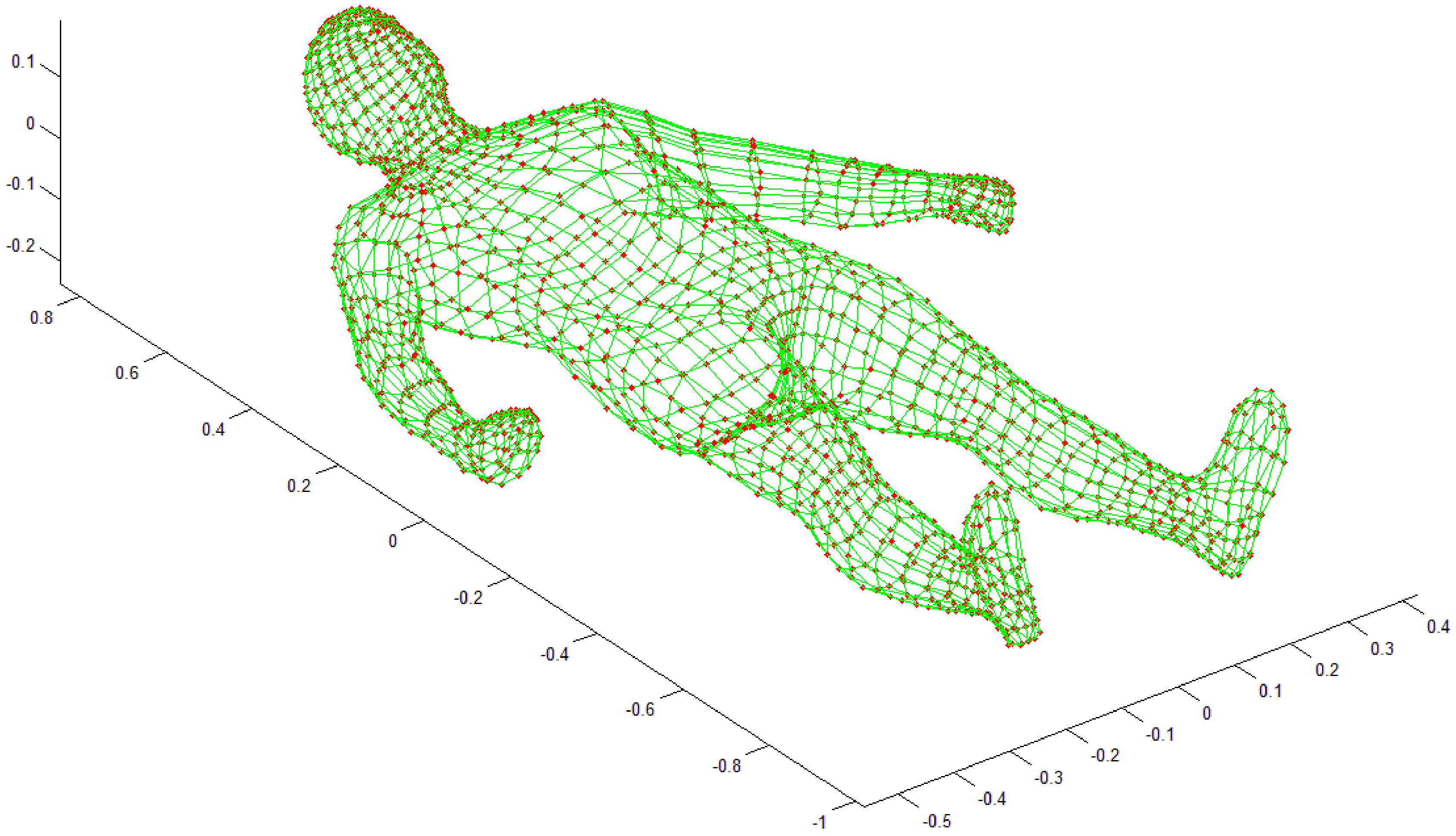


Coarsest graph

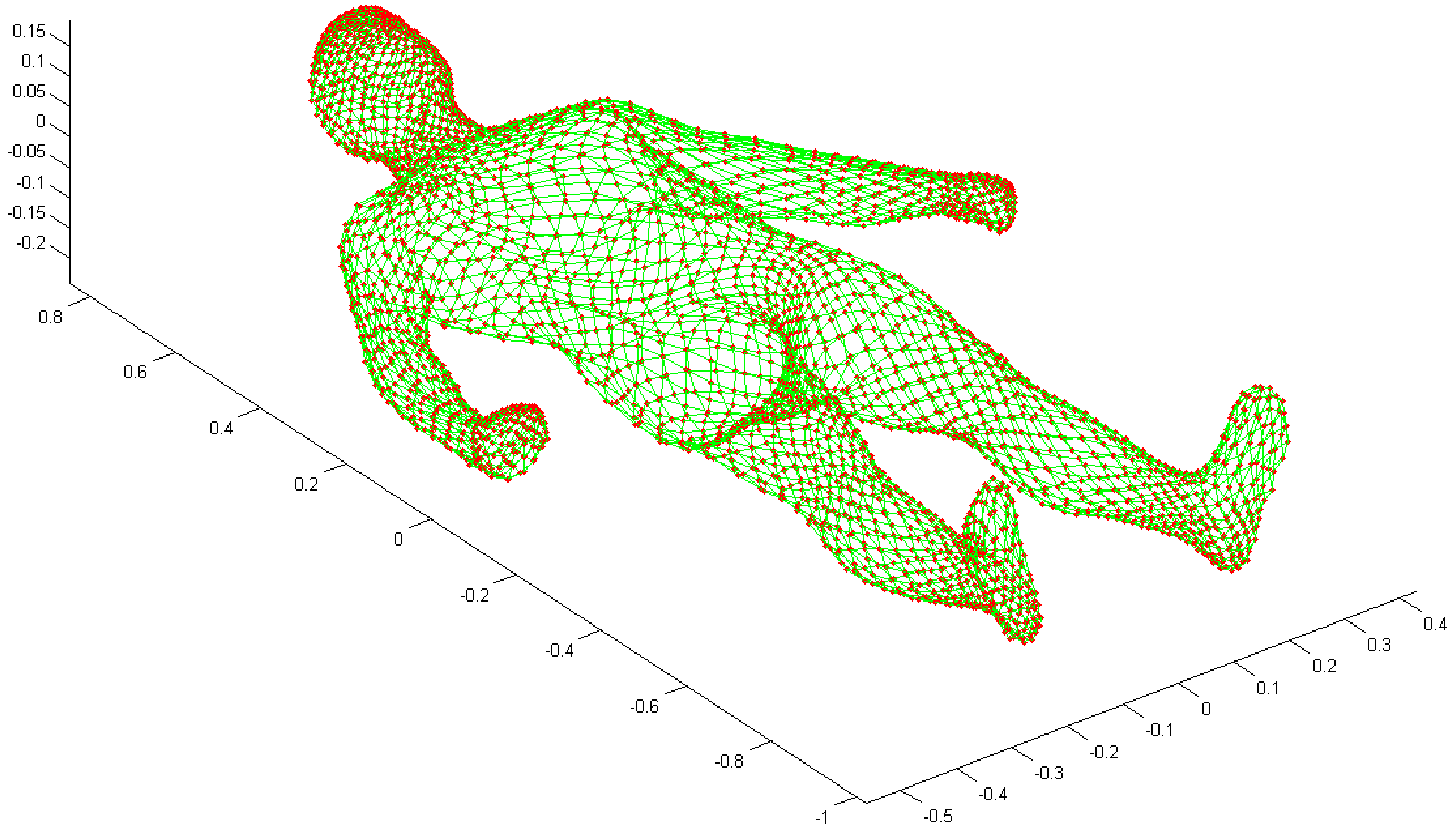
Multiresolution Graph



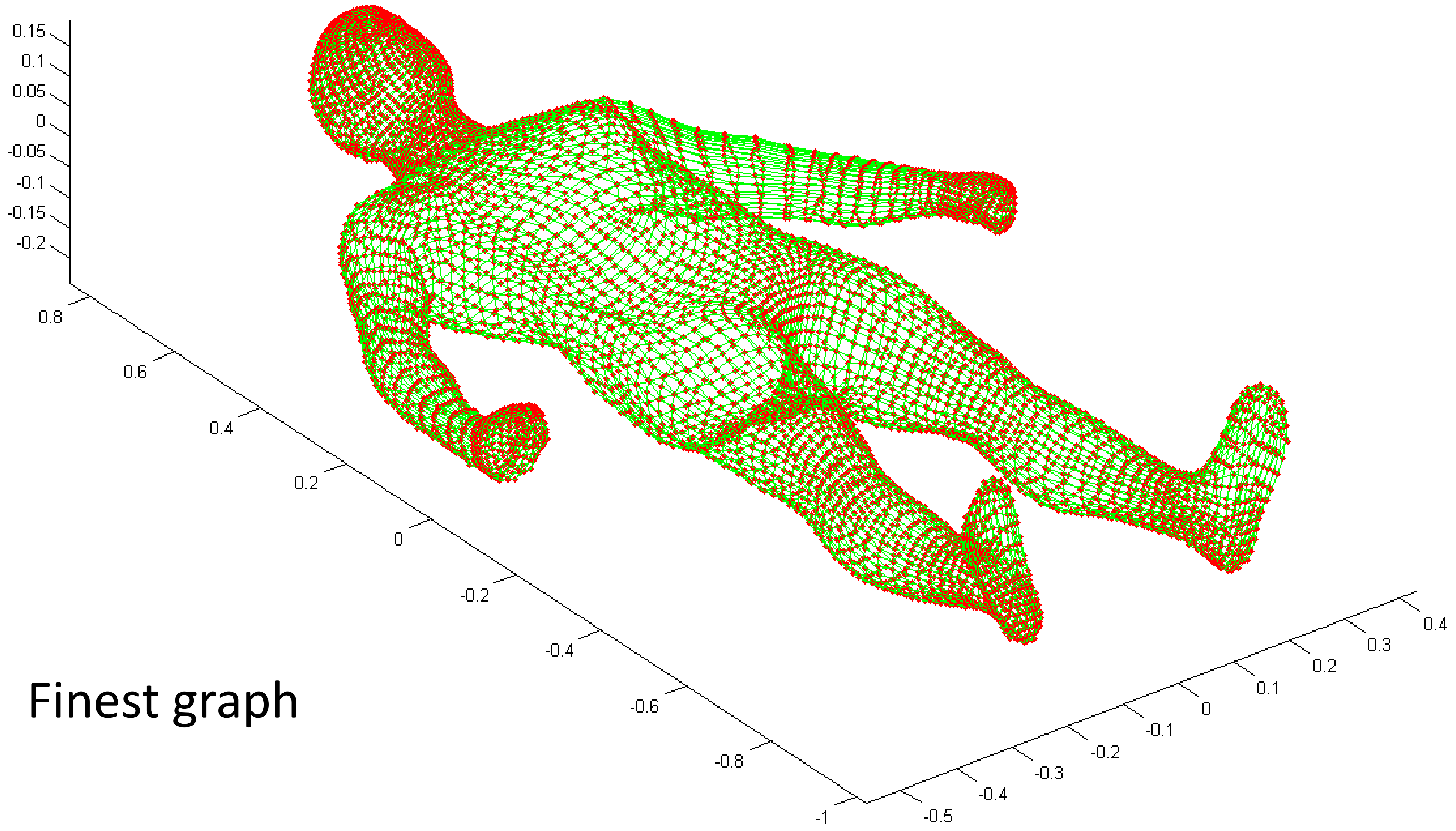
Multiresolution Graph



Multiresolution Graph

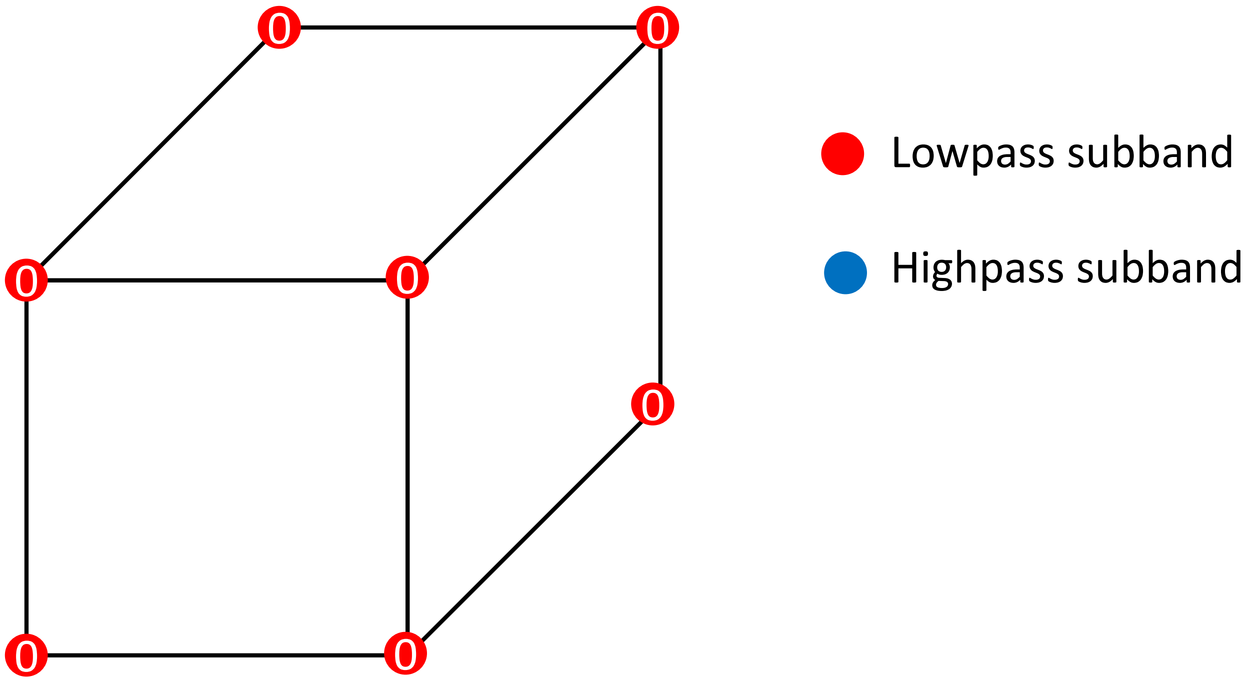


Multiresolution Graph

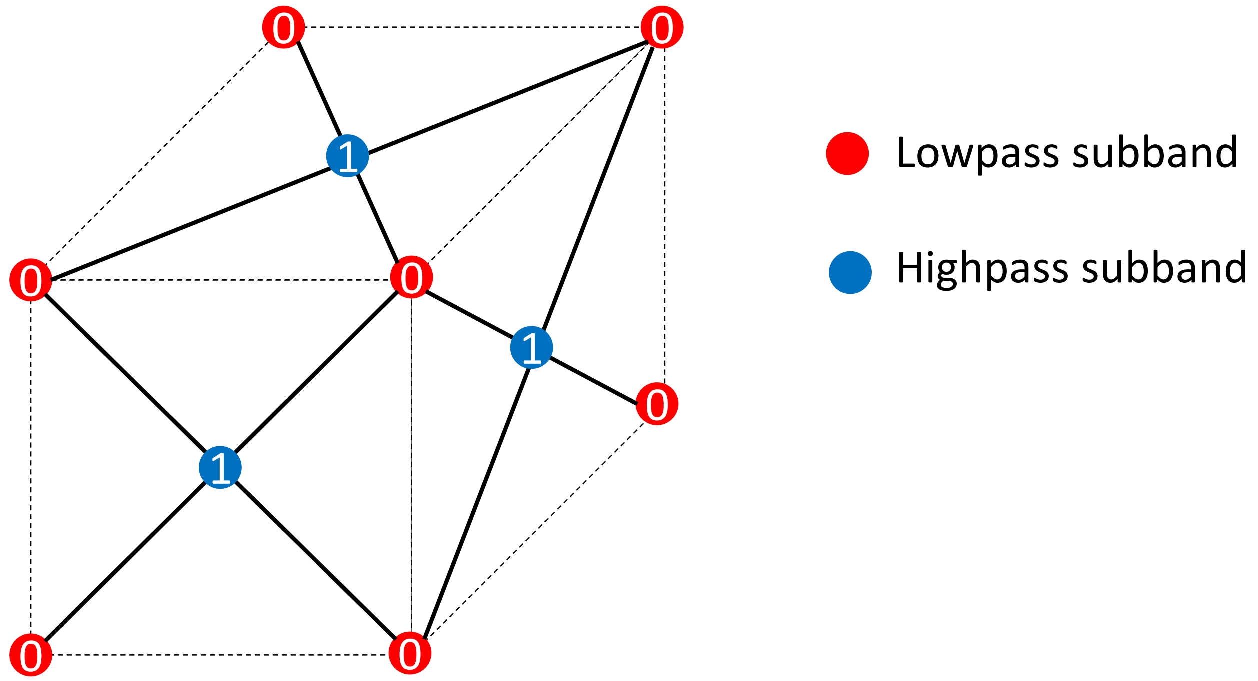


Finest graph

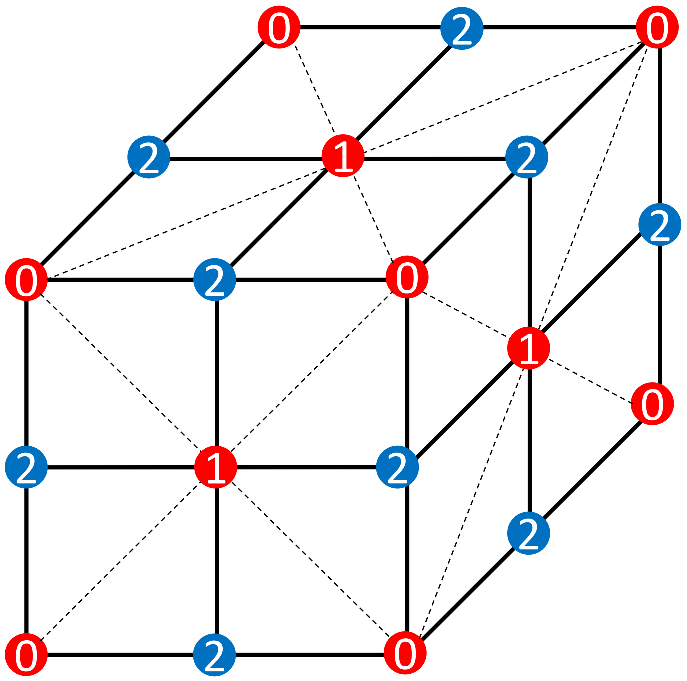
Catmull-Clark Subdivision



Catmull-Clark Subdivision



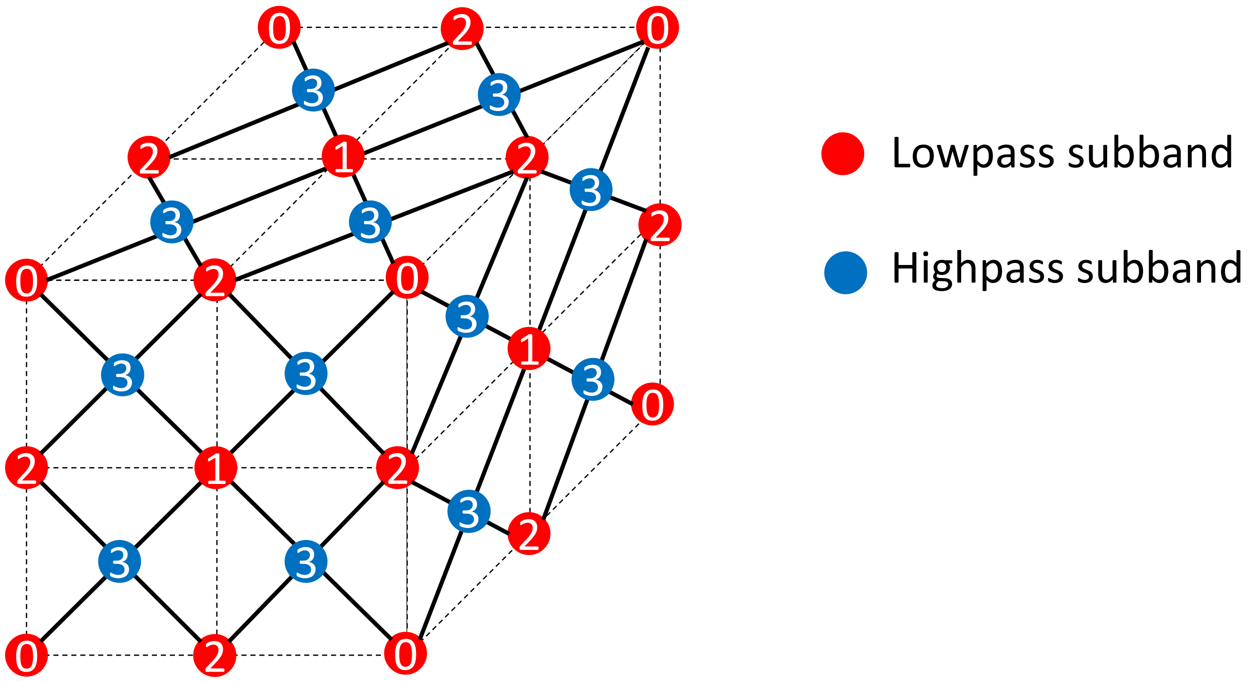
Catmull-Clark Subdivision



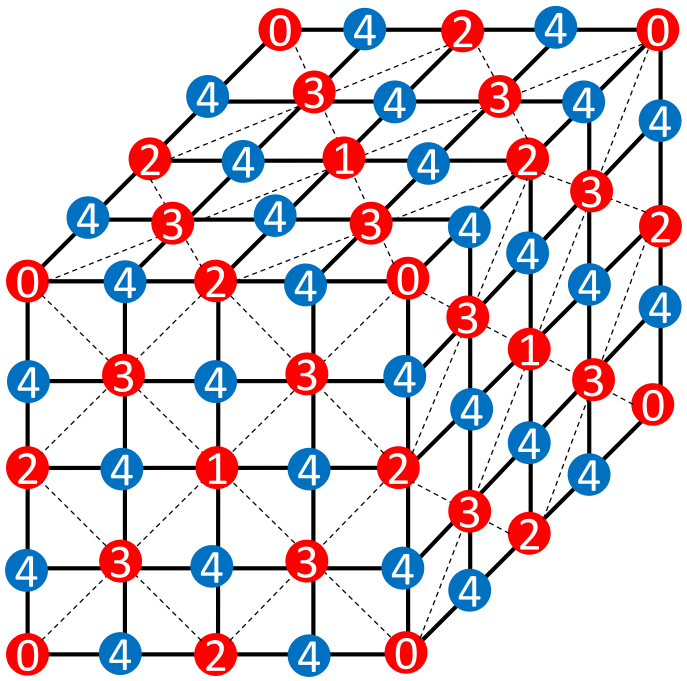
Lowpass subband

Highpass subband

Catmull-Clark Subdivision



Catmull-Clark Subdivision



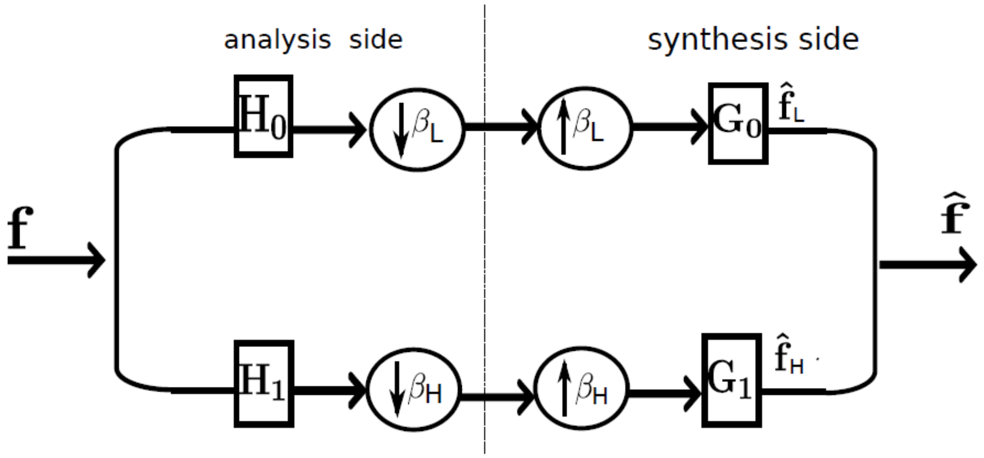
Lowpass subband

Highpass subband

Graph Wavelet Filter Banks (GWFBs)

Critical sampled, compactly supported, near-orthogonal, designed in spectral domain

Building block: two-channel filter bank on a bipartite graph



[Narang & Ortega 2013]

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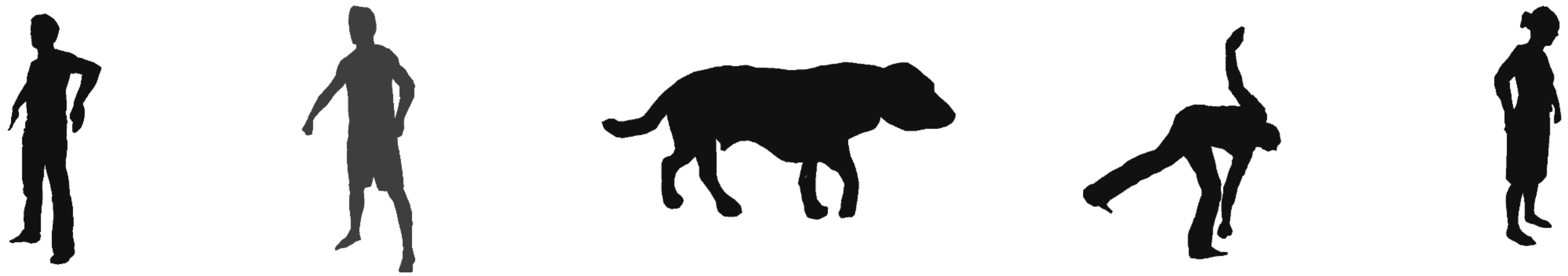
Dynamic geometry compression competition dataset[1]

Experiments carried over five sequences: Handstand, Dance, Dog, Wheel, Skirt

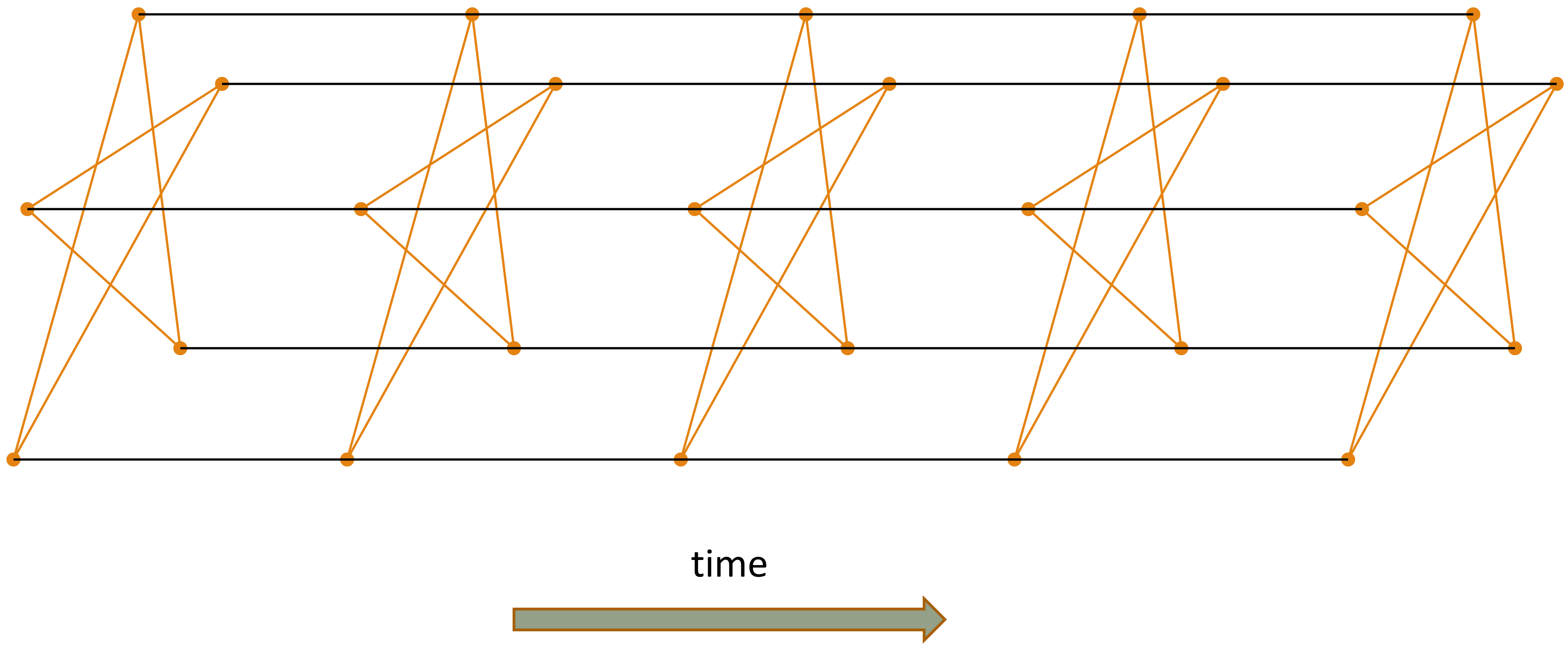


Performance measured in terms of Metro distance (RMS)

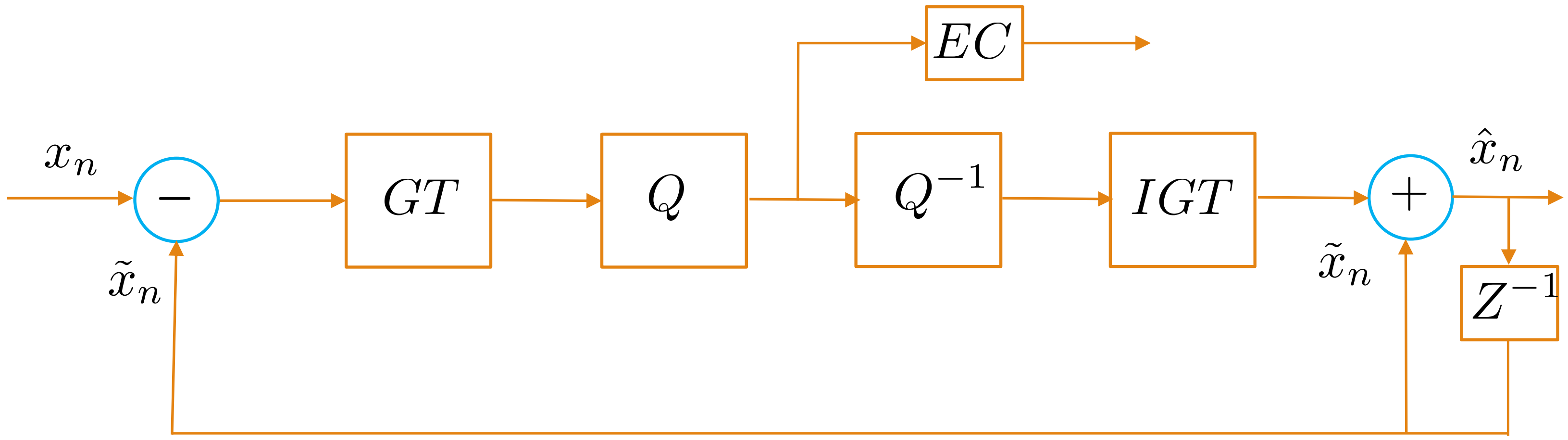
$$d_{RMS}(X,Y) = \max\{\underset{x\in X}{rms} \inf_{y\in Y} d(x,y), \underset{y\in Y}{rms} \inf_{x\in X} d(x,y)\}$$

[1] http://www.geometrycompression.org/

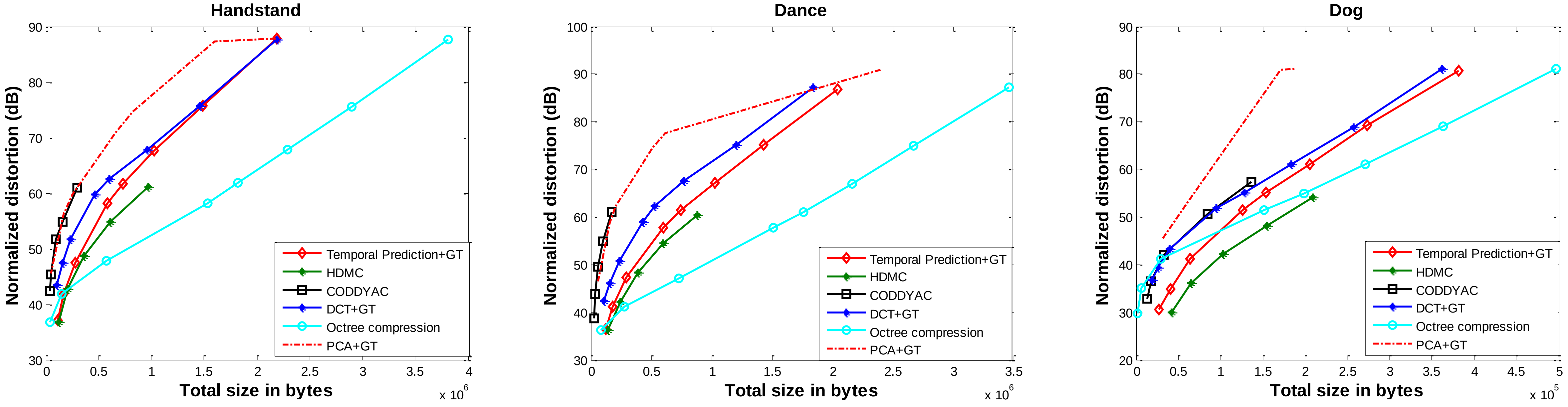
Trellis for Temporally Consistent Graphs



Predictive Transform Coding



Rate-Distortion Performance



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Motion Estimation and Compensation

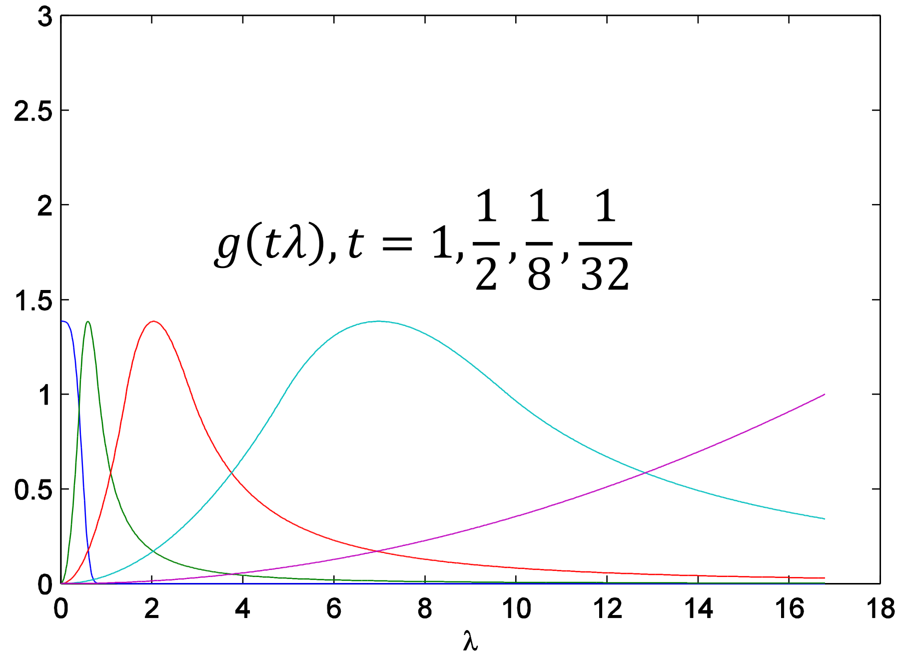
From graphs $\mathcal{G}_t$ and $\mathcal{G}_{t+1}$:

1. Extract feature vectors in each graph, f_m $(m \in \mathcal{G}_t)$ and $f_n(n \in \mathcal{G}_{t+1})$
2. Compute feature (dis)similarity, $score(f_m, f_n)$
3. Find best matching points, $m = n^*$ for each n
4. Keep matching pairs of interest, (n_i^*, n_i)
5. Compute sparse set of motion vectors, $mv_{n_i} = p_{n_i} - p_{n_i^*}$
6. Smooth to get dense motion field, $\widetilde{mv}_n$ for all $n \in \mathcal{G}_{t+1}$
7. Warp graph $\mathcal{G}_{t+1}$and connect to graph $\mathcal{G}_t$
8. Interpolate signals on $\mathcal{G}_{t+1}$from signals on $\mathcal{G}_t$

Features: Spectral Signatures on Graphs

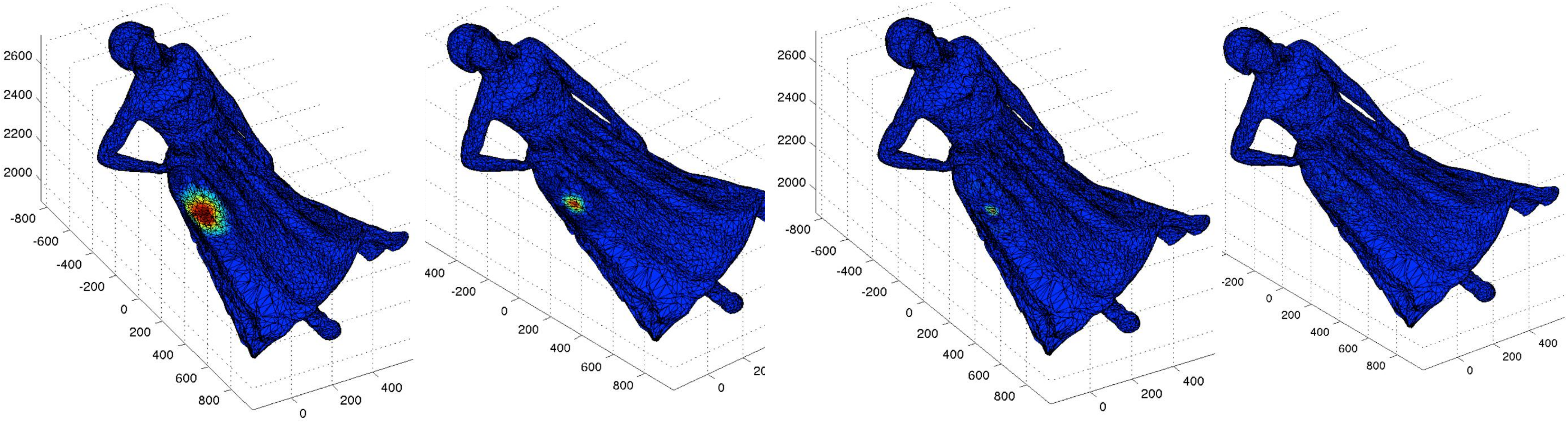
Based on spectral graph wavelets (SGWT)*: dilated, translated versions of a bandpass kernel designed in the graph spectral domain of the graph Laplacian

$$\psi_{t,n}(m) = \sum_{l=0}^{N-1} g(t\lambda_l)\chi_\ell^*(n)\chi_\ell(m)$$



*D. Hammond, P. Vandergheynst, and R. Gribonval, Wavelets on graphs via spectral graph theory, App. and Comp. Harm. Anal., 2011

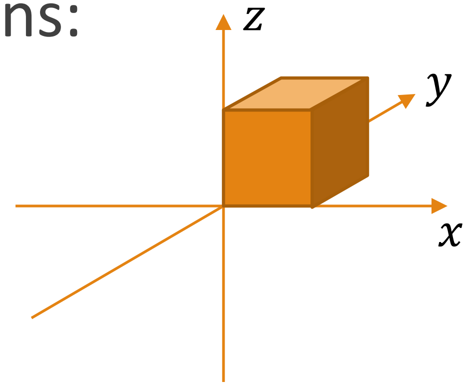
Spectral Graph Wavelets $\psi_{t,n}$ (example)



Can be efficiently implemented by approximating the wavelet operator with powers of the Laplacian (Chebyshev polynomials)

Feature Extraction

For each node $n \in \mathcal{G}$, define the octant indicator functions:



1. $o_{1,n}(k) = 1_{\{x(k)\geq x(n),\, y(k)\geq y(n),\, z(k)\geq z(n)\}}(k),\ k \in \mathcal{G}$
2. $o_{1,n}(k) = 1_{\{x(k)\geq x(n),\, y(k)\geq y(n),\, z(k)<z(n)\}}(k),\ k \in \mathcal{G}$

   …

8. $o_{1,n}(k) = 1_{\{x(k)<x(n),\, y(k)<y(n),\, z(k)<z(n)\}}(k),\ k \in \mathcal{G}$

For each color and geometry component $s \in \{r, g, b, x, y, z\}$ at that node compute the wavelet coefficients:

$$f_{n,t,o_i,s} = \langle s \cdot o_{i,n}, \psi_{t,n} \rangle \text{ for } i = 1, \dots, 8 \text{ and } t = t_1, \dots, t_{max}$$

Feature vector is concatenation of these wavelet coefficients: $f_n = \{f_{n,t,o_i,s}\}$

Feature (dis)similarity and matching

For all $m \in \mathcal{G}_t$ and $n \in \mathcal{G}_{t+1}$ compute the Mahalanobis distance

$$score(m,n) = (f_m - f_n)^T P (f_m - f_n)$$

where P is a covariance matrix estimated (i.e., trained) from features known to be in correspondence

For each $n \in \mathcal{G}_{t+1}$ define its best match in $\mathcal{G}_t$:

$$n^* = \arg\min_{m \in \mathcal{G}_t} score(m,n)$$

$$bestscore(n) = \min_{m \in \mathcal{G}_t} score(m,n)$$

Keep sparse set of matching points (n_i^*, n_i) s.t. each region in $\mathcal{G}_{t+1}$ has at least one point n_i and $bestscore(n_i) \leq thresh$.

Sparse set of motion vectors

Compute motion vectors for the sparse set of matching points:

$$mv_n^* = p_n - p_{n^*}$$

where $p_n = [x_n, y_n, z_n]^T$ is the position of vertex n

Approximate $score(m,n)$ for m near n^* in terms of $mv_n = p_n - p_m$:

$$score(m,n) \approx bestscore(n) + (p_n - p_m)^T M_n^{-1} (p_n - p_m)$$

where $M_n = \frac{1}{|\mathcal{N}_{n^*}^2|} \sum_{m \in \mathcal{N}_{n^*}^2} \frac{(p_m - p_{n^*})^T (p_m - p_{n^*})}{score(m,n) - bestscore(n)}$ and $\mathcal{N}_{n^*}^2$ is the two-hop neighborhood of n^*in $\mathcal{G}_t$

Smooth dense set of motion vectors

Letting $Q = \begin{bmatrix} M_1^{-1} & \cdots & 0_{3\times3} \\ \vdots & \ddots & \vdots \\ 0_{3\times3} & \cdots & M_N^{-1} \end{bmatrix}$ where $M_n^{-1} = 0_{3\times3}$ if $n \notin$ sparse set,

smooth the signal of motion vectors:

$$\widetilde{mv}^* = \arg\min (mv - mv^*)^T Q (mv - mv^*) + \lambda \sum_{i=1}^{3} (S_i mv)^T L_t (S_i mv)$$

where L_t is the graph Laplacian and S_i is a selection matrix

Penalize excess matching score on the sparse set

Impose smoothness of the motion vectors on the graph

Closed form solution

This has a closed form solution,

$$\widetilde{mv}^* = \left(Q + \lambda \sum_{i=1}^{3} S_i^T L_t S_i \right)^{-1} Qmv^*$$

which can be solved iteratively using MINRES-QLP for efficiency on large systems

S.-C.T Choi, and M.A. Saunders, MINRES-QLP for symmetric and Hermitian linear equations and least-squares problems, ACM Trans. Math. Softw., 2014.

Graph $\mathcal{G}_t$ warped to $\mathcal{G}_{t+1}$– Example 1

Graph $\mathcal{G}_t$ warped to $\mathcal{G}_{t+1}$– Example 2

previous frame current frame previous+current frame
previous frame+MC previous frame current frame+MC previous frame

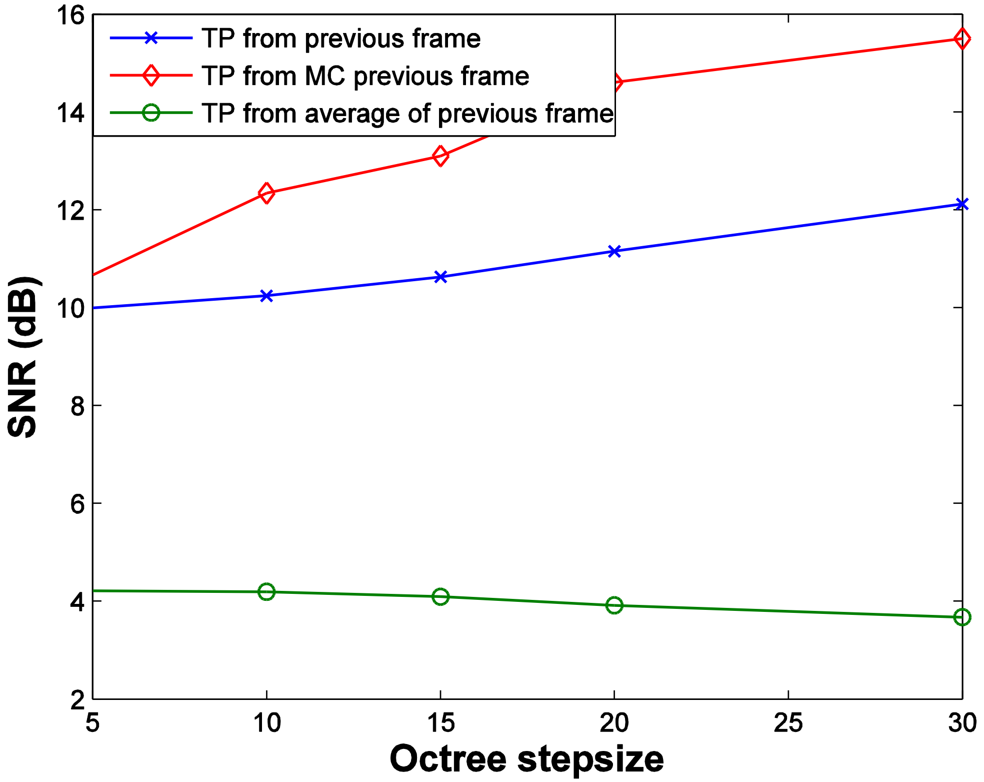
Graph $\mathcal{G}_t$ warped to $\mathcal{G}_{t+1}$– Example 3

previous frame
previous frame+MC previous frame
current frame
current frame+MC previous frame
previous_current frames

Color Prediction – Example 1

Prediction from previous frame

Prediction from motion compensated previous frame

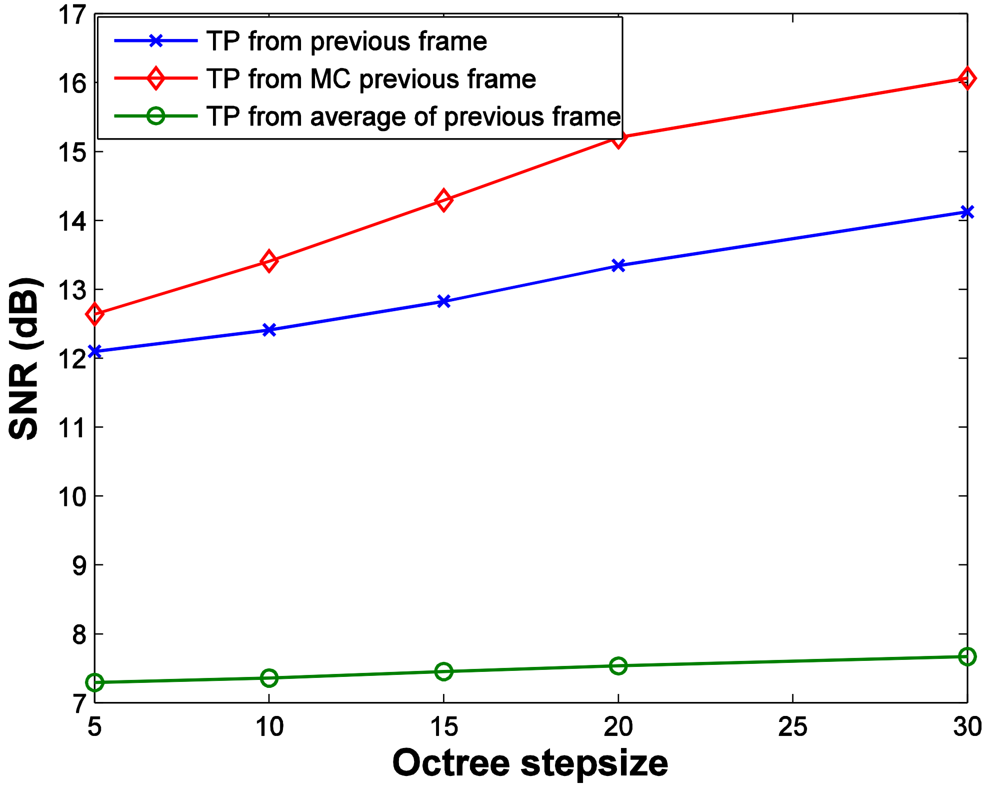


$$SNR = 10\log_{10}\left(\frac{\|\text{current\_color}\|^2}{\|\text{prediction\_error}\|^2}\right)$$

Color Prediction – Example 2

Prediction from the previous frame

Prediction from motion compensated previous frame



Conclusion

Geometry is a natural application of Graph Signal Processing

Dynamic Geometry, especially where topology is temporally inconsistent, is fertile ground for new problems in Graph Signal Processing