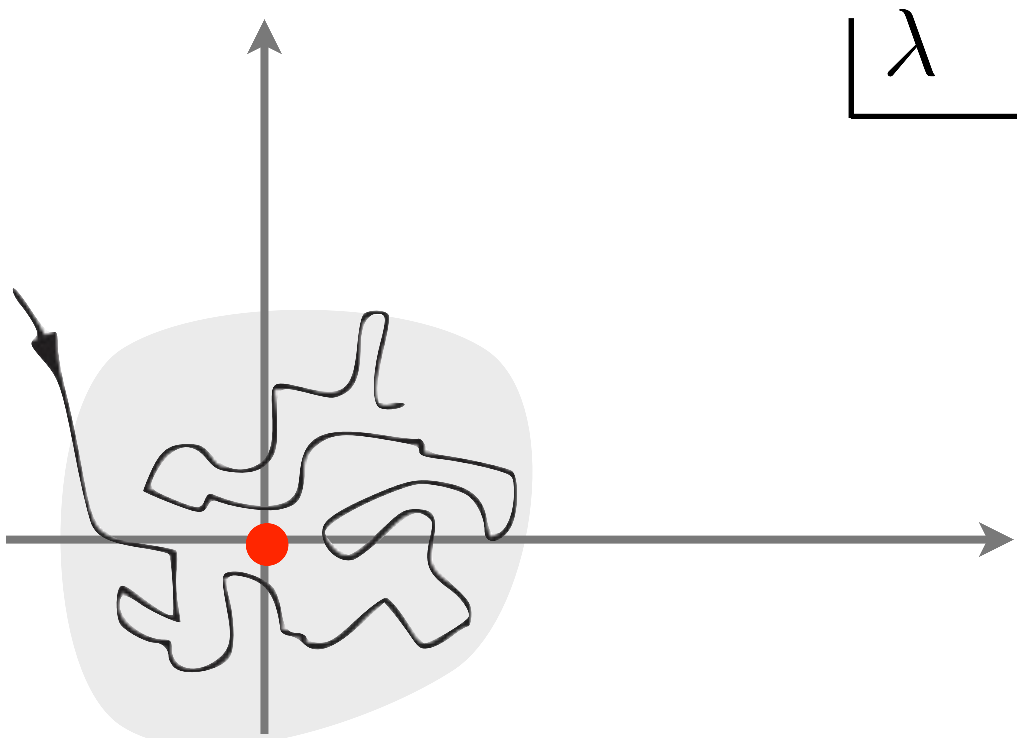
Lecture III

Constraining

- RG asymptotics in weakly coupled deformations of CFTs
- SFT asymptotics

Goal: study RG flows (perturbatively) near CFT fixed point

Ex: free field theory with small marginal couplings



$$\beta_I = b_{IJK}\lambda_J\lambda_K + \ldots$$

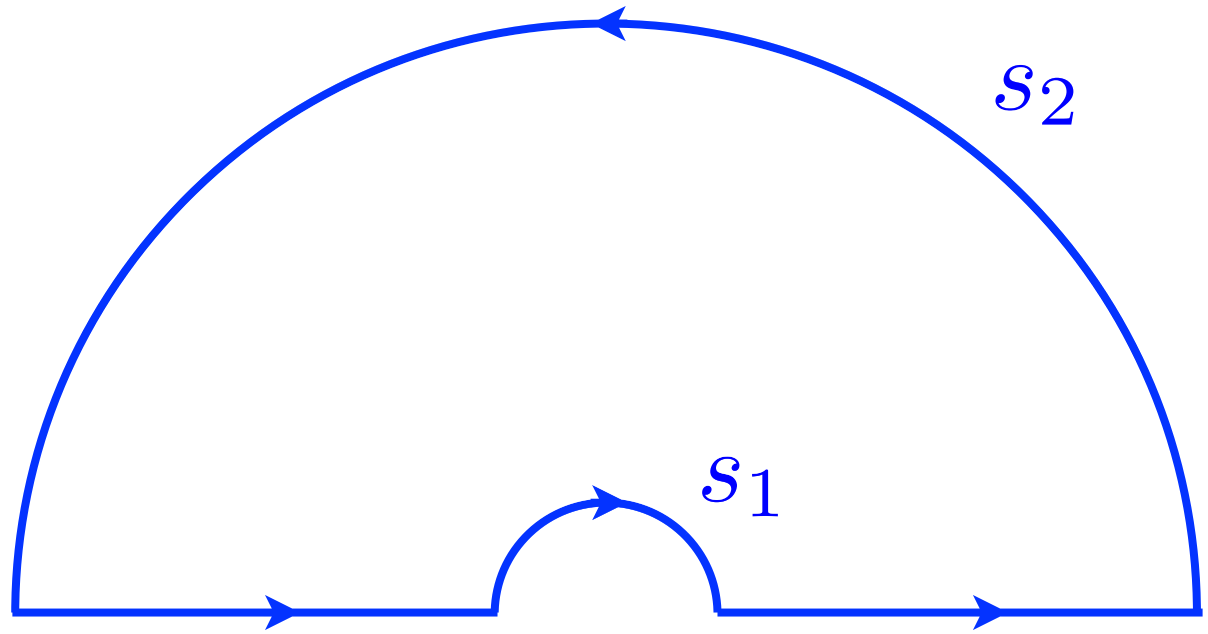
$$|\lambda_I| \ll 1$$

basic idea: $A(s)$ is finite, modulo CC term

more precisely: all UV divergences encountered in its computation must get reabsorbed in the running QFT couplings

$$A(s) = \alpha(s)s^2 \qquad\qquad \alpha(s) \equiv \alpha(\lambda(s))$$

$$\alpha(s) = -8a \qquad \text{in CFT limit}$$



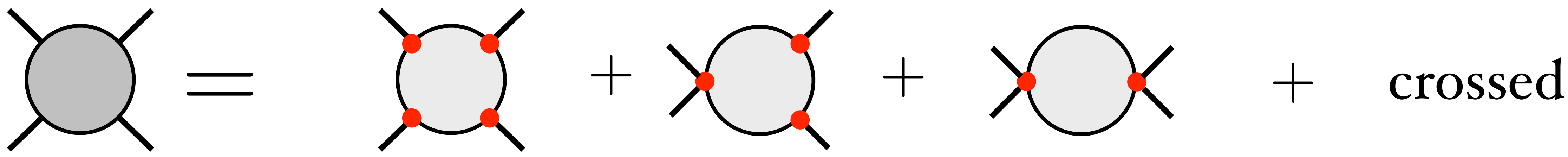
$$\bar{\alpha}(s) \equiv \frac{1}{\pi}\int_0^{\pi} d\theta\, \alpha(se^{i\theta})$$

$$\bar{\alpha}(s_2) - \bar{\alpha}(s_1) = \frac{2}{\pi}\int_{s_1}^{s_2} \frac{ds}{s}\, \mathrm{Im}\, \alpha(s)$$

$\mathrm{Im}\,\alpha(s) \rightarrow \geq 0$ by unitarity

absence of divergences ➡ $\lim_{s\to\pm\infty} \mathrm{Im}\, \alpha(s) = 0$

quickly drawing conclusions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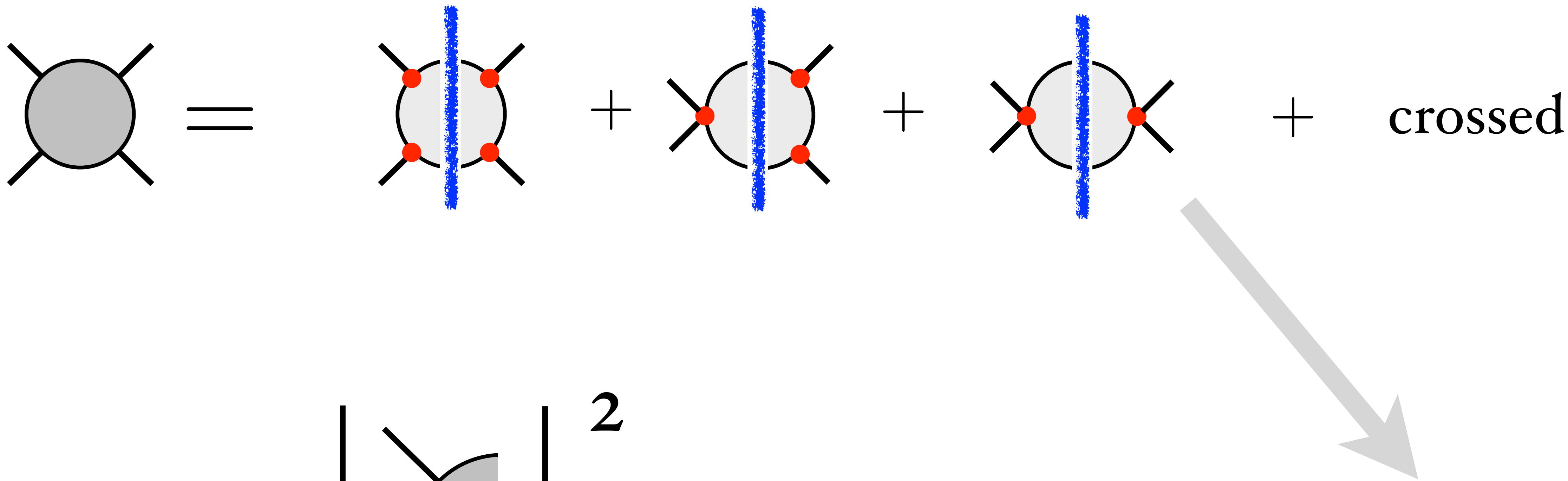
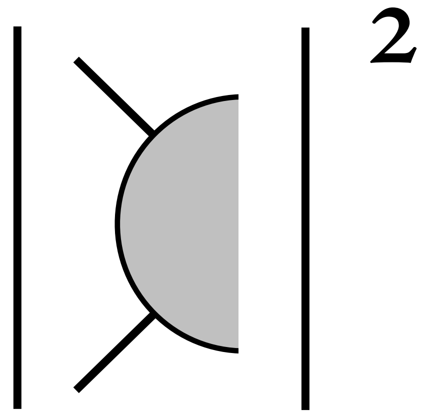
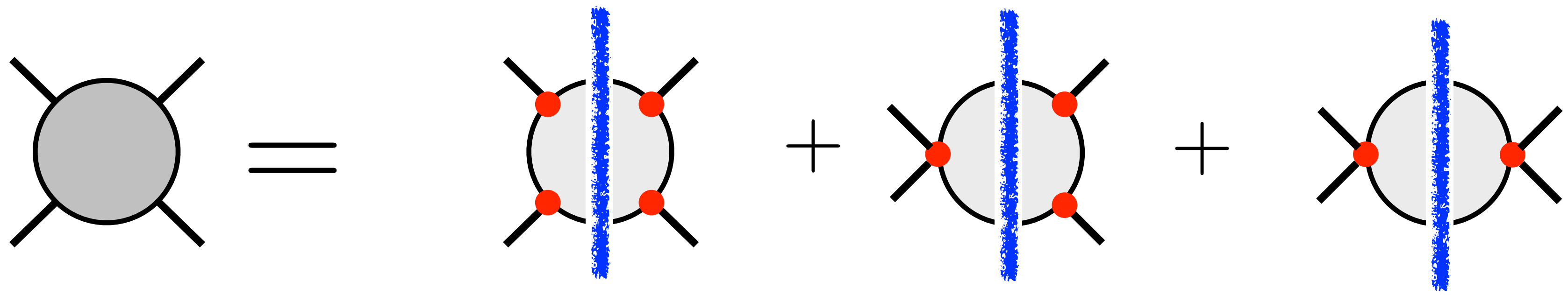


$\text{Im}\,\alpha(s)$ =

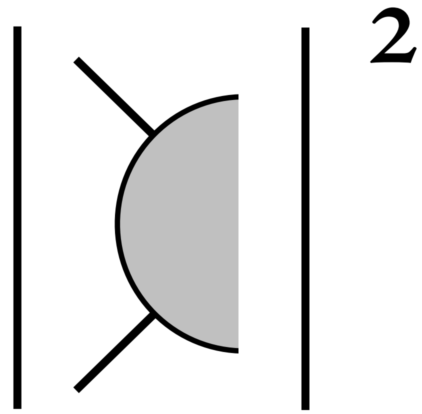


$$= \frac{1}{s^2}\sum_{\Psi}\left|\langle\Psi|\, T(p_1)T(p_2) + T(p_1+p_2)\,|0\rangle\right|^2$$



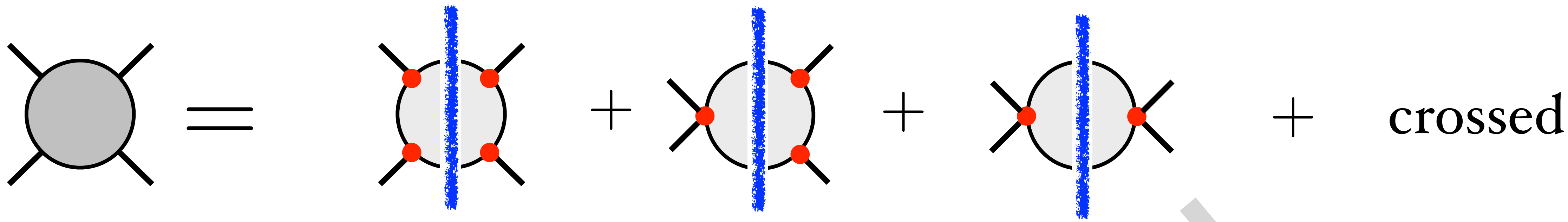
$+$ crossed

$\mathrm{Im}\,\alpha(s) =$

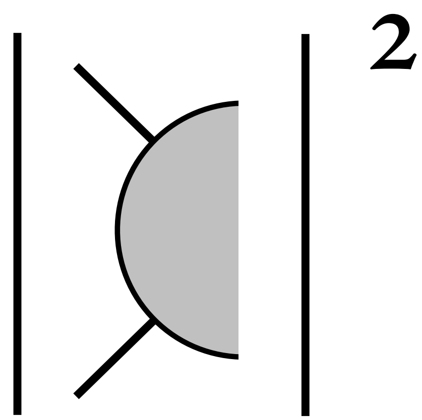


$$= \frac{1}{s^2}\sum_{\Psi}\left|\langle\Psi|\,T(p_1)T(p_2)+T(p_1+p_2)\,|0\rangle\right|^2$$

$$= \sum_I \beta_I \mathcal{O}_I$$



$$\operatorname{Im}\alpha(s) =$$



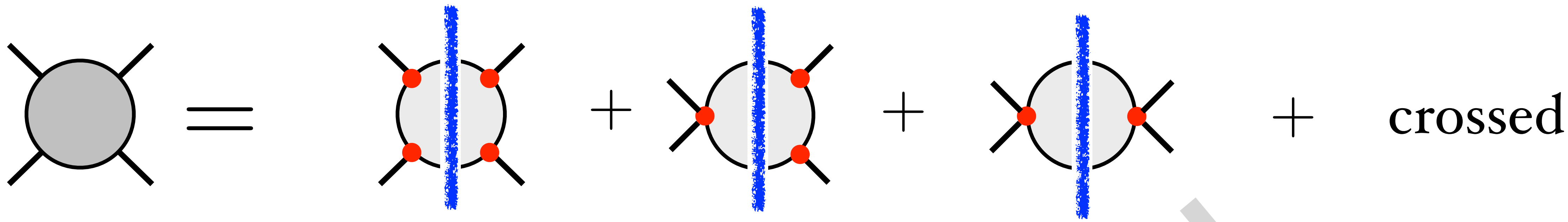
$$= \frac{1}{s^2}\sum_{\Psi}\left|\langle\Psi|\, T(p_1)T(p_2) + T(p_1+p_2)\,|0\rangle\right|^2$$

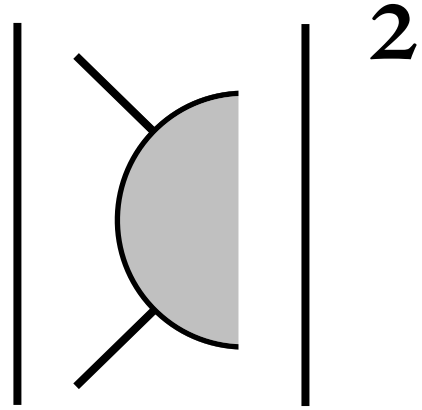
subleading if

$\lambda_I,\ \beta_J \ll 1$

$= \sum_I \beta_I \mathcal{O}_I$

qualification needed !

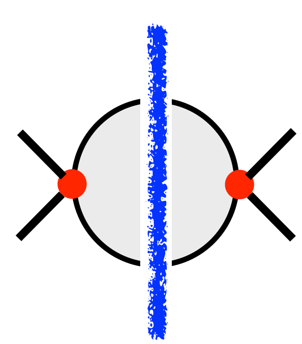


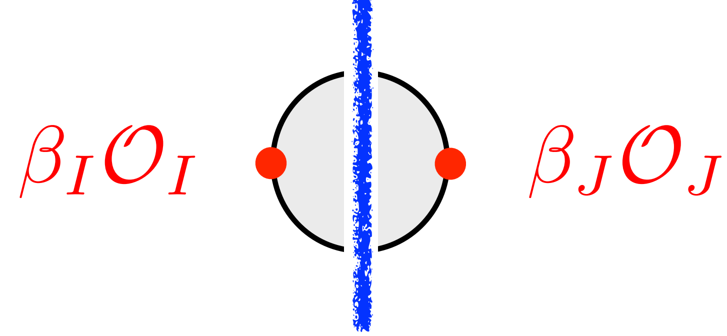
$\mathrm{Im}\,\alpha(s)$ =  = $\frac{1}{s^2}\sum_{\Psi}\left|\langle\Psi|\,T(p_1)T(p_2)+T(p_1+p_2)\,|0\rangle\right|^2$

subleading if $\lambda_I,\ \beta_J \ll 1$

$= \sum_I \beta_I \mathcal{O}_I$

qualification needed !

$\mathrm{Im}\,\alpha$ is dominated by  like in 2D proof !!



$$\mathrm{Im}\,\alpha = \sum_{IJ} \beta_I \beta_J \left[\underbrace{\frac{\mathrm{Im}\,\langle \mathcal{O}_I \mathcal{O}_J \rangle}{s^2}}_{C_{IJ}} + O(\lambda) \right]$$

C_{IJ} positive definite by unitarity

$\int \frac{ds}{s}\,\mathrm{Im}\,\alpha$ finite $\Longleftrightarrow$ $\beta_I \to 0$ asymptotically

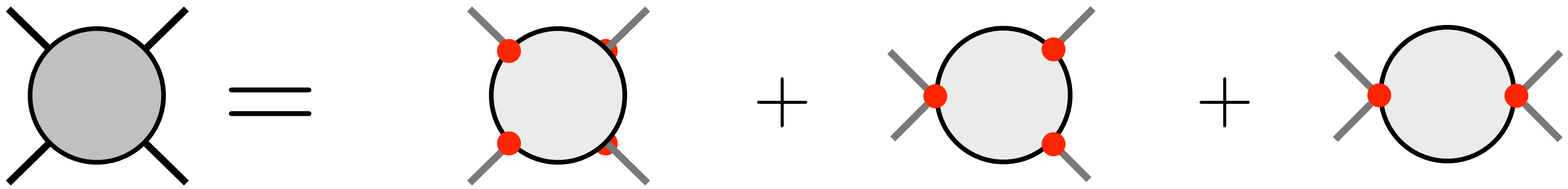
The theory necessarily asymptotes a CFT !

For instance, in the case of perturbations of free field theory the matrix C is given by

$$\mathcal{O}_1 = \frac{1}{4!}\Phi^4\,, \qquad c_{11} = \frac{1}{2^{10}(4!)^2\pi^6}$$

$$\mathcal{O}_2 = \Phi\bar{\Psi}\Psi\,, \qquad c_{22} = \frac{1}{2^4 4!\pi^4}$$

$$\mathcal{O}_3 = F_{\mu\nu}^2/4g^4\,, \qquad c_{33} = \frac{1}{2^5\pi^2 g^4}$$



$$\mathcal{M}(x_1,\ldots,x_4) \quad = \langle T(x_1)T(x_2)T(x_3)T(x_4)\rangle + \delta^4(x_1-x_2)\langle T(x_1)T(x_3)T(x_4)\rangle + \text{permutations}$$

$$+\,\delta^4(x_1-x_2)\delta^4(x_3-x_4)\langle T(x_1T(x_3)\rangle + \text{permutations}$$

naively one would proceed by substituting $\quad T = \sum_I \beta^I \mathcal{O}_I$

but we must apply more care....

I. Naive substitution $T = \sum_I \beta^I \mathcal{O}_I$ can only be correct when considering insertions at non-coinciding points: $x_i \neq x_j$

additional contact terms appear

$$T(x)T(y) = \sum_{IJ} \beta^I \mathcal{O}_I(x)\beta^J \mathcal{O}_J(y) + \delta^4(x-y) \times \ ?$$

indeed result dictated by dilation Ward id.

$$T = \partial_\mu S^\mu$$

$$\partial_\mu \langle S^\mu(x)\mathcal{O}(y)\dots\rangle = \delta(x-y)\langle \delta_S \mathcal{O}(x)\dots\rangle + \dots$$

II. In general there is more than just β's

$$T = \beta^I \mathcal{O}_I + \underbrace{S^A \partial_\mu J^\mu_A + t^a \Box \mathcal{O}_a}$$

d ~4 scalars that can mix in

$$T = \beta^I \mathcal{O}_I + S^A \partial_\mu J^\mu_A + t^a \Box \mathcal{O}_a$$

J^μ_A $\longleftrightarrow$ global symmetry G of fixed point

explicitly broken by marginal couplings λ^I

around free field theory: flavor group

$\dim \mathcal{O}_a \sim 2$ exists in

- theories with weakly coupled scalars
- supersymmetry with nearly conserved currents

Systematic treatment

Generating functional for composite operators

promote all relevant couplings to local sources

$$T_{\mu\nu} \leftrightarrow g_{\mu\nu}(x)$$

$$\mathcal{O}_I \leftrightarrow \lambda_I(x)$$

$$J_\mu^A \leftrightarrow A_\mu^A(x)$$

$$\mathcal{O}_a \leftrightarrow m_a(x)$$

$$\mathcal{O}_I(x) = \frac{\delta}{\delta\lambda_I(x)} W$$

etc ...

$$W \equiv W[g_{\mu\nu}, \lambda^I, A_\mu^A, m_a, \dots]$$

n-point correlators of T can be systematically written (in terms of correlators of the other operators) via the ***local Callan-Symanzik equation***

Jack, Osborn '90
Osborn '91

The local Callan-Symanzik equation

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$$\left(2g^{\mu\nu}\frac{\delta}{\delta g^{\mu\nu}} - \beta^I\frac{\delta}{\delta\lambda_I} - \rho_I^A\nabla_\mu\lambda^I\frac{\delta}{\delta A_\mu^A} + \dots\right)W = \mathcal{A} = \text{local}$$

basic idea

- by assigning suitable Weyl transformation properties to sources

$$\delta_W g^{\mu\nu} = 2\sigma g^{\mu\nu} \qquad \delta_W\lambda^I = \sigma\beta^I \qquad \dots$$

$W[\text{sources}]$ can be made Weyl invariant up to a local anomaly term

- integrating over spacetime, one recovers the usual, 'global', CS equation easy to prove in dimensional regularization

$$\mathcal{L}_0 = \mathcal{L}_0^{(1)} + \mathcal{L}_0^{(2)}$$

- depends on both sources and fields
- Weyl invariant

- depends on sources only
- not Weyl invariant

$$\delta_W W = \delta_W \mathcal{L}_0^{(2)} = \mathcal{A}$$

The unabridged local Callan-Symanzik equation

$$\int d^4x \Bigg\{ \sigma(x) \left[2g^{\mu\nu} \frac{\delta}{\delta g^{\mu\nu}(x)} - \beta^I \frac{\delta}{\delta \lambda^I(x)} - \rho_I^A \nabla_\mu \lambda^I \frac{\delta}{\delta A_\mu^A(x)} + \tilde{m}^a \frac{\delta}{\delta m^a(x)} \right] +$$

$$+\nabla_\mu \sigma(x) \left[\theta_I^a \nabla^\mu \lambda^I \frac{\delta}{\delta m^a(x)} - S^A \frac{\delta}{\delta A_\mu^A(x)} \right] - \Box \sigma(x) t^a \frac{\delta}{\delta m^a(x)} \Bigg\} W \quad =$$

$$= \quad \int d^4x \, \sigma(x) \, \mathcal{A}(x)$$

$$2\tilde{m}^a \; = \; 2m^b(\delta_b^a + \gamma_b^a) + \frac{1}{3}\eta^a R + d_I^a \Box \lambda^I + \frac{1}{2} \epsilon_{IJ}^a \nabla_\mu \lambda^I \nabla^\mu \lambda^J$$

schematically

$$\int d^4x\,\sigma(x)\left[2g^{\mu\nu}\frac{\delta}{\delta g^{\mu\nu}(x)} - \beta^I\frac{\delta}{\delta\lambda^I(x)} - \rho_I^A\nabla_\mu\lambda^I\frac{\delta}{\delta A_\mu^A(x)} + \dots\right] W \quad = \quad \int d^4x\,\sigma(x)\,\mathcal{A}$$

$$\left[\Delta_\sigma^g - \Delta_\sigma^\beta\right]\, W \quad = \quad \int d^4x\,\sigma\,\mathcal{A}$$

dilaton background

$$g_{\mu\nu} \;=\; \Omega^2\,\eta_{\mu\nu}$$

$$2g^{\mu\nu}\frac{\delta}{\delta g^{\mu\nu}(x)} \;\rightarrow\; \Omega\frac{\delta}{\delta\Omega}$$

by iterating CS eq. we obtain dilaton n-point amplitudes

Redundancies

$$\sigma(x)\left[2g^{\mu\nu}\frac{\delta}{\delta g^{\mu\nu}(x)}-\beta^I\frac{\delta}{\delta\lambda^I(x)}-\rho_I^A\nabla_\mu\lambda^I\frac{\delta}{\delta A_\mu^A(x)}+\tilde{m}^a\frac{\delta}{\delta m^a(x)}\right]+$$

$$+\nabla_\mu\sigma(x)\left[\theta_I^a\nabla^\mu\lambda^I\frac{\delta}{\delta m^a(x)}-S^A\frac{\delta}{\delta A_\mu^A(x)}\right]-\Box\sigma(x)t^a\frac{\delta}{\delta m^a(x)}$$

$$t^a \leftrightarrow t^a R\mathcal{O}_a$$

$$\theta_I^a \ : \ \ \mathcal{O}_I \ \to \ \mathcal{O}_I+\theta_{Ia}\Box\mathcal{O}_a$$

scheme choice ⇒

$$t^a=0$$

$$\theta_I^a=0$$

S^A : can be rewritten using Ward identity of explicitly broken global symmetry

$$\hat{S}\equiv S^A T_A$$

$$\int d^4x\left[\nabla_\mu(\sigma S^A)\frac{\delta}{\delta A_\mu^A}-\sigma(\hat{S}^A\cdot\lambda)^I\frac{\delta}{\delta\lambda^I}-\sigma(\hat{S}\cdot m)^a\frac{\delta}{\delta m^a}\right]W=0$$

Redundancies

$$\sigma(x)\left[2g^{\mu\nu}\frac{\delta}{\delta g^{\mu\nu}(x)} - \beta^I\frac{\delta}{\delta\lambda^I(x)} - \rho_I^A\nabla_\mu\lambda^I\frac{\delta}{\delta A_\mu^A(x)} + \tilde{m}^a\frac{\delta}{\delta m^a(x)}\right] +$$

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Redundancies

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$$\int d^4x\left[\nabla_\mu(\sigma S^A)\frac{\delta}{\delta A_\mu^A} - \sigma(\hat{S}^A\cdot\lambda)^I\frac{\delta}{\delta\lambda^I} - \sigma(\hat{S}\cdot m)^a\frac{\delta}{\delta m^a}\right]W = 0$$

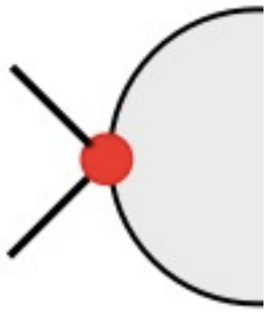
$$\int \sigma(x) \left[2g^{\mu\nu} \frac{\delta}{\delta g^{\mu\nu}(x)} - B^I \frac{\delta}{\delta \lambda^I(x)} - P_I^A \nabla_\mu \lambda^I \frac{\delta}{\delta A_\mu^A(x)} + \tilde{M}^a \frac{\delta}{\delta m^a(x)} \right] W = \int \sigma \mathcal{A}$$

$$\begin{aligned} B^I &= \beta^I - (\hat{S} \cdot \lambda)^I \\ \tilde{M}^a &= \tilde{m}^a - (\hat{S} \cdot \tilde{m})^a \\ P_I^A &= \rho_I^A + \partial_I S^A \end{aligned}$$

Notice: the trace of T is not controlled by naive β-function!

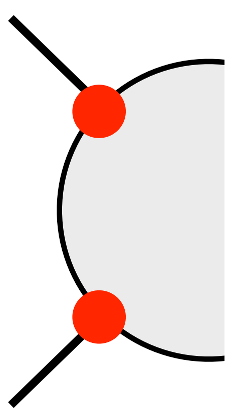
$$T_\mu^\mu = B^I \mathcal{O}_I + \ldots$$

$$T_\mu^\mu \neq \beta^I \mathcal{O}_I + \ldots$$



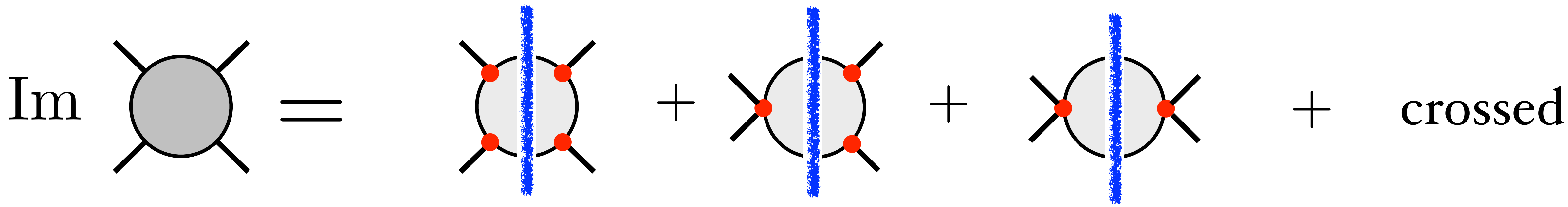
$$T(x) = B^I \frac{\delta}{\delta\lambda^I(x)} W$$

$$B^I \mathcal{O}_I(x)$$



$$T(x)T(y) = \left(B^I B^J \frac{\delta}{\delta\lambda^I(x)} \frac{\delta}{\delta\lambda^J(y)} + \delta(x-y)\, B^I \partial_I B^J \frac{\delta}{\delta\lambda^J(y)} \right) W$$

$$B^I B^J \mathcal{O}_I(x) \mathcal{O}_J(y) + \delta(x-y)(B^I \partial_I B^J)\mathcal{O}_J(x)$$



$$\mathrm{Im}\,\alpha(s) \;=\; \frac{1}{s^2}\sum_{\Psi}\left|\langle\Psi|B^I(\delta_I^J+\partial_I B^J)\mathcal{O}_J(p_1+p_2)+B^IB^J\mathcal{O}_I(p_1)\mathcal{O}_J(p_2)|0\rangle\right|^2$$

$$=\; B^I B^J\, G_{IJ}$$

$$G_{IJ} \;=\; \frac{1}{s^2}\sum_{\Psi}\langle 0|\mathcal{O}_I+\partial_I B^L\mathcal{O}_L+B^L\mathcal{O}_I\mathcal{O}_L|\Psi\rangle\langle\Psi|\mathcal{O}_J+\partial_J B^K\mathcal{O}_K+B^K\mathcal{O}_J\mathcal{O}_K|0\rangle \;\geq\; 0$$

$G_{IJ} \;>\; 0$ for a small perturbation of CFT

RG invariance

$$\mathrm{Im}\,\alpha(s) \;=\; B^I(\lambda(\mu))B^J(\lambda(\mu))\,G_{IJ}(\mu/\sqrt{s},\lambda(\mu)) \;=\; B^I(\lambda(\sqrt{s}))B^J(\lambda(\sqrt{s}))\,G_{IJ}(1,\lambda(\sqrt{s}))$$

$$\bar{\alpha}(s_2) \;-\; \bar{\alpha}(s_1) \;=\; \frac{2}{\pi}\int_{s_1}^{s_2}\frac{ds}{s}\,\mathrm{Im}\,\alpha(s)$$

$$s\frac{d\bar{\alpha}(s)}{ds} \;=\; B^I(s)B^J(s)G_{IJ}(s)$$

4D version of ‘local’ Zamolodchikov theorem

Remarkably

same equation obtained by Wess-Zumino consistency condition

Jack, Osborn '90

$$[\Delta^{CS}_{\sigma_1}, \Delta^{CS}_{\sigma_2}]\, W \;=\; 0$$

however without insight provided by dilaton trick was not obvious how to prove $G_{IJ} \geq 0$ is true beyond perturbation theory

- UV and IR asymptotics must satisfy $B^I = \beta^I - (\hat{S} \cdot \lambda)^I = 0$

★ these asymptotics are CFT's since $T^\mu_\mu = B^I \mathcal{O}_I$

- however a computation in a standard scheme RG-flow would look like a limit cycle $\beta^I = (\hat{S} \cdot \lambda)^I \neq 0$

confirmed by explicit computation, Fortin, Grinstein, Stergiou '12 illustration of β versus B in $O(N)$ scalar theory

$$\mathcal{L}_{int} \;=\; \frac{\lambda_{ijk\ell}}{4}\,\Phi_i\Phi_j\Phi_k\Phi_\ell \;\equiv\; \lambda_{ijk\ell}\mathcal{O}_{ijk\ell}$$

using operator language

$[\Phi_1 \ldots \Phi_n] \equiv$ renormalized composite operator

- $$T(x) \;=\; \beta_{ijk\ell}[\mathcal{O}_{ijk\ell}] + \Gamma_{ij}[\Phi_j \frac{\delta S}{\delta \Phi_i}] + S_{ij}\partial_\mu [J^\mu_{ij}] + a_{ij}\Box[\Phi_i \Phi_j]$$

- $$(N\cdot\lambda)_{ijk\ell}[\mathcal{O}_{ijkl}] + N_{ij}\left([\Phi_j \frac{\delta S}{\delta \Phi_i}] + \partial_\mu [J^\mu_{ij}]\right) = 0$$ Ward identity $$N_{ij} = -N_{ji}$$

coefficients defined modulo reparametrization

$$\beta_{ijk\ell} \to \beta_{ijk\ell} + (N\cdot\lambda)_{ijk\ell}\,, \qquad \Gamma_{ij} \to \Gamma_{ij} + N_{ij}\,, \qquad S_{ij} \to S_{ij} + N_{ij}$$

Osborn '91

$\exists$ family of Callan-Symanzik eqs. satisfied by same theory !

$= \delta_{ij} + \gamma_{ij}$

- $$T(x) \;=\; \beta_{ijk\ell}[\mathcal{O}_{ijk\ell}] + \Gamma_{ij}[\Phi_j \frac{\delta S}{\delta \Phi_i}] + S_{ij}\partial_\mu [J^\mu_{ij}] + a_{ij}\Box[\Phi_i \Phi_j]$$

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Osborn '91

$\exists$ family of Callan-Symanzik eqs. satisfied by same theory !

$$= \delta_{ij} + \gamma_{ij}$$

- $$T(x) \; = \; \beta_{ijk\ell}[\mathcal{O}_{ijk\ell}] + \Gamma_{ij}[\Phi_j \frac{\delta S}{\delta \Phi_i}] + S_{ij}\partial_\mu [J^\mu_{ij}] + \cancel{a_{ij}\Box[\Phi_i\Phi_j]}$$

- $$(N \cdot \lambda)_{ijk\ell}[\mathcal{O}_{ijkl}] + N_{ij}\left([\Phi_j \frac{\delta S}{\delta \Phi_i}] + \partial_\mu [J^\mu_{ij}]\right) = 0$$

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Osborn '91

∃ family of Callan-Symanzik eqs. satisfied by same theory !

‘fix gauge’ $$T(x) \;=\; B_{ijk\ell}[\mathcal{O}_{ijk\ell}] + G_{ij}[\Phi_j \frac{\delta S}{\delta \Phi_i}] + a_{ij}\Box[\Phi_i\Phi_j]$$

$$N_{ij} \;=\; -S_{ij} \qquad B_{ijk\ell} \;=\; \beta_{ijk\ell} - (S\cdot\lambda)_{ijk\ell} \qquad G_{ij} \;=\; \Gamma_{ij} - S_{ij}$$

- UV and IR asymptotics must satisfy $B_{ijk\ell} \;=\; \beta_{ijk\ell} - (S\cdot\lambda)_{ijk\ell} \;=\; 0$

★ these asymptotics are CFT’s

- however a computation in a standard scheme RG-flow would look like a limit cycle $\beta \;=\; (S\cdot\lambda) \;\neq\; 0$

confirmed by explicit computation, Fortin, Grinstein, Stergiou ’12

Corollary: perturbative SFTs are ruled out

An SFT would have the following coefficients in an arbitrary scheme

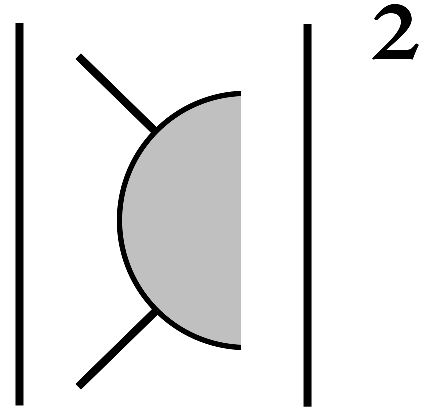
$$\beta_{ijk\ell} = (\tilde{S} \cdot \lambda)_{ijk\ell} \qquad\qquad S_{ij} \neq \tilde{S}_{ij}$$

can choose a 'gauge' where

$$T = 0 + \underbrace{(S - \tilde{S})_{ij} \partial_\mu [J^\mu_{ij}]}_{\equiv\ \partial_\mu V^\mu} + \text{e.o.m}$$

Non perturbative argument contra 4D SFTs

$\mathrm{Im}\, a(s)$ =



= $C = \text{const}$

absence of divergences

$$C \;=\; \frac{1}{s^2}\sum_{\Psi}\Big|\langle\Psi|\, T(p_1)T(p_2) + T(p_1+p_2)\,|0\rangle\Big|^2 \;=\; 0$$

by unitarity $$\mathrm{T}\{T(p_1)T(p_2)\} + T(p_1+p_2) \;=\; 0$$

- p_1 et p_2 are not arbitrary: $p_1^2 \;=\; p_2^2 \;=\; 0$

cannot yet directly infer $\mathrm{T}\{T(x_1)T(x_2)\} + \delta^4(x_1-x_2)\,T(x_1) \;=\; 0$

and conclude T is trivial

- yet the matrix elements should be very peculiar

$$\langle\Psi|\,T(p_1)T(p_2) + T(p_1+p_2)\,|0\rangle \;=\; 0$$

$\ell \;=\; 0,1,2,\ldots$ $\ell \;=\; 0$

$$\langle\Psi,\ell\geq 1|\,T(p_1)T(p_2)\,|0\rangle \;=\; 0$$

The importance of Unitarity

- Non-unitary SFT: massless vector without gauge invariance

$$S = \int d^4x \sqrt{-\hat{g}} \left(\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{h}{2} (\nabla_\mu A^\mu)^2 \right)$$

Coleman, Jackiw 1971
Riva, Cardy 2005

virial current $\quad V^\mu = h\, A_\nu F^{\mu\nu}$

partial cross section $\neq 0 \qquad \langle \Psi | T(p_1) T(p_2) + T(p_1 + p_2) |0\rangle \neq 0$

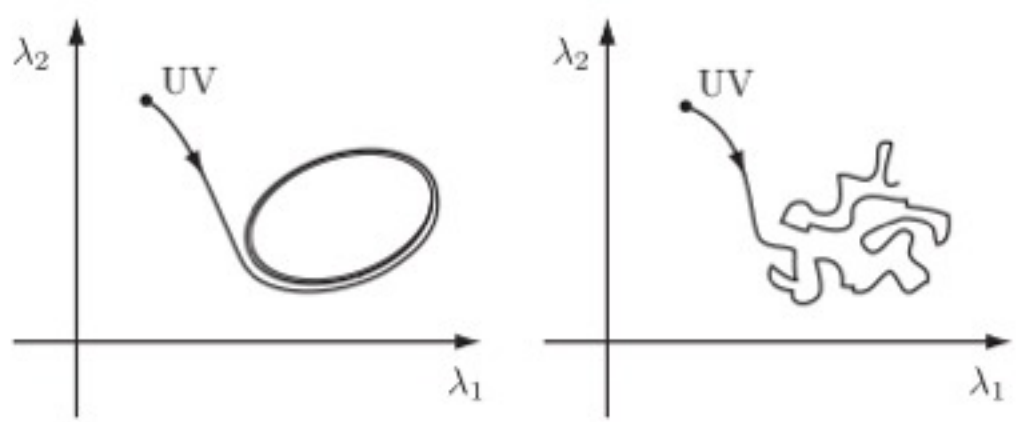
total cross section $= 0 \qquad \sum_\Psi \left| \langle \Psi | T(p_1) T(p_2) + T(p_1 + p_2) |0\rangle \right|^2 = 0$

Summary

- Finiteness of RG flow of dilaton scattering amplitude
- Unitarity

Powerful constraint on RG-flow

✦ Perturbative theories
✦ *Small* deformations of strongly coupled CFTs



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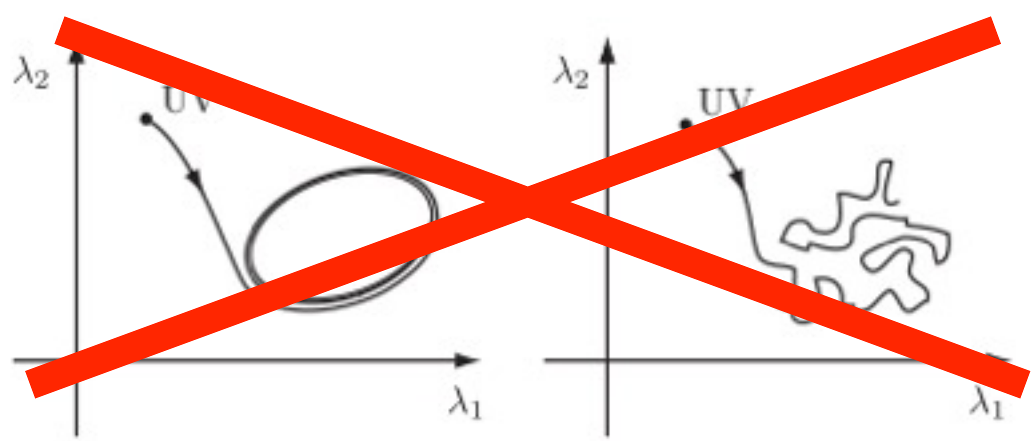

the only possible asymptotics are CFTs

✦ General case: $T \equiv T^{\mu}_{\mu}$ must be almost trivial

$$\langle\Psi|\, T(p_1)T(p_2) + T(p_1+p_2)\,|0\rangle \;=\; 0 \qquad \forall\,\Psi \qquad p_1^2 = p_2^2 = 0$$

very close to implying $T^{\mu}_{\mu} \;=\; 0$ but not there yet