Particle Physics
&
the Structure of 4D RG Flows

Riccardo Rattazzi

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

I. Particle Physics from RG flow perspective

II. Constraining the structure of RG flows in 4D

RR, S. Rychkov, E. Tonni, A. Vichi	**arXiv:0807.0004**
M. Luty, J. Polchinski, RR	**arXiv:1204.5221**
F. Baume, B. Keren-Zur, RR, L. Vitale	**in preparation**

Lecture I

Particle Physics from RG flow perspective

Effective Field Theory Paradigm

Whatever the description of physics at some high energy scale Λ_{UV} is (strings, discrete space-time, ...)

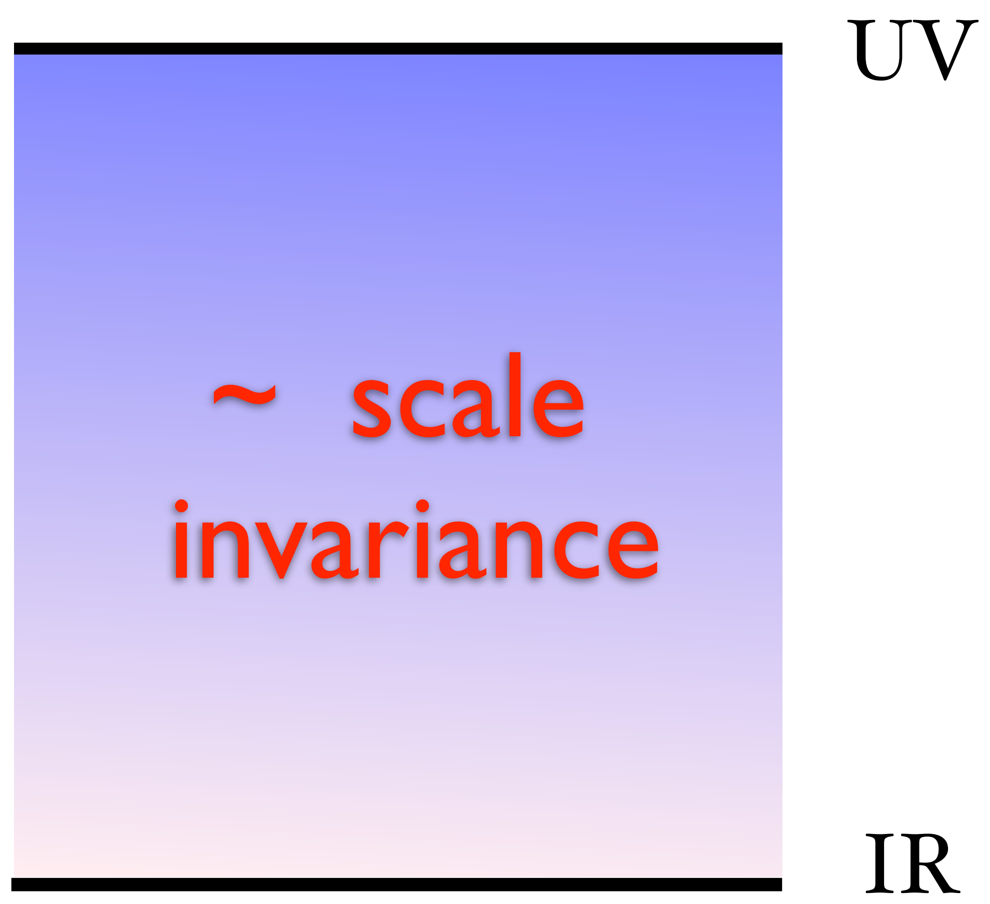
If long wavelength excitations exist

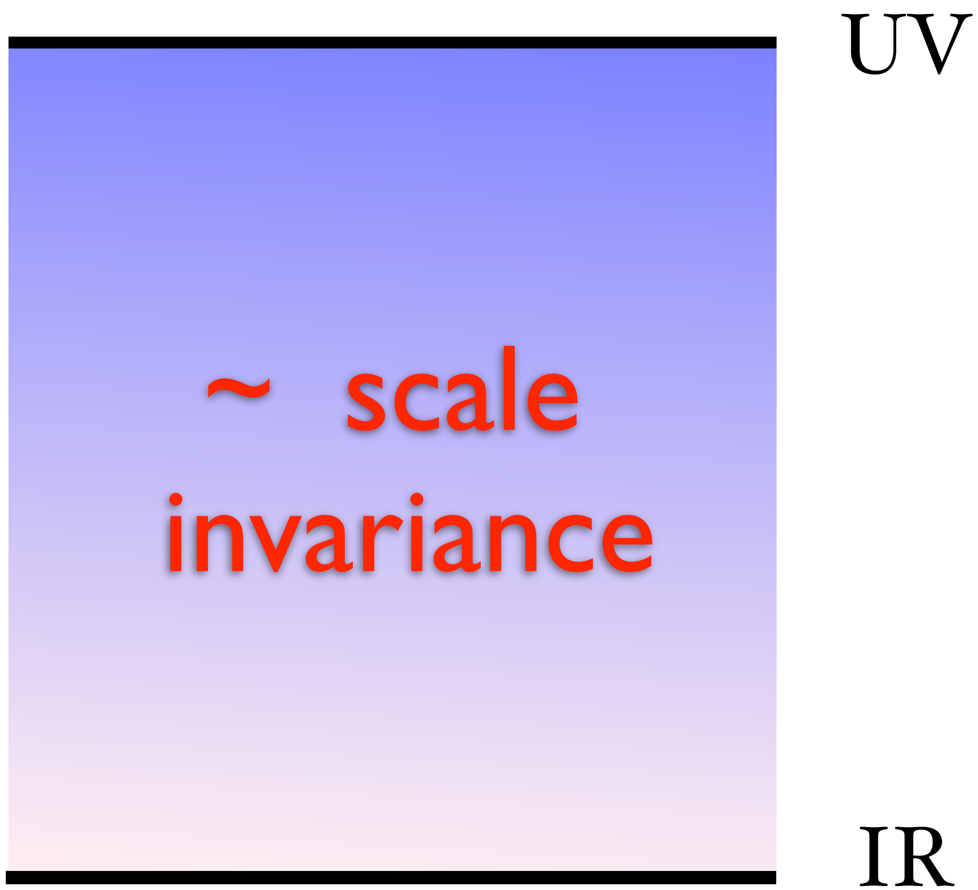
- low energy dynamics is described by an effective field theory $\equiv$ by solving an RG flow
- All structure present at UV scale decouples except for a finite number of relevant parameters

1. Decoupling of structure

UV

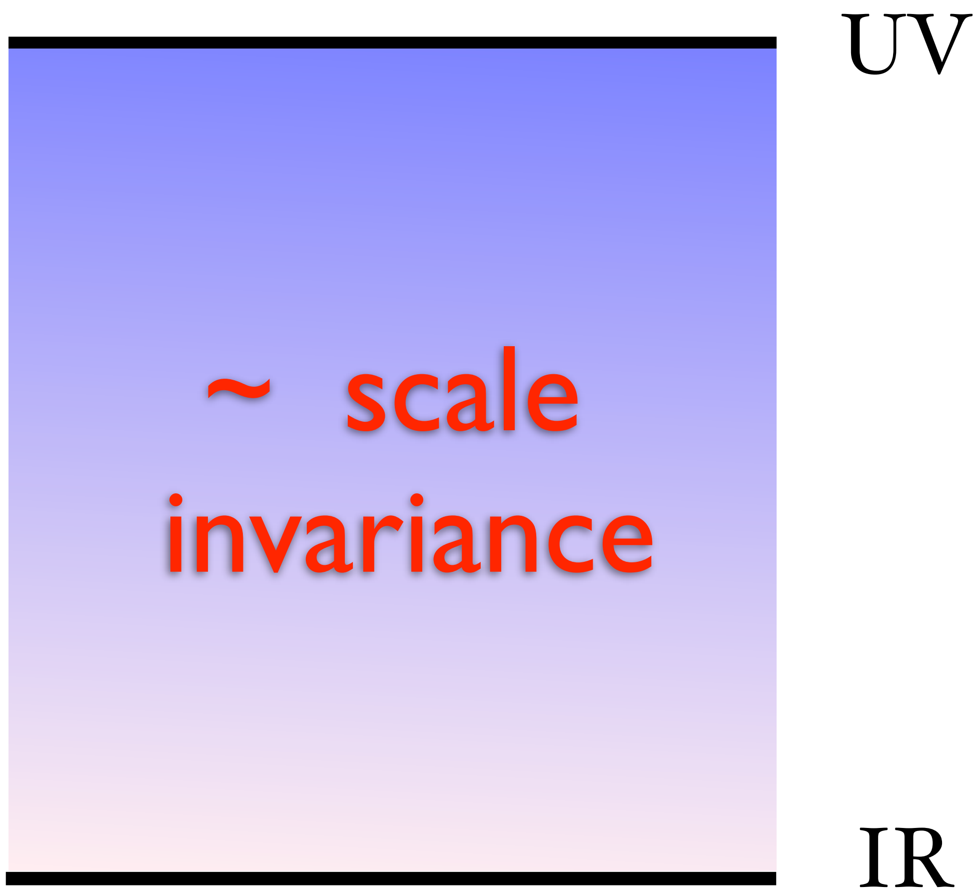
IR


UV
~ scale
invariance
IR



$$\mathcal{L} = \sum_i \lambda_i \mathcal{O}_i \qquad \dim \mathcal{O}_i = d_i$$

$$\bar{\lambda}_i(\mu) \sim \left(\frac{\mu}{\Lambda_{UV}}\right)^{d_i - 4} \bar{\lambda}_i(\Lambda_{UV})$$



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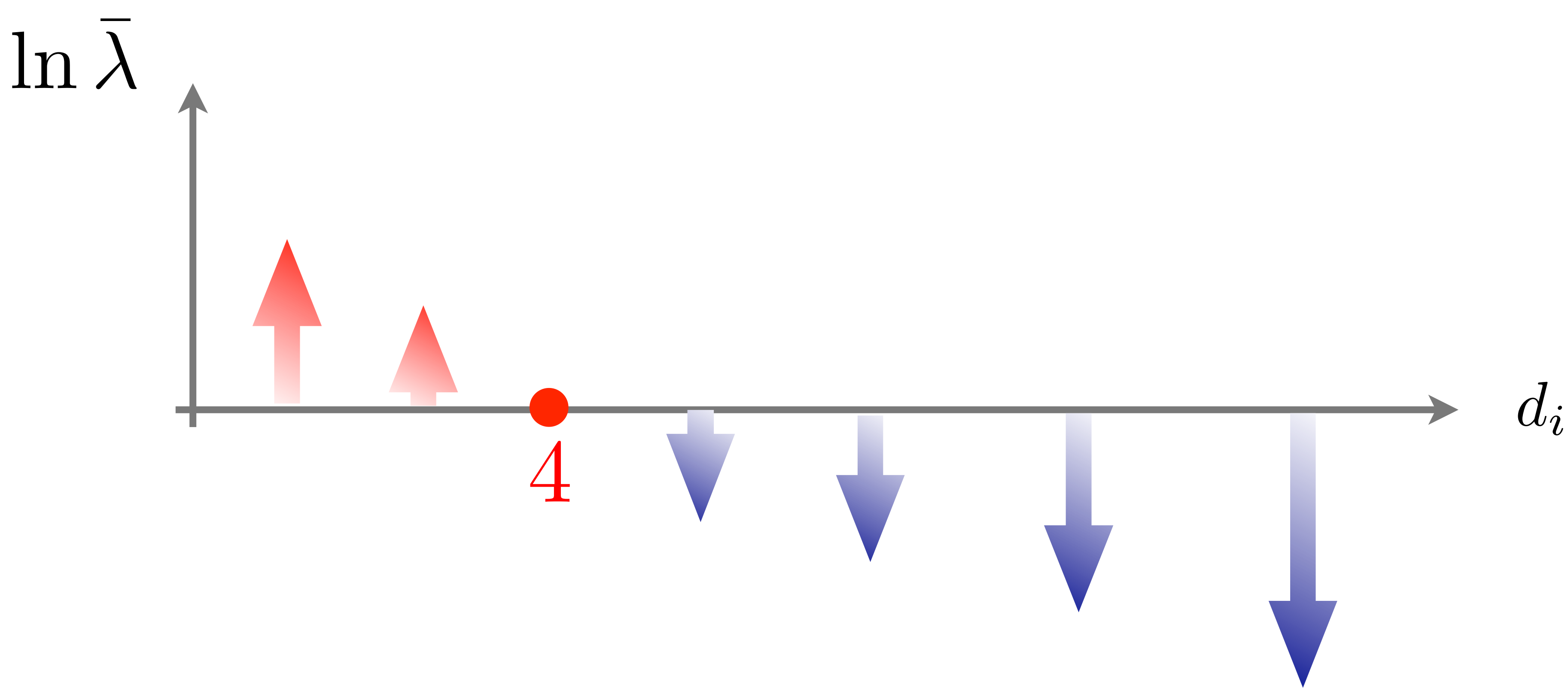
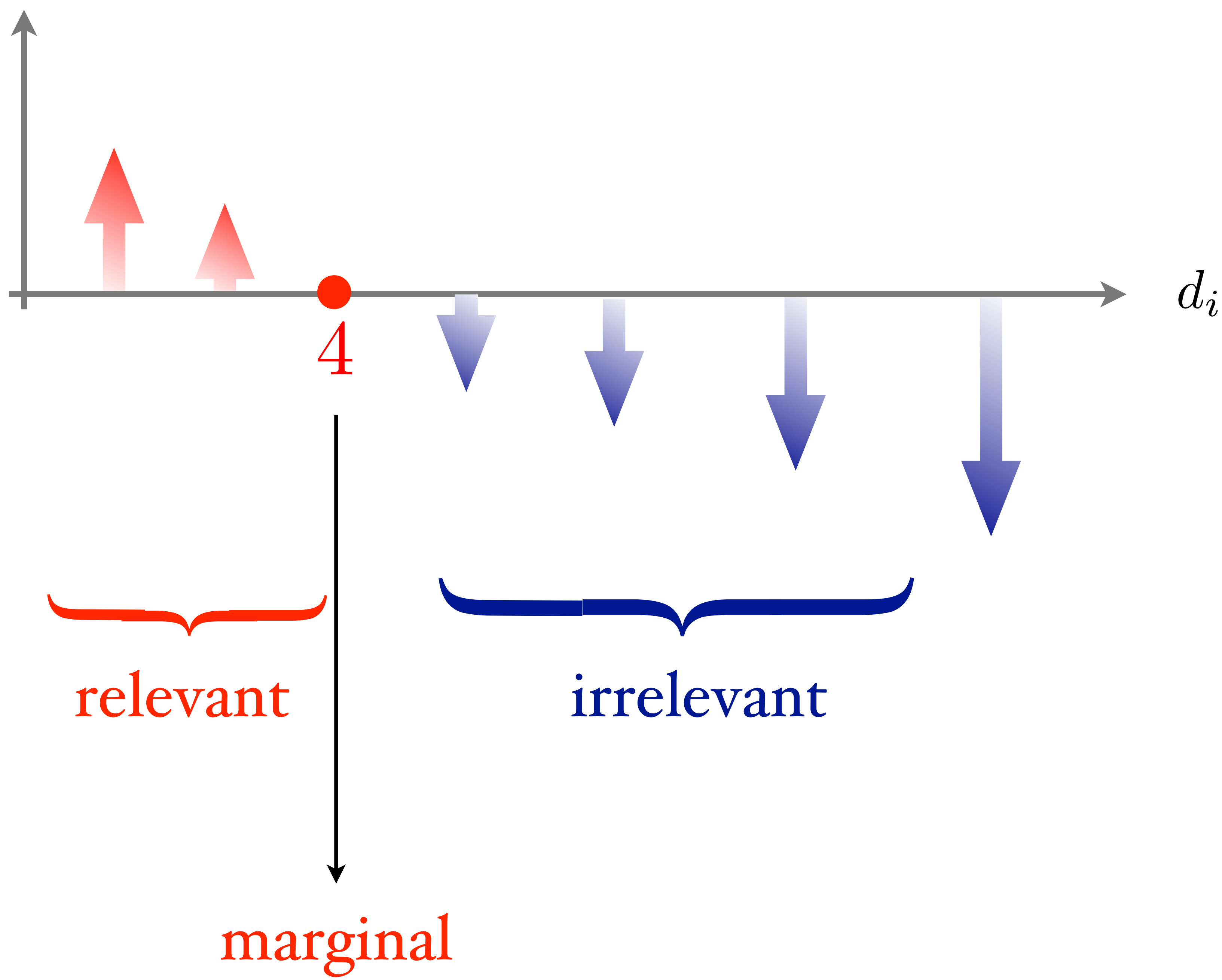
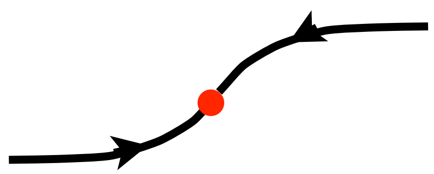
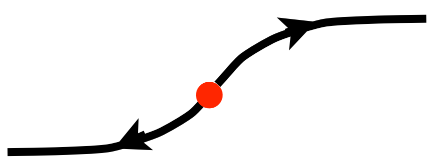



d_i
4
relevant
irrelevant
marginal

Can classify parameters according to their RG scaling $\mathcal{L} = \sum \lambda_i \mathcal{O}_i$



irrelevant $d_{\mathcal{O}} - 4 > 0$



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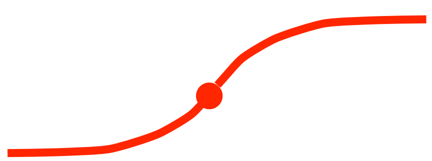


marginal $d_{\mathcal{O}} - 4 = 0$

- RG flow towards IR → infinite set of irrelevants is filtered out

- IR physics described by finitely many relevant plus marginal couplings

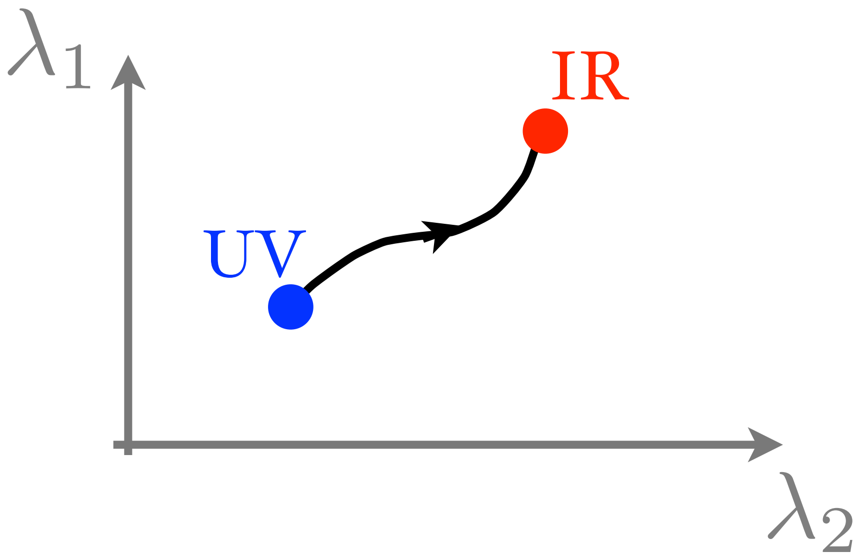
  (relevant plus marginal: renormalizable)

- occurence of accidental symmetries

- analogy with multipole expansion: every cow is spherical in first approx

2. The Origin of Mass Hierarchies (naturalness)

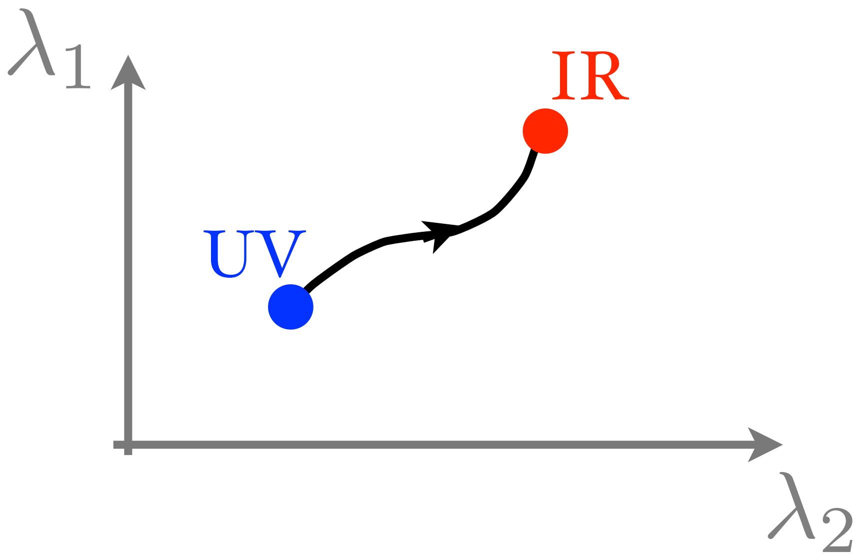
RG picture for the origin of Λ_{IR}



$$\Lambda_{IR} \sim$$

RG scale where 'distance' from UV point becomes O(1)

RG picture for the origin of Λ_{IR}



$\Lambda_{IR} \sim$ RG scale where 'distance' from UV point becomes O(1)

Ex

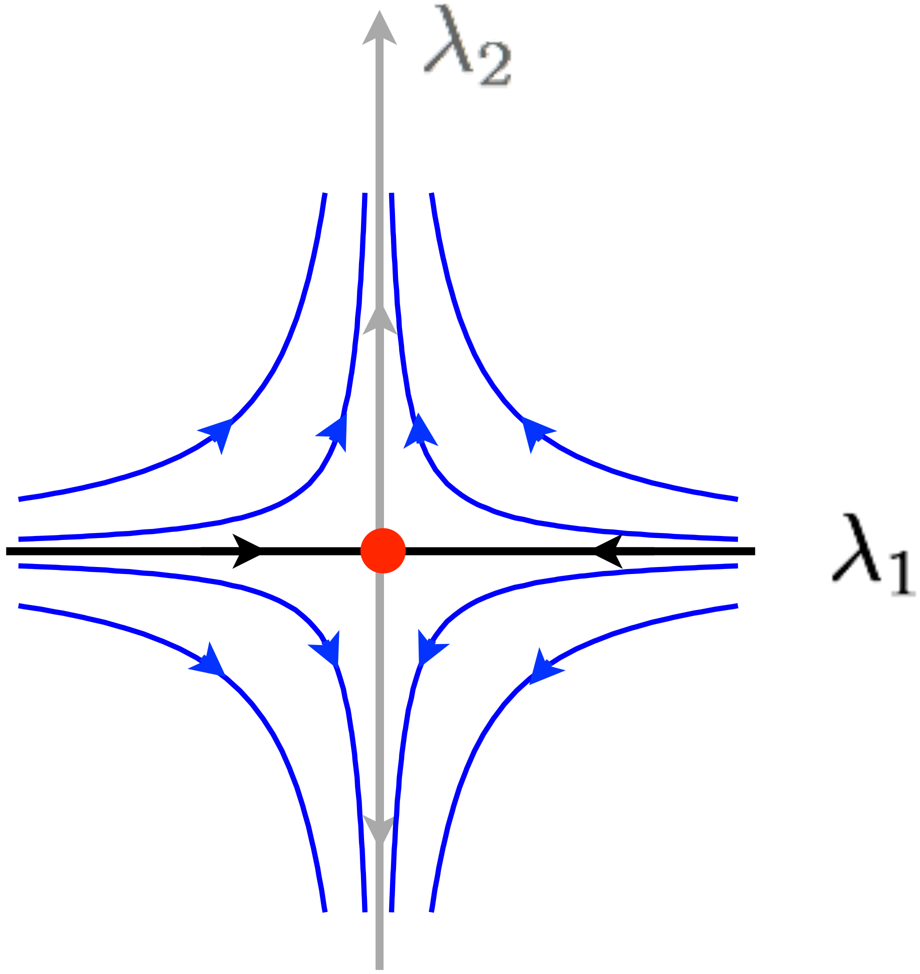
- scalar mass $\lambda(\mu) = \dfrac{m^2}{\mu^2}$

$$\mu \gg m \quad \rightarrow \quad \lambda \ll 1$$
$$\mu \sim m \quad \rightarrow \quad \lambda \sim 1$$

- QCD coupling $\lambda(\mu) = \dfrac{\alpha_s(\mu)}{4\pi}$

$$\mu \gg \Lambda_{QCD} \quad \rightarrow \quad \lambda \ll 1$$
$$\mu \sim \Lambda_{QCD} \quad \rightarrow \quad \lambda \sim 1$$

Un-natural hierarchy



$\lambda_2 \mathcal{O}_2 \qquad 4 - d_2 = O(1) > 0$

$$\lambda_2(\mu) = \lambda_2(\Lambda_{UV}) \left(\frac{\Lambda_{UV}}{\mu} \right)^{4-d_2}$$

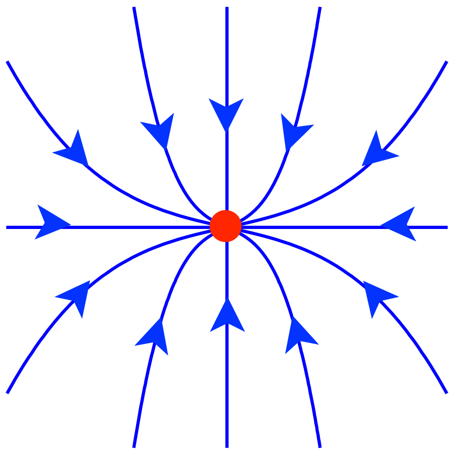
No hierarchy unless an UV parameter is tuned $\lambda_2(\Lambda_{UV}) \ll 1$

Ex: critical phenomena in thermal physics $\lambda_2(\Lambda_{UV}) \propto (T - T_c)$

Need lab technician to turn the knob and tune temperature

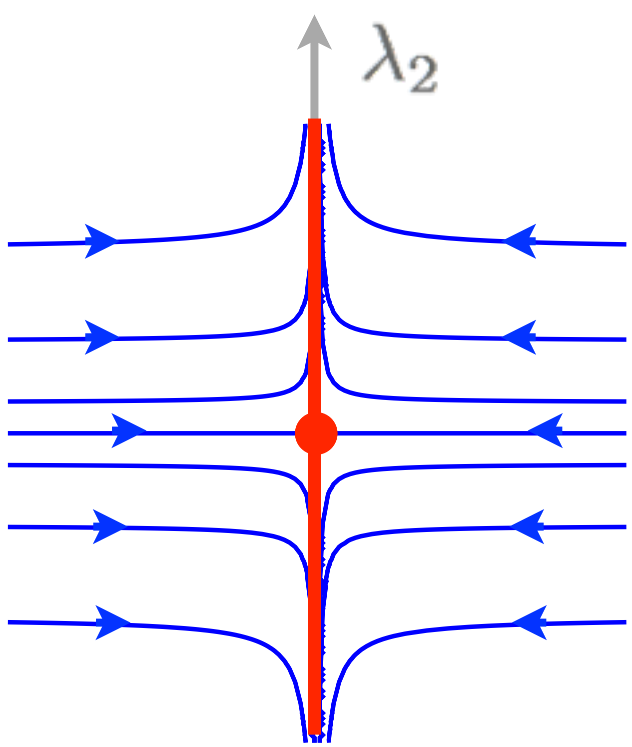
Natural hierarchy

infinite hierarchy $\Lambda_{IR} = 0$



- ✦ all coupling are irrelevant ➡ always flow to fixed point
- ✦ Ex: photons and phonons

dynamical hierarchy



$$\left.\begin{array}{l} \dfrac{d\ln\lambda_2}{d\ln\mu} = -\gamma \\ \\ 0 < \gamma \ll 1 \end{array}\right\}$$

marginally relevant coupling

λ_2 runs slowly ➡ exits fixed point at $\Lambda_{IR} \ll \Lambda_{UV}$

- Yang Mills theory
- Superconductor (BCS)

$$\gamma \sim \lambda_2$$

$$\Lambda_{IR} = \Lambda_{UV} e^{-1/\lambda_2(\Lambda_{UV})}$$

hierarchy from symmetry: All relevant couplings explicitly break some symmetry

- ✦ Couplings associated to broken symmetries can be conceivably be made naturally small, for instance through further dynamical hierarchies
- ✦ No need to turn the knob like for critical phenomena

Quantum ChromoDynamics	m_{quark}	chiral symmetry
Supersymmetric Standard Model	$m_{Sparticles}$	supersymmetry

Illustration: hierarchy from marginally relevant coupling

$$\Delta\mathcal{L} = \lambda\mathcal{O} \qquad \dim_{\mathcal{O}} = 4 - \epsilon$$

$$\lambda(\mu) = \lambda_0 \left(\frac{\Lambda_{UV}}{\mu}\right)^{\epsilon} \qquad \Lambda_{IR} \sim \lambda_0^{1/\epsilon}\,\Lambda_{UV}$$

for $\epsilon \ll 1$ a slight tuning $\lambda_0 \sim 0.1$ generates an exponential hierarchy

The Standard Model and the Hierarchy Paradox

$$\mathcal{L}_{SM} = \mathcal{L}_{kin} + gA_\mu \bar{F}\gamma_\mu F + Y_{ij}\bar{F}_i H F_j + \lambda(H^\dagger H)^2$$

d=4

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d=4

$$+ \frac{b_{ij}}{\Lambda_{UV}} L_i L_j H H$$

$$+ \frac{c_{ijkl}}{\Lambda_{UV}^2} \bar{F}_i F_j \bar{F}_k F_\ell + \frac{c_{ij}}{\Lambda_{UV}} \bar{F}_i \sigma_{\mu\nu} F_j G^{\mu\nu} + \ldots$$

$$+ \ldots$$

d>4

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$$+ \ldots$$

d>4

$\Lambda_{UV} \gg$ TeV (pointlike limit) nicely accounts for 'what we see'

$$+\theta\,\tilde{G}_{\mu\nu}\tilde{G}^{\mu\nu}$$

d=4

$$\mathcal{L}_{SM} \;=\; \mathcal{L}_{kin} + gA_\mu\bar{F}\gamma_\mu F + Y_{ij}\bar{F}_i H F_j + \lambda(H^\dagger H)^2$$

d=4

$$+ \quad \frac{b_{ij}}{\Lambda_{UV}} L_i L_j H H$$

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$$+ \quad \ldots$$

d>4

$\Lambda_{UV} \gg$ TeV (pointlike limit) nicely accounts for ‘what we see’

$$+ c\Lambda_{UV}^2 \, H^\dagger H$$

d=2

$$+ \theta \, \tilde{G}_{\mu\nu} \tilde{G}^{\mu\nu}$$

d=4

$$\mathcal{L}_{SM} \;=\; \mathcal{L}_{kin} + g A_\mu \bar{F} \gamma_\mu F + Y_{ij} \bar{F}_i H F_j + \lambda (H^\dagger H)^2$$

d=4

$$+ \quad \frac{b_{ij}}{\Lambda_{UV}} L_i L_j H H$$

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$$+ \quad \ldots$$

d>4

$\Lambda_{UV} \gg$ TeV (pointlike limit) nicely accounts for ‘what we see’

the three problems

$$+\Lambda_{UV}^4 \sqrt{g}$$ d=0

$$+c\Lambda_{UV}^2\, H^\dagger H$$ d=2

$$+\theta\, \tilde{G}_{\mu\nu}\tilde{G}^{\mu\nu}$$ d=4

$$\mathcal{L}_{SM} \;=\; \mathcal{L}_{kin} + g A_\mu \bar{F}\gamma_\mu F + Y_{ij}\bar{F}_i H F_j + \lambda (H^\dagger H)^2$$ d=4

$$+ \quad \frac{b_{ij}}{\Lambda_{UV}} L_i L_j H H$$
$$+ \quad \frac{c_{ijkl}}{\Lambda_{UV}^2}\,\bar{F}_i F_j \bar{F}_k F_\ell \;+\; \frac{c_{ij}}{\Lambda_{UV}}\bar{F}_i \sigma_{\mu\nu} F_j G^{\mu\nu} + \ldots$$
$$+ \quad \ldots$$
d>4

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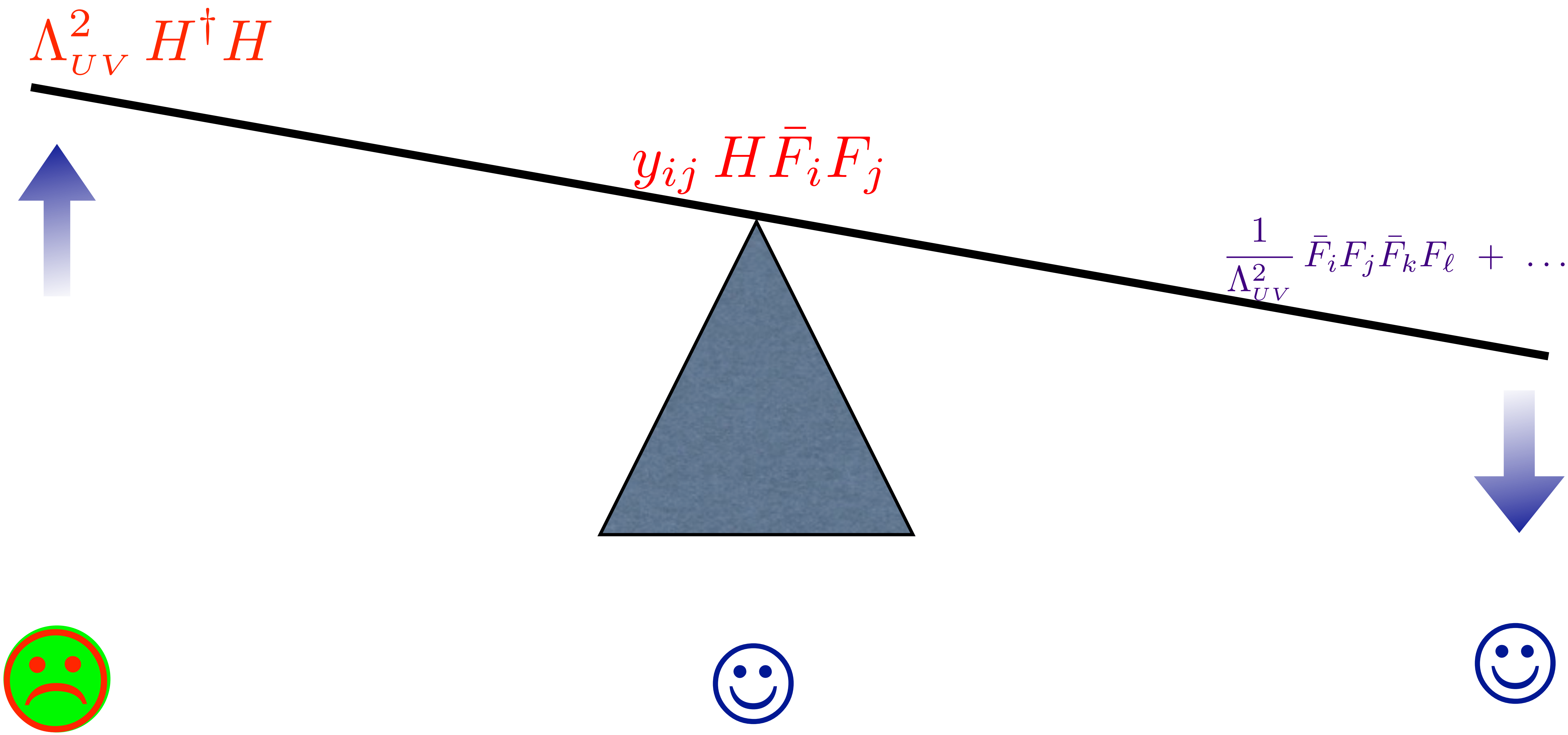
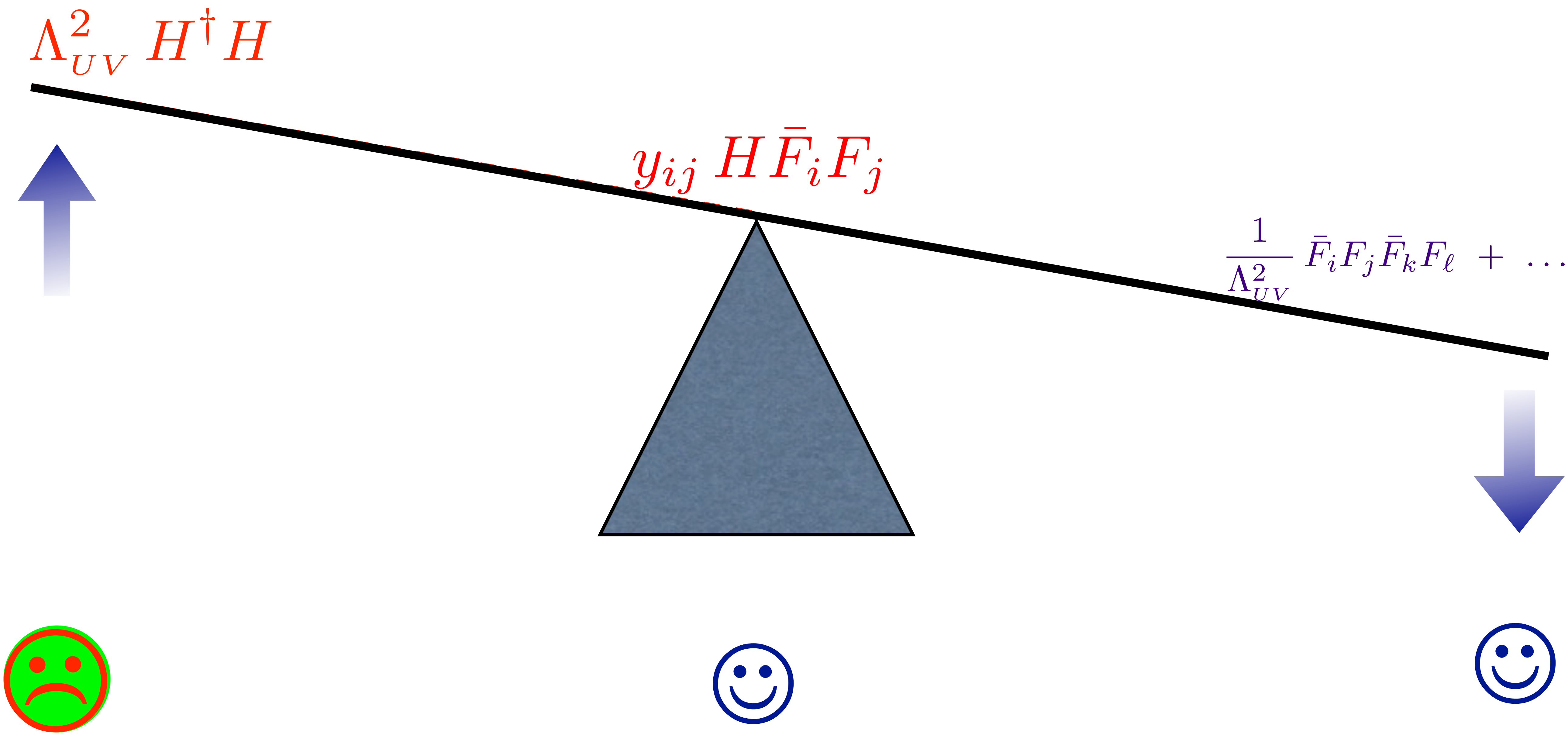
Hierarchy see-saw

Standard Model up to some $\Lambda_{UV}^2 \gg 1\,\mathrm{TeV}$



Hierarchy see-saw

Standard Model up to some $\Lambda^2_{UV} \gg 1\,\mathrm{TeV}$



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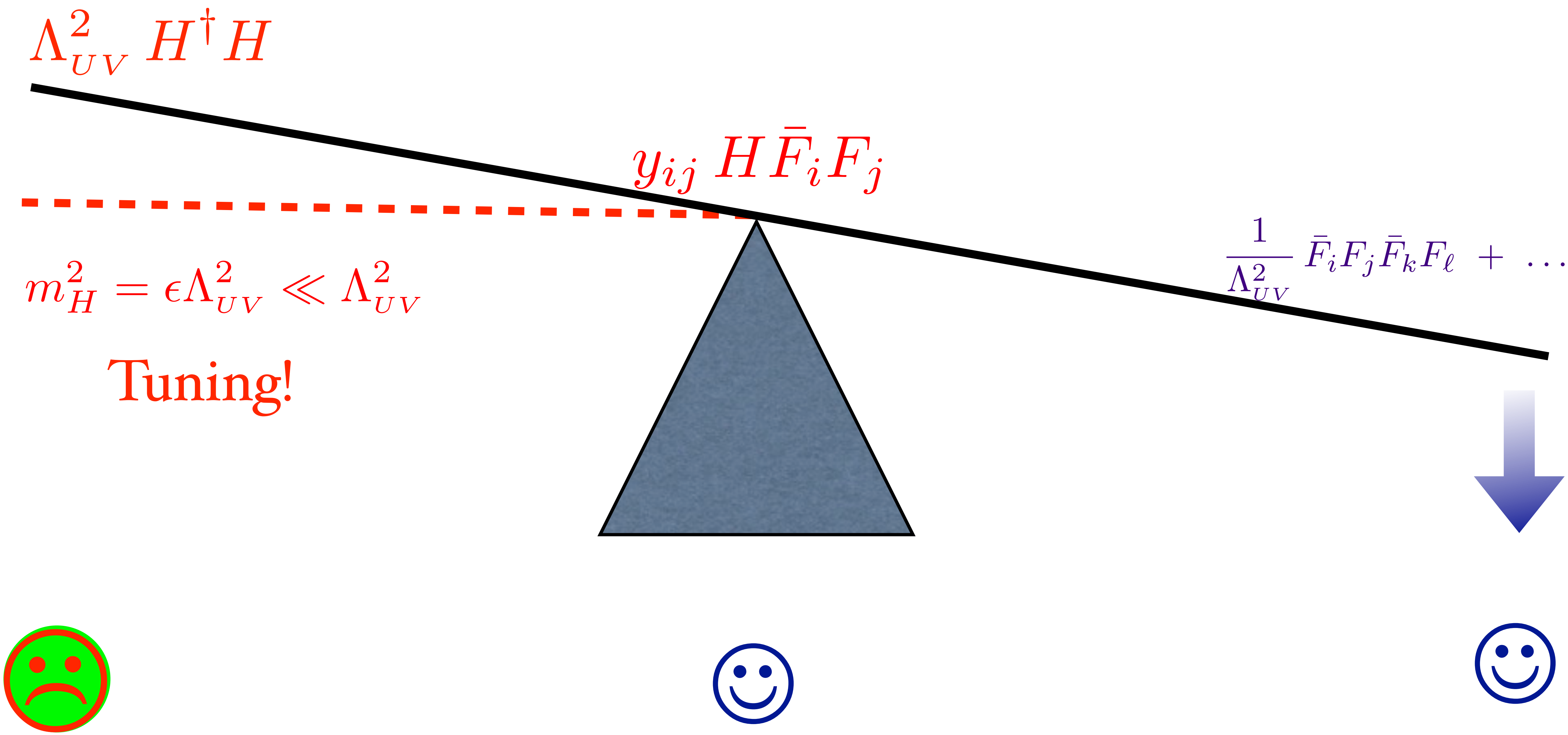
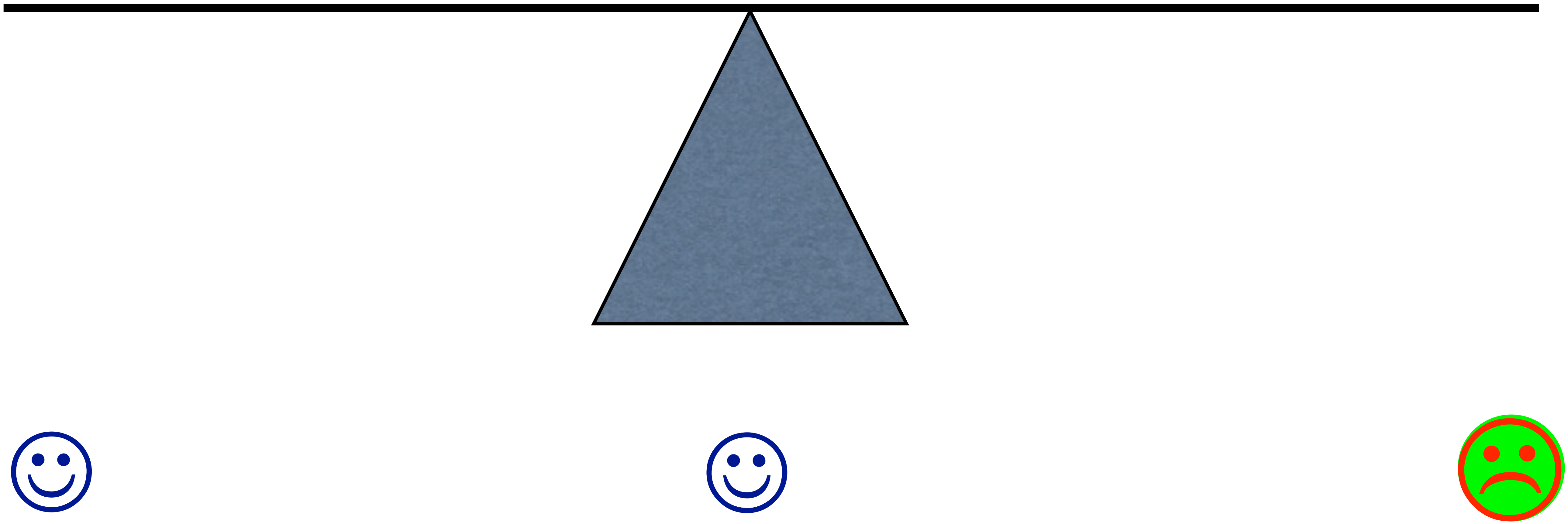


Natural SM : $\Lambda_{UV}^2 \lesssim 1\,\text{TeV}$

$\Lambda_{UV}^2\, H^\dagger H$ $\qquad$ $y_{ij}\, H \bar{F}_i F_j$ $\qquad$ $\frac{1}{\Lambda_{UV}^2} \bar{F}_i F_j \bar{F}_k F_\ell \; + \; \ldots$



The two possible microphysics scenarios

I. The SM is the correct description up to $\Lambda_{UV} \gg TeV$

- B, L and Flavor: beautifully in accord with observation
- Hierarchy remains a mystery, probably hinting that the question was not correctly posed
  - anthropic principle
  - failure of effective field theory ideology (UV/IR connection)

II. The SM is not the correct description already at $\Lambda_{UV} \sim 1\,\mathrm{TeV}$

- In the correct theory the hierarchy problem does not even arise (naturalness)
- What about B, L and Flavor? In practically all known models not nearly as nice as in SM

At $\mu \gg$ TeV the SM with elementary Higgs is approximately a free massless field theory

= approximately Conformal Field Theory

What other options for the UV asymptotics of particle physics?

- weakly coupled natural completion : Supersymmetry
- strongly coupled CFT
- scale but not conformally invariant QFT= SFT
- theory with (approximate) RG cycles

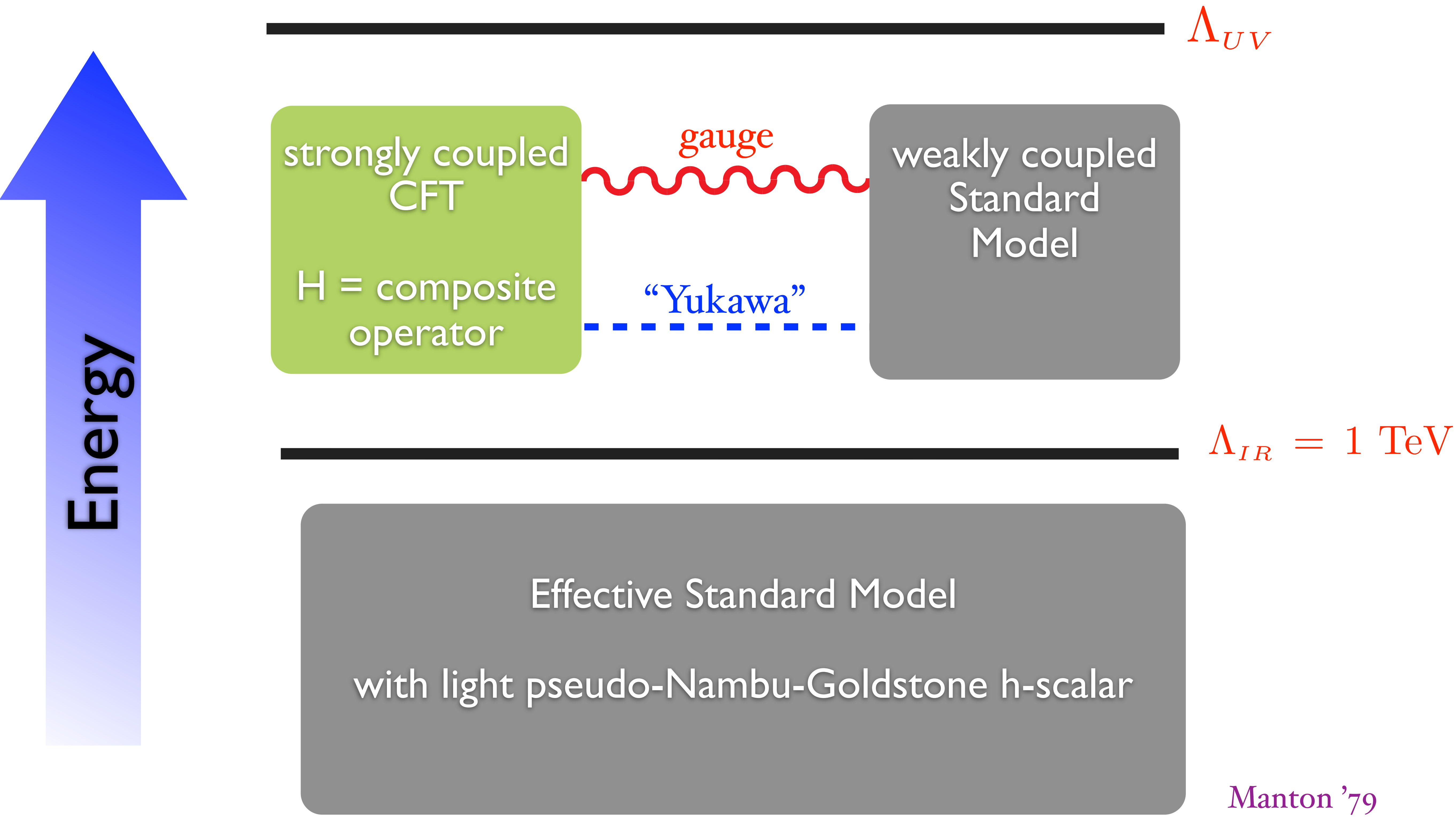
} ?

An example of a strongly coupled CFT:

Modern Composite Higgs Models

Holdom '86
....
Randall, Sundrum 99
Luty-Okui 04
Agashe, Contino, Pomarol 04
....

General Model Structure

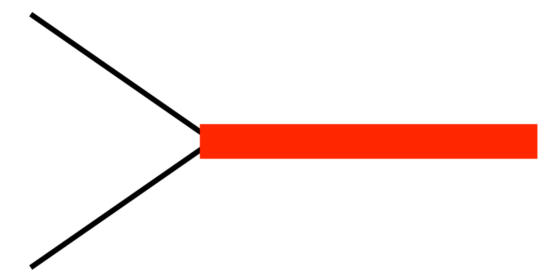


Manton '79
Hosotani '83
Georgi, Kaplan '84

Two Ways to Flavor

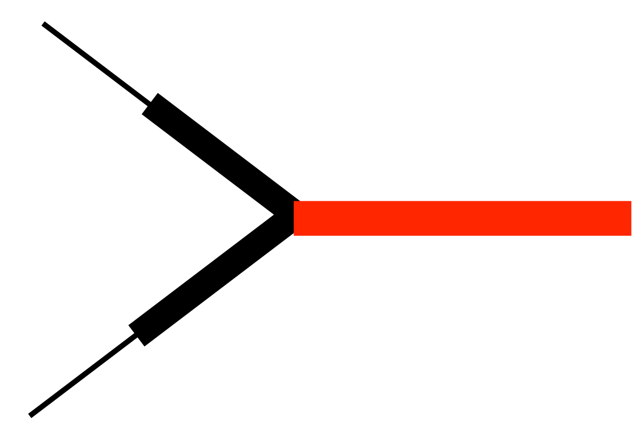
Bilinear: ETC, conformalTC

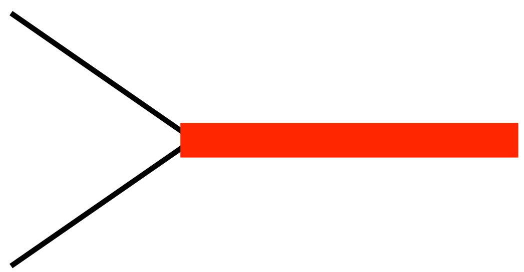
Dimopoulos, Susskind
Holdom
....
Luty, Okui



Linear: partial compositeness

D.B. Kaplan
....
Huber
RS with bulk fermions





Wishes ...

Flavor

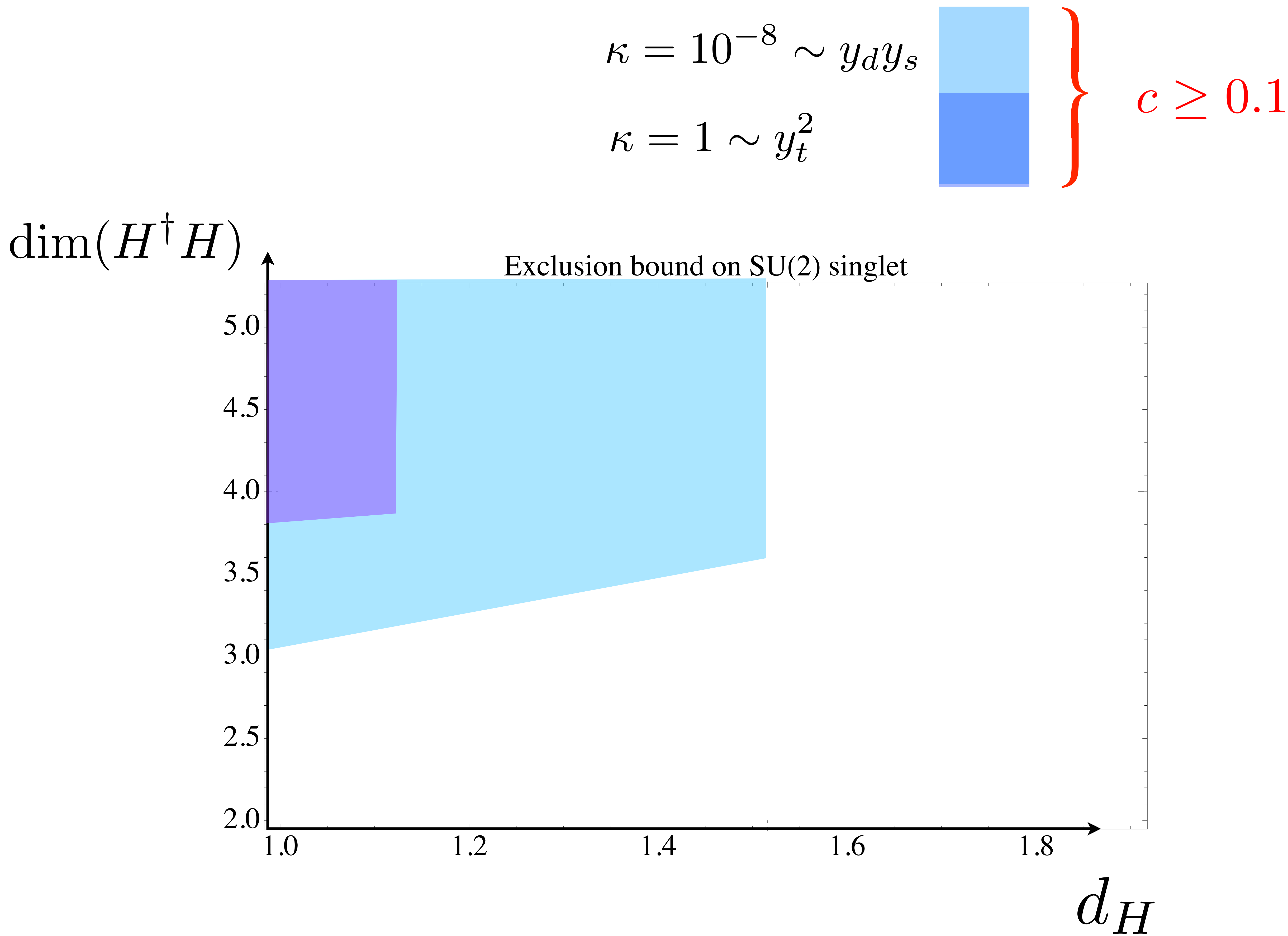
$$\frac{1}{\Lambda_{UV}^{d_H-1}} H\bar{F}F \; + \; \frac{\kappa}{\Lambda_{UV}^{2}}\bar{F}F\bar{F}F$$

wish d_H as close to 1 as possible

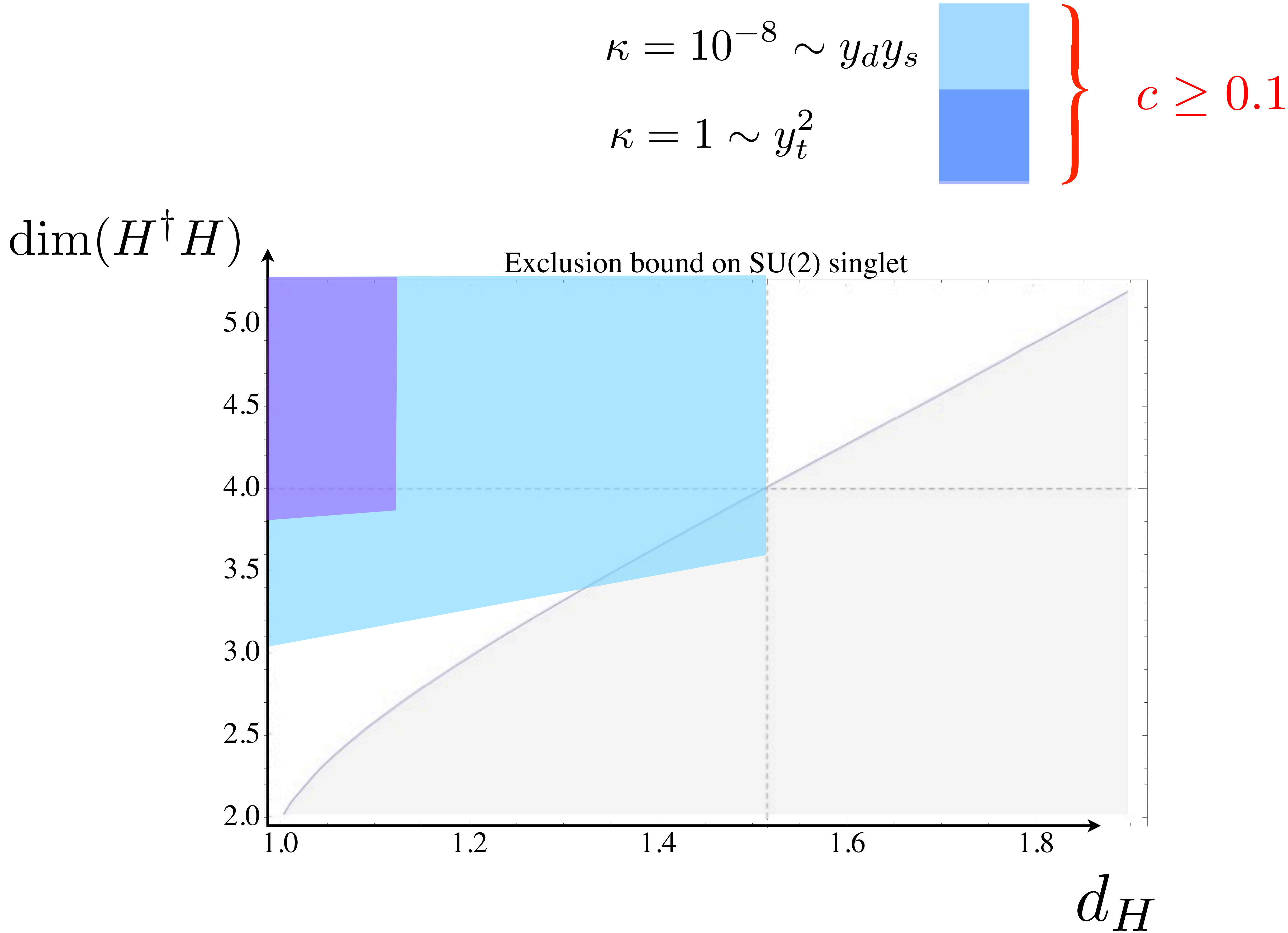
Hierarchy

$$c\;(\Lambda_{UV})^{\Delta-4}\,H^{\dagger}H \qquad \Delta \equiv \dim(H^{\dagger}H)$$

wish $\Delta > 4 - \varepsilon$



Rattazzi, Rychkov, Tonni, Vichi '08
Poland, Simmons-Duffin, Vichi '11



Rattazzi, Rychkov, Tonni, Vichi ’08
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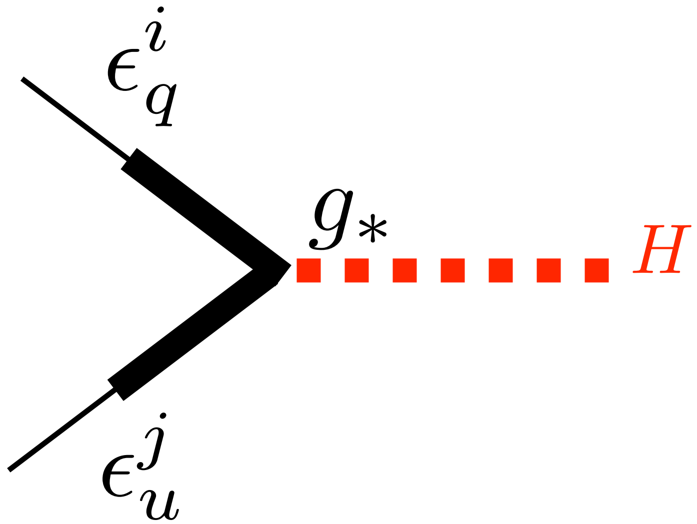
Flavor from partial compositeness

$$\mathcal{L}_{Yukawa} = \epsilon_q^i q_L^i \Psi_q^i + \epsilon_u^i u_L^i \Psi_u^i + \epsilon_d^i d_L^i \Psi_d^i$$



$$Y_u^{ij} \sim \epsilon_q^i \epsilon_u^j g_*$$

$$Y_d^{ij} \sim \epsilon_q^i \epsilon_d^j g_*$$

$d_\Psi \sim \frac{5}{2}$ ⇒ $\epsilon_q^i,\ \epsilon_u^i,\ \epsilon_d^i$ ~ dimensionless

all other flavor couplings decouple when $\Lambda_{UV} \to \infty$

- Problems of composite Higgs greatly alleviated, but not eliminated

Flavor from partial compositeness

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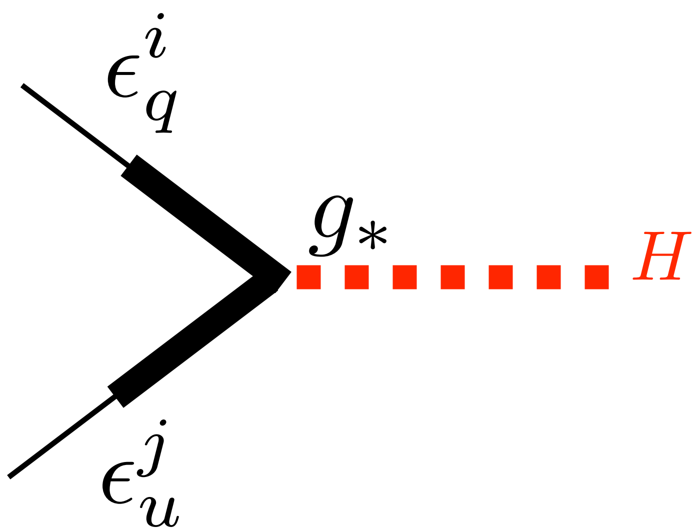

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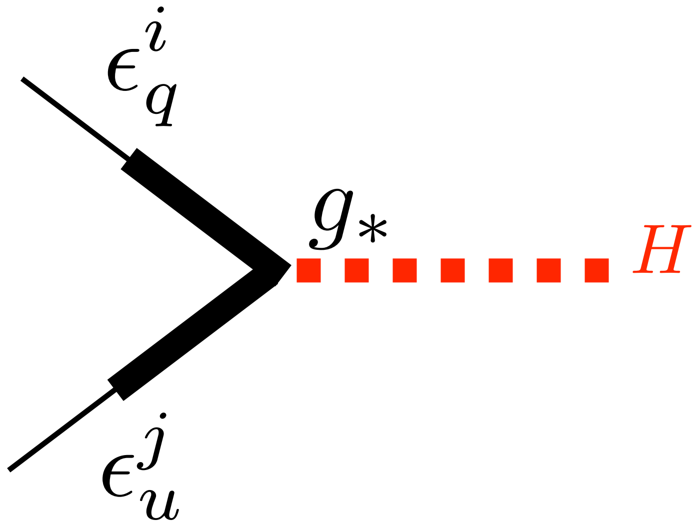

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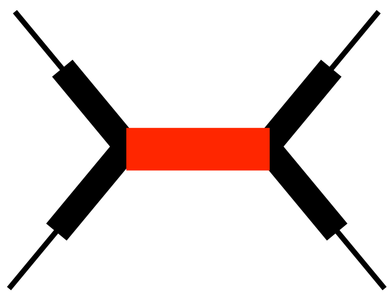
Flavor transitions controlled by selection rules

ΔF=2



$$\epsilon_q^i \epsilon_d^j \epsilon_q^k \epsilon_d^\ell \times \frac{g_*^2}{m_*^2} \quad (\bar{q}^i \gamma^\mu d^j)(\bar{q}^l \gamma_\mu d^\ell)$$

ΔF=1

$$\epsilon_q^i \epsilon_u^j g_* \times \frac{v}{m_*^2} \times \frac{g_*^2}{16\pi^2} \quad \bar{q}^i \sigma_{\mu\nu} u^j G_{\mu\nu}$$

Bounds & an intriguing hint

Davidson, Isidori, Uhlig '07
Csaki, Falkowski, Weiler '08

Keren-Zur, Lodone, Nardecchia, Pappadopulo, RR, Vecchi '12

ϵ_k	$m_\rho \gtrsim 10$ TeV
$\epsilon'/\epsilon, \quad b \to s\gamma$	$m_\rho \gtrsim \frac{g_\rho}{4\pi} \times (10-15)$ TeV
d_n	$m_\rho \gtrsim \frac{g_\rho}{4\pi} \times (20-40)$ TeV
CP violation in D decays $\Delta a_{CP} = a_{KK} - a_{\pi\pi} = -(0.33 \pm 0.12)\%$	If taken seriously ... $m_\rho \simeq \frac{g_\rho}{4\pi} \times 10$ TeV

- connection with weak scale not perfect
- Not crazy at all to see deviation in D's first
- d_n should be next

tuning

$$0.1\% \left(\frac{m_h}{125\,\mathrm{GeV}}\right)^2 \left(\frac{10\,\mathrm{TeV}}{m_\rho}\right)^2$$

$$\mu \to e\gamma \qquad\qquad \frac{\sqrt{m_\mu m_e}}{m_\rho^2}\,\bar{\mu}\sigma_{\alpha\beta}e\,F^{\alpha\beta}$$

MEG: Br($\mu \to e\,\gamma$) < 2.4 x 10^{-12} $\qquad$ $m_\rho \gtrsim 150\ \text{TeV}$

Partial compositeness clearly cannot be the full story

Must assume strong sector possesses some flavor symmetry

Range of possibilities

$U(1)_e \times U(1)_\mu \times (1)_\tau$

. . . .

SU(3) x SU(3) x ...

Redi, Weiler '11
Barbieri et al. '12