TP4: Measurement of the muon lifetime and g-factor

Instructions for students

1 Goal

The goal of the analysis is to first measure the lifetime of muons, and secondly measure the muon g-factor by repeating the experiment in the presence of a magnetic field.

2 Introduction

The muon is an elementary particle, a charged lepton of the second generation, with a mass 207-times bigger than the mass of the electron. They are produced in abundance in showers, caused by high-energetic cosmic rays at a height of about 15 km in the atmosphere. The so produced muons are highly energetic, with a mean energy of 6 GeV. Being relativistic is what allows them to reach the Earth's surface in large numbers, due to the time dilation effect. We observe a flux of 1 muon/cm^2/minute at sea level. We thus have a good sample of muons to measure in our experiment.

Task: *the mean lifetime of the muon is 2.2 μs. If they travel practically at the speed of light, what distance would we expect them to travel before decaying, if the time dilation effect is not taken into account? Taking time dilation into account, what is the mean lifetime we would observe (in our rest frame) if measuring muons travelling at 0.9998c?*

3 Muon lifetime

The muon almost exclusively decays to an electron, with neutrinos being emitted:

$$\mu^- \to e^- \overline{\nu}_e \nu_\mu \quad , \tag{1}$$

$$\mu^+ \to e^+ \nu_e \overline{\nu}_\mu \quad . \tag{2}$$

The muon decay is a process with a constant probability. This means that in any given time interval, the muon has the same probability to decay. The (differential) probability that a decay will occur in an infinitesimal time interval is thus a constant:

$$dP = \lambda dt \quad . \tag{3}$$

This leads to an exponential distribution $N = N_0 e^{-t/\tau}$, where N_0 is the number of muons at $t = 0$, and $\tau = 1/\lambda$ is the expected lifetime.

Task: *make sure you understand the above exponential law. Derive the equations for the distribution and the expected lifetime $\tau =< t >$.*

The mean lifetime of the muon, τ, is

$$\tau = 2.1969811 \pm 0.0000022 \mu\text{s} \quad . \tag{4}$$

4 Interaction of muons with matter

In order for us to measure the muon lifetime, it is necessary that the muon decays in our experimental apparatus. We thus wish to slow the muons and finally stop them in our detector, so that they decay inside.

Stopping the muons requires making them lose energy. When charged particles traverse matter, they interact with it and lose energy based on well known mechanisms. The primary mechanism by which muons lose energy in interactions with matter is ionisation, and the energy losses are described with the Bethe-Bloch formula. Most cosmic-ray muons have mean energy loss rates close to the minimum (they are MIPs - minimum ionising particles).

Task: *look up plots for Bethe-Bloch formula and make sure you understand it. You should be able to read e.g. the energy losses for a MIP for a material with a given density. What is the value for muons in copper? How about in plastic?*

The above mechanism holds for both negative and positive muons (antimuons). For negative muons, there is another important mechanism of interaction with matter. Muons can displace an electron in the atom, effectively taking its place. Such a bound muon can decay in the same way as a free muon. However, it is also possible that the muon is captured by the nucleus and binds with the proton to form a neutron

$$\mu^- + p \rightarrow n + \nu_\mu \quad . \tag{5}$$

Muon capture is only possible for negative muons; positive muons cannot be captured by the nucleus, they can only decay. For negative muons in material, there are two independent mechanisms for the muon to disappear: decay and capture, and they compete between each other. The rate of muon capture is heavily dependent on the Z of the matter. It scales as Z^4. Muon capture thus leads to a decreased lifetime in matter for negative muons, with the actual lifetime depending on the Z of the matter.

Task: *look up some values for muon lifetime in matter. In our experiment, the muons will be stopped in copper. What is the approximate lifetime of the muon in copper? You can either look it up directly in literature, or deduct it from another value in a specific material using the Z^4 dependence. What can you conclude about our experiment and the particles we will be measuring? Remember, positive muons are not affected by nuclear capture.*

5 Muon g-factor

Like the electrons, muons are fermions, particles with spin 1/2. The magnetic moment is proportional to the spin:

$$\boldsymbol{\mu} = g\frac{e}{2m_\mu}\mathbf{s} \quad , \tag{6}$$

where g is the muon g-factor. For leptons, Dirac theory supposes that g should be exactly 2, however, a full calculation taking into account higher order contributions and corrections, predicts a deviation from 2. We call this anomalous magnetic moment:

$$a = \frac{g-2}{2} \quad . \tag{7}$$

In a uniform magnetic field, the spin will begin a precession around the axis of the magnetic field. The frequency of the precession, named Larmor frequency, is

$$\omega_L = \frac{geB}{2m_\mu} \quad . \tag{8}$$

We see that the Larmor frequency is directly connected to the g-factor, and by measuring ω_L, we can infer g.

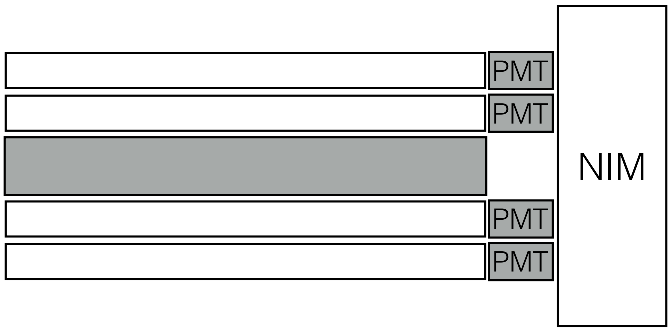

Figure 1: A general scheme of the experiment.

The cosmic muons come from pion decays and exhibit a general longitudinal polarisation. For positive muons, the preferred direction of the spin is antiparallel to the direction of the momentum in the pion rest frame. The asymmetry of the angular distribution of positrons from the muon decay is such that they are emitted preferentially in the direction of the spin of the positive muon.

In our experiment, we turn on an electric field in the horizontal direction. The observed muon decay distribution in the upward (top) or downward (bottom) direction will exhibit a modulation with a period, given by the Larmor frequency.

Task: *Review the theory of a particle of spin 1/2 in a magnetic field. Understand why the cosmic muons from pion decays are polarised and how (think about what is the pion spin, and what do we know about the neutrino...?) How about electrons/positrons from the muon decay? What is the relation between their momenta and the spin orientation of the muon? Infer how will the exponential distribution* $N(t) = N_0 e^{-t/\tau}$ *be changed in the presence of a magnetic field.*

6 Experimental setup

A basic sketch of the experimental setup is depicted in Fig 1. The muons are stopped in a copper plate. On both top and bottom of the copper plate, there are two plates of plastic scintillators, connected via light guides to photomultiplier tubes (PMTs). These are used to detect the passage of the muons, and of the decay products.

6.1 Scintillators

Scintillators are materials in which electrons, when decaying back to a low-energy state after being excited, emit photons. Excitations are caused by passage of a highly energetic particle. The identity of the particle cannot, however, be directly inferred.

Task: *make sure that you understand the basic principles of scintillators. The scintillators in our experiment are made of plastic. What is the approximate density? Is the time delay with which the scintillating light is emitted after the passage of a particle relevant for the measurement of the muon lifetime?*

6.2 Photomultiplier tubes

The PMTs detect the photons, emitted from the scintillators, and respond with a signal in the form of an electric current. Inside the PMTs, initial photons create electrons via the photoelectric effect, by hitting a photocathode. The electrons are then multiplied in several stages ($\mathcal{O}(10)$), hitting dynodes held at increasing voltages, in order to to produce a final electric current of a measurable magnitude. The final electric current on the anode raises it voltage for a time $\mathcal{O}(\mathrm{ns})$. The gain of the PMTs depends on the applied voltage.

Task: *look up a sketch of a photomultiplier tube. Make sure you understand the principle of operation. What would be the total multiplication factor of a 10-stage PMT, if the multiplication factor at every dynode was 6?*

6.3 High voltage supply

Supplies the voltage to the PMTs.

6.4 NIM

Nuclear instrumentation modules (NIM) are used to set up a trigger and data acquisition scheme.

6.4.1 Discriminator

The voltage pulses on the PMTs are analogue pulses, they vary in size. A discriminator converts them into digital signals when the input voltage satisfies the adjustable desired level (threshold). The threshold is adjusted to minimise background events. The digital signal also has an adjustable width.

6.4.2 Coincidence units

Coincidence units are used to perform simple logical operations such as "and" and "or".

6.4.3 Delay units

They are used to delay a signal for a desired time.

6.4.4 Gate

Produces a square signal of desired width.

6.5 Oscilloscope

The oscilloscope is used to observe analogue signals from the PMTs, as well as digital discriminated signals. It can be used to observe the outputs of the trigger scheme in each step as it is being build. It is also used to perform the actual time measurement and save data.

6.6 Trigger overlook

You will use the NIM modules to design a trigger - a logical scheme that starts the time measurement and ends it. The start of the measurement should correspond to the income of a muon that stops in the detector, and the measurement should stop when decay products are detected.

As explained before, the scintillators and PMTs do not provide information on the identity of the particle (to know when a muon has entered the detector, and when an electron leaves). There will be a lot of background producing signal from the PMTs, and not by far all signals will correspond to muon decays. Therefore, the events must be identified based on certain conditions, inferred from the knowledge of the event.

The first condition is that a muon must stop in the detector (more energetic muons will only traverse it, as they will not lose all energy). So, a muon must enter the detector, but not leave. This means that the top two scintillator plates must detect a passage of the particle, but the bottom two must not. Let's call such a signal "top" (and only top). Similarly, an electron is produced in the detector and it leaves it through either top or bottom scintillators. Let's call triggering only the bottom two scintillators "bottom" signal. A decay product of a muon will thus produce either a "top" or "bottom" signal.

As there will still be a lot of background that can produce such signals randomly, it is sensible to limit the time after the start of the measurement ("top" signal), and discard the measurement if a stop signal ("top" or "bottom") does not come in a reasonable time (reasonable based on the desired measurement - muon lifetime).

6.7 Magnetic field

A pair of Helmholtz coils is used to generate the magnetic field.

Task: *look up the formula for the magnetic field, generated by a Helmholtz coil, and make sure you can calculate the strength of the magnetic field in our experiment once the current is known.*

7 NOW LET'S GET TO WORK

7.1 Basic setup

Get familiar with the basic connections between the High voltage supply, the PMTs, the potentiometers, the readout of the PMT voltage and the anode output of the PMTs. Learn how to operate the HV supply, how to change the voltage on the PMTs and how to read it with the voltmeter.

7.2 Get to know the signals

Observe the signals from each photomultipliers (anode output) on the oscilloscope. Then put the signals through a discriminator and observe the output signals. Get familiar with them. Observe and understand the effects of the threshold and width settings.

7.3 Calibrate the PMTs

The first major task is to find good settings for the PMT voltages. In every experiment, the detectors need to be calibrated prior to the beginning of the data taking, in order to ensure and set that their output is what we expect it to be, what we want it to be, and reliable. We wish to find a voltage for each PMT where it is in a stable operating regime, and to ensure that all PMTs are set as to have similar responses.

To do that, you can use a Counter unit, that counts the signals that are output by the discriminator. Observe the number of counts in a time interval with dependence on the voltage on the PMT. Observe the effect of the threshold setting.

When you are familiar with what's going on, perform a detailed measurement of the response of the PMTs in dependence on the voltage applied. Do not exceed an absolute value of the voltage of 2.4kV.

Choose a voltage for each PMT in a stable regime, so that all PMTs have approximately the same response.

7.4 Calibrating the threshold

A sensible threshold setting must be chosen, to suppress background, but signal (muon decays) should pass it.

Make a measurement of the counts in dependence of the threshold settings of the discriminators. To calibrate the threshold (understand on an absolute scale to what a certain setting corresponds), you can use a well-known radioactive source, such as ^{60}Co. Reflect and choose an appropriate threshold level based on the setting of the experiment (muon decay), and the observation of background.

You might need to re-calibrate the PMTs due to a change of the threshold.

7.5 Set up the trigger

A "top" signal will be used as "start" of the measurement on the oscilloscope. It also opens a gate. If a "stop" signal comes while the gate is open, the measurement should be recorded. The gate needs to be opened with a certain delay with regard to the "start" signal, as the signal itself has a certain width an could cause an immediate "stop". The "stop" signal is if either a "top" or "bottom" signal comes while the gate is open. If the case that "bottom" would produce a "stop", the signal can be further delayed by a larger time in order to separate the peaks from "top" and "bottom", so that they can be analysed separately (important especially for the second part of the measurement). Alternatively, the two signals for stop (from

”top” and ”bottom”) can be separately guided to the oscilloscope and the two measurements performed separately.

As some NIM units might not work at all, or not work properly (resulting in ”weird” behaviour) always test at every stage that the output looks proper. Build your trigger step-by-step and make sure you understand the output at each stage.

7.6 Decay time measurement

Configure the oscilloscope to measure the time between the ”start” and ”stop” signal. Let the measurement run for an extended period of time ($\sim$1 week) to record enough data for a good statistics.

7.7 Extract the mean liftetime

Use a program of your choosing, like e.g. MATLAB, to fit the data. Add a term to the exponential distribution to describe possible additional background. Think also about the negative muons - should their presence be taken into account, and how?

7.8 Set up for the measurement of the g-factor

The trigger and data acquisition settings are the same as for the previous measurement, with the addition that the magnetic field is turned on while collecting data.

Measure the magnetic field in several places on the scintillators to ensure it is uniform. Compare the measurement to the calculation of the magnetic field.

7.9 Extract the g-factor

Fit an appropriate modification of the exponential function to account for the modulations, and extract the Larmor frequency. Think about the previous measurement of the lifetime. Can that measurement help you to achieve better precision now?