Imaging Optics 2021 - Course Note 6 - Lecture 9 and/or 10

Interferometry and Coherence

1. Interference
2. Michelson interferometer
3. Temporal coherence
4. Other interferometers
5. Young's double slit interferometer
6. Spatial coherence
7. Imaging with incoherent light

> With regard to wave superposition, Richard Feynman wrote:[2]
>
> No-one has ever been able to define the difference between interference and diffraction satisfactorily. It is just a question of usage, and there is no specific, important physical difference between them. The best we can do is, roughly speaking, is to say that when there are only a few sources, say two, interfering, then the result is usually called interference, but if there is a large number of them, it seems that the word diffraction is more often used.

Interferometry

Interferometry is the most common method for measuring the phase of an optical field. A wide variety of interferometers exist. We will start with the Michelson interferometer shown in Figure 1.

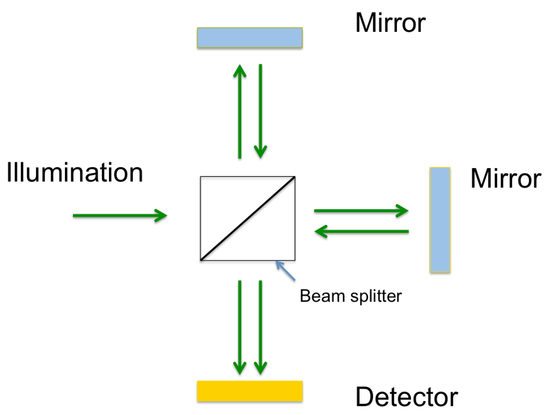


Figure 1 Michelson inteferometer

The Michelson is widely used to measure the shape of a surface, characterize the coherence properties of a light source, or measure phase shifts in the space between the two arms. The signal detected by the detector is

$$I_{\det} = |E_1 + E_2|^2 = |E_1|^2 + |E_2|^2 + 2|E_1||E_2|\cos(\phi_1 - \phi_2) = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\phi_1 - \phi_2)$$

The fact that we have two separate waves overlapping or interfering in the detector plane results in an intensity pattern that contains information about the phase. Specifically we obtain information about the phase *difference* between the two beams of the interferometer. We can think of one of the two beams as the known reference and then we can deduce from the interference pattern the phase of the other beam. In Figure 3 we see the phase on a Michelson interferometer with one of the two mirrors being spherical.

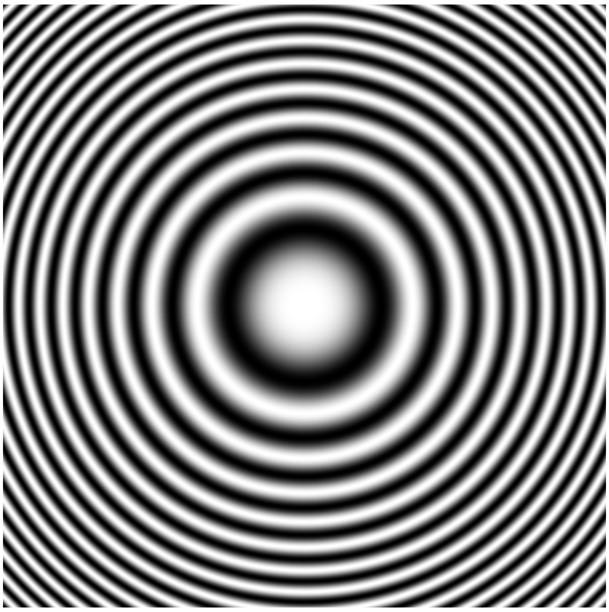


Figure 3

The recorded pattern in this case is of the form

$$I_{\det}(x,y) \sim \cos[\phi_1 - \beta(x^2 + y^2)]$$

This pattern is called a *zone plate* in optics and it comes up often since it is essentially the interferometric recording of the impulse response of free space propagation. If the shape of the mirror were not spherical and unknown we might try to deduce its shape from the interference measurement. For a spherical mirror it seems natural to interpret the zone plate as corresponding to a spherical surface shape for the second mirror. If the shape is not regular however we may have a problem of interpretation. The fact that we observe a signal that is proportional to the cosine of the phase introduces an ambiguity since $\cos\phi = \cos(\phi + 2m\pi)$ for any integer m. This is the *phase unwrapping* problem. It is an easy problem in 1D if we assume continuity of the surface being imaged but the problem becomes a complex optimization exercise in 2D. The phase ϕ_1 can be a constant or it can be a known function of x and y. We will see later on as we talk about holography that making the reference a linear or quadratic function of position brings up interesting possibilities.

Michelson invented and used this interferometer in 1887 (together with Morley) to measure the speed of light as the earth rotates around the sun (Figure 4). It was thought at the time that there was an "aether" everywhere filling in what we now know to be vacuum. It was also believed that the eather moved with respect to the earth. If this were true the speed of light would depend on whether the light travelled "down-wind" or "up-wind". Michelson showed that there was no difference in the speed of light up or down wind and this put in question the existence of the aether. Michelson got the Nobel prize.

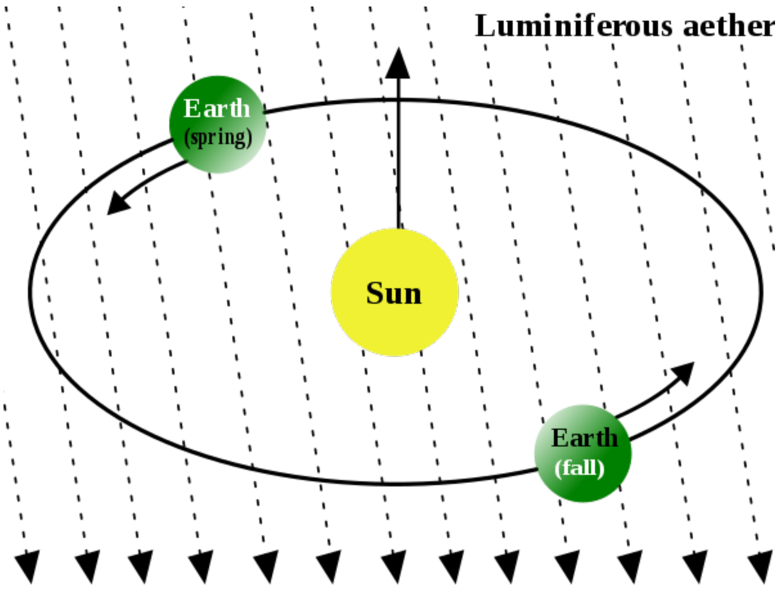
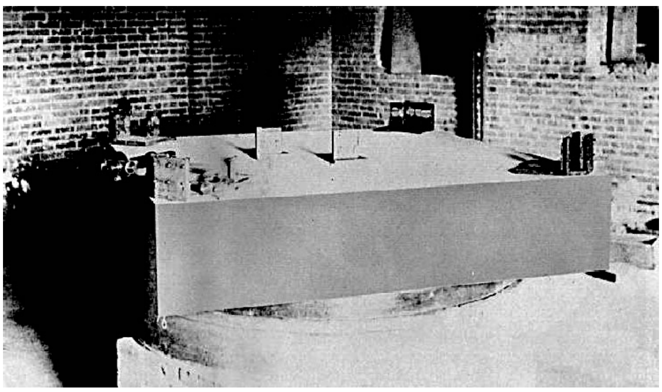
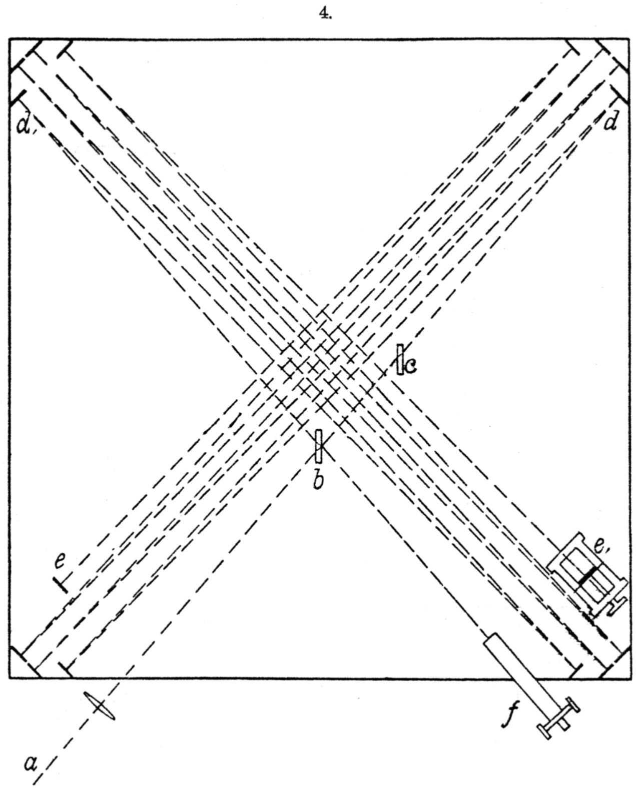


Figure 4. The velocity of the aether would change the speed of light as the velocity of the wind changes the speed of sound.





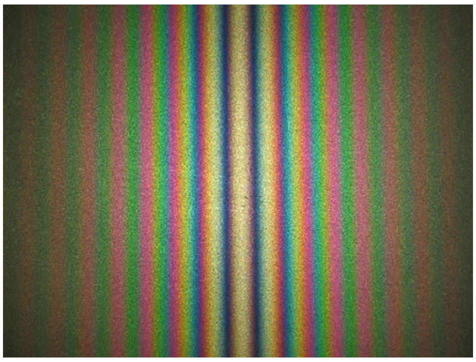


Figure 5. The interferometer used in the Michelson-Morley experiment (photograph and schematic diagram) and the fringes obtained with white light. Notice that the middle fringe is black-and-white. A change in velocity of the light shifts the position of this fringe.

The pattern in Figure 5 is the interference between two white light plane waves. We can understand the rainbow effect by looking at the interference pattern of two monochromatic plane waves at wavelength λ.

$$\left|e^{-j2\pi\sin\theta x/\lambda}+e^{+j2\pi\sin\theta x/\lambda}\right|^2=2+2\cos(4\pi\sin\theta x/\lambda)$$

At x=0 the position of the central fringe does not depend on wavelength whereas at x>0 the position depends on the wavelength. Therefore with the white light source the central fringe is white and the rest are rainbow colored. Michelson used this effect to track the central fringe.

More recently an upgraded version of the Michelson interferometer was constructed (Figure 6) and used to detect gravity waves. The leaders of this project at Caltech and MIT received the Nobel prize in 2017.

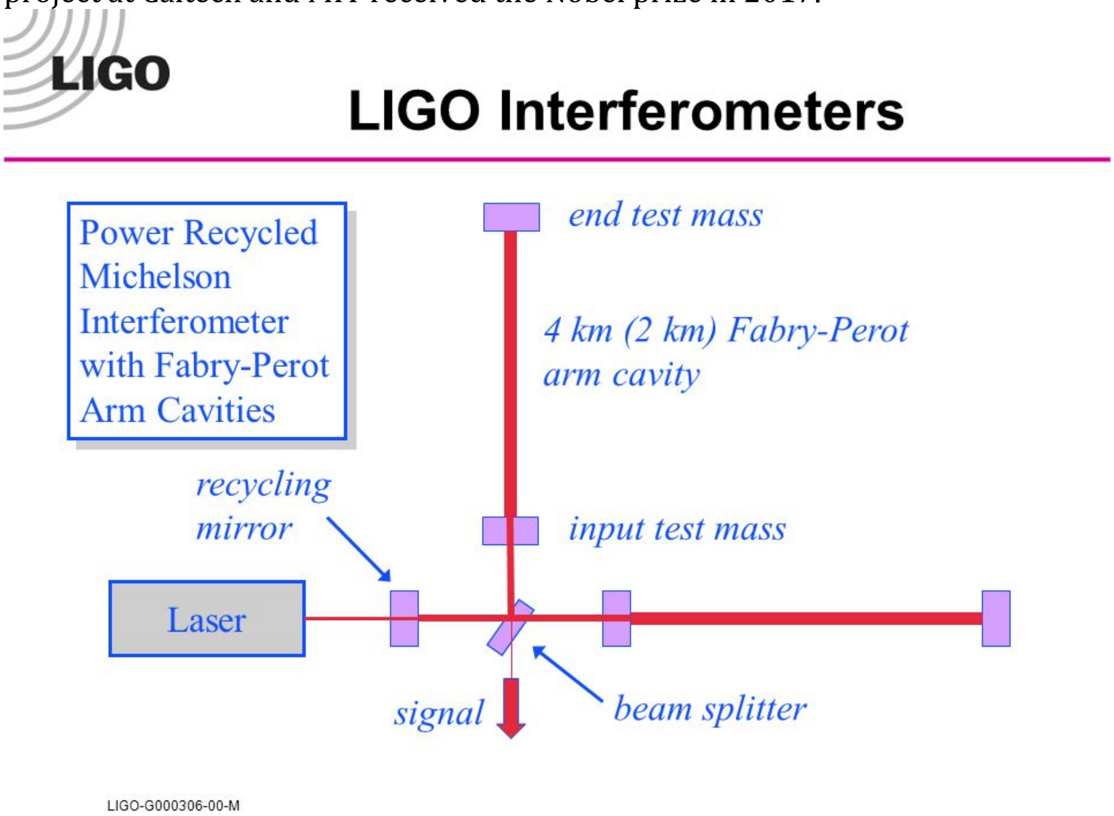


Figure 6 Schematic diagram of the Michelson interferometer used in the gravity wave detection experiment (LIGO).

<u>Source coherence</u>

In the discussion so far we have been assuming a perfectly coherent light source (time dependence $e^{j\omega t}$). If we do not have a purely monochromatic source then our ability to observe interference is generally impaired. Consider for example the Michelson interferometer with a source whose light amplitude can be described by $E(t) = a(t)e^{j\omega_0 t}$ where a(t) is the slow varying temporal envelope of the source. Suppose that the two paths of the interferometer differ by a distance d and both mirrors are perfectly flat. Then the signal detected is

$$
\begin{aligned}
I_{\det} &= \int |a(t)|^2 \, dt + \int |a(t-\tau)|^2 \, dt + 2\,\mathrm{Re}\{\int a(t)a^*(t-\tau)dt\}\cos\omega\tau \\
&= 2R(0) + 2R(\tau)\cos\omega\tau
\end{aligned}
$$

where $\tau = d/c$ and $R(\tau)$ is the auto-correlation of the source (*a(t)*). This is also called the temporal coherence function. We know from the correlation theorem that the Fourier transform of the correlation is the power spectrum. See Figure 4. The wider the bandwidth (*W*) of the light source (while light source) the shorter its correlation length. The coherence length is $\Delta d = c\,\Delta\tau = \frac{c}{W}$. In order to observe interference between two optical beams the difference in the optical paths they follow to reach the detector must be less than the coherence length of the source.

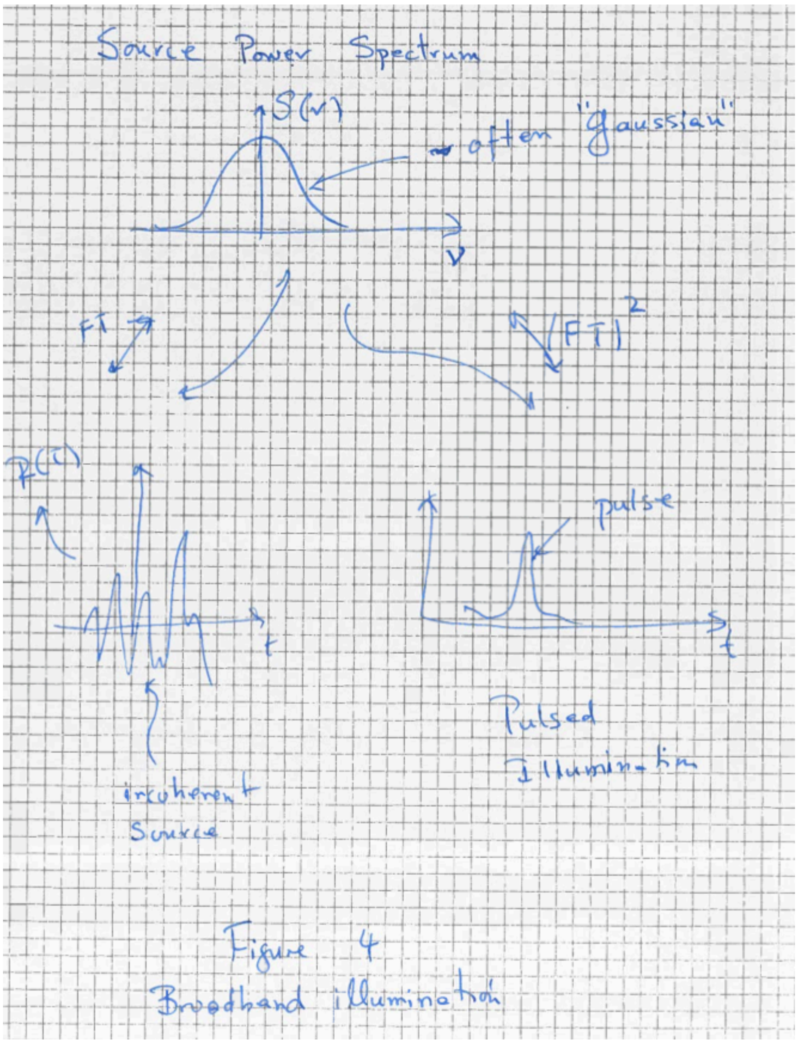


Figure 7

Optical Coherence Tomography (OCT)

If a source with low coherence is used then Δd can be very short. Practically a micrometer is readily obtainable. A Michelson interferometer can then be used as an imaging device by scanning the reference mirror on one arm and the object on the other. When a scatterer on the objects reflects light in one arm an interferometric sinusoidal signal is detected only if the signal reflected from the mirror on the other arm is at the same distance within one coherence length. In this way scanning the object in 3D and detecting the interferometric signal a full 3D image of the object can be formed. See Figure 5 for an example of an OCT image of a retina.

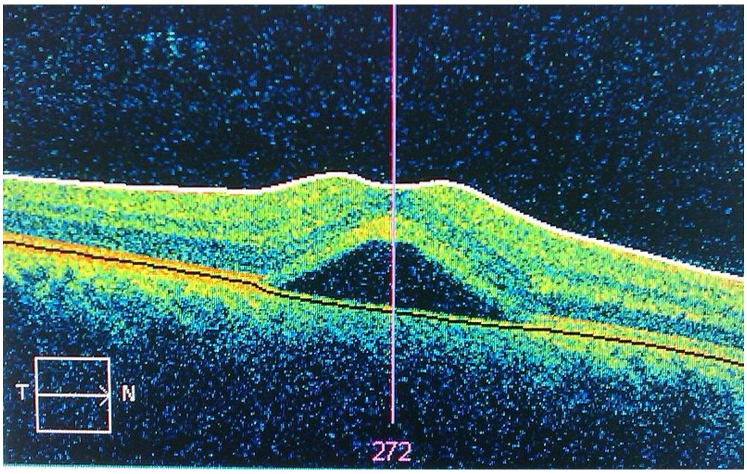
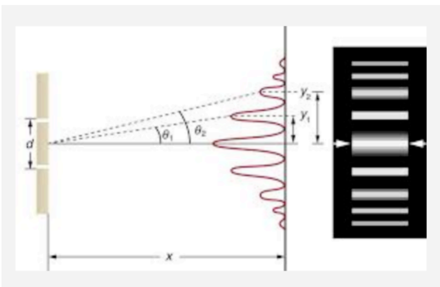


Figure 8. OCT image of a retina

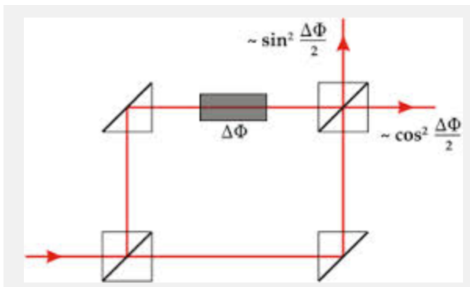
Other interferometers

The drawing of four different interferometers are shown in Figure 6: The Mach-Zhender, a shearing interferometer, Youngs double slit, and the pinhole interferometer.

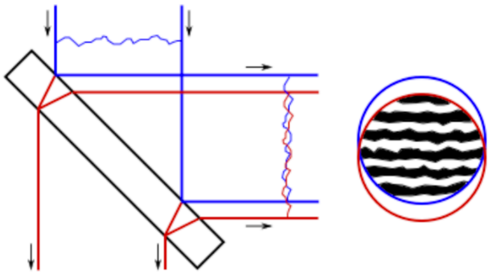
The pinhole interferometer is particularly interesting. A pinhole in a semitransparent medium smaller than the diffraction resolution limit provides a reference to the wavefront that goes through the entire semitransparent mask . The transmittance of the mask can be written as $t(x) = \frac{1}{2} + \frac{1}{2} rect(\frac{x}{w})$. The constant background provides the pathway for the signal whereas the small square aperture samples the field and scatters it into a spherical wave which acts as a reference. See Figure 7. This system is similar in some respects to dark field imaging.



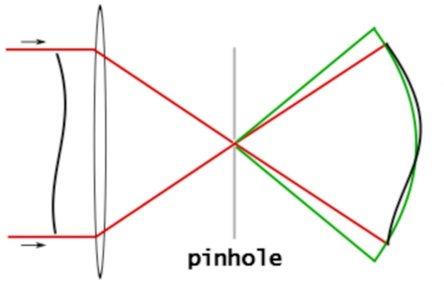
Young's slit interferometer



Mach Zehnder interferometer



Shearing interrerometer



Pinhole interferometer

Figure 8 Interferometers

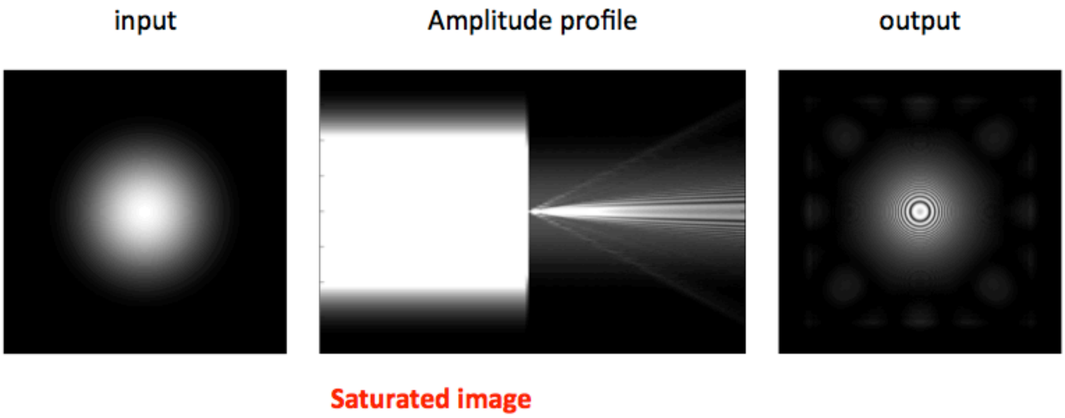


Figure 9. Pinhole interferometer

<u>Wavefront measurement</u>

A diagram of the Shack Hartman interferometer is shown in Figure 10. An array of small lenses (a lenslet array) is used. Each lenslet samples the local wavefront which appears as a plane wave to the small aperture of the lens. The local direction of the plane wave is mapped to position in the back focal plane of the lenslet. Measurement of the position is an estimate of the local phase gradient from which we can obtain the phase profile by integration.

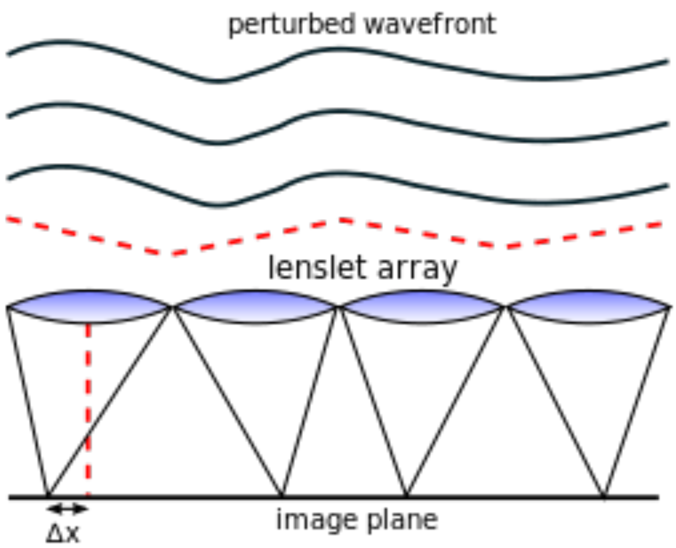


Figure 10. Shack-Hartman interferometer

1. **<u>Spatially Modulate Illumination:</u>**

We have assumed that the input object in an imaging system is a thin transparency illuminated by a monochromatic plane wave. This is sometimes true, particularly in a laboratory setting, but often this is not the case. The illumination maybe polychromatic and not collimated and objects may be the reflectance of a scattering surface. In most cases we can model such more complex situations by selecting a “structured illumination” properly. If we illuminate the input transparency with a monochromatic plane wave travelling at an angle instead of along the z-axis, the pass-band of the imaging system is shifted. This is demonstrated in Figure 6.7 for different angles of illumination. We can see that distortion a low pass filtering is introduced because only one of the diffracted orders of the higher spatial frequencies makes it through the aperture of the lens and when we plot amplitude or intensity these frequencies do not show up.

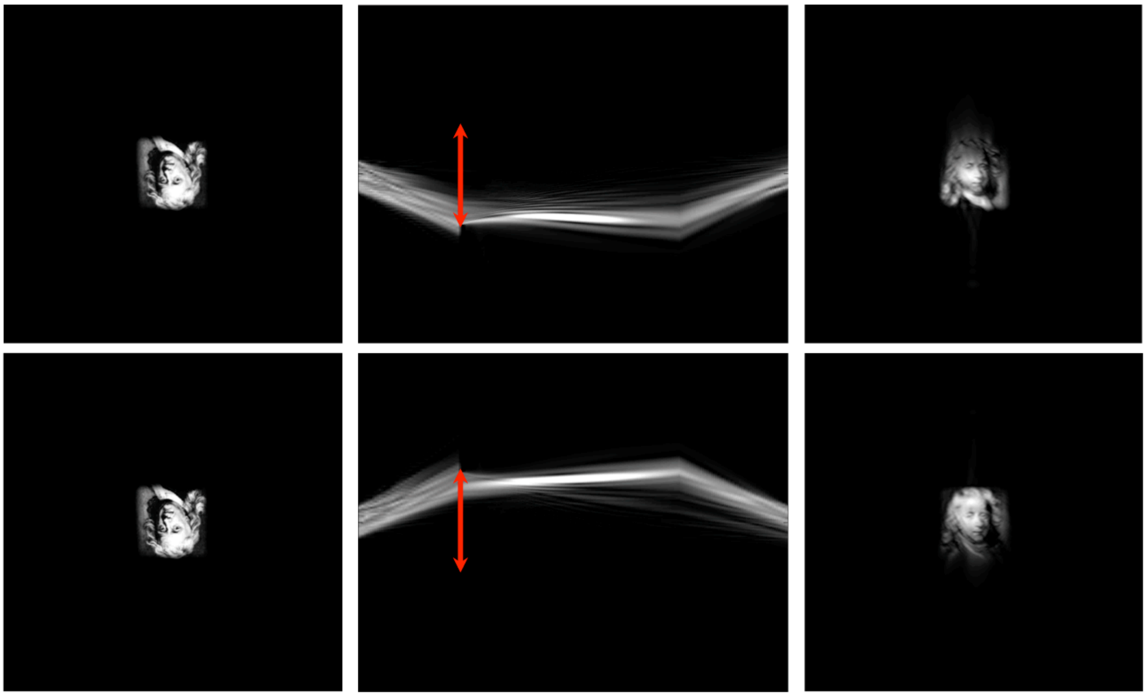


Figure 11

We can try a variation on this theme where two off-axis plane waves are used for illumination at this time and in this way we can improve the resolution by capturing higher spatial frequencies (Figure 12).

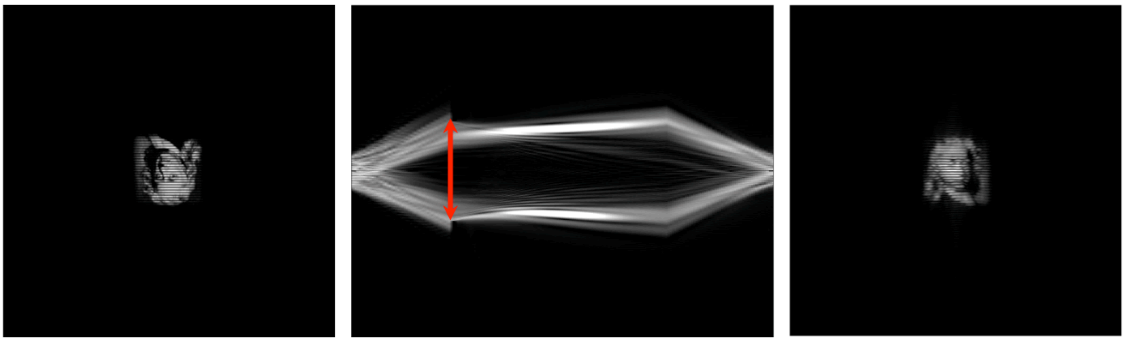


Figure 12

Another possibility is to have a spherical wave illumination (Figure 13). For a converging wave this can help keep the light near the optical axis and reduce aberrations.

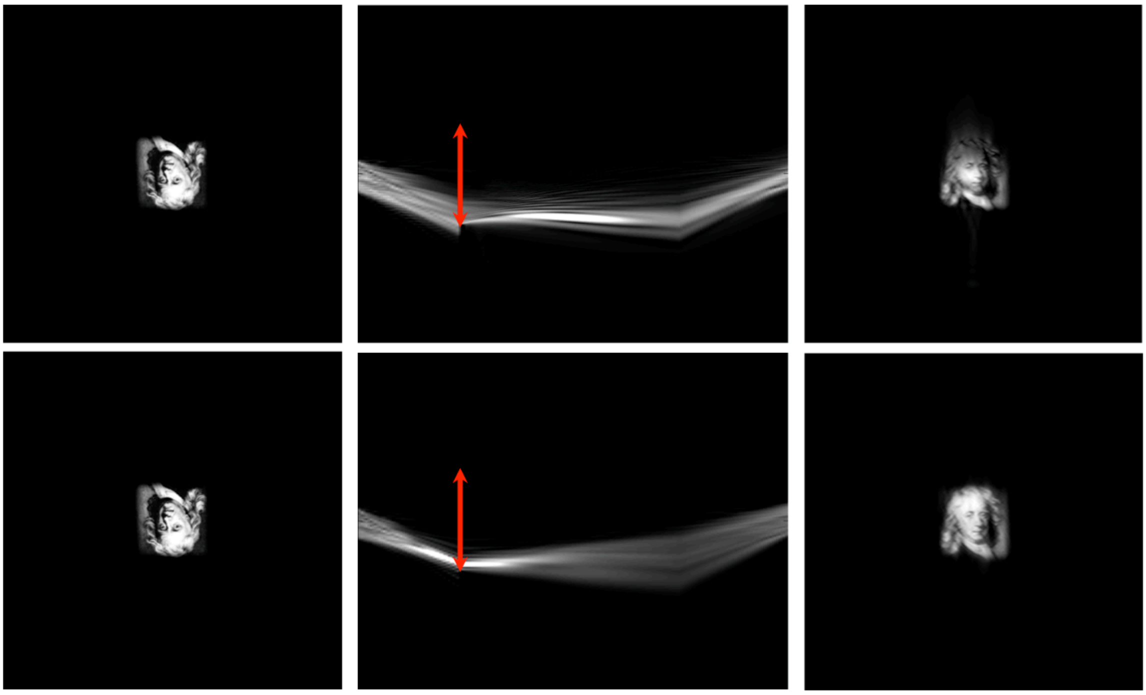


Figure 13

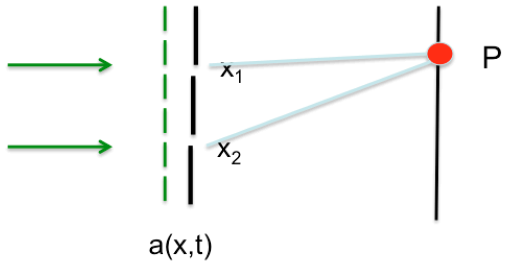
<u>Incoherent Illumination</u>:

If the illumination is not monochromatic then in principle it is straightforward to analyze the optical system one wavelength at a time as a monochromatic system as we have been doing. In the end we can superimpose all the frequency components to obtain the polychromatic response. This is precisely the approach if the temporal variation of the polychromatic light illumination is known deterministically. In general this would be a pulsed light source. The other possibility is to have an "incoherent" of a "partially coherent" illuminating source. In this case the amplitude of the illuminating field is modulated by a quasi-monochromatic wave $a(x, y, z = 0, t)$ where the time modulation is a random process that is not deterministically known but instead it can only be characterized by its time averaged properties such as the time averaged intensity

$$\bar{I}(x, y, z = 0) = \frac{1}{T}\int_T |a(x, y, z = 0, t)|^2 \, dt$$

or its power spectrum. By quasi-monochromatic we mean that the response of the optical system does not vary with wavelength of frequency over the bandwidth of the monochromatic component under consideration. We can take the spatial Fourier transform in x and y of the sample realization of the random process $a(x, y, z = 0, t)$ to obtain a spectrum of plane waves with random amplitudes propagating at different angles. We can best analyze the effects of the coherence properties of the illumination on the diffraction pattern by considering how light originating from different points on the wavefront will interfere downstream. See Figure 12.

Spatial coherence



$$E(P)=a(x_1,t-\tau_1)e^{j(\omega t-kr_1)}+a(x_2,t-\tau_2)e^{j(\omega t-kr_2)} \qquad \tau = kr/\omega = r/v$$

$$I(x')=\frac{1}{T}\int\left|a(x_1,t-\tau_1)\right|^2 dt+\frac{1}{T}\int\left|a(x_2,t-\tau_2)\right|^2 dt$$

$$+\frac{1}{T}\int\left[a(x_1,t-\tau_1)a^*(x_2,t-\tau_2)\right]dt\, e^{-jk(r_1-r_2)}+c.c.$$

$$= \quad I_1+I_2+2\,\mathrm{Re}\,\frac{1}{T}\int\left[a(x_1,t-\tau_1)a^*(x_2,t-\tau_2)\right]dt\,\cos(k(r_1-r_2))$$

$$\textit{Coherence function}: \Gamma(x_1,x_2)$$

Figure 14. Spatial coherence

For a purely spatially **incoherent** light source light will not interfere with light coming from any other place:

$$\Gamma(x_1,x_2,y_1,y_2)=\frac{1}{T}\int_T a(x_1,y_1,t)a^*(x_2,y_2,t)dt=I(x_1)\delta(x_1-x_2)$$

If we assume spatially incoherent illumination at the input of a single lens imaging system, we obtain.

$$I_g(x',y')=\frac{1}{T}\int_T g(x',y',t)g^*(x',y',t)\,dt$$

$$=\frac{B^2}{T}\int_T dt\left[\iint\iint a(x,y,t)a^*(\xi,\eta,t)f(x,y)f^*(\xi,\eta)\right]$$

$$\times \quad \mathrm{sinc}\left[\frac{B(x-x'/M)}{\lambda z_1}\right]\mathrm{sinc}\left[\frac{B(y-y'/M)}{\lambda z_1}\right]\mathrm{sinc}\left[\frac{B(\xi-x'/M)}{\lambda z_1}\right]\mathrm{sinc}\left[\frac{B(\eta-y'/M)}{\lambda z_1}\right]dxdyd\xi d\eta$$

The next step it to carry out the integration over time (t) first which gives us $I_0\delta(x-\xi)\delta(y-\eta)$ (where I_0 is the average illumination intensity) and then we get the final expression for the output intensity as

$$I_g(x',y')=\iint f(x,y)f^*(x,y)\,\mathrm{sinc}^2\left[\frac{B(x-x'/M)}{\lambda z_1}\right]\mathrm{sinc}^2\left[\frac{B(y-y'/M)}{\lambda z_1}\right]dxdy$$

$$=\iint I_f(x,y)\,\mathrm{sinc}^2\left[\frac{B(x-x'/M)}{\lambda z_1}\right]\mathrm{sinc}^2\left[\frac{B(y-y'/M)}{\lambda z_1}\right]dxdy$$

This simple result tells us that the system which with coherent illumination is linear in complex field amplitude and has an impulse response equal to a sinc function now it is a linear system in light intensity instead and it has an impulse response equal to

sinc². The result is quite intuitive since each point at the input reads out the impulse response of the coherent system and the overall intensity is recorded as the sum of the intensities rather the sum of the amplitudes which are then squared.
The intensity of the incoherent imaging system also in the form of a convolution between the input intensity and sinc squared. This means we can determine the frequency response for the incoherent imaging system which is the Fourier transform of sinc squared which is a triangle function (the autocorrelation of a square). The intuitive explanation for this is described in Figure 13.

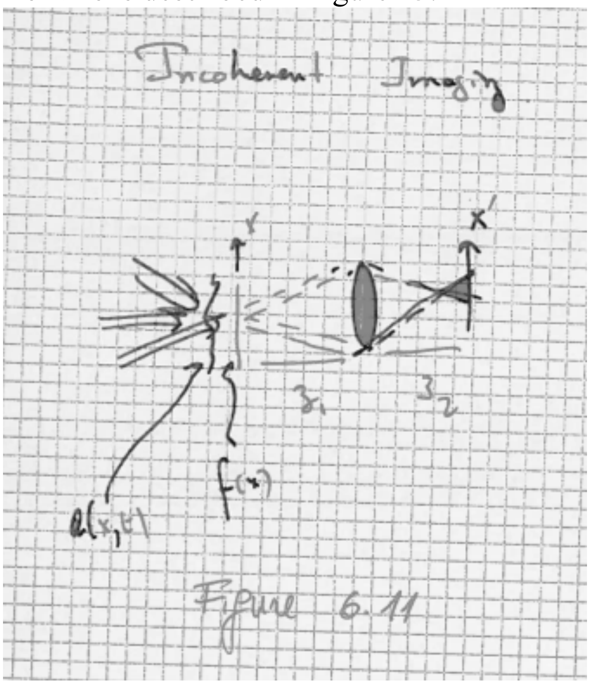


Figure 15

Diffuse illumination:

Diffuse illumination is the opposite of collimated. Light propagates in all directions. Spatially incoherent light is diffuse as we just saw but is also possible to have coherent light (laser light) that is diffuse. This is the case when we bounce a laser off the (rough) surface of a wall. Let us assume the amplitude reflectivity of the wall is f(x,y) and the roughness introduces a phase $e^{-j\phi(x,y)}$ then the input (the object) to the imaging system is the product of the two. The effect of the diffuse illumination is similar to the incoherent light illumination discussed above except now at the output we have

$$|g(x',y')|^2 = B^2\left[\iint\iint e^{-j\phi(x,y)}e^{+j\phi(\xi,\eta)}f(x,y)f^*(\xi,\eta)\right.$$

$$\left.\times\quad \operatorname{sinc}\left[\frac{B(x-x'/M)}{\lambda z_1}\right]\operatorname{sinc}\left[\frac{B(y-y'/M)}{\lambda z_1}\right]\operatorname{sinc}\left[\frac{B(\xi-x'/M)}{\lambda z_1}\right]\operatorname{sinc}\left[\frac{B(\eta-y'/M)}{\lambda z_1}\right]dxdyd\xi d\eta\right]$$

This is very similar to the incoherent case but there is an important difference. In the incoherent case the randomness was due to the unknown temporal oscillation of the light which was averaged out by the detector integration. In this case the randomness (the unknown) is the exact shape of the wall. We take the shape wall to be a random variable with some probability distribution. In order to obtain the average we would have to repeat the experiment many-many times on different walls or at least on different positions on the wall and then compute things like the average (the expected value) and the standard deviation. For a single measurement the coherent scattering from the wall is very noisy (speckle; see Figure 16 below) because this averaging has not been done. If we model the roughness of the wall as a random variable we can calculate the mean and standard deviation of the image intensity and then divide the two to obtain the signal to noise ratio. When we do that we obtain SNR=1.

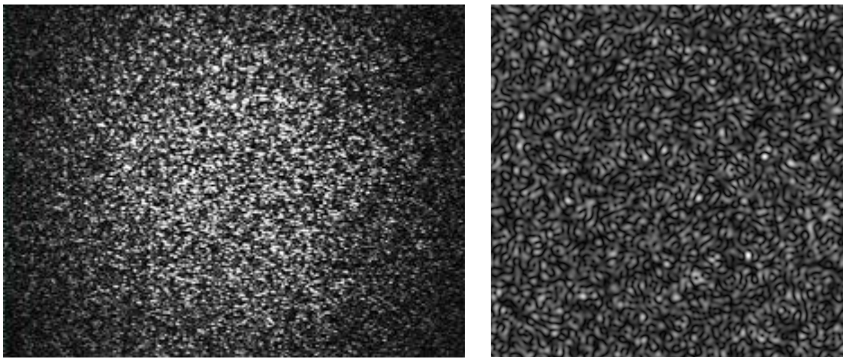


Figure 16.