Imaging Optics 2021

Course Note 5

Lecture 9 and/or 10

Outline

1. Lens waveguide; Hermite polynomials
2. Propagation in inhomogeneous media-split step method (Prism; GRIN lens)
3. Single mode fiber
4. Coupling into a single mode fiber from the end or the side
5. Radiation modes, scattering in, scattering out.
6. Multimode fibers
7. Speckle correlation with multimode fiber; spectrometer

In this lecture we will discuss optical waveguides, a topic of practical importance because optical fibers are used widely in communications. For the first time in the course we will propagate in true inhomogeneous media where the complex dielectric constant ε is continuously varying in space instead of having thin transparencies separated by segments of homogeneous space. Before we examine true waveguide structures we will start with the lens waveguide.

Lens waveguides:

A diagram of the lens waveguide is shown in Figure 1. The system is a cascade of imaging systems, each feeding its successor. If each imaging stage has unit magnification and distortions due to the finite aperture, aberrations or scattering can be ignored, then any input field is replicated periodically in z. In this way the light is "guided" along the z axis. It is possible to have stable waveguiding structures of this type where the magnification of an individual stage is not one but the magnification of each stage is compensated by demagnification of one or multiple subsequent stages.

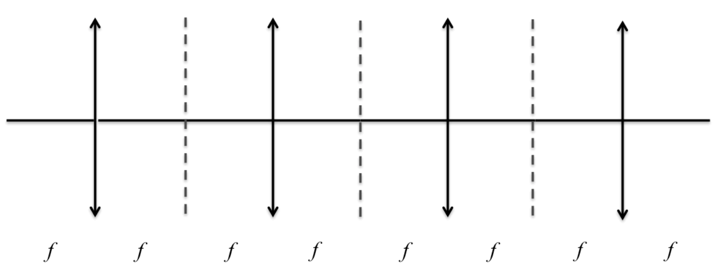


Figure 1

Another interesting configuration is a unit magnification 4F imaging system (two Fourier Transforming lenses in a row). This is the system drawn in Figure 1. Equivalently, we can think of this system as a sequence of FT lenses with each lens in the sequence taking the FT of the output of it predecessor. One way that this system can guide light is if the Fourier transform of the pattern midway between two lenses is equal to the pattern itself.

In other words if the pattern is an eigenmode of the FT operator. We have already seen that the FT of a Gaussian is also a Gaussian.

$$FT\left[e^{-\pi x^2}\right]=\left[e^{-\pi u^2}\right]$$

When we take the FT with a lens, x is the real spatial coordinate measured in units of length and the frequency variable u is also displayed in space with $u = x'/\lambda F$. In order to find a Gaussian eigenfunction of the FT lens we need to use the FT scaling theorem and find the properly scaled Gaussian at the input that will be replicated at the output.

$$FT_{lens}[e^{-\pi\left(\frac{x}{w_0}\right)^2}] = e^{-\pi(\frac{x' w_0}{\lambda F})^2} = e^{-\pi\left(\frac{x'}{w_0}\right)^2} \Rightarrow w_0 = \sqrt{\lambda F}$$

The Gaussian is not the only eigenfunction of the Fourier transform operator. There is an entire family of functions called the Hermite-Gauss polynomials that satisfy this requirement. These functions are shown in Figure 2.

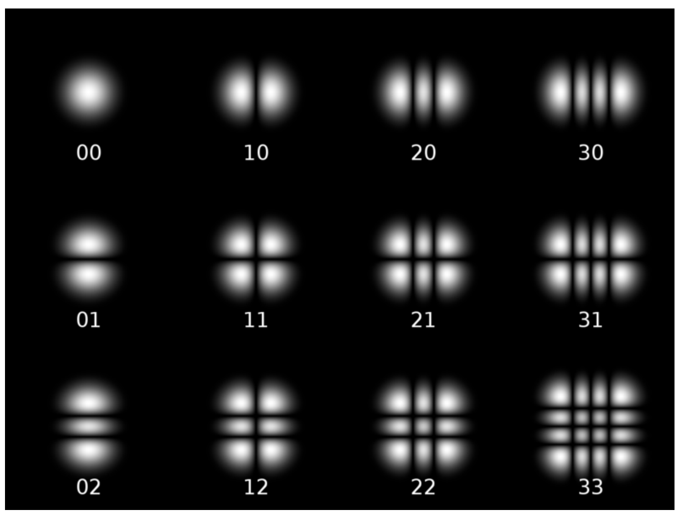


Figure 2

It is interesting to contemplate why the FT of these functions is the same as the function itself. For the simple Gaussian beam the basic idea is that the FT of a “blob” is also a blob. A blob contains mostly lower frequencies and for higher frequencies there is a decreasing representation in the make-up of the blob. The Gaussian beam is the blob for which this argument is exact. For the higher order Hermite polynomials the argument is a bit more interesting. The shape of the beam is reminiscent of a wavelet, a topic you might have learned about in signal processing classes. For the first order we have one cycle of a sine-wave within a Gaussian window.

Looking at the diffraction of this pattern (Figure 3a and 3b) we see that the diffraction to the far field has a zero at x=0 (on axis) because of the symmetry of the + and – contributions of the incident field. It has constructive interference at two symmetric off-axis positions in the far field where the extra path difference between the two input cycles is half a wavelength. In this way the far field pattern (which is the FT of the input) reproduces the shape of the input. An alternative way is to think of the input as the product of a Gaussian and a sine wave. The FT of that is two Gaussians side-by-side (using the convolution theorem) and once again the input is reproduced in the Fourier domain. The shapes of the Hermite polynomial modes are shown in Figure 3a.

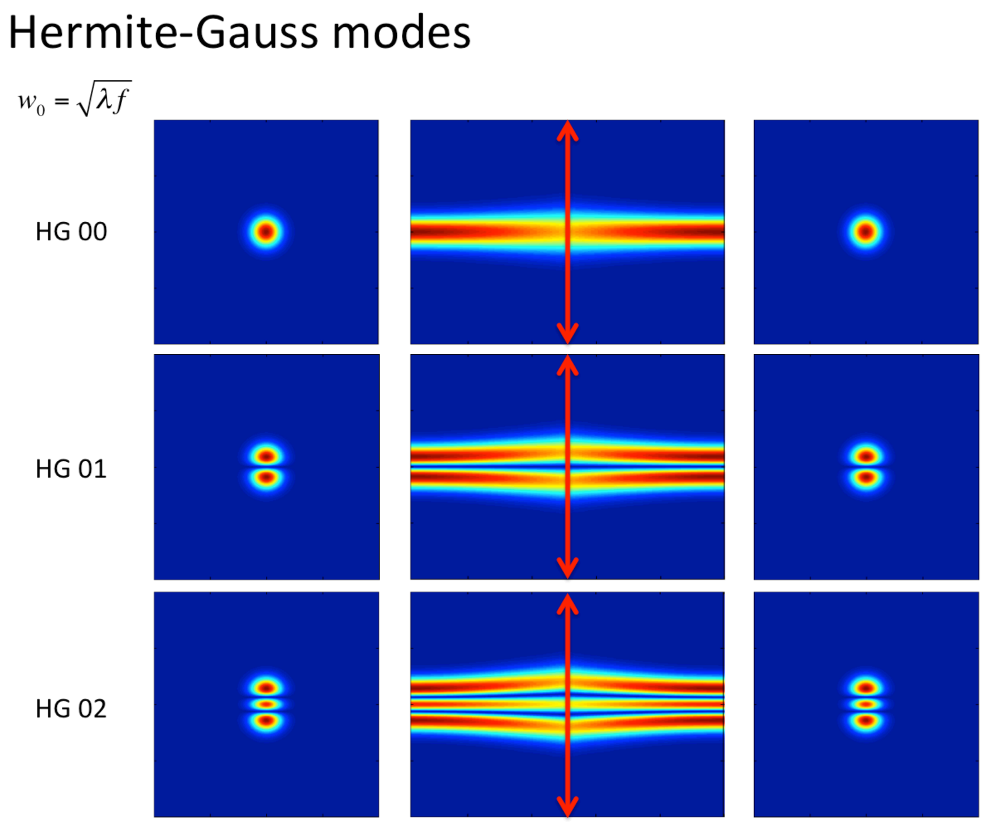
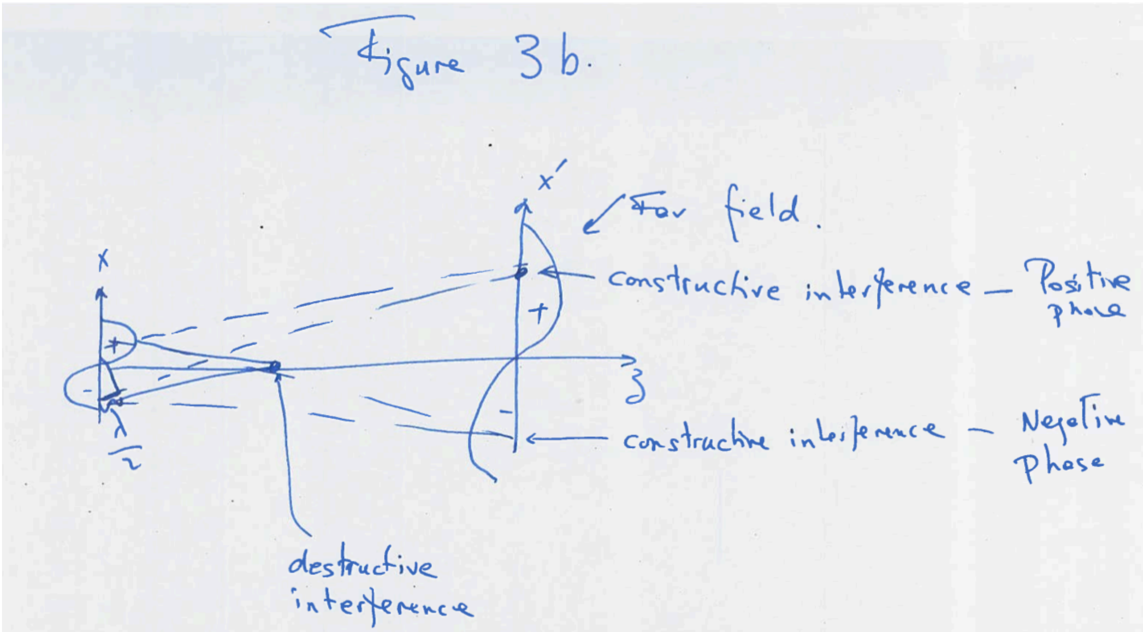


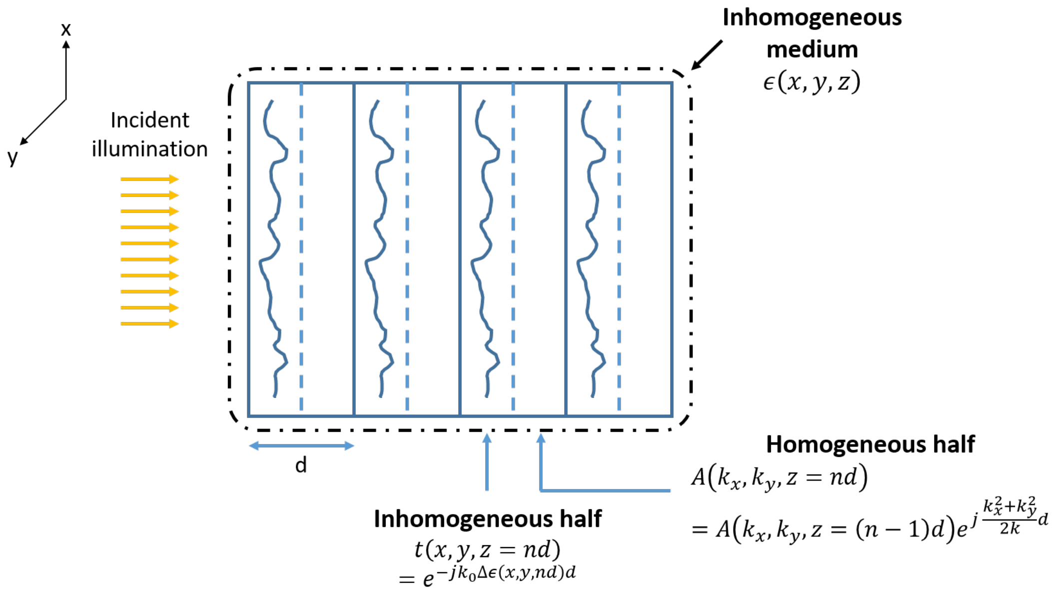
Figure 3a



Split Step Method:

So far we have been looking at combinations of long stretches of homogeneous media separated by thin transparencies. In such systems we were able to apply the BPE once for each chunk of homogeneous medium and once for each thin transparency. The difference in applying the beam propagation method in continuously varying inhomogeneous media is that we have to do it one thin slice at a time. (Figure 4). We consider each slice as consisting of a "half" that is free space and the other "half" being a thin transparency. It the slice is thin enough then the effects of diffraction and light modulation by ε(x,y,z) can be treated sequentially rather than simultaneously.

Split step method



Let's consider the propagation of a Gaussian beam through a GRIN lens. We expect that a collimated Gaussian beam will become a focusing spherical wave as it propagates through the lens. We can study that with the BPE and experiment with the consequences of the choice of the step size in z. Figure 5. The thinner we make Δz, the more "accurate" the simulation becomes and eventually as we decrease Δz further the result no longer changes . In fact this is exactly how we determine that the sampling is fine enough: Decreasing Δz no longer changes (improves) the simulation result. Notice that the focused spot in Figure 5 is sharper (more focused) for larger Δz and then spreads as the sampling in z becomes finer. Why? When we start with coarse steps we effectively treat the GRIN lens as a thin transparency neglecting the effects of diffraction through the finite thickness of the lens. The index modulation of the GRIN lens has the right phase modulation to produce a converging spherical wave. The effect of the finite thickness acts as an aberration that broadens the focused spot.

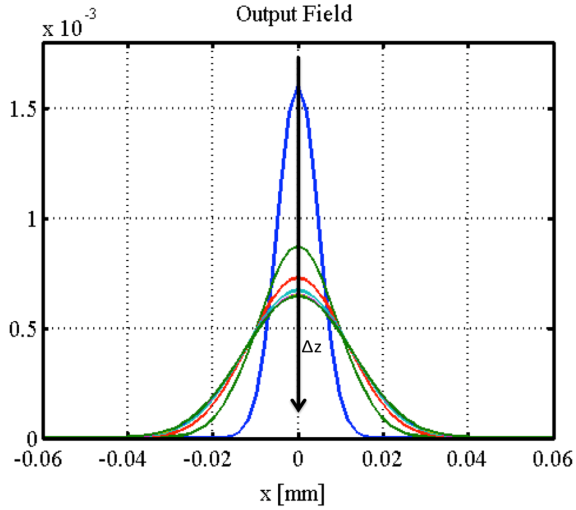
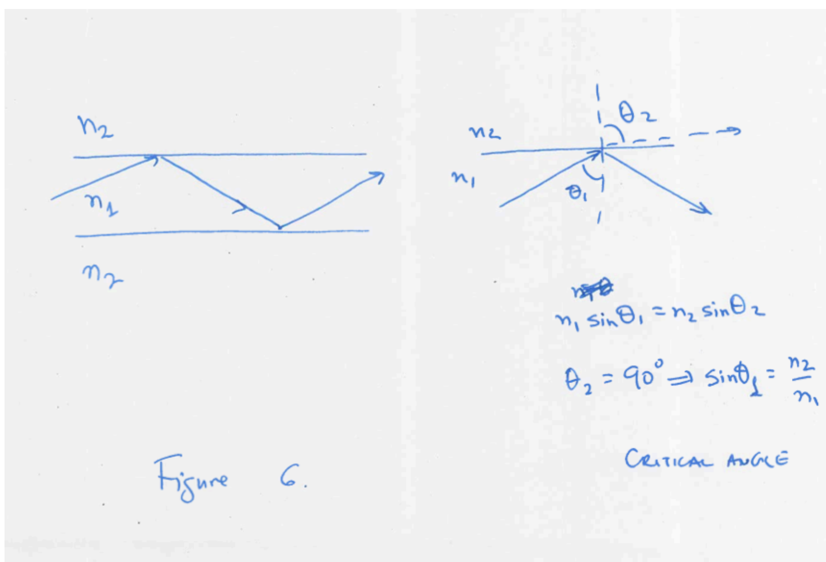


Figure 5

Dielectric Waveguides

The schematic diagram of a planar dielectric waveguide is shown in Figure 6.



A dielectric layer of thickness d and index n_1 is sandwiched between two dielectrics of index $n_2<n_1$. Intuitively, this structure can guide light because rays are reflected into the core through total internal reflection at the interface of the two dielectrics . A ray incident on a dielectric interface will undergo total internal reflection if the incidence angle θ_{inc} is smaller than the critical angle. The critical is obtained from Snell's law. When θ_2 becomes 90 degrees (see Figure 3) then θ_1 is at the critical angle sin $\theta_c = n_2/n_1$. The incidence angle is $\theta_{inc}=90^0- \theta_1$. Therefore if the two indices are not too far apart we expect the propagation inside the waveguide to be paraxial.

In Figure 7 we show the propagation of light in such a "slab" waveguide for varying distances d when the structure is illuminated with a Gaussian beam. As d gets smaller the light becomes regular and does not change as it propagates in z. This is a mode of the waveguide and at the smallest scale for d the waveguide only allows one mode to propagate. Any further decrease in d causes the light to leak out. If d is increased then multiple modes can propagate at different phase velocities. The interference among the modes is responsible for the observed variation in z.

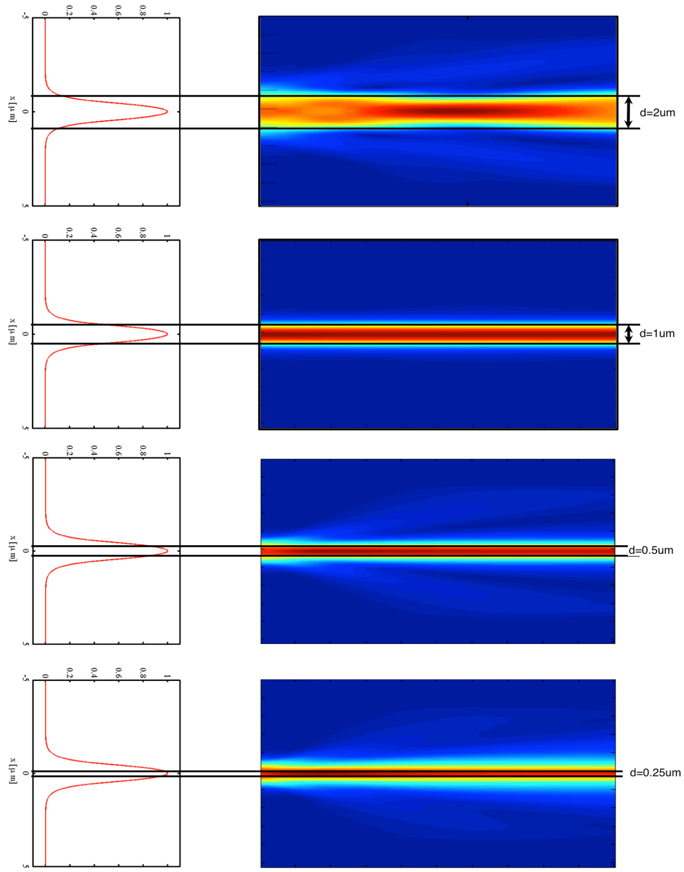
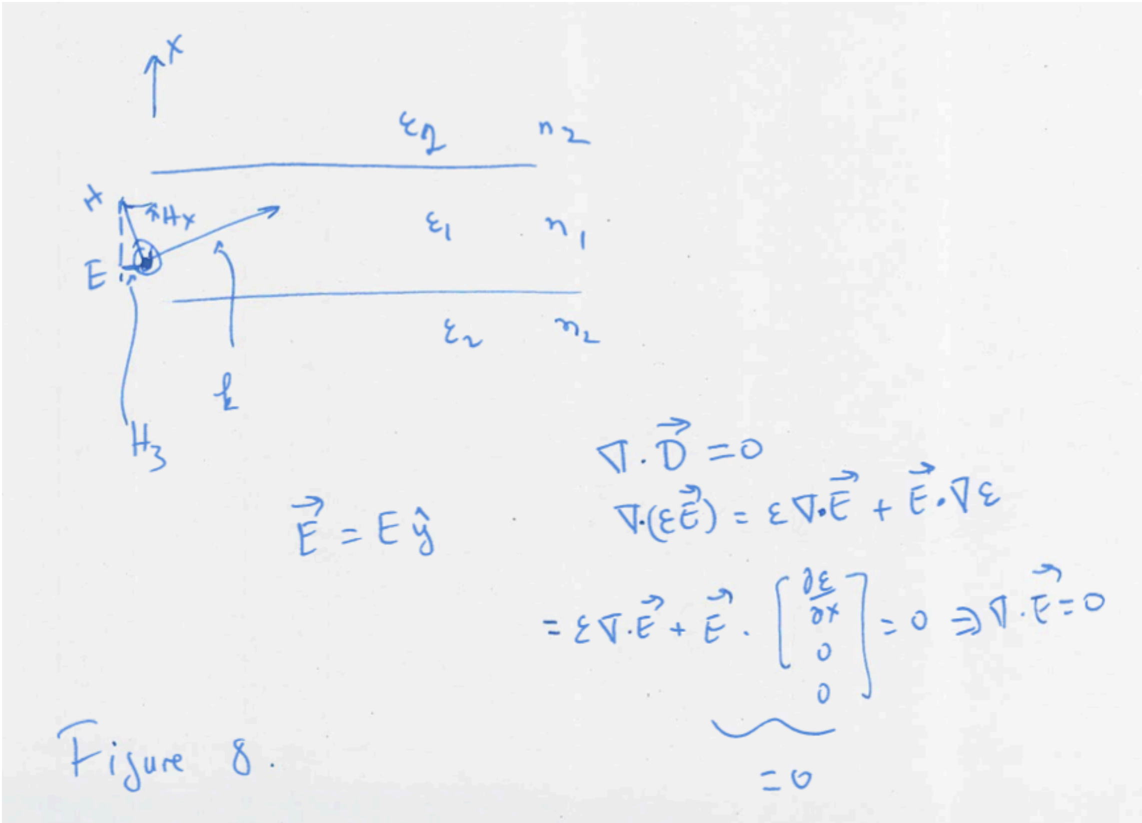


Figure 7

This cut-off behavior of dielectric waveguides is only true for TE modes. TE stands for Transverse Electric field which means the electric field does not have an E_z component but the magnetic field H does. For a TE mode we can show that the divergence of the E field is zero in a slab waveguide and most other dielectric waveguide structures. See Figure 8. We used the zero divergence condition to derive the wave equation and hence the BPE. If $\nabla \cdot \vec{E} \neq 0$ then the BPE we have been using is no longer valid and we have to use the "vectorial" version of the BPE or some other numerical method to simulate Maxwell's equations (finite element methods) that can account for the vectorial nature of the optical field. In this class we will stick with scalar diffraction theory which makes it possible to examine propagation phenomena is a more transparent and intuitive way. The intuition we will gain generally transfers to the more complex cases that require vectorial analysis/simulation even though the details may require more complex computational tools than the BPE we use.



In Figure 9 we show the diffraction pattern of a thin cross section of the waveguide structure. This is a slice of the waveguide in one step of the BPE. We can see that the diffraction focuses the light, similar to the way the rings we saw earlier focus the light on-axis by diffraction. The diffraction process is in general a defocussing or light spreading process. Therefore the dielectric waveguide is not too different from a lens waveguide.

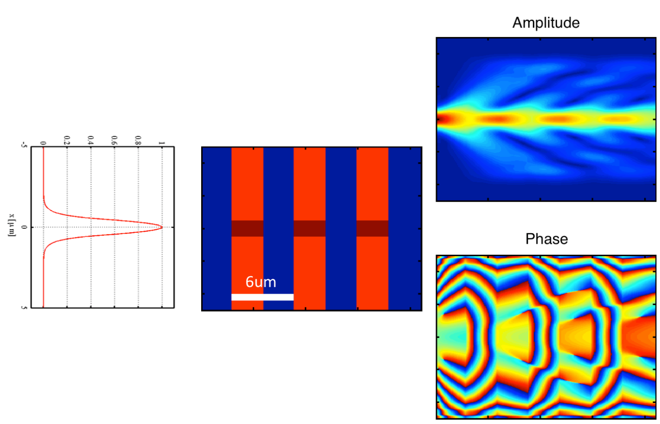


Figure 9

Waveguide modes:

We saw that for smaller values of d the light distribution inside the high index region becomes Gaussian-like and the distribution in the low index regions decays rapidly (exponentially) with increasing x. We can attribute the rapid decay in the low index regions to the total internal reflection of light incident from the high index side. How about the light distribution inside? If d is too small then there is no light propagation possible because the light will diffract at high angles and not be trapped by the total internal reflection at the boundary between the two dielectrics. This "cut-off" as d gets smaller depends on the wavelength since the diffraction angles depend on the wavelength (smaller wavelengths diffract less therefore a small waveguide can guide short wavelengths).

As d increases we can fit a larger single Gaussian-like mode at the entrance which propagates. At the larger d values we can also fit modes with multiple bumps. Why is that? We can think of the input having a certain numerical aperture (the divergence due to diffraction $\Delta\theta \approx \lambda / w$ where w is the input spot size) and then this traps the light due to total internal reflection. If w gets smaller then the light cannot be trapped. If w gets larger then we can fit a bigger Gaussian beam which diffracts at smaller angles. If we imagine these bigger beams being modulated by a grating in x then the light will diffract at a larger angle but it will still be trapped. Therefore a waveguide with larger d will be able to carry more modes (modes with different shapes).

What is a mode? When we consider an inhomogeneous medium that does not vary in z (the propagation direction) then we expect the solution to Maxwell's equations to be in the form $E(x,y,z) = A(x,y)e^{-jk_z z}$ because of translational invariance in z. If we shift the medium by d then the solution must be the same except for a multiplication by a constant since a constant multiplier in front is always possible as a solution to the wave equation. Therefore in a system like this obeys the following:

$$E(x,y,z=d) = A(x,y)e^{-jk_z(z+d)} = E(x,y,z=o)e^{-jk_z d}$$

Therefore $A(x,y)$ is the eigenfunction and e^{-jkz} is the eigenvalue. We expect from our discussion above that waveguides with larger d will support modes. But how do we determine how many modes? In free space the modes were plane waves and there is an infinite continuum of them. In this case the modes are discrete. One way to find that out is go back to Maxwell's equations. Within each of the 3 regions the medium is homogeneous (ε is constant) and therefore the wave equation as previously derived holds. The solution to the equation also holds but only over the limited region of constant ε. The additional constraint is that at the boundaries between regions the continuity of the electric and magnetic fields across the boundaries must be satisfied. These constraints give us a discrete set of solutions for the modes of the waveguide. The equations for the modes for a dielectric slab and a cylindrical dielectric waveguide (optical fiber) are shown in Figure 10.

Single mode slab waveguide

$$E_y(x, z, t) = E_m(x) \exp[i(\omega t - \beta z)]$$

$$E_m(x) = \begin{cases} A \sin hx + B \cos hx, & |x| < \frac{1}{2}d \\ C \exp(-qx), & x > \frac{1}{2}d \\ D \exp(qx), & x < -\frac{1}{2}d \end{cases}$$

Multimode fiber

$$E_y = \begin{cases} AJ_l(hr)e^{il\phi} \exp[i(\omega t - \beta z)], & r < a \\ BK_l(qr)e^{il\phi} \exp[i(\omega t - \beta z)], & r > a \end{cases}$$

Figure 10

If we know the modes of the waveguide (or any other structure whose ε does not vary in z we can generally take any input light distribution and express it as a linear combination of the eigenmodes, then propagate each mode with its own eigenvalue (or phase delay due to its distinct k_z) and then sum up the fields at the end. This is analogous to the plane wave decomposition of an input field in homogeneous media. Remember that a plane wave is the eigenmode of homogeneous linear medium.

If we illuminate a slab waveguide from the side then most of the light goes through it. See Figure 11. Temporarily some light gets trapped but then it leaks back out The modes that are not supported inside the waveguide are called radiation modes.

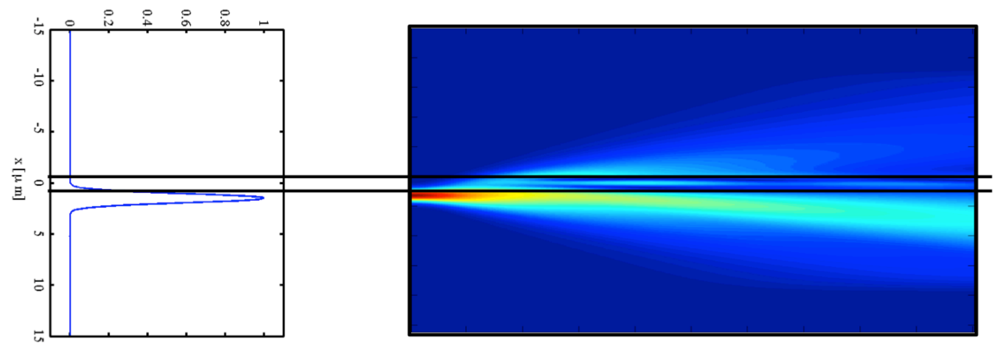


Figure 11

Optical fibers are generally illuminated from the edge. If the illumination at z=0 matches one of the modes then all or most of the energy is coupled to this one mode. Otherwise in a multimode fiber the input energy is coupled to multiple modes. See Figure 12.

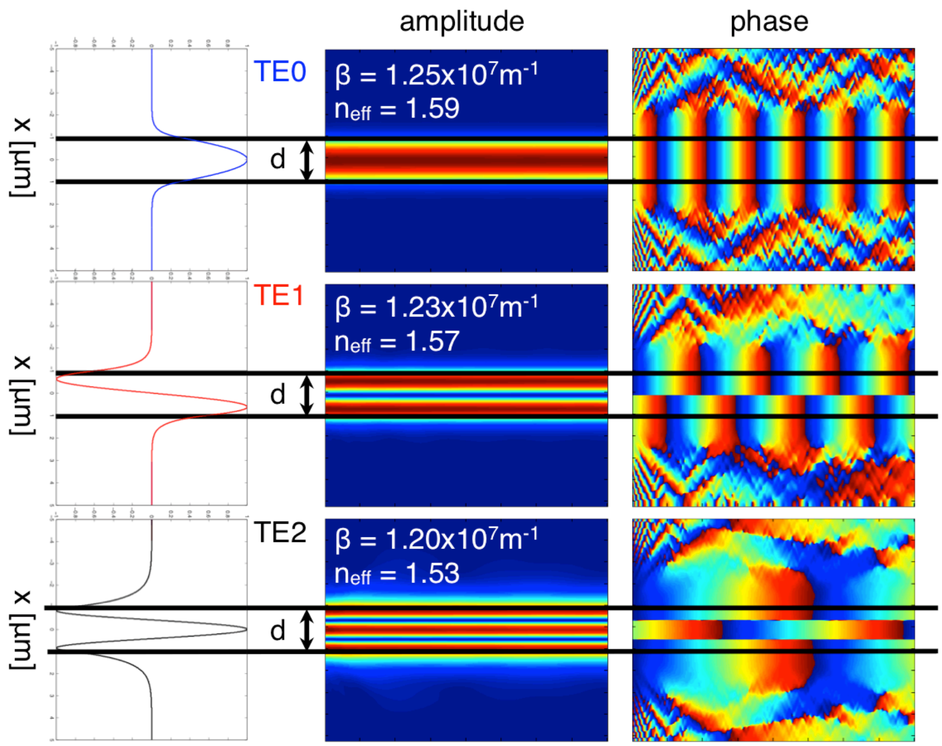


Figure 12

Another way to couple light into optical waveguides is via 'evansescent' coupling. Each of the modes in a fiber has its own propagation constant k_z. Therefore if we can find a way to introduce an optical field that has the same component k_z then idt will launch the corresponding mode selectively Figure 13 .

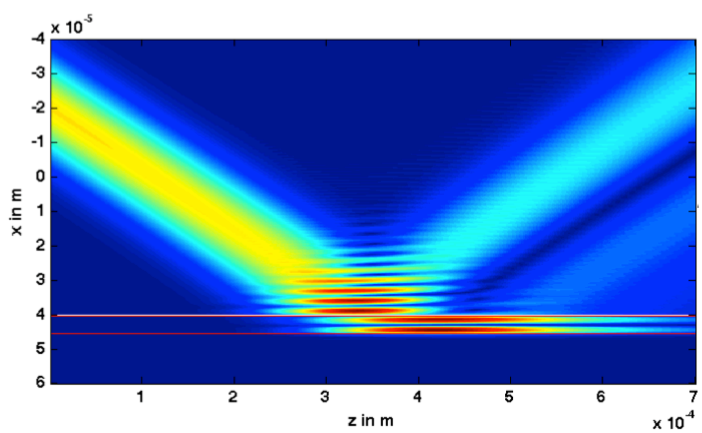


Figure 13

Speckle:

In a fiber with many modes we can obtain speckle at the output. Speckle is a random pattern that forms when mutliple beams with unrelated phases interfere. In Figure 14 the illumination of a large diameter fiber (in other words a fiber that can support many modes) is illuminated with a Gaussian beam. We can see the result at the output face of the fiber is a speckle pattern.

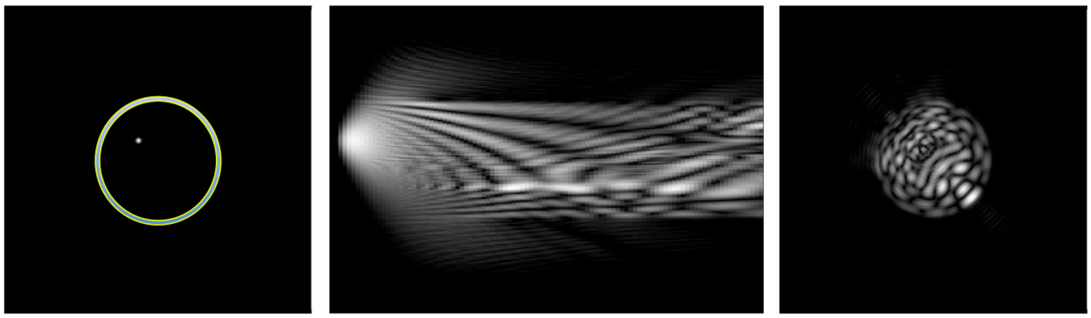


Figure 14

If we allow this pattern to diffract in free space we obtain rings. See Figure 15. Why? If you look at the equations for the mode of a cylindrical waveguide the mode is a Bessel beam. The same Bessel beam that we encountered earlier that diffracts slowly. The far field of Bessel beams are rings.

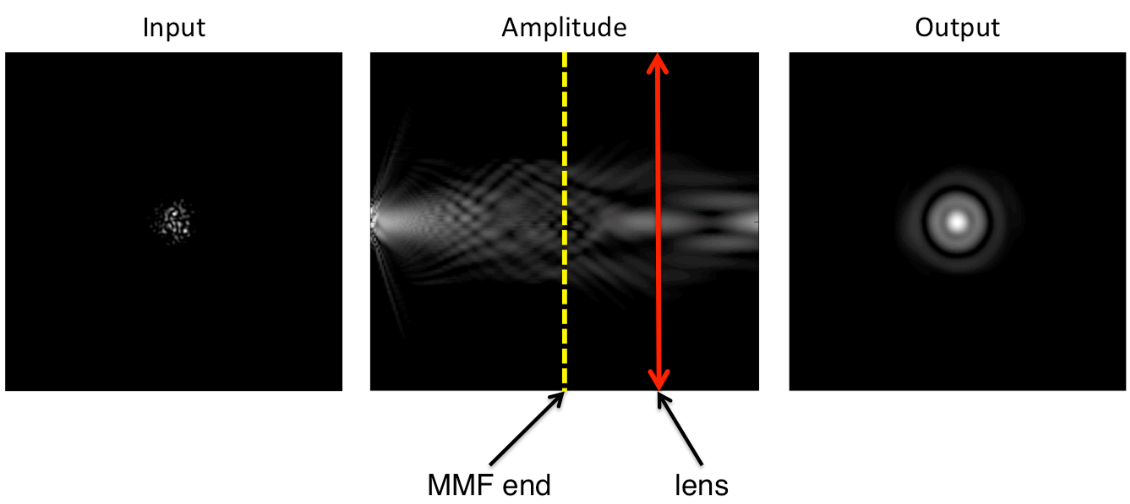


Figure 15

If we start with a point source at one end of a multimode fiber then we get a speckle pattern at the output. We can make an imaging device out of a multi-mode fiber by illuminating with point sources one at a time on a regular grid at the input and recording the complex field at the output (the speckle pattern). When we put an arbitrary pattern as an input we can recover the input by forming inner products of the resulting speckle pattern with all the stored speckle patterns Figure 16.

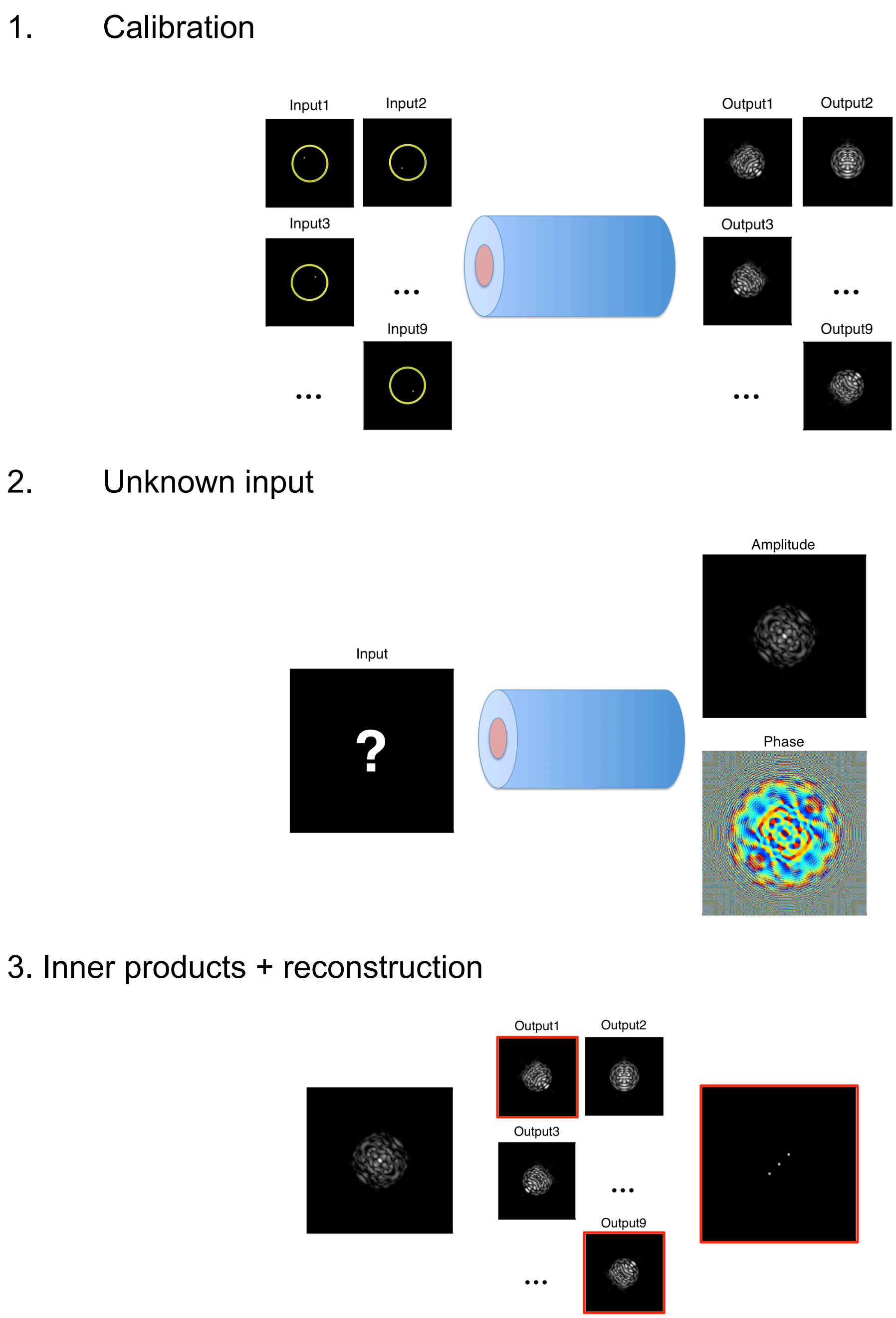


Figure 16