Imaging Optics - Lecture 4 – 19/3/2020

Thin transparencies, lenses, imaging

Outline

1. Inhomogeneous media
2. Thin transparencies
3. Gratings
4. Lenses

   Imaging sections:

5. Imaging in animals
6. Lens and diffraction
7. Imaging as a filtering operation
8. Telescope
9. Resolution; point spread function
10. FT lens
11. 4F system
12. Phase contrast

So far we looked at wave propagation in homogeneous media such as air, free space. We have implicitly assumed that the 2D distribution of light was modulated somehow at the input plane in amplitude and phase and we were able to predict how light propagates in arbitrary distances through free space once this initial 2D spatial modulation is established. In real life the modulation of light takes place through its interaction with matter. Often the devices that are used to imprint a desired amplitude, phase, time, polarization and color modulation on an optical beam, are thin elements that modulate the light at a single plane. Liquid crystal devices are widely used for this purpose in television sets and also projectors. The old fashion black board is another example. Such thin transparencies can be used not only as input devices in optical systems but also in the optical train as light modulating elements that define the functionality of the optical system. Lenses (thin lenses in particular) are the best known and the most important optical device of this type since lenses enable imaging in our eyes, cameras, telescopes, microscopes, and many other optical instruments. In order for a device to spatially modulate the properties of a light beam (write an image) it must be constructed with an inhomogeneous medium. In other words its optical properties are different from one spatial location to another. We will first modify the BPE to accommodate propagation in inhomogeneous media and we will then we will use it to investigate thin transparencies.

Interaction of light with matter:

Light interacts with matter by exerting electromagnetic forces on charged particles inside matter (electrons and ions). These interactions can have a profound effect on the matter. Two examples are ablation and laser tweezing where the solid matter is completely removed or mechanically moved around by the light.

Laser Ablation video:

http://www.youtube.com/watch?v=bVDFw_eHfvc

Optical Tweezers video:

http://www.youtube.com/watch?v=ju6wENPtXu8

The interaction of light with matter can also have a very strong influence on the light. Light can be absorbed, amplified, reflected, refracted, scattered, or phase modulated (among other things) by matter. If we return to Maxwell's equations (Lecture 1) and look for where the material properties appear, we will find them hiding in the variables ε and μ. The permittivity of the material (or its dielctric constant) and its magnetic permeability. The conductivity of the material is another property that affects optical light propagation ($Conductivity\,(\sigma)$) if there are free charges and currents in the medium. In the optical domain the interaction of light with free carriers in metals is often called plasmonics. For this lecture we will consider materials with no free charges and therefore no currents. Glasses and dielectrics fall in this category. Metals and semiconductors do not. The interaction of light with the bound charges inside matter (the electrons) leads to absorbtion of incident photons and a change in the speed of light through the material. In Maxwell's equations this is accounted for by defining a compex dielectric constant:

$$\varepsilon = \varepsilon' + j\varepsilon''$$

notice that if we had a material with constant (uniform) imaginary dielecric constant then the propagation of light through this material in the z-direction would be a plane wave decaying exponentially in z. The wavevector $k_z = \omega^2 \mu \varepsilon$ is complex in this case leading to the decaying wave.

What if ε is a function of space? In other words, what if the medium is inhomogeneous? Going back to the derivation of the BPE little would change in the derivation (see Lecture 1 power point presentation). There is a step where we set $\nabla \cdot \vec{E} = 0$ which was made possible through the $\nabla \cdot \vec{D} = \nabla \cdot \varepsilon \vec{E} = 0 = \frac{\partial \varepsilon}{\partial x} E_x + \varepsilon \frac{\partial E_x}{\partial x}$ equation. Normally we can neglect the derivative of epsilon with respect to x if the spatial modulation of $\varepsilon = \varepsilon_0 + \Delta\varepsilon(x,y,z)$ is weak ($\Delta\varepsilon \langle\langle \varepsilon_0$). Then the wave equation for the inhomogeneous case becomes the same as before except the dielectric constant is spatially varying. Assuming a solution in terms of the slowly varying envelope $E = e^{j(\omega t - kz)} A(x,y,z)$ and substituting in the inhomogeneous wave equation we obtain after dropping the second derivative of A with respect to z as before:

$$2jk\frac{\partial A}{\partial z}=\frac{\partial^2 A}{\partial^2 x}+\frac{\partial^2 A}{\partial^2 y}+\omega^2\mu\Delta\varepsilon(x,y,z)A$$

This is quite interesting since we have the effects on propagation due to the inhomogeneity being "added" to the same BPE. In other words, A(x,y,z) changes as light propagates in z due to diffraction as before and in addition it changes because of the inhomogeneity. If the transverse spatial variation of the light is small and the transverse spatial derivatives can be neglected then

$$\frac{\partial A}{\partial z}=\frac{-j\omega^2\mu\Delta\varepsilon(x,y,z)}{2k}A$$

For a thin transparency (Figure 1) we can neglect any dependence of Δε on z and in this case

$$A(x,y,z=\Delta z)=A(x,y,z=0)e^{\frac{-j\omega^2\mu\Delta\varepsilon(x,y,z=0)}{2k}z}=A(x,y,z=0)t(x,y)$$

If Δε is real then we have a thin phase transparency. If it is imaginary then we have an absorption of scattering thin transparency. A hybrid amplitude and phase is also possible. The characteristic feature of a thin transparency is that the field immediately after the transparency is just the product of the incident field and the transmittance t(x,y). This implies that the diffraction of the field is negligible while propagating through the thickness Δz of the transparency. (Homework).

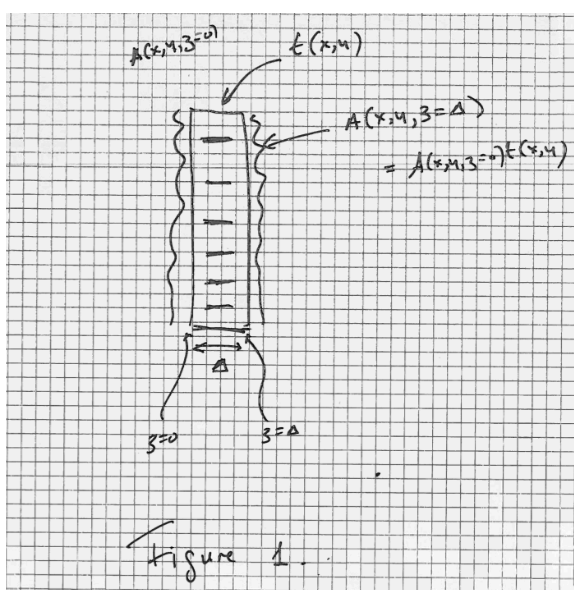


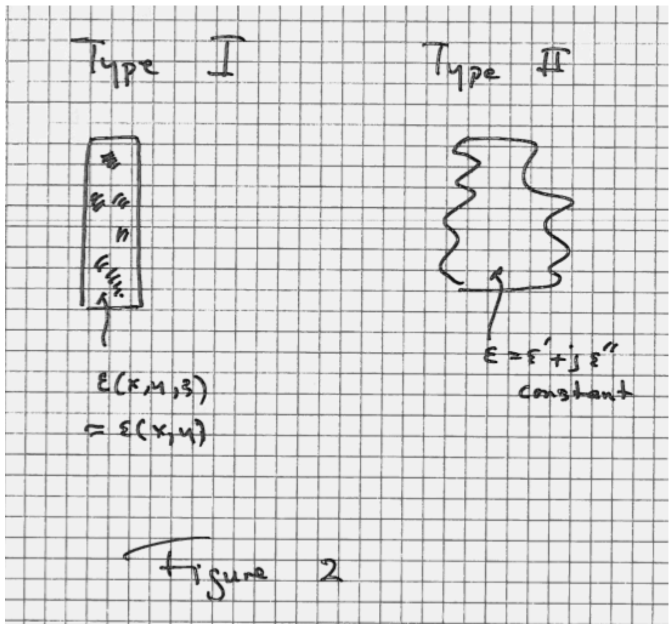
Figure 1.

Types of thin transparencies: There are two basic ways to record a thin transparency (Figure 2) :

Type I. Spatial modulation of the complex index

Type II. Spatial modulation of the shape of the medium

The first category includes photographic film, liquid crystal displays, phase or amplitude gratings, etc. The second category includes most lenses, blazed gratings, binary optics, etc.



Gratings

Amplitude grating: (Figure 3a)

$$\varepsilon = j\varepsilon'' \text{ and } t(x,y) = e^{\frac{\pm j\omega^2\mu(j\varepsilon')}{2k}z} = \frac{1}{2} + \frac{1}{2}\cos(Kx)$$

The +/- comes from the $k^2 = \omega^2\mu\varepsilon$ relationship. Either sign would satisfy the relationship. For real ε the two opposite signs represent forward and backwards propagation. For imaginary ε they represent attenuation versus gain. If we know there is no gain in the system, we pick the sign (+ in this case) that gives decay. Notice that there is no simple relationship between ε' and the desired transmittance. A calibration process is normally required to obtain the desired result. For a photographic film the absorption is determined by the density and thickness of the absorbing molecules. For a metallic coating amplitude modulation can be achieved by varying the thickness of the metal coating. Otherwise techniques such as half-toning can be used.

Phase Gratings: (Figure 3b)

A phase grating can be realized by periodic modulation of the dielectric constant (or index) of the material.

$$t(x,y) = e^{\frac{-j\omega^2\mu\varepsilon''}{2k}z} = e^{\frac{-j\omega^2\mu\varepsilon_0\sin(Kx)}{2k}z} = e^{-j\alpha\sin(Kx)}$$

The phase grating has a well known series expansion with the coefficients being Bessel functions. Therefore phase gratings have many diffraction orders.

Another way to realize a phase grating is by periodically varying the thickness of a homogeneous medium. (Figure 3c). We can write the transmittance of a phase grating of this type as

$$t(x,y) \sim e^{\frac{-j\omega^2\mu\varepsilon_0'}{2k}z_0\sin(Kx)} = e^{-j\beta\sin(Kx)}$$

which has the same form as the index modulated phase grating.

Phase Wrapping: It is possible that the combination of the thickness z of the grating and the change in dielectric constant $\Delta\varepsilon = \varepsilon'' - \varepsilon_0$, with respect to the free space dielectric constant ε_0 is sufficiently large so that the exponent of the transmittance of the thin phase transparency ($t(x,y) = e^{\frac{-j\omega^2\mu\varepsilon''}{2k}z}$) exceeds 2π. This introduces an ambiguity when we attempt to measure the phase of the function of the transparency but it can be useful in designing thin optical elements. For instance, when the optical phase element is normally fabricated by varying the thickness of a homogeneous dielectric (lens, prism) then a thin version of the same element can be fabricated by recognizing that we never need a material thickness more than the thickness necessary to reach an optical equal to 2π to synthesize a phase wavefront. The thin version of the conventional glass lens is the Fresnel lens and the thin equivalent of the prism is the blazed grating (Figure 4).

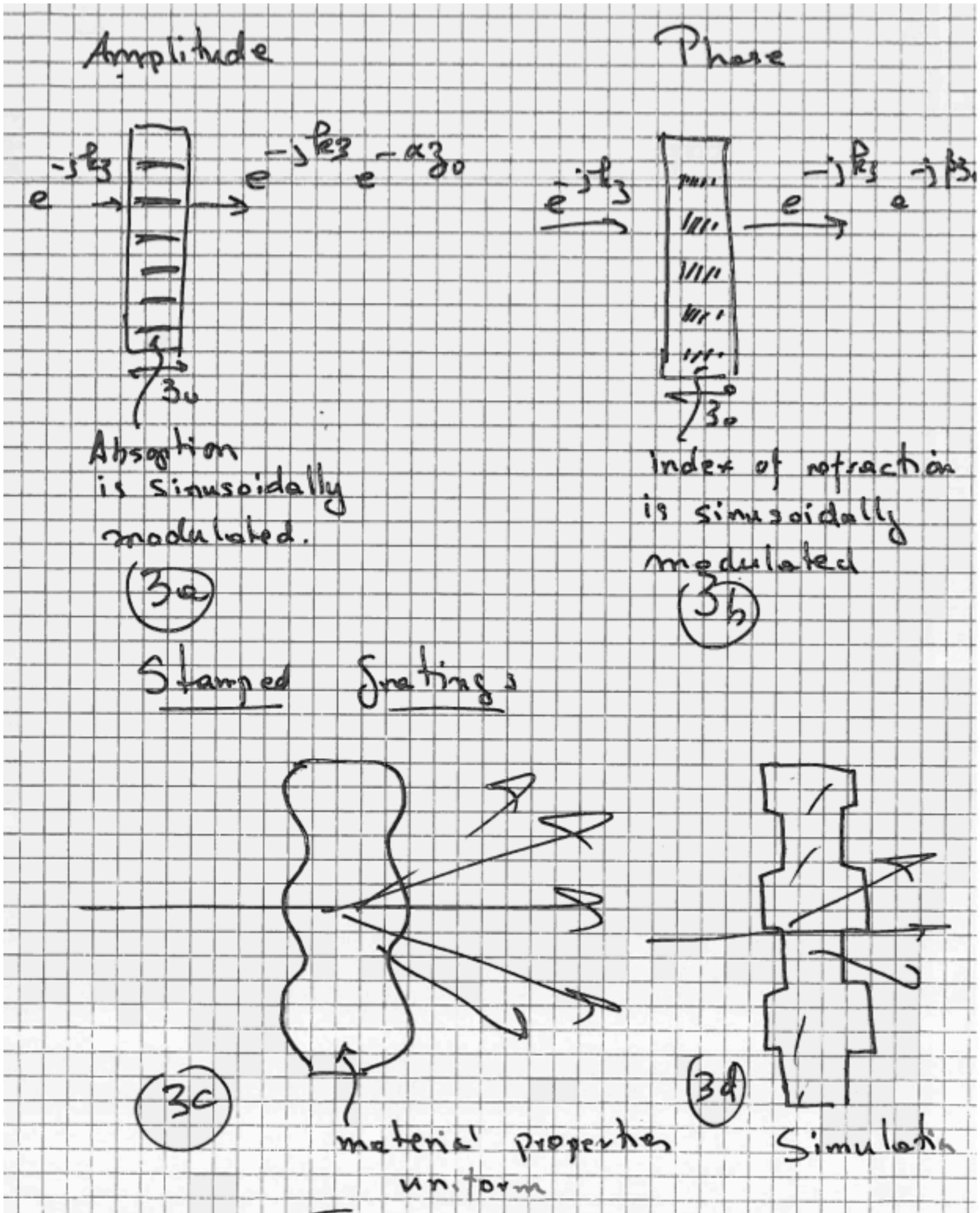
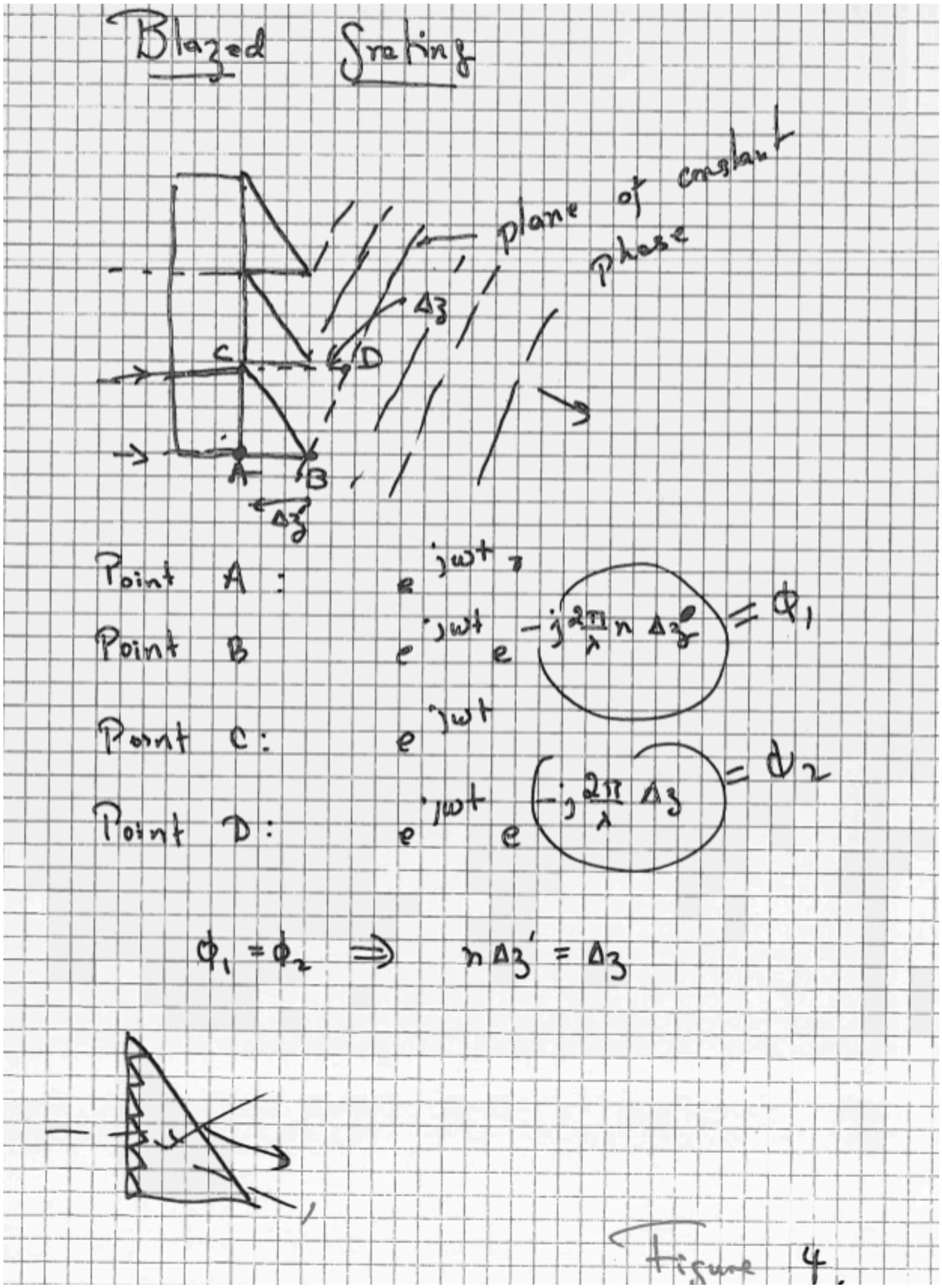
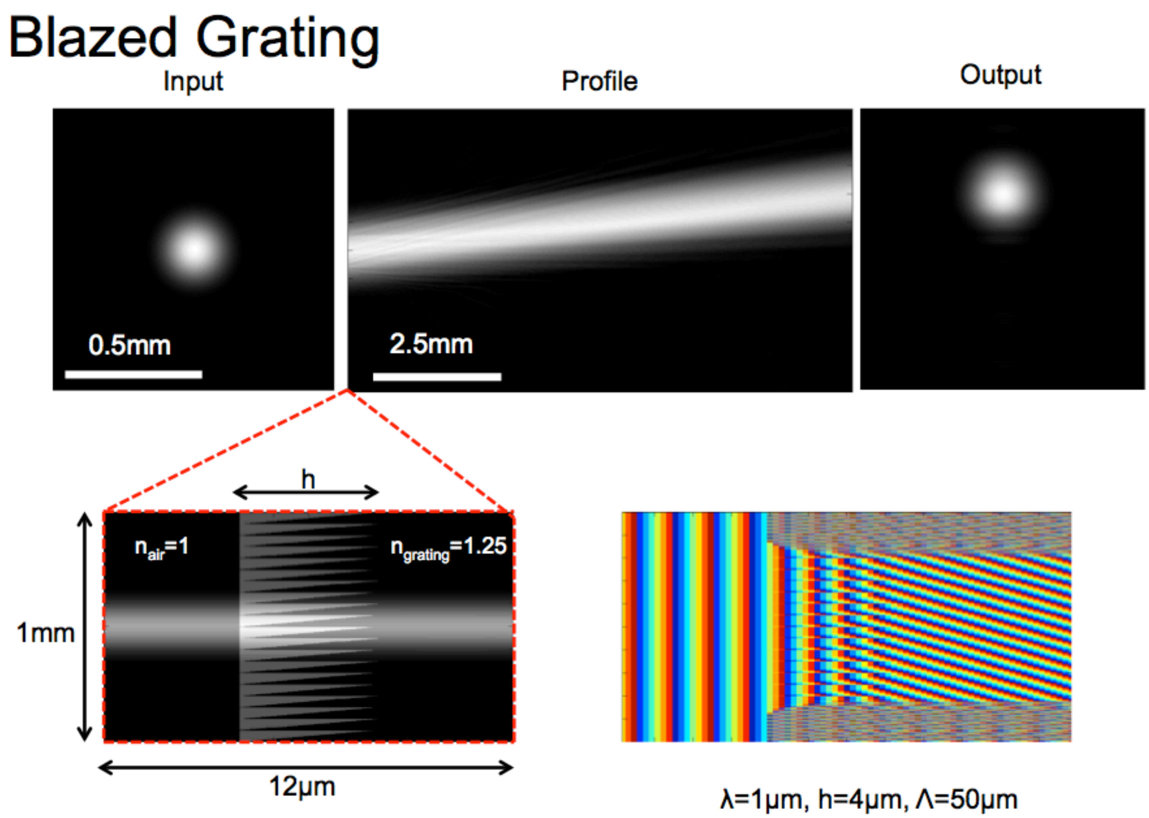

Amplitude
Phase
e^{-jk_z}
$e^{-jk_z} e^{-\alpha z_0}$
z_0
Absorption is sinusoidally modulated.
3a
e^{-jk_z}
$e^{-jk_z} e^{-j\beta z_0}$
z_0
index of refraction is sinusoidally modulated
3b
Stamped Gratings
3c
material properties uniform
3d
Simulation

Figure 3.

Blazed Grating

Figure4





Lens.

The transmittance of a normal convex lens is

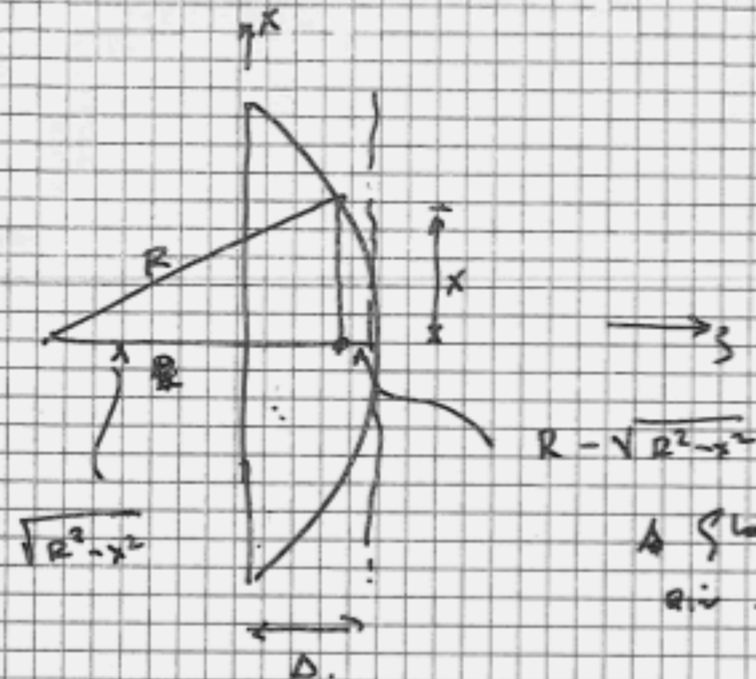
$$t_{lens} \sim e^{+j\frac{\pi}{\lambda F}(x^2+y^2)}$$

See Figure 5 for the derivation. The lens is a very special transparency in optics. This is because in the paraxial approximation it is the complex conjugate of the impulse response of the Fresnel diffraction operator. Recall that the impulse response of the Fresnel diffraction is the paraxial form of a spherical wave:

$$E_{Sph} \sim e^{-jkr} \cong e^{-jkz_1 - \frac{j\pi(x^2+y^2)}{\lambda z_1}}$$

$$r = \sqrt{z_1^2 + x^2 + y^2} \cong z_1 + \frac{x^2+y^2}{2z_1}$$

Lens



$\xi_{lens} = \Delta - \left(R + \sqrt{R^2-x^2}\right)$

$\Delta \quad D$

in glass: $e^{-j\frac{2\pi}{\lambda} n \left(R - \sqrt{R^2-x^2}\right)}$

in air $e^{-j\frac{2\pi}{\lambda}\left(\Delta - \sqrt{R^2-x^2} - R\right)}$.

Overall:

$$\simeq e^{-j\frac{2\pi(n-1)}{\lambda}\left(R - R + \frac{x^2}{2F}\right)}$$

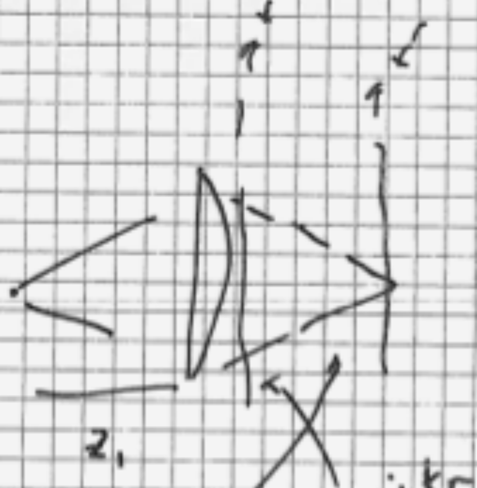
$$e^{-j\frac{2\pi}{\lambda}\Delta}\, e^{-j\frac{2\pi}{\lambda}}\, e^{-j\frac{2\pi}{\lambda}(n-1)\left[R + \sqrt{R^2-x^2}\right]}$$

$$\sim e^{+j\frac{\pi(x^2+y^2)}{\lambda F}}$$

Fresnel lens

Figure 5.

The fact that the diverging spherical wave has minus sign as it diverges while the lens function has a plus implies that the lens can undo the divergence of blurring due to propagation. (Figure 6).



$$e^{jkr} = \underbrace{e^{-j\frac{\pi}{\lambda z_1}x^2} \, e^{+j\frac{\pi}{\lambda F}x^2}}$$

$$\int \underbrace{\quad} \; e^{j\frac{\pi}{\lambda z_2}(x-x')^2} \, dx$$

$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{F} \qquad \text{imaging.}$$

Figure 6

Imaging

Animal eyes

A diagram of the human eye is shown in Figure 5.1. It has a lens and curved cornea. The two surfaces combine to focus the light. The lens of the human eye has considerable flexibility allowing us to focus far and near except as you get older when the material hardens and the muscle that drives it both deteriorate. The detector array (the "sensor") of the eye is the retina. It is an amazing device in all respects. It has excellent resolution (around 80 cycles per degree) near the center (phobea) and then it can move around to point to the region of interest. The lens is similar in shape to the lenses we see in cameras. This is because this shape is what is needed to make a transparency that focuses a diverging spherical wave.

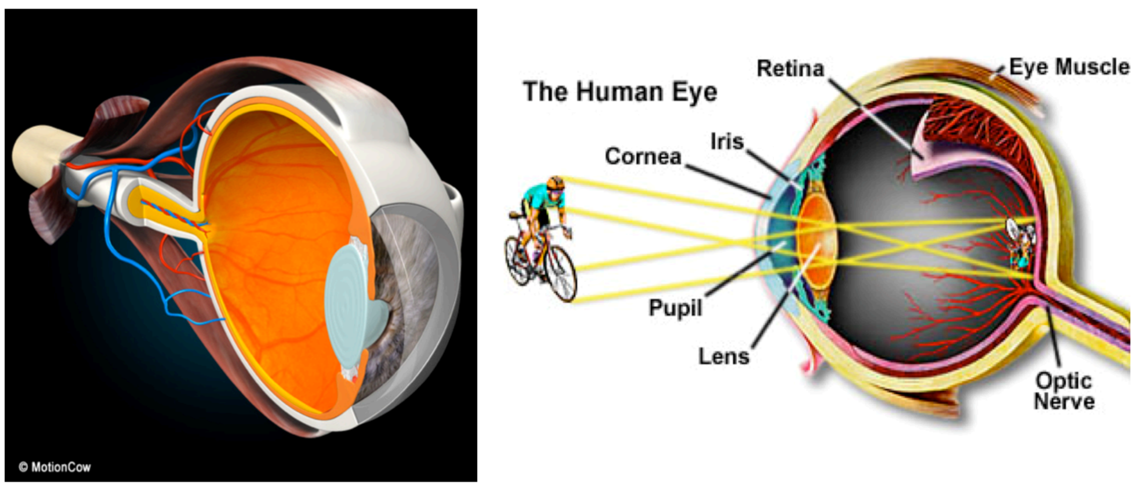


Figure 5.1

The development of eyes in animals started with single light sensors connected to the nervous system simply to report bright or dark. Then multiple sensors developed on the surface of the animal to improve the quality of vision. Animals then acquired the capability to have angular or spatial resolution by having multiple sensors on ***curved*** surfaces. Two main options developed. An alignment of detectors on a surface with outward curvature led to insect vision. An inward curvature led to "pinhole lenses" and then real lenses. See Figure 5.2.

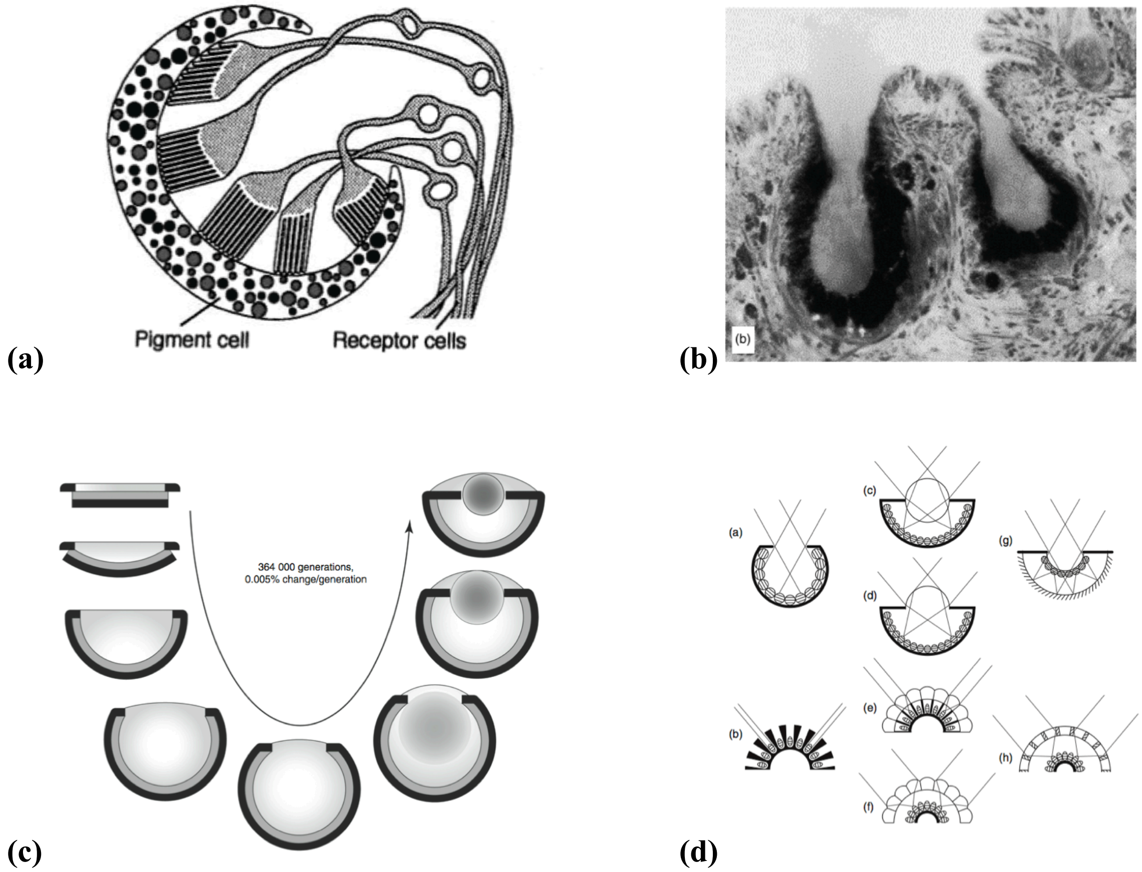
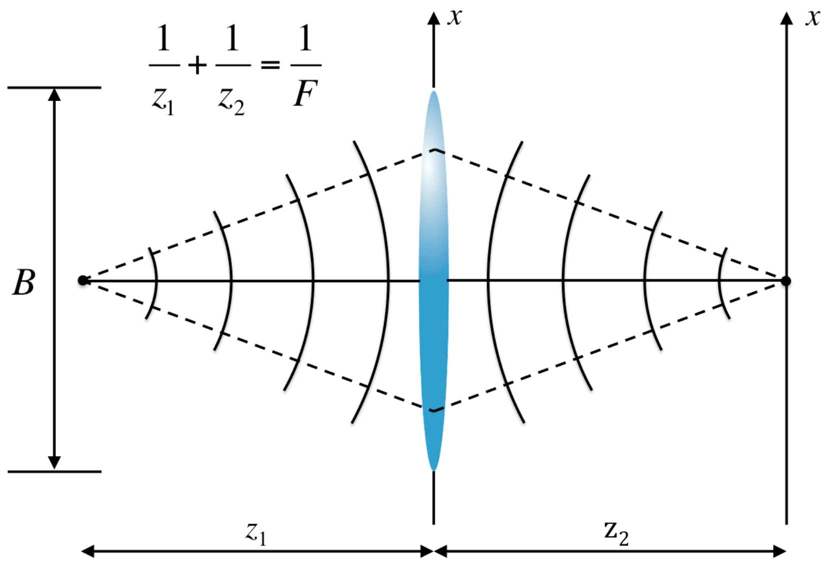


Figure 5.2

<u>Single lens imaging</u>

A single convex lens focuses a spherical diverging from a point source to another spot. This gives us the imaging condition: $\frac{1}{z_1}+\frac{1}{z_2}=\frac{1}{F}$. See Figure 5.3.



Incident field: $E_{sph} = e^{-jkr} \approx e^{-jkz} e^{-j\frac{\pi}{\lambda z_1}x^2}$

Lens transmittance: $t_{lens}(x) = e^{+j\frac{\pi}{\lambda F}x^2}$

Field at the output:

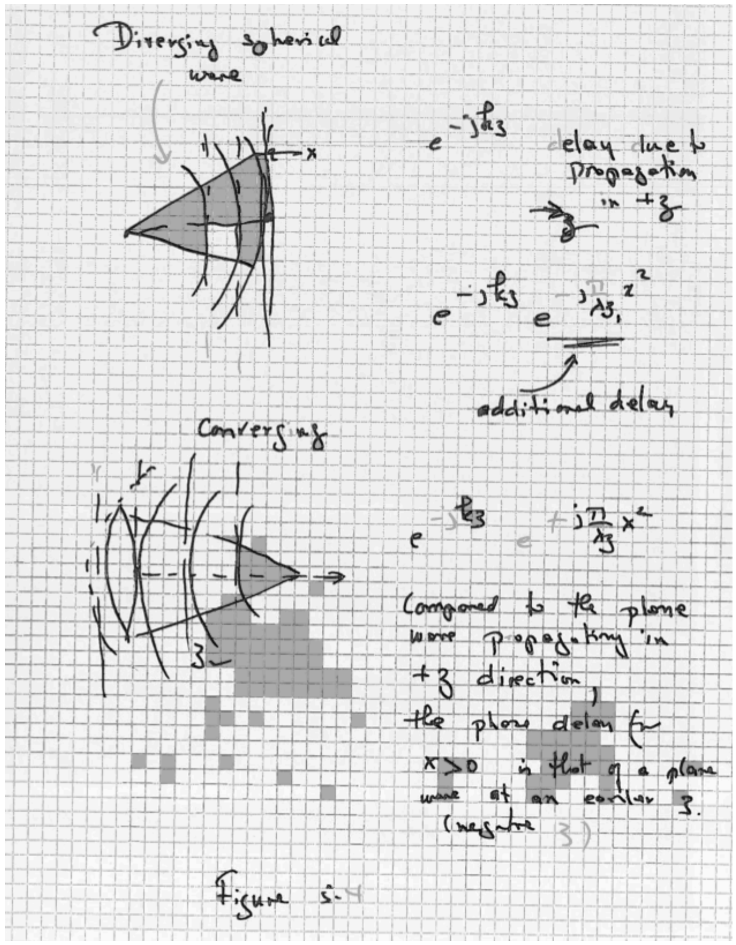
$$g(x') = \int e^{-j\frac{\pi}{\lambda z_1}x^2} e^{+j\frac{\pi}{\lambda F}x^2} e^{-j\frac{\pi}{\lambda z_2}(x-x')^2} dx =$$

$$= e^{+j\frac{\pi}{\lambda z_2}x'^2} \int_{-B/2}^{B/2} e^{+j\frac{2\pi x x'}{\lambda z_2}} dx = e^{+j\frac{\pi}{\lambda z_2}x'^2} B \operatorname{sinc}\left[B\frac{x'}{\lambda z_2}\right]$$

Figure 5.3

In our notation $e^{j(\omega t - kz)}$ is a plane wave travelling in the +z direction. Adding a distance Δz in the propagation path would add a delay (a negative phase) to the temporal sine wave. A spherical wave with a minus sign is a diverging wave. On axis (x=y=0) the spherical wave is like the plane wave acquiring negative phase as z increases. Off axis, as x or y increases, the expanding spherical wave picks up additional phase delay (or negative phase) as if a plane wave had propagated further in z. (See Figure 5.4).

For a converging spherical wave, the correct expression is $e^{j(\omega t - kz + \frac{\pi(x^2+y^2)}{\lambda z})}$. In this case the spherical wave at x>0 has the at the phase of the plane wave at distance z<0. See figure 5.4.



In Figure 5.5 we use the BPM to simulate a single lens imaging system with varying apertures and divergence of the object beam. Deterioration in the sharpness of the image quality can be seen. There are two reasons for this. The first is lens aberrations. The paraxial approximations we made for the light propagation and the lens are not satisfied as we move to the edge of the lens (high angles) and this introduces distortions. Normally lenses can be carefully designed using ray tracing to correct for aberrations. The other source of distortion is more fundamental. The finite aperture of the lens blocks some of the light from the object and the distortion observed is due to this missing light. We will see that the missing light is essentially the loss of the high spatial frequency components of the object. Therefore the image appears smoothed out, blurry.

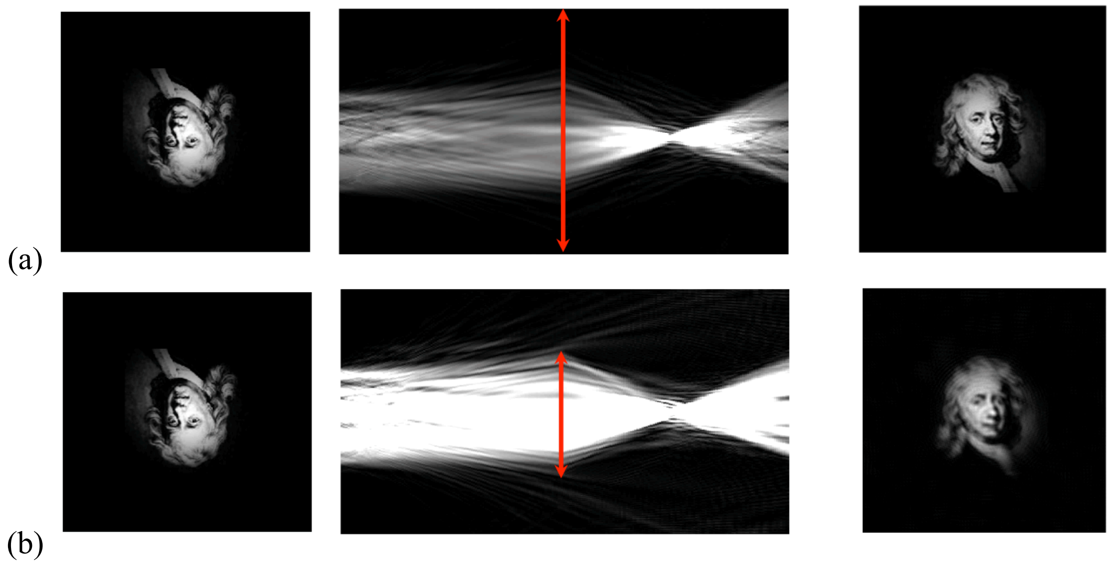
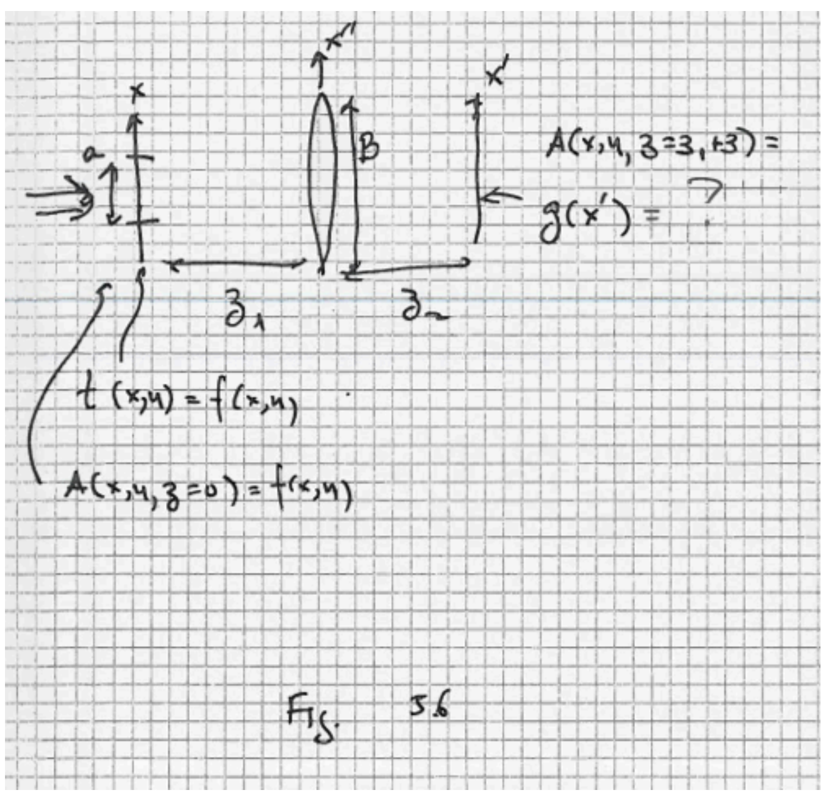


Figure 5.5

The single lens imaging system is drawn in Figure 5.6. A thin transparency with transmittance $t(x,y) = f(x,y)$ is placed at z=0 and illuminated with a plane wave propagating in the +z direction. The field immediately after z can be calculated as follows.



$$A(x,y,z=0)=f(x,y)$$

$$@z=z_1^-:\quad \iint f(x,y)e^{-j\frac{\pi\left[(x-x")^2+(y-y")^2\right]}{\lambda z_1}}dxdy$$

$$@z=z_1^+:\quad \left[\iint f(x,y)e^{-j\frac{\pi\left[(x-x")^2+(y-y")^2\right]}{\lambda z_1}}dxdy\right]e^{+j\frac{\pi(x^2+y^2)}{\lambda F}}$$

$$@z=z_1+z_2:\quad g(x',y')=\iint_B\left[\iint f(x,y)e^{-j\frac{\pi\left[(x-x")^2+(y-y")^2\right]}{\lambda z_1}}dxdy\right]e^{+j\frac{\pi(x^2+y^2)}{\lambda F}}e^{-j\frac{\pi\left[(x'-x")^2+(y'-y")^2\right]}{\lambda z_2}}dx"dy"$$

$$=\iint f(x,y)\left[\iint rect(\frac{x"}{B})rect(\frac{y"}{B})e^{-j\frac{\pi\left[(x"-x)^2+(y"-y)^2\right]}{\lambda z_1}}e^{+j\frac{\pi(x^2+y^2)}{\lambda F}}e^{-j\frac{\pi\left[(x'-x")^2+(y'-y")^2\right]}{\lambda z_2}}dx"dy"\right]dxdy$$

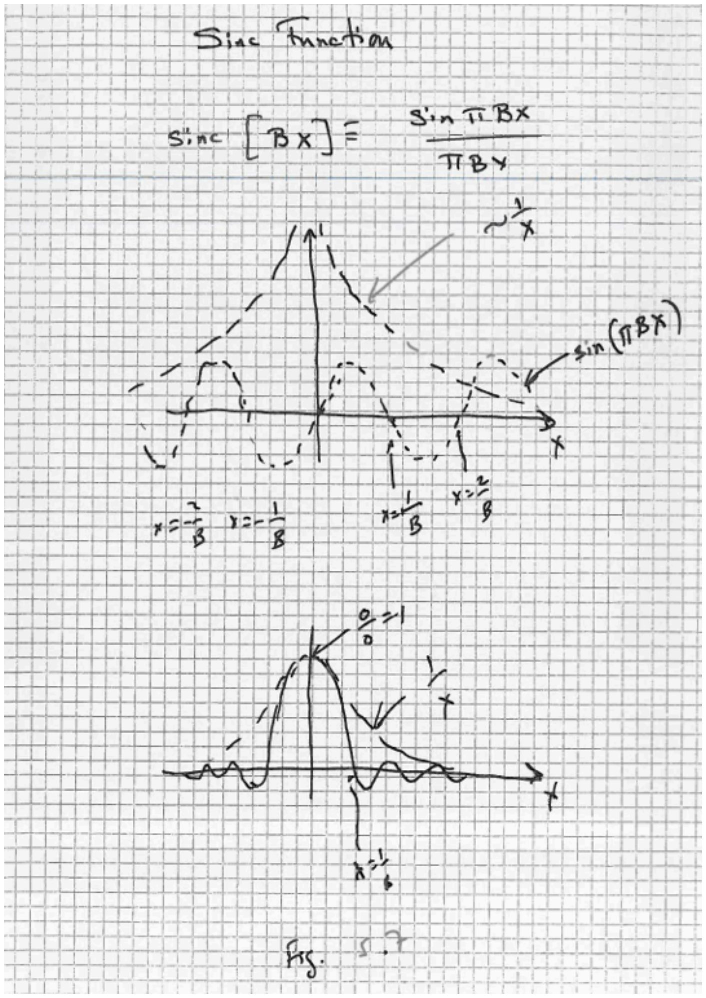
$$with\ \ \frac{1}{z_1}+\frac{1}{z_2}=\frac{1}{F}$$

$$g(x',y')=e^{-j\frac{\pi(x^2+y^2)}{\lambda z_2}}\iint f(x,y)e^{-j\frac{\pi(x^2+y^2)}{\lambda z_1}}\left[\iint\ rect(\frac{x"}{B})rect(\frac{y"}{B})e^{+j\frac{2\pi xx"}{\lambda z_1}}e^{+j\frac{2\pi x'x"}{\lambda z_2}}dx"dy"\right]dxdy$$

$$=B^2e^{-j\frac{\pi(x^2+y^2)}{\lambda z_2}}\iint f(x,y)e^{-j\frac{\pi(x^2+y^2)}{\lambda z_1}}\sin c\left[\frac{B}{\lambda z_1}(x+x'/M)\right]\sin c\left[\frac{B}{\lambda z_1}(y+y'/M)\right]dxdy$$

where $M=z_2/z_1$ *is the magnification*

We used above the fact that the Fourier transform of $f(x)=rect(x/B)$ is $F(u)=B\,\mathrm{sinc}(Bu)$ with the spatial frequency $u=\dfrac{x+x'/M}{\lambda z_1}$ and the same for the y dimension. The sinc function is plotted in Figure 5.7.



As B gets large then the sinc function tends to a delta function and the output g(x', y') becomes f(x'/M, y'/M) times the two phase factors. In other words the output is a magnified version of the input. For a finite B however the situation is more complex. We would like to get rid of the quadratic term inside the integral since without it, the image g(x',y') becomes the convolution of f(x,y) with a sinc function. This would lead us to the very simple description of the imaging system as a low pass filter, as we will see a bit later. Under what conditions can we ignore the quadratic term? One possibility is to have

$$e^{-j\frac{\pi(x^2+y^2)}{\lambda z_1}} \cong 1$$

This implies $x^2 << \lambda z_1$ for the maximum value of x which is equivalent to saying that the lens is in the far field of the input aperture. This is generally the case for a telescope (Figure 5.8).

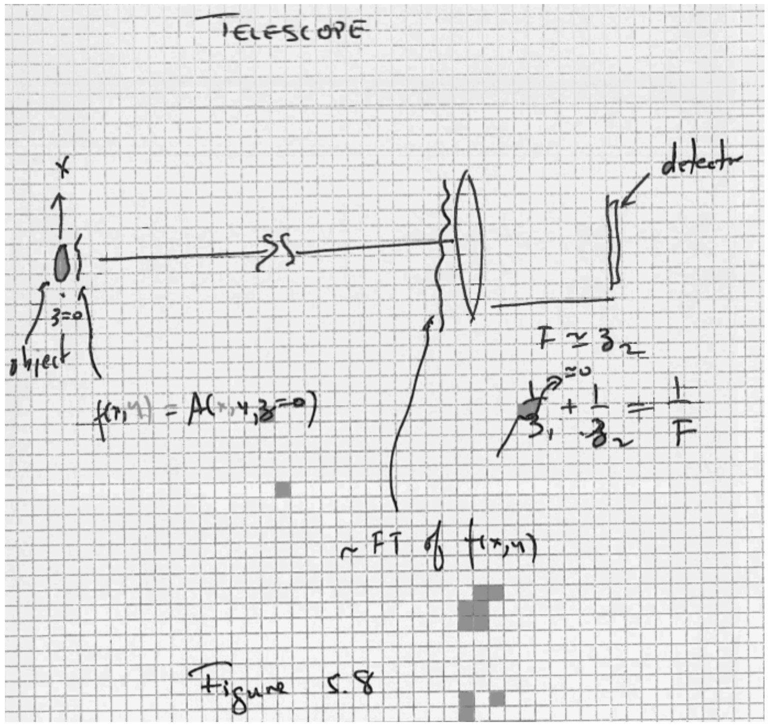


Figure 5.8

The light coming from the distant object with near field distribution A(x,y,z=0)=f(x,y) becomes at z=z_1 the Fourier transform of f(x,y):

$$A(x",y",z=z_1) = g(x",y") = e^{-j\pi(x"^2+y"^2)/\lambda z_1} \iint f(x,y) e^{+j2\pi(xx"+yy")/\lambda z_1}\, dx\, dy$$

Notice that a spatial frequency $u = K_x / 2\pi$ of the object will diffract light at $x" = u\lambda z_1$ at the plane of the lens. For example a grating with period Λ=1cm that is at z_1=1km from the lens would produce a spot of light centered at x"=10cm. Therefore we would need a lens with aperture B>10cm to be able to see the grating (see Figure 8). Another test would be to consider two points at z=0 separated by d=1cm. We can model the two point sources as two delta functions:

$$f(x,y) = \delta(x-d/2)\delta(y) + \delta(x+d/2)\delta(y)$$

With this as the input we obtain at the plane of the lens:

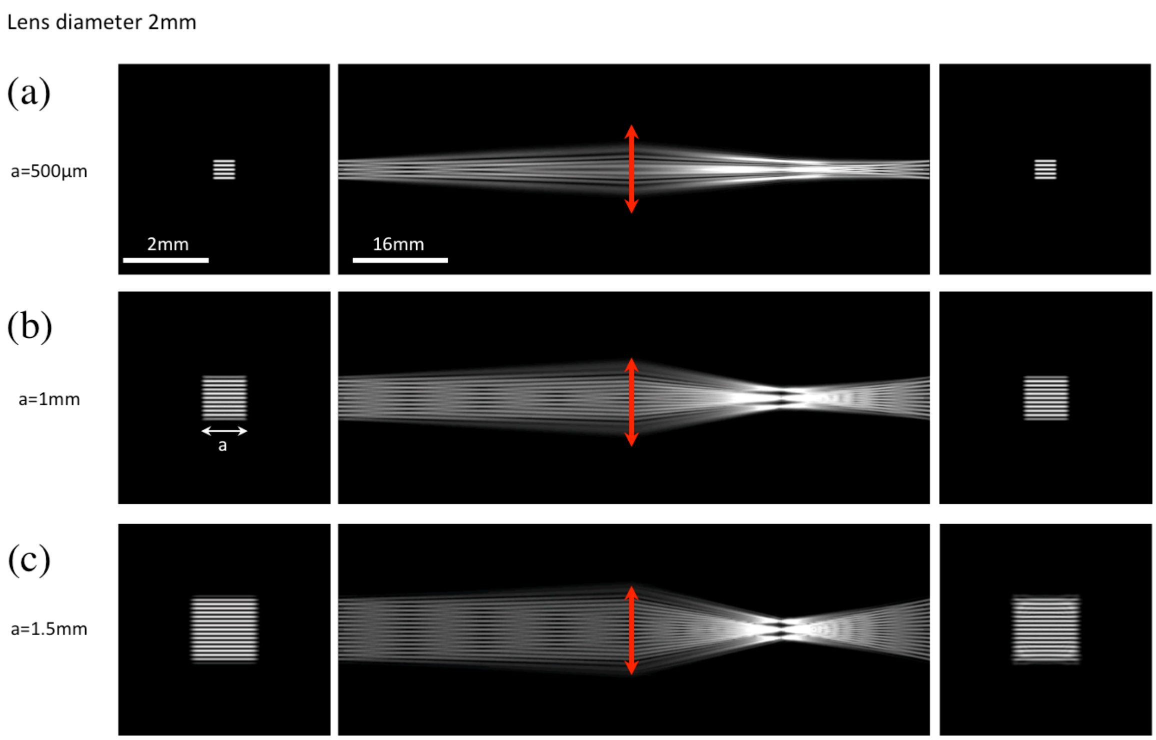
$$g(x",y") = e^{-j\pi(x"^2+y"^2)/\lambda z_1}\left[e^{+j\pi dx"/\lambda z_1} + e^{-j\pi dx"/\lambda z_1}\right] = 2e^{-j\pi(x"^2+y"^2)/\lambda z_1}\cos(\pi dx"/\lambda z_1)$$

In order for the telescope to sense that there are two point sources in the distance, the sinewave incident onto the aperture of the lens much have at least one full period fitting into the aperture of the lens. This condition gives us the same answer for the required aperture (B=10cm) in order to see a feature of size d=Λ=1cm. Since the field incident on the lens in this case is the Fourier transform of the input and then the lens must take the Fourier transform again to reproduce the object. Indeed the light distribution at the back focal plane (z_2~F) is the Fourier transform of the field near the front of the lens. More on this a little later (homework).

If the object is not in the far field it is difficult to say analytically what it the effect of the quadratic term. The width of the main lobe of the sinc function is $\lambda z_1 / B$. If this is much narrower than the fastest oscillation in the quadratic term then we can think of this term as reading out the quadratic phase term without any further impact on the image quality. In other word if the instantaneous spatial frequency (the derivative of the phase) is never bigger than $B / \lambda z_1$ then it is safe to bring the quadratic phase term outside the integral. The instantaneous frequency is $2x / \lambda z_1$ and it attains its maximum value at x=a/2. With this criterion we have the requirement that a<<B and then the single lens imaging relationship can be written as follows:

$$g(x',y') \sim \iint f(x,y)\sin c\left[\frac{B}{\lambda z_1}(x+x'/M)\right]\sin c\left[\frac{B}{\lambda z_1}(y+y'/M)\right]dxdy$$

If the condition is not satisfied we can use the BPM to study the effects (Figure 5.9 a,b,c,d,e,f).



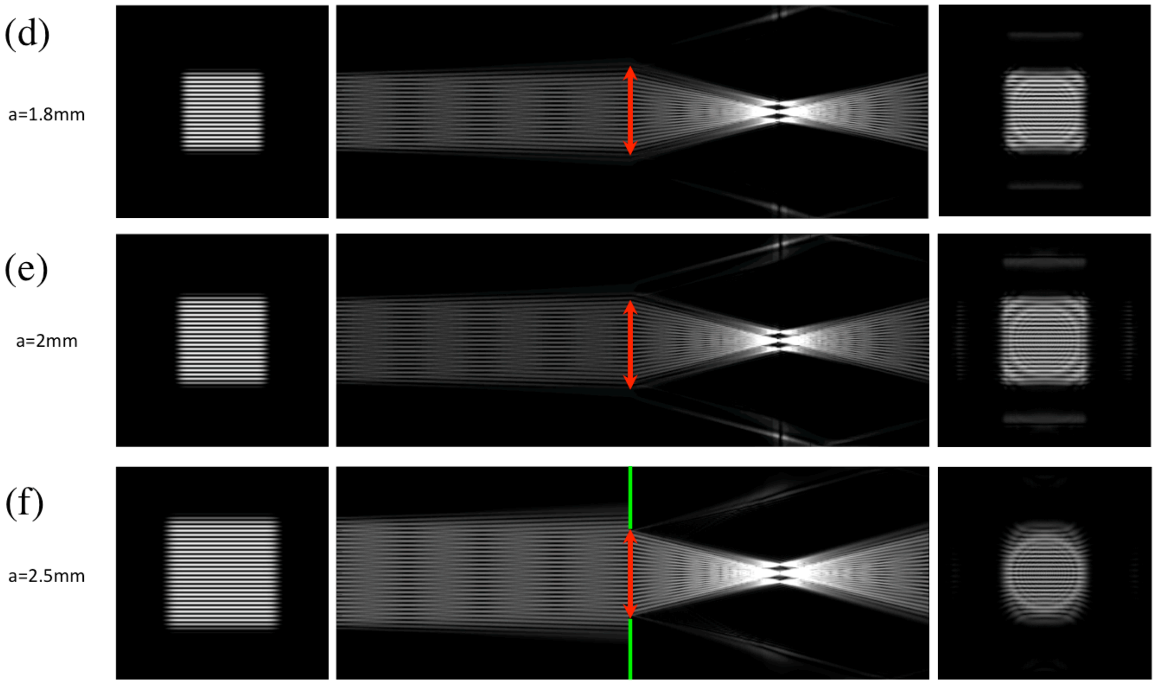


Figure 5.9

The equation above has the form of a convolution. Now we can talk about the frequency response and the resolution of the optical imaging system.

Resolution and Point Spread function

If f(x,y)=δ(x-x_o, y-y_o) then the output is just the sinc function centered at x_o, y_o. It would be the Airy function if the aperture if the lens were spherical (as is commonly the case) rather than square. This is the *"point spread function"* in optics because it tells us how a single point at the input spreads at the output. In linear systems it is called the impulse response. You can think of the PSF as the result of a measurement with a fixed point detector and moving point source at the input or a fixed source and a moving detector. For an ideal imaging system the PSF might be a delta function. The finite aperture spreads the PSF and this becomes the "diffraction limit" of resolution. The resolution is generally taken to be the width of the main lobe of the PSF. For the sinc function it can be taken as

$$\Delta x = \frac{\lambda z_2}{B} \sim \frac{\lambda_0}{2n\sin\theta} = \frac{\lambda_0}{2\,N.A.} \qquad where\ \ N.A. = n\sin\theta\ \ (numerical\ aperture)$$

Frequency response:

Taking the Fourier transform of the expression for the single lens imaging system we obtain:

$$G(u,v) = F(u,v) \quad rect(\frac{u}{B/\lambda z_1})rect(\frac{v}{B/\lambda z_1})$$

The band-pass nature of image formation is illustrated in the BPE simulation of Figure 5.10.

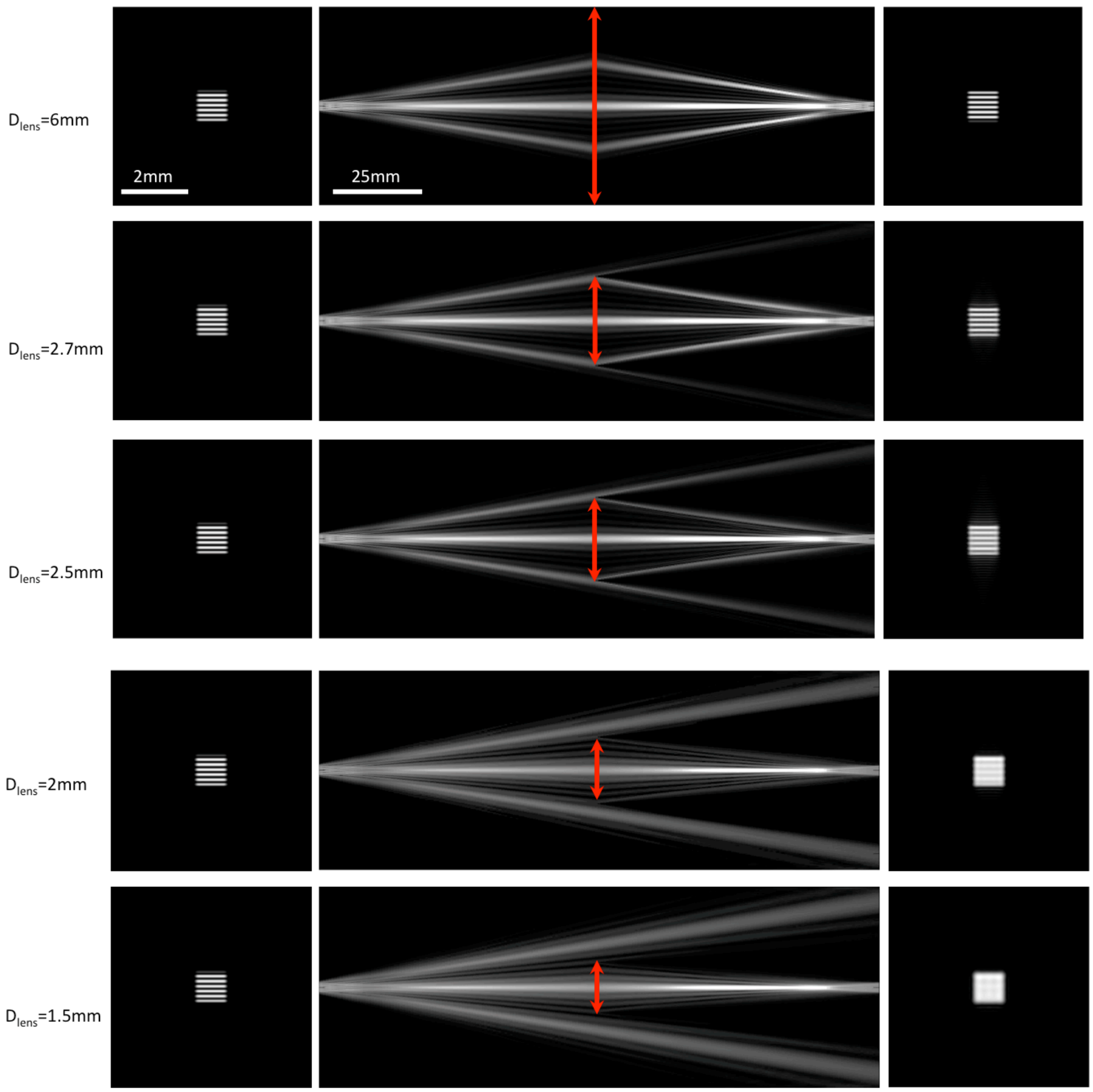


Figure 5.10

Fourier Transform lens

Homework Derive the fact that the light distribution at the back focal plane is the Fourier transform of the distribution incident on the other side of the lens. Use the code to assess the impact of the finite input and lens apertures. Spectrometer resolution.

4F Imaging System:

See Figure 5.11.

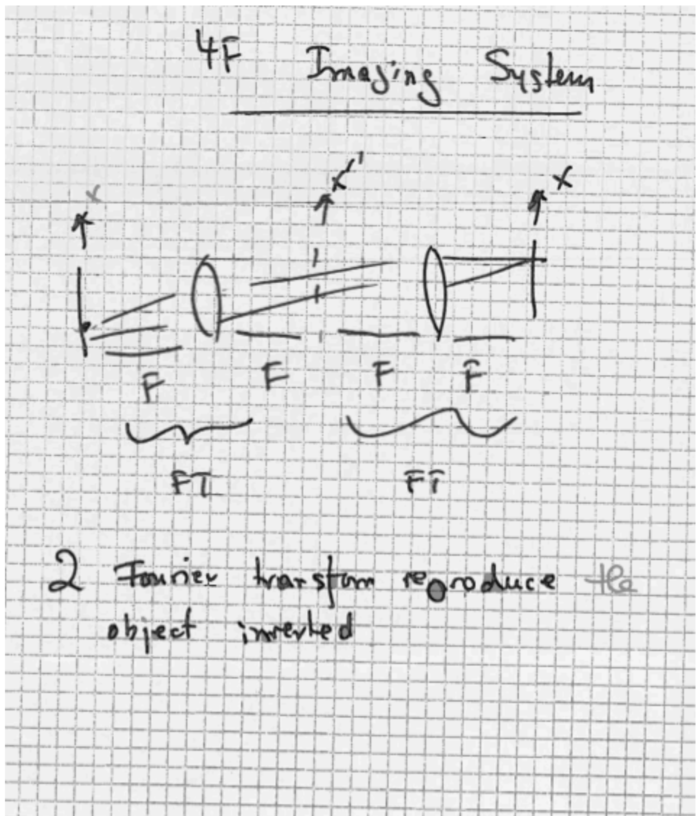


Figure 5.11

Dark field and Zernike phase microscopy

Many objects of interest in microscopy are largely transparent. When light passes through such an object, the main effect is a spatially varying phase shift that, as we saw in class, is not directly observable with a conventional microscope. In practice, if we have a phase object with a small phase variation, we can observe that it will not produce an amplitude difference if detected using a classical 4F imaging system. Indeed, for $\phi(x)$ relatively small, we can write:

$$U_{obj}(x) = e^{j\phi(x)} \approx 1 + j\phi(x)$$

$$\Rightarrow \left|U_{out}(x)\right|^2 = \left|1 + j\phi(x)\right|^2 = 1 + \phi^2(x) \approx 1$$

If we use the *Zernike phase method*, we are going to insert a phase mask in the Fourier plane of the 4F system in order to shift the zero-order light by π/2 or 3π/2.

What will happen to the field using a π/2 shift mask ($M_{\pi/2}(k)$) is the following:

$$U_{obj}(x) = e^{j\phi(x)} \approx 1 + j\phi(x)$$

$$M_{\pi/2}(k) = \begin{cases} e^{j\frac{\pi}{2}} & \text{if} \quad k = 0 \\ 1 & \text{if} \quad k \neq 0 \end{cases}$$

At the Fourier plane we will have:

$$\tilde{U}_{obj}(k) = \delta(k = 0) + j\tilde{\phi}(k)$$

$$\Rightarrow \tilde{U}_{obj}(k) M_{\pi/2}(k) = \delta(k = 0) e^{j\frac{\pi}{2}} + j\tilde{\phi}(k) = j\left(\delta(k = 0) + \tilde{\phi}(k)\right)$$

Thus, at the output plane:

$$U_{out}(x) = j\left(1 + \phi(x)\right) \Rightarrow \left|U_{out}(x)\right|^2 = \left|j\left(1 + \phi(x)\right)\right|^2 =$$
$$= 1 + 2\phi(x) + \phi^2(x) \approx 1 + 2\phi(x)$$

So on our detector we will have a field that is linearly proportional to the phase of our input object. The term $\phi^2(x)$ has been canceled being small by definition. Inserting, instead, a 3π/2 shift, we will have a contrast inversion. In fact the output will be:

$$U_{out}(x) = j\left(-1 + \phi(x)\right) \Rightarrow \left|U_{out}(x)\right|^2 = \left|j\left(-1 + \phi(x)\right)\right|^2 =$$
$$= 1 - 2\phi(x) + \phi^2(x) \approx 1 - 2\phi(x)$$

We can observe this effect using the BPM code. As input a Gaussian beam with a FWHM=1200μm was used (Figure 5.12).

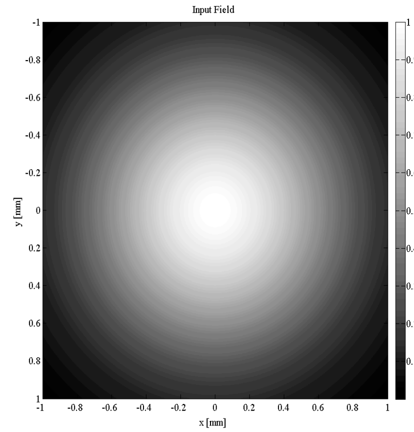


Figure 5.12

In Figure 5.13 the result obtained using a $\pi/2$ phase mask is shown. In particular I took the square of the output given by the code itself in order to obtain the intensity of our field. In Figure 5.14 I subtract the intensity output, the intensity of the Gaussian beam without any object inserted at the Fourier plane in order to better appreciate the achievable contrast.

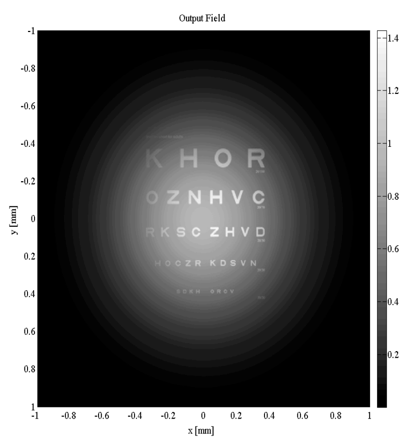


Figure 5.13

Figure 5.14

In Figure 5.15 and 5.16 I did exactly the same using a $3\pi/2$ phase mask. The resulting contrast, as expected, is comparable with the previous one.

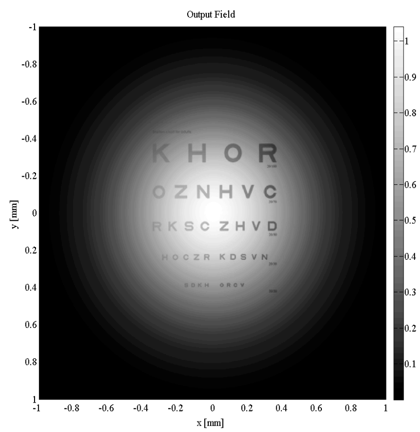


Figure 5.15

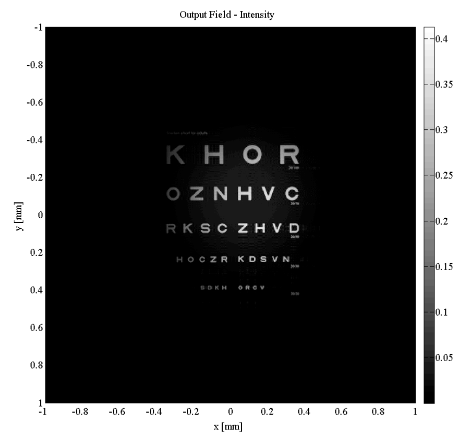


Figure 5.16

In both cases the phase mask consists in a small circle (diameter equal to 35μm) that is able to phase shift just the zero-order of the Fourier transform of the input given by the first lens of the imaging system.

The second method exploited in order to observe a phase object is the central dark ground method, also known as dark field microscopy. As for the first method, “something” has to be inserted at the Fourier plane of the 4F imaging system: in this case a transmission mask that blocks the zero-order light.

In Figure 5.17 the result given by this method is shown. Even if the phase object seems to be quite visible, looking at the colorbar on the right of the image, we can see that the maximum value of the output is almost 0.07, very low compare to the previous technique. So also the contrast is very low. Indeed if we consider our transmission mask at the Fourier plane being:

$$M(k) = \begin{cases} 0 & \text{if} \quad k = 0 \\ 1 & \text{if} \quad k \neq 0 \end{cases}$$

$$U_{obj}(x) = e^{j\phi(x)} \approx 1 + j\phi(x)$$

$$\tilde{U}_{obj}(k) = \delta(k=0) + j\tilde{\phi}(k)$$

$$\Rightarrow \tilde{U}_{obj}(k) M(k) = +j\tilde{\phi}(k)$$

$$U_{out}(x) = j\phi(x) \Rightarrow \left|U_{out}(x)\right|^2 = \left|j\phi(x)\right|^2 = \phi^2(x)$$

It means that our output is proportional to $\phi^2(x)$, the same term that we canceled using the Zernike method because it was “small”.

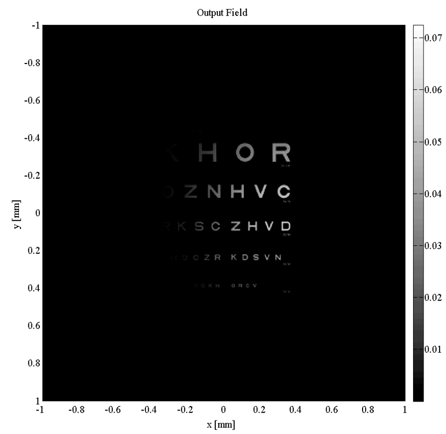


Figure C.6