Superoscillatory diffraction-free beams

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It is theoretically shown that by superimposing diffraction-free solutions of the Helmholtz equation, one can construct localized diffraction-free beams that pass through predetermined points on subwavelength distances. These beams are based on the phenomenon of superoscillations and thus do not contain any evanescent waves. The effect of an aperture and noise is examined in specific examples where truncated beams with $\lambda/3$ subwavelength features can propagate into the far field.

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The advance of the nanofabrication techniques during the last decades has made possible to easily realize optical nanostructures [1,2]. As a result, the study of the near field attracts a lot of interest due to potential applications in imaging and biology. Superresolution methods [1–4] is also an intense field of research, since it extends the traditional imaging below the diffraction limit. Since the spatial dimensions of such structures are far smaller than the wavelength of the optical field, the study of evanescent waves is inevitable. Evanescent waves decay very fast and as a result restrict the subwavelength information to the near field only. Various tools and methods are used to extract this information and transfer it to the far field. In particular, the most known way is the use of nanotips [1,2] that scan the near field. In all cases evanescent waves are present. A recently proposed way of achieving superresolution in the near field is to use the counterintuitive effect of superoscillations [5–14]. In this framework, an optical field can exist that contains subwavelength variations (always in the low intensity regions) but no evanescent waves. Superoscillations have been recently introduced and experimentally observed in the optical domain [5,8,10–13] in the context of superresolution [10,11], and far-field subwavelength focusing [12,13]. A question of interest is whether superoscillatory beams can transfer subwavelength information to the far field by simple propagation (no focusing, no lenses or any other device).

In this Letter, we show how to create superoscillatory diffraction-free beams [15–19] by superimposing already known diffraction-free solutions of the Helmholtz equation [15–17]. Such optical beams do not contain any evanescent waves and can propagate practically undistorted carrying these subwavelength features into the far field.

Let us consider the two-dimensional Hemlholtz equation in the free space that is given by

$$\frac{\partial^2 U}{\partial z^2}+\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}+k_0^2 U=0, \tag{1}$$

where U is the electric field envelope, $k_0=2\pi/\lambda_0$, and λ_0 is the free-space wavelength. Any linear superposition of the waves with the same propagation constant results in a diffraction-free beam. A celebrated example is that of the mth order Bessel beam [15,16] which is given by the expression $U(r,\phi,z)=J_m(ar)e^{im\phi+i\beta z}$, where the dispersion relation is $a^2+\beta^2=k_0^2$ and β the propagation constant of the beam. From the dispersion relation, we can deduce that $0<a<k_0$. This means that the lobes of any Bessel beam have a width of the order of λ_0. We are interested to see if a stationary diffraction-free solution of Eq. (1) of the general form $U(x,y,z)=g(x,y)e^{i\beta z}$ can be superoscillatory. For this to happen, we force this solution to pass through a set of predetermined points $P_m(\vec{r}_m), m=1,\ldots,N,\ \vec{r}_m=(x_m,y_m)$ separated by subwavelength distances. We also assume that our solution is a superposition of N diffraction-free beams

$$g(x,y)=\sum_{m=1}^{N}c_m f_{m-1}(x,y), \tag{2}$$

where the coefficients c_m are unknown. Note that all these N beams have the same propagation constant β and no evanescent waves. Therefore Eq. (2) describes a stationary superposition and not a periodic interference pattern. The solution of the problem is given by the relation

$$\vec{c}=\overleftrightarrow{M}^{-1}\bullet\vec{G}, \tag{3}$$

where $M_{ij}=f_{j-1}(\vec{r}_i)$, $\vec{c}=[c_1c_2\ldots c_N]^T$, $\vec{G}=[g(\vec{r}_1)g(\vec{r}_2)\ldots g(\vec{r}_N)]^T$, and $i,\ j=1,\ldots N$. The numerical solution of the above problem gives us the coefficients $\vec{c}$, and thus the superoscillatory diffraction-free beam. At this point we have to note that our method is quite general, as it is not an optimization technique, does not involve any focusing of the obtained beam and also does not depend on any fixed distance between the source and the target. Furthermore, the generated subwavelength regions are not the outcome of any optimization algorithm for the focal spot, but inherent properties of the general superoscillatory solution and as such remain invariant for any propagation distance z. Therefore, they can be located in multiple parts of the final fully coherent beam (center and/or tails). This is in sharp contrast with superoscillations in speckle beams, where subwavelength features appear in random positions of a partially coherent beam. As we will see below, the required number of diffraction-free beams is very small, and the subwavelength features have a width of the order of $\lambda/3$ (at least). As a result,

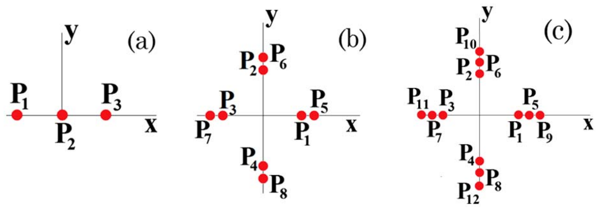

Fig. 1. (Color online) Schematic diagrams of three different patterns (a), (b), and (c) of predetermined points (red dots) in the x–y plane. The distances between the dots are subwavelength.

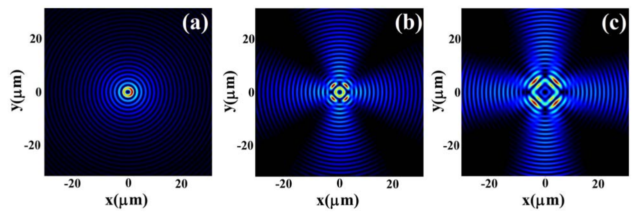

Fig. 2. (Color online) Transverse intensity profiles of the resulted diffractionless beams that are based on the patterns depicted in Figs. 1(a)–1(c), respectively.

free-space transfer of subwavelength images (embedded in a superposition of diffractionless beams) to the far field is possible with our method.

From now on, we consider for simplicity only superpositions of Bessel beams $f_n = J_n(ar)e^{in\phi}$, even though f_n can be any family of stationary diffractionless beams. For better understanding, we apply the above formulation to the simplest analytically solvable case of a beam that passes through three points on the x axis [Fig. 1(a)]. The two points P_1 and P_3 are located at $x = -\delta$ and $x = \delta$, respectively, while the third one P_2 is at the origin. Since all these points are located near zero, we can use the asymptotic expansion of the Bessel functions to calculate the superposition coefficients of the three Bessel beams, which are $c_1 = g(\vec{r}_2) \equiv G_2$, $c_2 = [G_3 - G_1](a\delta)^{-1}$, and $c_3 = 4[G_1 + G_3 - 2G_2](a\delta)^{-2}$. If, for example, $\vec{G} = [0 \;\; 1 \;\; 0]^T$ then $\vec{c} = [1 \;\; 0 \;\; -8/(a\delta)^2]^T$ and the total field can be written as $g(r,\phi) = J_0(ar) - [8/(a\delta)^2]J_2(ar)e^{i2\phi}$. When $a\delta \ll 1$, which means that $\delta \ll \lambda_0$, the ratio $r_I = [\max(|U|^2)]_{\text{soregion}}/I_{\max}$, between the intensity at the subwavelength regions and the maximum intensity, approaches high values, a result typical of superoscillations. In all our calculations, the wavelength is $\lambda_0 = 1\,\mu$m and the spatial frequency of the Bessel beams is $a = 2\,\mu\text{m}^{-1}$. For this transverse wavenumber, one can show that for linearly polarized beams the adopted scalar approximation is pretty accurate and vectorial effects can be neglected. As a first example, we choose the three points of Fig. 1(a) such that the distance between them is $d(P_2, P_3) = 245$ nm and $d(P_1, P_2) = 490$ nm. The transverse intensity profile of the resulted superoscillatory beam is depicted in Fig. 2(a). The FWHM of the intensity at the formed subwavelength region is $w \sim 400$ nm, with $r_I \sim 1/70$, and the beam is a superposition of J_0, J_1, J_2. Moreover, for the eight points cross pattern shown in Fig. 1(b), we get a superposition of J_2, J_6 [Fig. 2(b)], with a ratio of $r_I \sim 1/85$ and $w \sim 350$ nm. Also, for the 12-point pattern of Fig. 1(c), we have a superposition of J_2, J_6, J_{10} [Fig. 2(c)], with a ratio of $r_I \sim 1/600$ and $w \sim 300$ nm. Note that we can force the superposition to pass through any pattern in the transverse plane, even that of a subwavelength image. This precise control is what makes the suggested method a possible tool for subwavelength imaging. By increasing the number of points we increase the number of the superimposed beams. Also by decreasing the distances between these points, we decrease dramatically the ratio r_I and make the experimental observability of the imposed superoscillations more difficult.

Let us now examine in more detail the nature of a superoscillatory diffractionless beam. By imposing alternate sign $[1 \;\; -1 \;\; 1 \;\; -1]$ prescribed values of the field at the middle points P_5, P_6, P_7, P_8 of the pattern of Fig. 1(c), we get the beam with the intensity profile of Fig. 3(a). A cross section along the x axis of the field (absolute value) of such a beam is given in Fig. 3(b), where the superoscillatory regions are located inside the red squares. Again we have a superposition of J_2, J_6, J_{10}, but with a ratio of $r_I \sim 1/300$ and $w \sim 300$ nm. The phase $\theta(x,y)$ of the transverse field is presented in Fig. 3(c), where we can clearly see the existence of multiple optical vortices [20], not only on the subwavelength regions but all over the beam. The phase also gives us valuable insight since we can calculate the local wavenumber $k_l \equiv \nabla\theta(x,y)$ [5,6,8], and see that in the superoscillatory regions the k_l exceeds the maximum transverse wavenumber k_0. When this happens, $\nabla^2\rho/\rho > 0$ in these regions [8], where $\rho(x,y) = |g(x,y)|$. Indeed this is the case for the field of Fig. 3(b) as we can see in Fig. 3(d), where the y-cut of the $\nabla^2\rho/\rho$ is plotted versus the transverse coordinate x. Clearly $\nabla^2\rho/\rho > 0$ on the vicinity of the P_1, P_5, P_9 points where the field is forced to pass. This proves that at this region the beam is superoscillatory. In other words, our beam carries subwavelength features in predetermined regions without any evanescent waves.

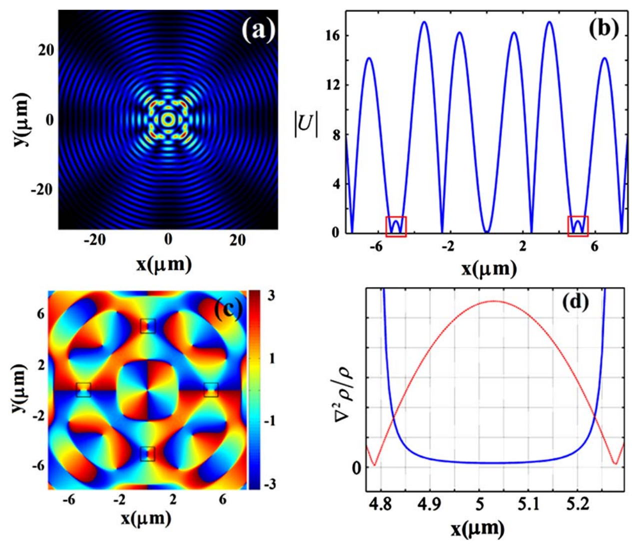

Fig. 3. (Color online) (a) Intensity transverse profile, (b) y-cut of the $|U|$ in (a), (c) phase of the field in (a), and (d) y-cut of the $\nabla^2\rho/\rho$ of the field in (a). The red squares in (b) and the black in (c), denote the location of the superoscillatory regions. The red (upper) line in (d) is the y-cut of $|U|$ around the superoscillatory region located at $x \sim 5\,\mu$m.

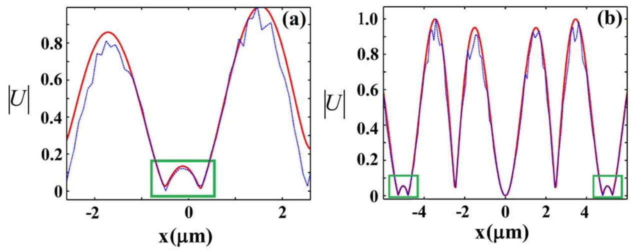

Fig. 4. (Color online) Normalized $|U|$ (solid red lines) of the truncated beam at $y=0$ for (a) Fig. 2(a) as an input after $z = 250\,\mu$m, and (b) Fig. 3(a) as an input after $z = 150\,\mu$m, and under the presence of noise. The jagged blue lines represent the fields at the input, and the green squares the superoscillatory regions.

The effect of an aperture and noise is examined to understand if truncation of such beams will allow the embedded subwavelength features to propagate undistorted into the far field. We apply the Rayleigh–Sommerfeld diffraction integral formulation [21] to examine their diffraction properties. A supergaussian type of aperture (with a FWHM $\sim 160\,\mu$m) is assumed, and 20% noise is added in both amplitude of the input beams of Figs. 2(a) and 3(a). The absolute values of the fields at a distance of $z = 250\,\mu$m and $z = 150\,\mu$m are depicted in Figs. 4(a) and 4(b) (solid lines), respectively. The input field profiles (jagged lines) are also shown for comparison. We can clearly see that the field can propagate undistorted for a distance of $150\lambda_0 \gg \lambda_0$, carrying subwavelength features of the order of 300 nm into the far field. This distance can be much greater by choosing a wider aperture at the input.

In conclusion, we have theoretically studied the properties of diffractionless beams with subwavelength features that propagate undistorted into the far field. These beams do not contain any evanescent waves and are superoscillatory.

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