



## Huygens–Fresnel diffraction and evanescent waves

Konstantinos G. Makris\*, Demetri Psaltis

School of Engineering, Swiss Federal Institute of Technology Lausanne (EPFL), CH-1015 Lausanne, Switzerland

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### ABSTRACT

Based on a modified Huygens–Fresnel principle valid in the near field, we analytically verify the exponential wave decay of evanescent waves. This field decay along the propagation direction can be considered as the result of interference between elementary oscillating dipole sources rather than point sources. Specific examples where the modified Huygens–Fresnel principle is applicable are also examined. In particular, the diffraction through a single nano-slit, as well as, the propagation of an engineered input leading to a subwavelength focus are presented.

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A central topic in optics is light propagation through apertures. This is the phenomenon of optical diffraction [1,2]. Scalar diffraction theory [3] has been extensively studied both experimentally and theoretically. In this case, every component of the electromagnetic field is assumed to be uncoupled from the rest, and therefore it satisfies the Helmholtz wave equation. Based on the integral representation of the solutions of this equation, the Huygens–Fresnel principle (HF) can be derived [2]. According to this principle every secondary wavefront can be considered as the result of interference between elementary point sources distributed in the primary wavefront. Despite the fact that light propagation in a homogeneous medium can be completely analyzed through plane-wave decomposition and application of the Helmholtz equation, the corresponding integral form of the Huygens–Fresnel representation has found wide-spread use.

Over the last few years, there is a growing interest in the diffraction and scattering of light from subwavelength structures [4]. Since the spatial dimensions of such systems are smaller than the operational wavelength, scalar diffraction theory fails to accurately describe the field distribution in such structures. In examples like metallic nano-holes arrays [5], subwavelength apertures [6], nano-tips [7], the full Maxwell's equations need to be solved. In particular, all the field components must be known in the near field of the object, where the contribution of evanescent waves cannot be longer ignored.

Even though scalar diffraction theory cannot accurately predict the field generated by a sub-wavelength object, it can account for the propagation of that field for arbitrarily distances away from the obstacle. For sub-wavelength propagation distances the Huygens–Fresnel principle must be modified, in order to describe the non-paraxial near field regime. Such a modified Huygens–Fresnel formula (MHF) that remains valid for short propagation distance  $z$ , was considered several years ago [8,9]. It was only recently however, that

the physical meaning of this principle was clearly explained as interference between dipoles and not point sources, as the traditional Huygens–Fresnel formula assumes [10].

The purpose of this paper is the application of this modified Huygens–Fresnel diffraction formula to problems that are inherently non-paraxial and involve evanescent waves. Specifically, we show that evanescent wave propagation is the result of interference between oscillating dipoles with an appropriate phase distribution.

Let us first recall the Huygens–Fresnel formula [2] that is valid for far field propagation distance ( $z \gg \lambda$ , where  $\lambda$  is the free-space wavelength of light). The field amplitude  $U(x, y, z)$  after distance  $z$  is given by

$$U^{HF}(x, y, z) = \frac{1}{2i\lambda} \iint_S [1 + \cos\theta] \frac{e^{ikr}}{r} U(x', y', 0) dx' dy', \quad (1)$$

where  $r = \sqrt{(x-x')^2 + (y-y')^2 + z^2}$  and the obliquity factor is  $\cos\theta = z/r$ , where  $\theta$  is the angle between the unit vector  $\hat{\eta}$  normal to the screen and the unit vector  $\hat{r}$  parallel to the line that connects  $(x, y, z)$  and  $(x', y', z')$  points. The wavenumber of free space is denoted by  $k$  and is given by  $k = 2\pi/\lambda$ . With  $S$  we denote the optical aperture or the integration surface in general. In the near field ( $z \ll \lambda$ ) the appropriate modified Huygens–Fresnel diffraction integral [8–10] is:

$$U^{MHF}(x, y, z) = -\frac{1}{4\pi} \iint_S \left[ ik(1 + \cos\theta) - \frac{\cos\theta}{r} \right] \frac{e^{ikr}}{r} U(x', y', 0) dx' dy' \quad (2)$$

Notice the inclusion of the term  $\cos\theta/r$  which accounts for the dipole radiation [10]. We are interested in the propagation of a single plane wave that is generated, for example, by total internal reflection along an interface for incidence angles higher than the critical one. Let's assume the field at  $z=0$  to be  $U(x', y', 0) = A \exp(ik_e x')$ , with  $k_e$  the corresponding wavenumber. In other words, this is the phase modulation of the elementary dipoles that are normally oriented with respect to the  $z=0$  plane. Now we ask what is the field distribution

\* Corresponding author.

E-mail address: [kgmakris78@yahoo.com](mailto:kgmakris78@yahoo.com) (K.G. Makris).

right after  $z=0$  for  $z \ll \lambda$ . The answer to this is well known. It is either a propagating or an evanescent wave depending on the value of the  $k_e$ . The novel result of this paper is the derivation of the evanescent wave propagation in the near field through the use of Eq. (2). Without loss of generality we perform the field calculation on-axis, which means that  $x=0=y$ . We change to polar coordinates  $x'=\rho \cos \phi, y'=\rho \sin \phi, dx' dy'=\rho d\rho d\phi$ , where  $\rho \in [0, +\infty), \phi \in [0, 2\pi]$  and we use the integral representation of Bessel function  $J_n(z) = \frac{1}{2\pi i^n} \int_0^{2\pi} e^{izc\cos\phi} e^{in\phi} d\phi$  [11]. Substitution of the input field  $A \exp(ik_e x')$  at  $z=0$ , into Eq. (2) gives us:

$$U^{MHF}(z) = -\frac{A}{2} \int_0^{+\infty} \left[ ik \left( 1 + \frac{z}{\sqrt{\rho^2 + z^2}} \right) - \frac{z}{\rho^2 + z^2} \right] \frac{e^{ik\sqrt{\rho^2 + z^2}}}{\sqrt{\rho^2 + z^2}} J_0(k_e \rho) \rho d\rho. \quad (3)$$

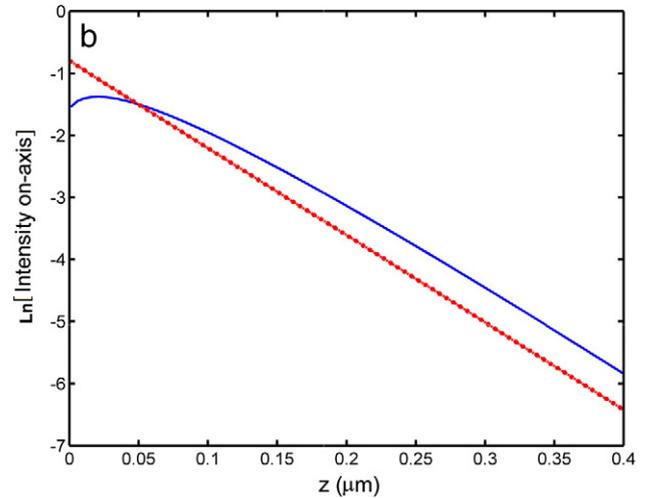
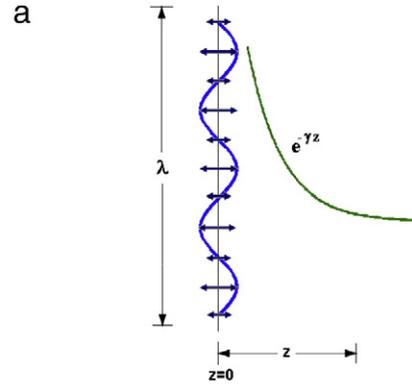
This integral can be written as the summation of three separated integrals  $I_1, I_2, I_3$ , namely  $U^{MHF}(z) = -\frac{A}{2}(I_1 + I_2 + I_3)$ , where  $I_1(z) = ik \int_0^{+\infty} \frac{e^{ik\sqrt{\rho^2 + z^2}}}{\sqrt{\rho^2 + z^2}} J_0(k_e \rho) \rho d\rho$ ,  $I_2(z) = ikz \int_0^{+\infty} \frac{e^{ik\sqrt{\rho^2 + z^2}}}{\rho^2 + z^2} J_0(k_e \rho) \rho d\rho$ , and  $I_3(z) = -z \int_0^{+\infty} \frac{e^{ik\sqrt{\rho^2 + z^2}}}{(\rho^2 + z^2)^{3/2}} J_0(k_e \rho) \rho d\rho$ . These three integrals, are closely related and we can show that they satisfy the condition  $\frac{1}{ik} \frac{dI_1}{dz} = I_2 + I_3$ . Based on this relation, the field at  $z$  can be written as  $U^{MHF}(z) = -\frac{A}{2} \left( I_1 + \frac{1}{ik} \frac{dI_1}{dz} \right)$ . By changing again variables  $u^2 = z^2 + \rho^2$ , and using the formula  $\int_z^{+\infty} e^{iku} J_0(k_e \sqrt{u^2 - z^2}) du = i \frac{e^{iz\sqrt{k^2 - k_e^2}}}{\sqrt{k^2 - k_e^2}}$  [11], we arrive at the diffracted near field ( $z \ll \lambda$ ) analytical expression for both propagating and evanescent waves:

$$U^{MHF}(z) = \begin{cases} \frac{A}{2} \left[ 1 + \frac{k}{\sqrt{k^2 - k_e^2}} \right] e^{i\sqrt{k^2 - k_e^2} z}, & \text{when } k_e < k \\ \frac{Ak_e}{2\gamma} e^{-\gamma z} e^{i\phi}, & \text{when } k_e > k \end{cases} \quad (4)$$

In the above equation the decay factor is  $\gamma = \sqrt{k_e^2 - k^2}$  and the phase shift is  $\phi = \tan^{-1}(-k/\gamma)$ . This phase shift  $\phi$  takes values from  $-\pi/2$  to 0 as  $k_e$  varies from  $k$  to infinity and is the interplay between two different phase factors. The one is the phase difference of  $\pi/2$  between the two first terms and the third one of Eq. (2) and the second is the phase of the input field. From Eq. (4) we can see that, when  $k_e < k$  the two phases cancel each other, whereas they result in a net phase factor for the case where  $k_e > k$ . Interestingly enough, if  $k_e > k$  the field is an evanescent wave and decays in the  $z$ -direction, as expected [12]. One question that naturally arises is what would be the result if one uses the classical HF principle instead of the modified one. For the case of evanescent waves ( $k_e > k$ ) and the same input field at  $z=0$ , it can be shown that the field distribution on-axis is given by the following closed form expression:

$$U^{HF}(z) = -\frac{A}{2} ik \left\{ \frac{e^{-\gamma z}}{\gamma} + z \int_0^{+\infty} \frac{e^{ik\sqrt{\rho^2 + z^2}}}{\rho^2 + z^2} J_0(k_e \rho) \rho d\rho \right\} \quad (5)$$

We numerically evaluate Eq. (5), and the  $\ln[|U^{HF}(z)|^2]$  is plotted as a function of the propagation distance  $z$  in Fig. 1(b) (blue solid line). As we can see the two curves are quite different in the near field, while they converge to the same result for  $z > \lambda$ . More specifically, the functional form of the diffracted field for the HF integral does not correspond to the linear line  $-\gamma z$  of the exponential wave decay. Only the MHF-integral gives the correct evanescent wave decay tail while the HF principle fails. As a result, an evanescent wave cannot be considered the outcome of point sources interference but that of radiating dipole interference.



**Fig. 1.** (a) Distribution of the oscillating dipoles normally oriented with respect to the dielectric interface. The resulting field decay due to interference is also depicted. (b) The calculated intensity on-axis (in natural logarithmic scale) of the diffracted field is shown for the case of an incident plane wave parallel to the dielectric interface when  $k_e > k$ . The calculation is based on the classical Huygens-Fresnel principle (solid line) and the modified Huygens-Fresnel principle (dotted line).

The MHF formulation of evanescent wave propagation, suggests that near field propagation can be considered as the interference of dipoles distributed on the incident wavefront. A similar effect has been studied in the microwave regime, where surface waves are formed under specific conditions in resonant circular arrays of cylindrical dipoles [13]. Even though we derived this only for the specific case of a plane wave, this point of view may be useful in the understanding of other near field problems. In particular, the diffraction of an incident evanescent wave from a circular aperture or the calculation of the field in subwavelength distances from a curved surface, are few of the cases where the MHF principle can be applied. As far as the limitations of this scalar approach are concerned, they are the same with the limitations of the scalar diffraction theory in general [3]. More specifically, if the medium is linear, isotropic, homogeneous nondispersive, nonmagnetic, then all the components of the electromagnetic field are uncoupled and obey a single scalar wave equation. Therefore scalar diffraction integrals, that are valid in the near field, can accurately describe the propagation of subwavelength beams. Since the field components are known (through a full vectorial eigenmode or scattering analysis) at the edge of a nanostructure, then every component can be separately propagated by scalar diffraction theory. More specifically, many recent studies [14–24] of wave propagation in realistic nanostructures, both theoretical [14,15,22,23] and experimental [16–21,24], have demonstrate the validity of scalar diffraction theory in the near field. The agreement

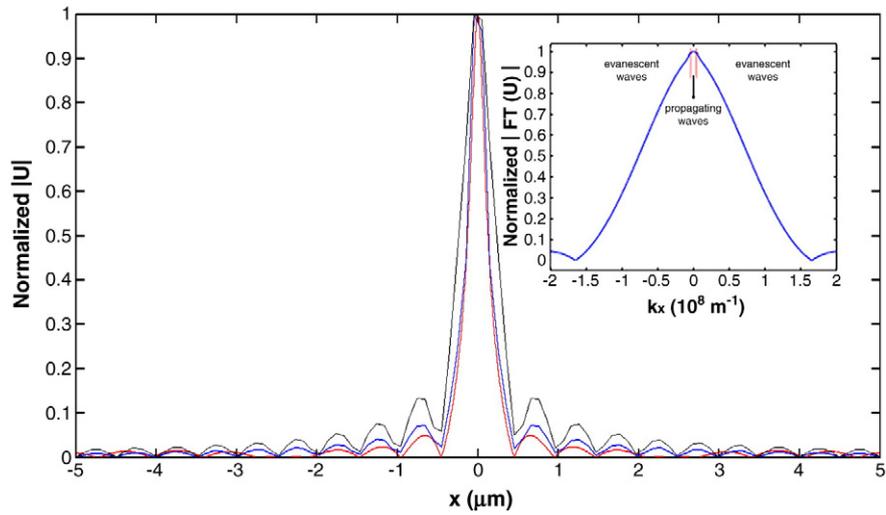


Fig. 2. (a) Absolute value of diffracted field from a 50 nm nanoslit at 100 nm away from the screen, calculated with FEMLAB (red line), classical Huygens–Fresnel principle (black line), and modified Huygens–Fresnel integral (blue line). In the inset the Fourier spectrum of generated field on the screen at  $z=0$  is depicted.

between numerical and experimental data is almost excellent and this proves that scalar diffraction theory describes the basic near field diffraction well enough to be widely applied in many different near field optics problems.

A similar derivation shows that the Rayleigh–Sommerfeld diffraction integral of the first kind [2] also yields an evanescent wave solution. The Rayleigh–Sommerfeld formulation is accurate for all propagation distances  $z$  (including  $z=0$ ), but it does not lead us to the

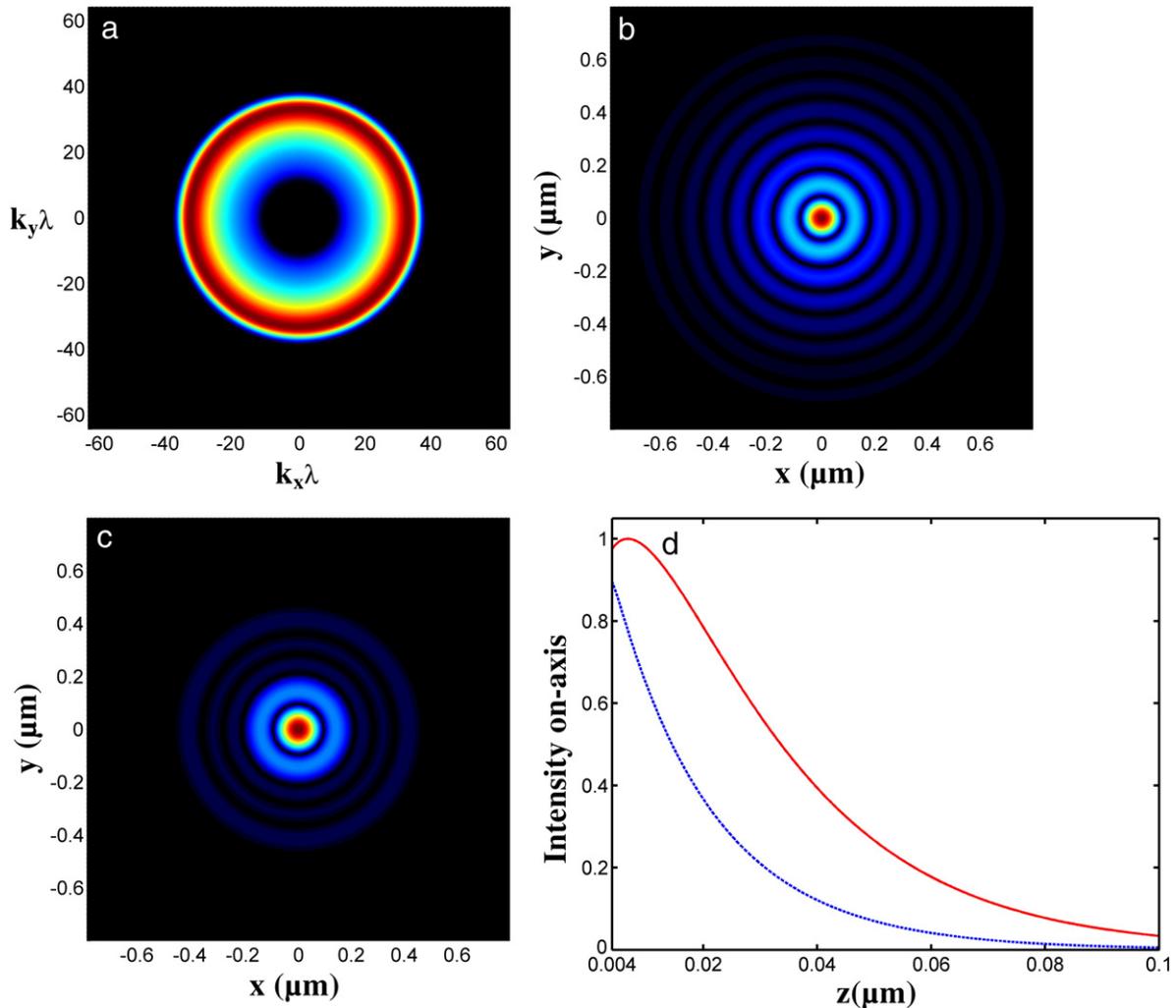


Fig. 3. (a) Evanescent wave spectrum, and (b) field profile, at  $z=0$ . (c) Absolute value of field distribution at the focus calculated with the MHF formula. (d) Comparison of the on-axis intensity of the diffracted evanescent wave field calculated with MHF integral (dotted line), HF formula (solid line).

satisfying physical picture of wave propagation via radiating dipoles. It is also restricted to planar geometries. In any case, the derivation in this paper is valid for both diffraction integrals (see Appendix A).

A first application example of the MHF principle in near-field propagation is the wave diffraction from a nano-aperture. In particular, we consider the radiation from a point-source propagating through a one-dimensional subwavelength slit of 50 nm width ( $\lambda = 1\mu\text{m}$ ). The field right after the aperture were calculated by using the COMSOL package and then were propagated for a distance of 100nm by scalar diffraction integrals. The comparison results are shown in Fig. 2. We can see that the MHF principle (blue curve) gives results very close to the FEMLAB simulation (red curve). On the contrary, the classical HF integral (black curve), gives less accurate results. In particular, the full width at half maximum of the diffracted beam is  $\sim 0.125\mu\text{m}$  based on MHF principle, while it is  $\sim 0.25\mu\text{m}$  according to the HF principle. The inset of Fig. 2 depicts all the spectral components of the field at  $z = 0$ . As we can see, the spatial wavenumbers that correspond to evanescent waves dominate over that of the propagating waves in the Fourier spectrum of the field. This means that mostly evanescent waves are involved in the propagation and thus classical HF cannot be applied.

A second example is that of subwavelength focusing as a result of diffraction of optical beams that contain only evanescent components [22]. It was shown both theoretically [22,23] and experimentally [24] in the microwave regime, that beams created as a superposition of evanescent waves can diffract in the near field in such a way that subwavelength focusing is possible. Here we demonstrate this effect through the MHF formulation. By using an evanescent wave computer generated hologram, we can create an optical beam that contains only evanescent wave components. The spatial frequency spectrum of such a beam at  $z = 0$  is depicted in Fig. 3(a), and the corresponding field profile is illustrated in Fig. 3(b). The diffracted wave after a propagation distance of  $\lambda/10$ ,  $\lambda = 1\mu\text{m}$  is given in Fig. 3(c), and we can clearly see that near-field focusing occurs. The results of the comparison of the on-axis intensity as a function of the propagation distance  $z$ , between classical HF (solid line), MHF principle (dotted line) are shown in Fig. 3 (d). The difference in the results is obvious. We also note that the MHF integral leads to more accurate description of the diffracted near field, since we get almost identical results when we evaluate the on-axis intensity by applying the Rayleigh–Sommerfeld integral.

In conclusion, we have analytically showed that an evanescent wave can be considered the result of interference between elementary dipoles normally oriented along the propagating wavefront. This was done by applying the modified Huygens–Fresnel principle valid in the near field, where dipoles act as secondary sources to produce, through interference, the diffracted wave. Specific examples of light propagation in the near field were also examined.

## Appendix A

The Rayleigh–Sommerfeld diffraction integral of first type, is defined by the following relation [2]:

$$U^{RS1}(x, y, z) = -\frac{1}{2\pi} \iint_S \frac{\partial}{\partial z} \left( \frac{e^{ikr}}{r} \right) U(x', y', 0) dx' dy' \quad (\text{A1})$$

Since  $\frac{\partial}{\partial z} \left( \frac{e^{ikr}}{r} \right) = \frac{e^{ikr}}{r} \left[ ik - \frac{1}{r} \right] \cos\theta$ , the above expression can be rewritten in a more convenient form:

$$U^{RS1}(x, y, z) = -\frac{1}{2\pi} \iint_S \cos\theta \left[ ik - \frac{1}{r} \right] \frac{e^{ikr}}{r} U(x', y', 0) dx' dy' \quad (\text{A2})$$

which is nothing more than the last two terms of the diffraction integral (near field terms) of Eq. (2) multiplied by a factor of 2. By assuming the same input field  $U(x', y', 0) = A \exp(ik_e x')$ , and following the same way of calculating the field on-axis, we get:  $U^{RS1}(z) = -A(I_2 + I_3)$ , where the integrals  $I_2(z), I_3(z)$  are defined in the main text. We already know that  $I_2 + I_3 = \frac{1}{ik} \frac{dI_1}{dz}$ . But we calculated before the closed formed expression of  $I_1(z)$  integral, which is:  $I_1(z) = -k \frac{e^{-\gamma z}}{\gamma}$ , so  $I_2 + I_3 = -e^{-\gamma z}$ . This means that  $U^{RS1}(z) = Ae^{-\gamma z}$ . This means that the evanescent wave tail can be also derived by the Rayleigh–Sommerfeld integral of the first kind (formally equivalent to the scalar Helmholtz equation [25], which also gives us the same  $Ae^{-\gamma z}$  result). But there are two main differences with respect to the main derivation of this paper. Firstly, this result does not lead us to the interesting physical picture of any evanescent wave, which is that of the interference between oscillating dipoles. Secondly, the Rayleigh–Sommerfeld integral was derived for flat surfaces only, whereas MHF formulation is valid for any curved surface.

## References

- [1] A. Sommerfeld, *Mathematical Theory of Diffraction*, Birkhäuser, Boston, 2003.
- [2] M. Born, E. Wolf, *Principles of Optics*, Pergamon, Cambridge, 1999.
- [3] J. Goodman, *Introduction to Fourier Optics*, McGraw Hill, 1996.
- [4] L. Novotny, B. Hecht, *Principles of Nano-Optics*, 2006 Cambridge.
- [5] T.W. Ebbesen, et al., *Nature* 391 (1998) 667.
- [6] H.A. Bethe, *Phys. Rev.* 66 (1944) 163.
- [7] L. Novotny, D.W. Pohl, B. Hecht, *Opt. Lett.* 20 (1995) 970.
- [8] C.L. Andrews, *Phys. Rev.* 71 (1947) 777;  
C.L. Andrews, *Am. J. Phys.* 19 (1951) 280.
- [9] S.D. Peterson, J.E. Kasper, *Am. J. Phys.* 40 (1972) 1274.
- [10] D.A. Miller, *Opt. Lett.* 16 (1991) 1370;  
F. Depasse, M.A. Paesler, D. Courjon, J.M. Vigoureux, *Opt. Lett.* 20 (1995) 234.
- [11] I.S. Gradshteyn, I.M. Ryzhik, *Table of integrals, series and products*, Academic press, 2000.
- [12] P. Yeh, *Optical waves in crystals*, Wiley-Interscience, 2005.
- [13] G. Fikioris, K. Matos, *IEEE Antennas Propag. Mag.* 50 (2008) 97.
- [14] M.W. Kowarz, *Appl. Opt.* 34 (1995) 3055.
- [15] F.M. Huang, N.I. Zheludev, *Nano Lett.* 9 (2009) 1249.
- [16] A.J. Huber, B. Deutsch, L. Novotny, R. Hillenbrand, *Appl. Phys. Lett.* 92 (2008) 203104.
- [17] L. Feng, K.A. Tetz, B. Slutsky, V. Lomakin, Y. Fainman, *Appl. Phys. Lett.* 8 (2008) 2479.
- [18] R. Zia, M.L. Brongersma, *Nat. Nanotechnol.* 2 (2007) 426.
- [19] J.Y. Laluet, E. Devaux, C. Genet, T.W. Ebbesen, J.C. Weeber, A. Dereux, *Opt. Express* 15 (2007) 3488.
- [20] G. Gay, O. Alloschery, B. Viaris de Lesegno, C. O'Dwyer, J. Weiner, H.J. Lezec, *Nat. Phys.* 2 (2006) 262.
- [21] H.J. Lezec, T. Thio, *Opt. Express* 12 (2004) 3629.
- [22] R. Merlin, *Science* 317 (2007) 927.
- [23] A. Grbic, R. Merlin, *IEEE Trans. Antennas Propag.* 56 (2008) 3159.
- [24] A. Grbic, L. Jiang, R. Merlin, *Science* 320 (2008) 511.
- [25] G.C. Sherman, *J. Opt. Soc. Am.* 57 (1967) 546.