Multiple-invariant space-variant optical processors

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Multiple invariant optical correlators are considered. By multiple invariance, we mean invariance to more than one distortion parameter per axis of the processor. Space variant optical processors using coordinate transformations and a new phase detection scheme are used to realize such correlators. A theoretical analysis and experimental verification are included.

I. Introduction

The parallel processing and real-time features of optical processors have not been sufficient alone for these systems to see extensive use in pattern recognition.[1] One reason for this lack of practical use for optical correlators has been their lack of flexibility and the limited number of operations achievable in these processors.[2] Hybrid optical/digital processors[3] and space variant optical processors[4,5] have increased the flexibility of these systems and included nonlinear operations and space variant systems to the repertoire of optical processors. We recently reported on a space-variant, distortion-invariant optical correlator using coordinate transformations that allows correlation of two functions that are distorted versions of one another.[4] However, this approach could only be applied to functions that were distorted by at the most, two separate distortions (for a 2-D system). Since more than two distortions occur in practice in image pattern recognition problems, optical processors must address such real issues if they are to be viable candidate pattern recognition systems. In this paper, we consider the formulation of a multiple-invariant, space-variant optical processor that is invariant to multiple distortions. A general theoretical analysis is presented first, followed by several specific cases and implementation methods. We conclude with experimental confirmation of the fundamental principle.

II. Space-Variant Processors

Since these multiple-invariant systems will be formulated using space-variant processing methods, specifically by use of coordinate transformations, we review the general formulation of a distortion-invariant, space-variant processor. A 1-D example is considered for simplicity. Let $f(x)$ be the original undistorted function. The distorted function $f'(x)$ is described by

$$f'(x) = f(x') = f[g(x,a)], \tag{1}$$

where $g(x,a)$ is the distorting function, and a is the unknown distortion parameter. To realize a space-variant processor [invariant to the $g(x,a)$ distorting function], a coordinate transformation $\xi = h^{-1}(x)$ is applied to both functions. This produces two new functions $f_1(\xi)$ and $f_1'(\xi)$, which can be used in a conventional space-invariant correlator to achieve correlation invariant to the distortion $g(x,a)$.

The coordinate transformation is chosen to convert the distortion into a shift ξ_0 for all values of a. Given $g(x,a)$, the coordinate transformation can be found from[6]

$$\xi = h^{-1}(x) = -\frac{d\xi_0}{da}\int_{-\infty}^{x} \frac{\dfrac{\partial g(x,a)}{\partial x}}{\dfrac{\partial g(x,a)}{\partial a}}\,dx. \tag{2}$$

These new coordinate transformed functions $f_1(x)$ and $f_1'(x)$ are shifted versions of one another. Thus a conventional space-invariant system can be used to correlate them. The intensity of the output correlation peak will be independent of a, and the location of the output correlation peak will be proportional to a. These basic principles are used in several stages in the present multiple invariant system.

Prior space-variant processors using coordinate transformations can accommodate only one distortion parameter per axis (one distortion parameter for a 1-D system or two distortions for a 2-D system or in general one distortion parameter per axis). If the distorting function depends on more than one parameter, the distorted function is

$$f'(x) = f(x') = f[g(x,a_1,a_2)], \tag{3}$$

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Received 30 June 1977.
0003-6935/78/0215-0655$0.50/0.

where 1-D functions are used, and the distorting function g is described by two distortion parameters a_1 and a_2. Then from Eq. (2), we see that since $h^{-1}(x)$ depends on $\partial g/\partial a_i$ and since

$$(\partial g)/(\partial a_i) \neq (\partial g)/(\partial a_j), \tag{4}$$

each distortion parameter requires a different coordinate transformation $h^{-1}(x)$. It thus follows that the maximum number of distortion parameters that coordinate transformation processing can accommodate equals the number of dimensions of the processing system. In this context, a distortion is multiple if it is described by more parameters than coordinate transformation processing can accommodate.

III. Prior Multiple-Invariance Approaches

When the distorted function is described by Eq. (3), we can achieve multiple invariance by (a) scanning through all values of the additional distorting parameter or (b) elimination of the additional distorting parameter by filtering in the transform plane.

The first approach involves construction of a number of systems invariant to one of the parameters (say a_1), with each system corresponding to one value of the second parameter a_2. If these systems cover all values of a_2, the output of one of them will be the same for all values of a_1 and a_2 for all input functions related by Eq. (3). This approach has been used in multiple holographic matched spatial filter systems,[7] conventional multichannel Doppler signal processors,[8] and in optical correlators using mechanical movement of components to effect a scale or rotational search.[9] However, these solutions require a large processing space (if parallel) or are slow (if sequential).

The second approach is more attractive. It uses the fact that a shift in the input coordinates can be transformed into a linear phase factor in Fourier transform space. Elimination of this phase factor also eliminates any shifts in the input coordinates. Until now only the formation of the magnitude of the Fourier transform has been suggested as a method of achieving multiple invariance.[2,10] We consider this formulation and its shortcomings in detail below.

With the reference function $f(x)$ and the distorted input function $f'(x)$ defined by Eq. (3), we can realize multiple invariance by first applying the transformation $x = h(\xi)$ to $f'(x)$. This transformation must satisfy

$$g(x,a_1a_2) = g[h(\xi - \xi_0),a_2], \tag{5}$$

where $\xi_0 = \xi_0(a_1)$ is a constant depending only on the unknown distortion parameter a_1. The transformation $x = h(\xi)$ is determined from Eq. (2). This yields a new function

$$f''(\xi - \xi_0, a_2) = f\{g[h(\xi - \xi_0),a_2]\}. \tag{6}$$

If we attempted to correlate f and f'' in a conventional space-invariant processor, the peak intensity of the correlation will fluctuate as a_2 changes. Using a 2-D multichannel processor, each channel can be adjusted to correspond to different a_2 values. This will solve the 1-D problem presently being discussed but not the 2-D version of it.

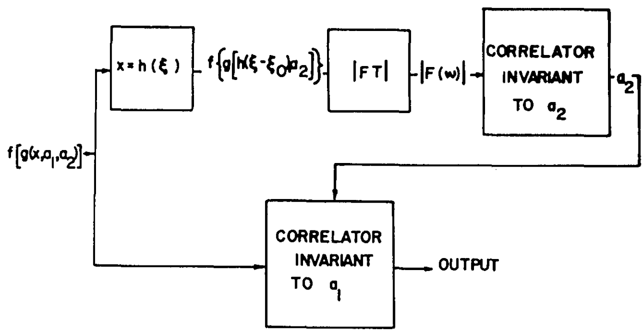


Fig. 1. Schematic block diagram of a multiple-invariant, space-variant optical correlator using the magnitude of the Fourier transform.

To address the real problem, elimination of the distortion parameter a_1, we form the Fourier transform of f and f''. This yields

$$\mathcal{F}\,[f''(\xi - \xi_0,a_2)] = \exp\,(-j\omega\xi_0)F''(\omega), \tag{7}$$

where F'' is the Fourier transform of f'' and ω is the radian spatial frequency variable in the Fourier transform plane. If we form the magnitude of the Fourier transform of f'', we will have removed the dependence on a_1. We can then retransform $F''(\omega)$ and recover f'' independent of a_1. This new f'' function and f can then be correlated, except that we have lost the phase of $F(\omega)$ and hence potentially useful information about f''. The magnitude of the Fourier transform represents the frequencies present in the input, and the phase of the Fourier transform is affected by the distribution of these frequencies in the input. Thus some information is lost, since two functions with the same frequency content but different spatial distributions of these frequencies could produce identical results. This could lead to false correlations for such functions. The extent of this as a problem depends upon the specific functions and application.

However, two processors can be cascaded as in Fig. 1. The first processor consists of a coordinate transformation that converts a_1 to a shift by ξ_0 as before. The magnitude of the Fourier transform of this function (together with f subjected to the same transformation and Fourier transform magnitude operation) can then be used as the input to a correlator invariant to a_2 (as a_2 affects the magnitude of the Fourier transform). The location of the output correlation peak will then be proportional to a_2 as in conventional space-variant processing by coordinate transformations.

f and f' (the original distorted function) are then used as inputs to a second space-variant correlator (invariant to a_1). This second correlator is made adjustable so that it responds to different a_2 values. The output of the second correlator then provides a correlation invariant to a_1 and a_2, and the location of the output correlation peak is proportional to a_1. Thus a_1 and a_2 can be determined and multiple distortions accommodated by this system.

This method has several limitations besides the loss of phase information associated with the formation of the magnitude of the Fourier transform. First, the coordinate distortion described by a_2 may or may not be preserved as a coordinate transformation in Fourier space. Furthermore, in the presence of noise, the magnitude of the Fourier transform is not completely unaffected by a shift since interference occurs between the signal and the noise.

IV. Phase Extraction

The major problem with achieving multiple invariance by forming the magnitude of the Fourier transform of the coordinate transformed function was the loss of phase that occurred. We have recently[11] reported on and demonstrated a method whereby the phase can be extracted from a complex wavefront. For this present application, we describe this phase extraction process as follows.

At the input plane we record $f(x - x_0) + f(-x + x_0)$ and detect the magnitude of its Fourier transform on a TV camera. After thresholding the output signal with a limiter and normalizing the width of all fringes by a monostable in the video line, we obtain a function $T(\omega)$ whose derivative is

$$\frac{dT(\omega)}{d\omega} = [\sin(q)] \left[\frac{d\phi(\omega)}{d\omega} + x_0\right], \tag{8}$$

where $\phi(\omega)$ is the phase of the Fourier transform $F(\omega)$ of $f(x)$, and q is a constant.

Recall that the phase of the Fourier transform of the function in the multiple-invariant correlator contains a portion that is nonlinear in ω (due to the spatial distribution of the function itself) and a portion that is linear in ω (due to the location of the function in the input plane). By taking the Fourier transform of Eq. (8), the dc term is proportional to the shift in the input coordinates. This value can then be used to position the input function properly. Alternatively, the derivative of Eq. (8) can be formed (this eliminates the constant term x_0 and all linear phase terms in the transform) and integrated twice. This yields the desired phase function free of all linear terms. This phase function can then be combined with the magnitude of the Fourier transform and inverse transformed to reconstruct the input function free from a shift.

V. Multiple-Invariant Correlation

The schematic block diagram of a multiple-invariant correlator that avoids the phase loss problems of the prior system is shown in Fig. 2. Recall that the object of this system is to correlate two functions: $f'(x)$ defined by Eq. (3) and the undistorted function $f(x)$ and to determine the unknown distortion parameters a_1 and a_2. For simplicity, only the $f'(x)$ channel of this system is shown in Fig. 2. The undistorted reference function $f(x)$ is operated upon similarly.

As shown in Fig. 2, a coordinate transformation $x = h_1(\xi_1)$ is applied to the distorted input function $f'(x) = f(x')$, where from Eq. (3)

$$x' = g(x,a_1,a_2). \tag{9}$$

The coordinate transformation $\xi_1 = h_1^{-1}(x)$ affects only the distortion parameter a_1 and converts its effect to a shift by ξ_0 in the ξ_1 coordinate, such that

$$g[h_1(\xi_1),a_1,a_2] = g_1(\xi_1 - \xi_{01},a_2), \tag{10}$$

where $\xi_{01} = \xi_{01}(a_1)$ is a constant dependent on the unknown parameter a_1 only. The Fourier transform of

$$f_1(\xi_1 - \xi_{01},a_2) = f[g_1(\xi_1 - \xi_{01},a_2)] \tag{11}$$

is $F(\omega) \exp(j\omega\xi_{01})$. Using the linear phase extraction scheme[11] of Sec. IV, we can determine $\xi_{01}(a_1)$ and hence a_1.

If we use the $\xi_{01}(a_1)$ value found from the phase extraction system to shift $f_1(\xi_1 - \xi_{01},a_2)$, we obtain $f_1(\xi_1,a)$. Applying the inverse transform $\xi_1 = h_1^{-1}(x)$ to this function we obtain

$$f\{g_1[h_1^{-1}(x),a_2]\} = f[g(x,a_2)], \tag{12}$$

which is now independent of a_1.

We can now apply conventional space-variant correlation[4,6] by coordinate transformation to this function and $f(x)$. We achieve this by applying the coordinate transformation $\xi_2 = h_2^{-1}(x)$ to both $f(x)$ and $f[g(x,a_2)]$, where ξ_2 is selected to satisfy Eq. (2). These two new coordinate transformed functions $f(\xi_2)$ and $f(\xi_2 - \xi_{02})$ [where $\xi_{02} = \xi_{02}(a_2)$ is a constant dependent on the unknown a_2 only] are then used as inputs to a conventional space-invariant correlator. The output correlation peak is invariant to the distortion described by Eqs. (3) and (9). The location of the output correlation peak is proportional to ξ_{02} and hence to a_2. Thus the desired distortion invariant correlation has been realized and the unknown distortion parameters a_1 and a_2 determined.

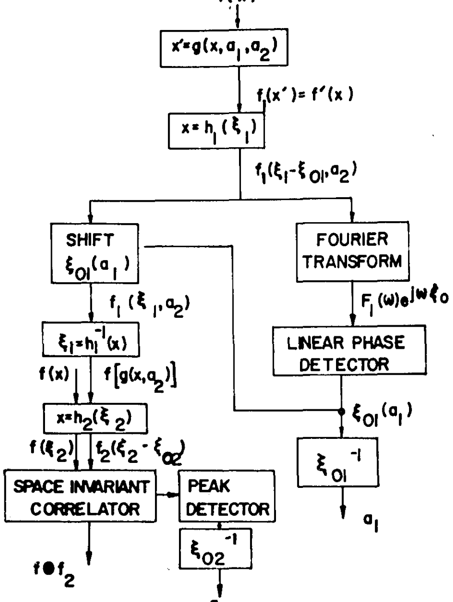


Fig. 2. Schematic block diagram of a multiple-invariant, space-variant optical correlator with no phase loss of information.

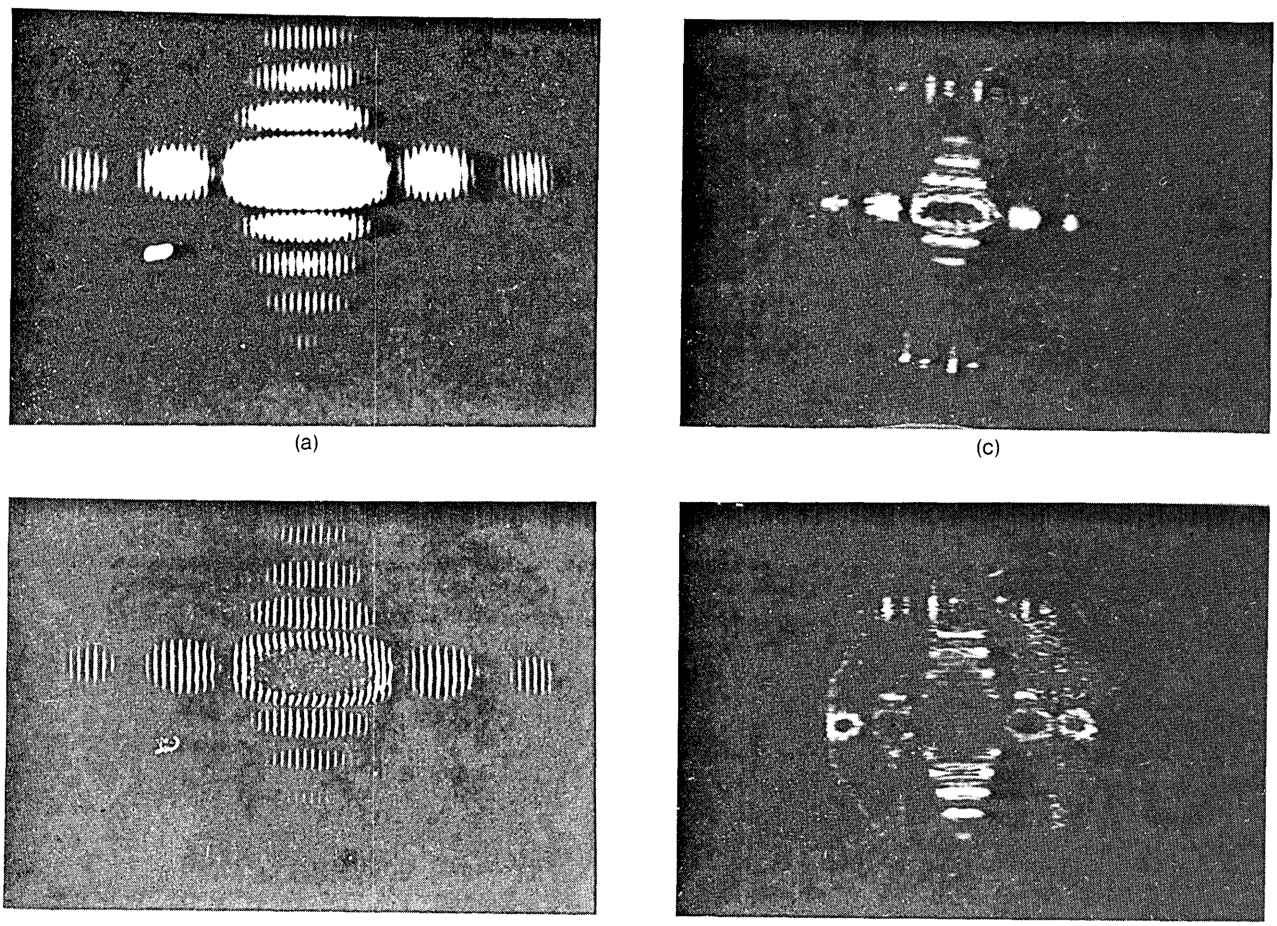
VI. Experimental Demonstration

To demonstrate this multiple-invariant correlator and its use, we consider the case of a multiple distortion consisting of a shift in the input coordinate (i.e., the input object can occur anywhere within the input plane) and another distortion such as a scale or rotational change between the input and reference functions. Since a shift of the input object in one coordinate occurs independently of the other coordinate in a 2-D image, two separate parameters describe the shift distortion. If any other type of distortion exists simultaneously, space-variant processing by coordinate transformation alone is not sufficient, and multiple-invariant, space-variant processing is needed since the limit of one distortion per axis has been exceeded. (The shift alone involves one distortion per axis.)

The phase extraction scheme described in Sec. IV can be used to extract the shift in the input function. We demonstrate this experimentally below. This shift information can then be used to center automatically the reference and input functions properly to allow space-variant processing by coordinate transformation to be used as in Fig. 2.

To demonstrate the key issue, extraction of the shift information, two rectangular inputs (1 mm × 2 mm) separated by 10 mm were used. Their interference pattern was formed by a 762-mm focal length lens and detected by a vidicon camera. This pattern, photographed from a monitor, is shown in Fig. 3(a). The effect of the magnitude of the Fourier transform of the rectangular inputs is visible as a variation of the intensity of the fringes in the main and side-lobes and across all lobes. The thresholded and pulse normalized version of this pattern is shown in Fig. 3(b). The effect of the amplitude modulation has now been removed, the fringe width is now uniform, and the fringe intensity far more uniform. [A peak detector circuit was used in conjunction with the threshold circuit to insure that the fringes in Fig. 3(b) occur at the center of the fringes in Fig. 3(a).] Since the input rectangular function is symmetric, its Fourier transform is real, and the frequency of the fringes is constant and determined by the separation or shift in the inputs.

The derivative of the first diffracted order of the pattern in Fig. 3(b) is shown in Fig. 3(c). This pattern is constant and proportional to the separation or shift $2x_0$ between the two inputs.



(a) (c)

(b) (d)

Fig. 3. Experimental demonstration of shift or phase extraction in a multiple-invariant, space-variant optical correlator: (a) magnitude of the Fourier transform of the interference pattern; (b) thresholded and pulse width normalized version of (a); (c) derivative of the first diffracted order of the pattern in (b) showing the input phase data; (d) same as (c) but for the pattern in (a) showing the need to remove the amplitude modulation.

The spacing between the two inputs $2x_0 + x_1$ (where $2x_0$ is the separation for zero shift and x_1 is the shift) was varied. For each case the intensity $(x_0 + x_1)^2$ of the derivative of the function like the one in Fig. 3(b) was detected. In Fig. 4, we show a plot of the actual and experimentally determined values of the input shift x_1. The departure of the two curves at large x_1 is due to departures in the performance of the lens at large apertures and higher space bandwidths. To demonstrate the necessity to eliminate the magnitude of the Fourier transform from the interference pattern, we directly differentiated the pattern in Fig. 3(a). The result, shown in Fig. 5(c), clearly shows the effects of amplitude modulation. Detection of the phase data from this latter pattern is clearly impossible.

VII. Discussion

As noted before, if a shift in the input and another distortion are simultaneously present, a multiinvariant processor is needed. We can apply this to functions distorted as

$$f[g(x,a_1,a_2)] = f[g(x - a_1,a_2)], \tag{13}$$

as noted earlier. In this case, the shift in the input coordinate is eliminated first, and a space-variant system invariant to a_2 is then used. The scale, rotation, and shift-invariant problem[10] addressed earlier is a practical 2-D example of such a problem.

In instances in which a shift in the input coordinates cannot be expressed by Eq. (13), such as

$$g(x,a_1,a_2) = x^{a_1} + a_2, \tag{14}$$

we first eliminate the distortion parameter a_1. This is accomplished[12] by applying the coordinate transformation $\xi_1 = \ln(lnx)$ to the function. Equation (14) then becomes

$$g[\exp(\exp\xi_1),a_1,a_2] = \exp[\exp(\xi_1 + \ln a_1)] + a_2 \tag{15}$$
$$= g_1(\xi_1 - \xi_{01},a_2),$$

where the effect of the coordinate exponentiation distortion parameter a_1 has been converted to a shift by $\xi_{01}(a_1) = -\ln a_1$. This shift can be eliminated as noted earlier. The effective distortion then reduces to

$$g[\exp(\exp\xi_1),1,a_2] = \exp(\exp\xi_1) + a_2$$
$$= g_1(\xi_1,a_2). \tag{16}$$

An inverse coordinate transform $\xi_1 = \ln(\ln x)$ is now applied to Eq. (16) to yield

$$g(x,1,a_2) = x + a_2. \tag{17}$$

The resultant function can now be processed in a conventional shift-invariant, space-invariant optical correlator.

It should be noted that this multi-invariant optical correlator is not limited to just two distortion parameters per axis. Consider the distorted function

$$f[a_1(x - a_2)^{a_3}]. \tag{18}$$

The parameters a_2 and a_1 can be eliminated as noted earlier, and a system such as the one just discussed, invariant to the distortion parameter a_3, used to produce a 1-D optical correlator invariant to all three distortion parameters and in which all three unknown distortion parameters can be determined.

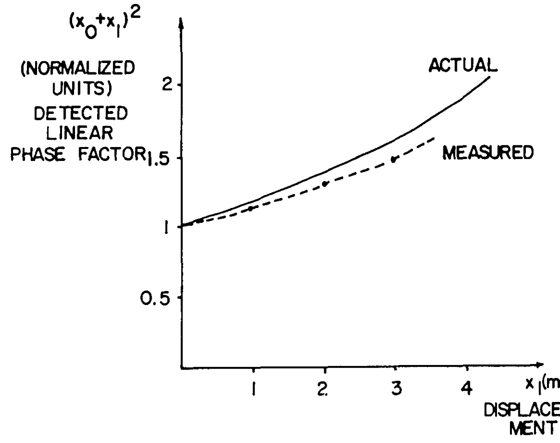


Fig. 4. Plot of actual vs experimentally determined values of the shift of the input function.

VIII. Summary

A multiple-invariant, space-variant optical processor has been described in which two functions described by any number of separate distortion parameters can be correlated with no loss in SNR of the correlation. The unknown distortion parameters can also be determined in this scheme. Experimental confirmation of the key step (determination of the non-linear phase portion of a complex optical transform) has been provided.

The support of the Ballistic Missile Defense Advanced Technology Center on contract DASG-60-77-C-0034 and the Air Force Office of Scientific Research, Air Force Systems Command, on contract AFOSR-75-2851 which made this work possible is gratefully acknowledged.

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