The departure of the VCM result from the exact CMT result for $\theta_o \leqslant 3\theta_c$ is to be expected since radiation escaping at small angles relative to the axis of the fiber would be most influenced by the presence of the core–cladding interface, which is neglected in the VCM. However, for $\theta_o > 3\theta_c$, excellent agreement between CMT and the approximate technique is displayed.

Thus, the VCM can be said to be an excellent approximate technique for the calculation of radiation mechanisms whose radiation is not paraxially directed. For paraxially directed radiation, the exact CMT must be used.

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Space–bandwidth product and accuracy of the optical Mellin transform

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The optical Mellin transform has recently been successfully realized[1] and used to construct useful scale invariant optical pattern recognition systems,[2] in Doppler signal processing,[3] and in the correlation of nonvertical imagery,[4] among other applications. The Mellin transform of $f(x)$ can be synthesized optically by applying a coordinate transformation $\xi = \ln x$ to the input coordinate x and an optical Fourier transform to $f[\exp(\xi)]$. We now examine the space–bandwidth product requirements N' in the coordinate transformed space and derive expressions for their relationship to the input space–bandwidth product N and the accuracy of the optical Mellin transform.

We denote the minimum resolution element size in x and ξ space by Δx and $\Delta\xi$, respectively. We consider an input function defined over $0 \leq x < x_{\max}$. The corresponding function in ξ space is defined over $-\infty \leq \xi \leq \ln x_{\max}$ for full accuracy recording. By virtue of the required input coordinate transformation, the portion of the input function lying at or near $x = 0$ maps into $\xi = -\infty$ in coordinate transformed space. To avoid the difficulty associated with this region, we restrict the domain of the input function to $\delta \leq x \leq x_{\max}$. If we denote the number of resolution elements in the interval $0 \leq x < \delta$ by M, we find the spatial extent of the input function in terms of its space–bandwidth N to satisfy

$$M\,\Delta x \leq x \leq N\Delta x. \tag{1}$$

The resolution $\Delta\xi$ in ξ space is related to Δx by

$$\Delta\xi = \frac{d(\ln x)}{dx}\quad \Delta x = \frac{\Delta x}{x}. \tag{2}$$

Table I. Space–Bandwidth Requirements for an Optical Mellin Transform as a Function of Accuracy and Scale Change

N Input space–bandwidth	$100\,M/N$ % accuracy	$100\,a$ Max. % scale change	N' Mellin space–bandwidth
500	1%	100%	2650
500	1%	200%	2850
500	2%	100%	2300
500	2%	200%	2500
1000	1%	100%	5300
1000	1%	200%	5700
1000	2%	100%	4600
1000	2%	200%	5000

Since the resolution in ξ space is in uniform in practice, the worst-case resolution $\Delta\xi$ is

$$\Delta\xi = \frac{\Delta x}{x_{\max}} = \frac{\Delta x}{N\Delta x} = \frac{1}{N}. \tag{3}$$

The spatial extent of the function in ξ space is

$$\ln(N\Delta x) - \ln(M\Delta x) = \ln(N/M). \tag{4}$$

Combining Eqs. (3) and (4), we find the space–bandwidth N' in ξ space to be

$$N' = [N \ln(N/M)]. \tag{5}$$

From Eq. (5), we see that N' is larger than N and that N' decreases as M increases. However, the ratio M/N is the portion of the input lying in the interval $0 \leq x < \delta$ that was omitted. Thus, the accuracy of the system is inversely proportional to M/N, and accuracy and space–bandwidth requirements are interrelated. A scaling of the input function by a does not affect N, but it will change the number of resolution elements lying in the interval $0 \leq x < \delta$. As a increases, M will increase proportionally, and vice versa. Thus N' is determined not only by N and by the accuracy requirement (a function of M/N) but also by the desired range of the scale factor a (smaller values of a correspond to larger input functions).

The space–bandwidth N' is listed in Table I for various input space–bandwidths N, for various accuracy ratios (M/N), and for several scale ranges. From these data we see that one requires over five times the input space–bandwidth to achieve 1% accuracy and accommodate invariance for a 200% scale change. Far lower space–bandwidth requirements would be needed if less accuracy and less of a scale change are acceptable.

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