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Letters to the Editor

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Doppler signal processing: a new technique

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Realization of an optical Mellin transform has recently been reported and applications of its scale invariance in image processing suggested.[1–3] We now report on extensions of these operations to signal processing. Since the effect of the Doppler frequency shift that arises when there is relative motion between the source and receiver is equivalent to a scaling of the time axis from t to at,[4] this appears to be a viable signal processing area in which Mellin transform and correlation techniques are applicable. Although Mellin-type matched filtering operations can be realized by digital or optical techniques, the optical implementation will be emphasized because of the parallel processing and high speed advantages of an optical system.

For backscattered radiation in radar, the detected frequency ω_d is related to the emitted radiation ω_o and the radial velocity v_s of the source object by

$$\omega_d = \omega_o(1 + 2\dot{R}_o/c)\,, \tag{1}$$

where $v_s = \dot{R}_o$ is the rate of change of range R_o and $v_o = c$ is the velocity of propagation of the waves in the medium. This Doppler frequency shift is equivalent to a scaling of the time axis from t to bt, where $b = 1 - 2\dot{R}_o/c$.

It has been demonstrated that the magnitudes of the Mellin transforms M_1 and M_2 of two scaled functions $f_1(x,y)$ and $f_2(x,y) = f_1(bx,by)$ are identical.[1] This property has been used to correlate scaled input imagery with no loss in the SNR of the correlation peak from the autocorrelation case.[2] Of particular interest in the case of Doppler signal processing is the fact that the correlation peak in Mellin correlation is shifted from its on-axis location by $K \ln b$, where K is a constant.[2] Thus, the scale change between f_1 and f_2 (or in this case the Doppler shift and, hence, the velocity of the target) can be determined from the location of the correlation peak. The Mellin transform of $f(x)$ can be realized by forming the Fourier transform of the scaled function $f(\exp x)$, where the scaling is obtained by a log module in the deflection system of the input transducer on which $f(x)$ is recorded.[1]

To demonstrate the use of Mellin transforms in Doppler signal processing, the logarithmically scaled input signal f_1 was recorded on nine lines in the left half of the input plane and the nine logarithmically scaled Doppler shifted versions f_2 of it on the same nine lines but in the right half of the input plane. The transmittance of the input plane is then described by

$$\sum_n [f_{n1}(x + x_o, y - ny_o) + f_{n2}(x - x_o + x_n, y - ny_o)]\,, \tag{2}$$

where $n = 9$ rows, the separation between rows is y_o, and the spacing between signals on a given line is $2x_o$. The one-dimensional Fourier transform of this input distribution was recorded on film, and a second one-dimensional transform of this film pattern obtained. This output pattern contains the correlations $f_{n1} \times f_{n2}$ at $x_n' = 2x_o - x_n$. This is a modified version of a joint-transform correlator.[5]

A photograph of the region of the output plane around $x' = 2x_o$, $y' = 0$ (displayed on a thresholded monitor) is shown in Fig. 1. The relative location x_n' of the correlation peak on each line is proportional to the logarithm of the scale change between the signals on line n in the input and, thus, the Doppler shift between these signals. All correlation peaks are of the same intensity, and only their location now contains information. For convenience, a constant Doppler increment was applied to each of the nine reference signals f_{n2}.

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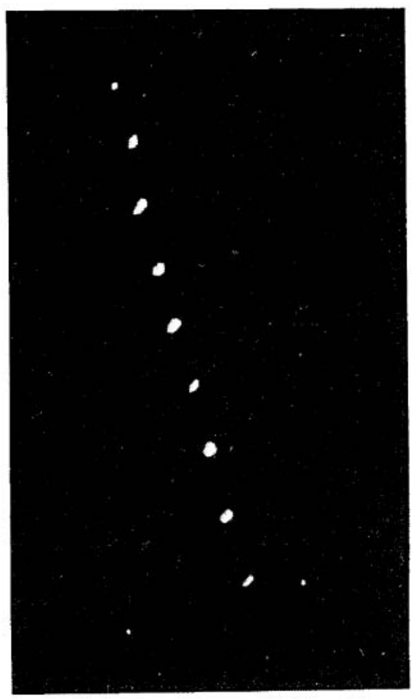

Fig. 1. Output plane showing the Mellin correlation of an input signal and nine Doppler-shifted versions of it. The horizontal coordinates of the correlation peaks are proportional to the logarithm of the Doppler shift between the signals.

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