Energy spectrum of a quantum well (QW) vs quantum dot (QD)

We aim calculating the energy spectrum of a AlGaAs-GaAs-AlGaAs mesa shaped quantum dot with a diameter d = 100 nm, a thickness L_{GaAS} = 5 nm and a thickness of the potential barrier L_{AlGaAS} = 4 nm . The typical carrier concentration is about n = 10^{18} cm^{-3} and the effective mass is about 0.07 of the free electron mass.

a) Calculate the QD energy splitting due to the vertical and lateral confinement.

b) Calculate the same energy splitting in the case of a quantum well, i.e. d = 10 μm

c) We can describe the drain-AlGaAs-GaAs-AlGaAs-source sandwich as a classical capacitor (actually two capacitors connected in series). Assuming a relative dielectric constant of $\varepsilon_r = 5$ ($\varepsilon_o = 8.9\ 10^{-12}$ F) for the AlGaAs barrier, calculate the energy splitting due to the charging energy (Coulomb blockade) for the quantum dot and the quantum well

d) Can you always observe these energy spectra? Which is the maximum temperature at which the energy spectra can be resolved?

e) What do you have to modify in the dot size if you want to observe the coulomb blockade at room temperature?

Solution: energy spectrum of a quantum dot

a) The problem can be schematized by a particle in a 3D box with infinite potential on the box walls. Due to the different vertical and lateral size of the quantum dot the wave vector spectrum is quantized following:

$$k_z = \frac{n\pi}{L}; k_{x,y} = \frac{m\pi}{d}$$

The energy spectrum and the level spacing are given by:

$$E = \frac{\hbar^2 k^2}{2m}; \quad \Delta E = \frac{\hbar^2}{2m}(2k\Delta k + \Delta k^2) \approx \frac{\hbar^2}{2m} 2k\Delta k; \quad \Delta k_z = \frac{\pi}{L}; \quad \Delta k_{x,y} = \frac{\pi}{d}$$

Because the excited electrons are the ones close to the Fermi level, the *k* wave vector has to be evaluated at E_F. Assuming a spherical Fermi surface we have

$$N = 2\frac{4/3\,\pi k_F^3}{\frac{\pi}{L}\frac{\pi}{d}\frac{\pi}{d}} \Rightarrow n = \frac{N}{V} = \frac{8}{3}\frac{k_F^3}{\pi^2}$$

giving $k_F = 1.5\ 10^8\ m^{-1}$.
Substituting in the previous formula we obtain:
$\Delta E_{zQD} = 100$ meV and $\Delta E_{xQD} = 5$ meV

b) $\Delta E_{zQW} = 100$ meV and $\Delta E_{xQW} = 0.05$ meV

c) The AlGaAs barrier represent the dielectric material filling the space between the two sides of a plate capacitor (drain contact-GaAs surface). The level spacing in the coulomb blockade regime is given by:

$$\Delta E = \frac{e^2}{C}; \quad C = \varepsilon_0 \varepsilon_r \frac{d^2}{L}$$

$$\Delta E_{QW} = 1.4 10^{-4} meV; \quad \Delta E_{QD} = 1.4 meV$$

d) In order to observe and resolve the energy spectrum the kT << ΔE (kT = 0.1 ΔE), which implies for the energy splitting:

$$T_{zQW,QD} = 230K; \quad T_{xQD} = 12K; \quad T_{xQW} = 120mK$$

and for the coulomb blockade splitting:

$$T_{QD} = 1.6K; \quad T_{xQW} = 0.16 mK$$

N.B: It is now clear that to observe a quantum blockade in the case of a QW is a quite hard job!!!!

d) In order to resolve at room temperature the coulomb blockade spectrum of a QD the following relation has to be fulfilled:

$$d^2 = \frac{e^2 L}{10 \varepsilon_0 \varepsilon_r kT}; \quad or \quad d = 7.5 nm$$

i.e. we have to reduce the QD lateral size