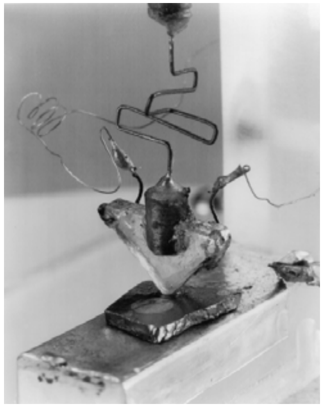
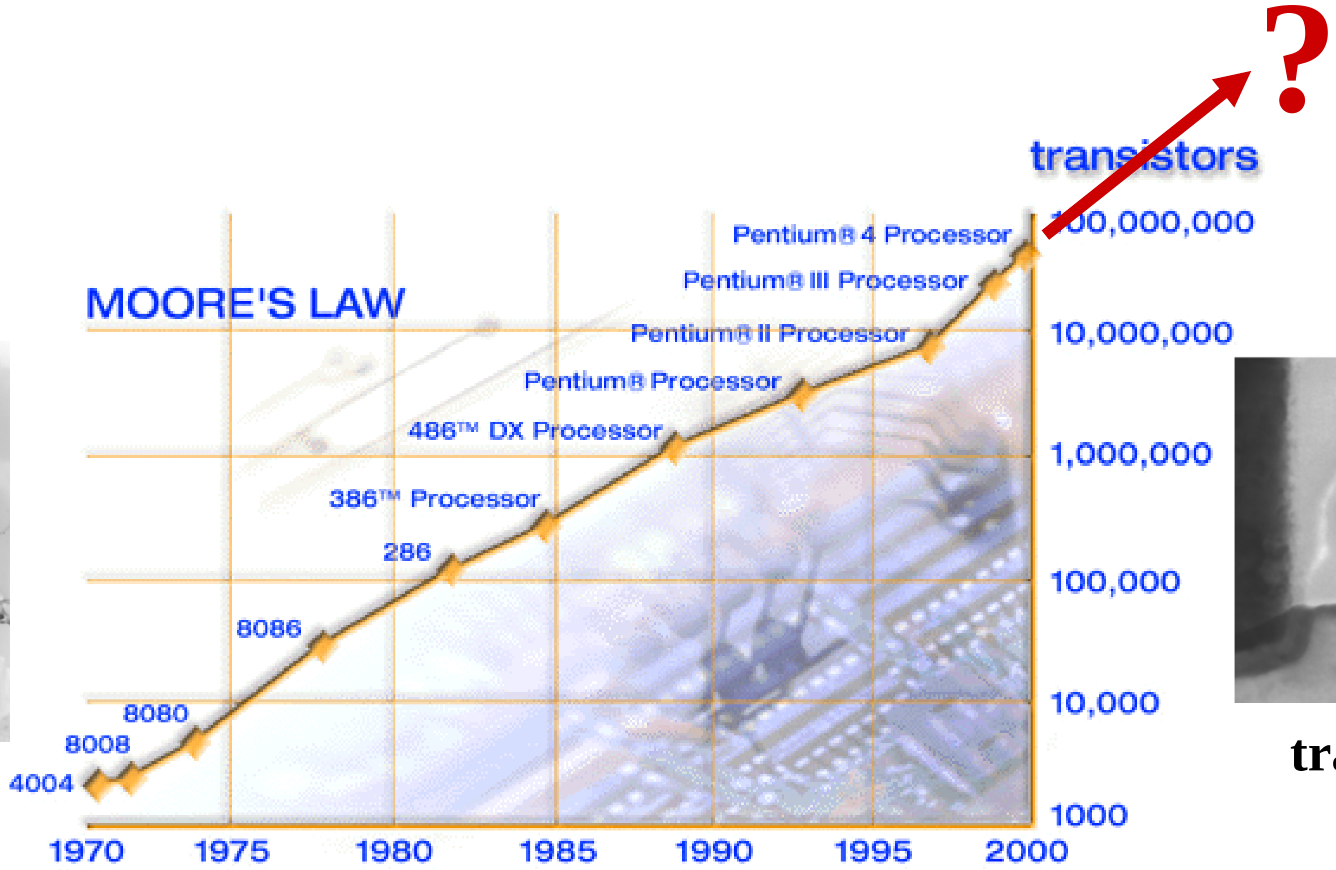
Frontiers in
Nanoscale Science I

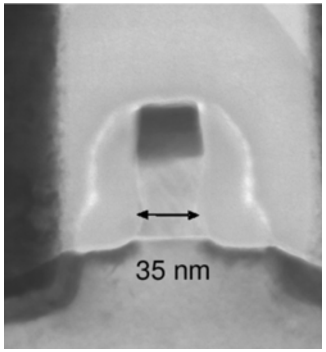
Incredibly Shrinking Transistor
Semiconductor Nanostructures
Quantum Transport
Molecular Electronics

The incredible shrinking of the transistor



transistor
1947

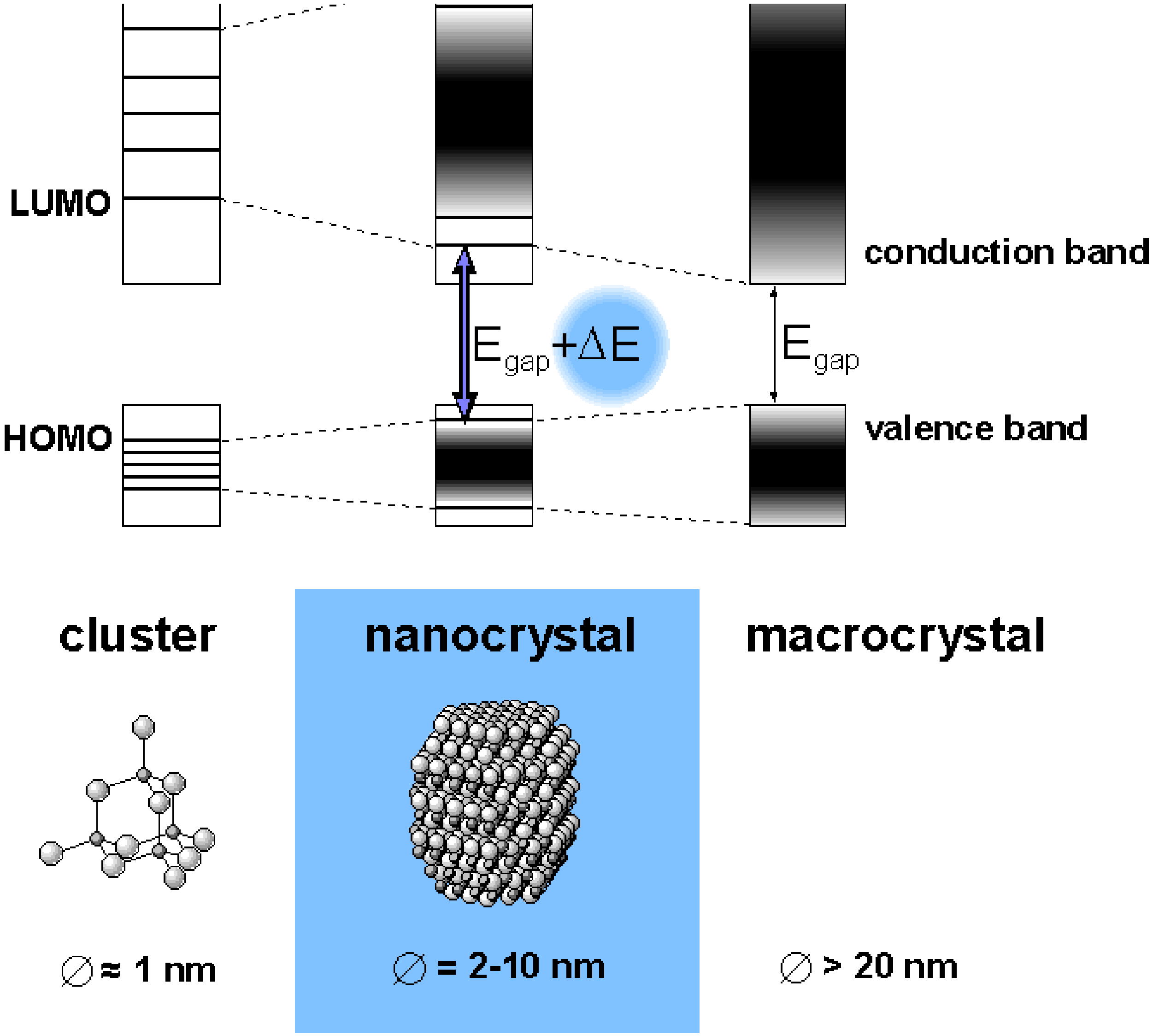




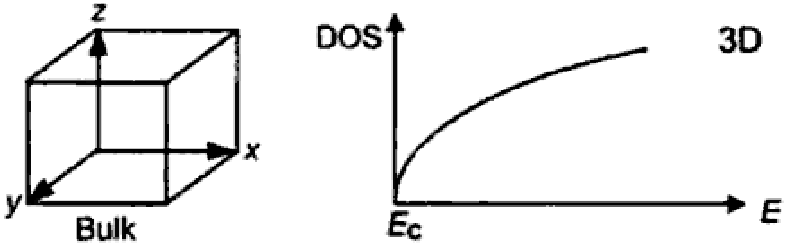
transistor
2005

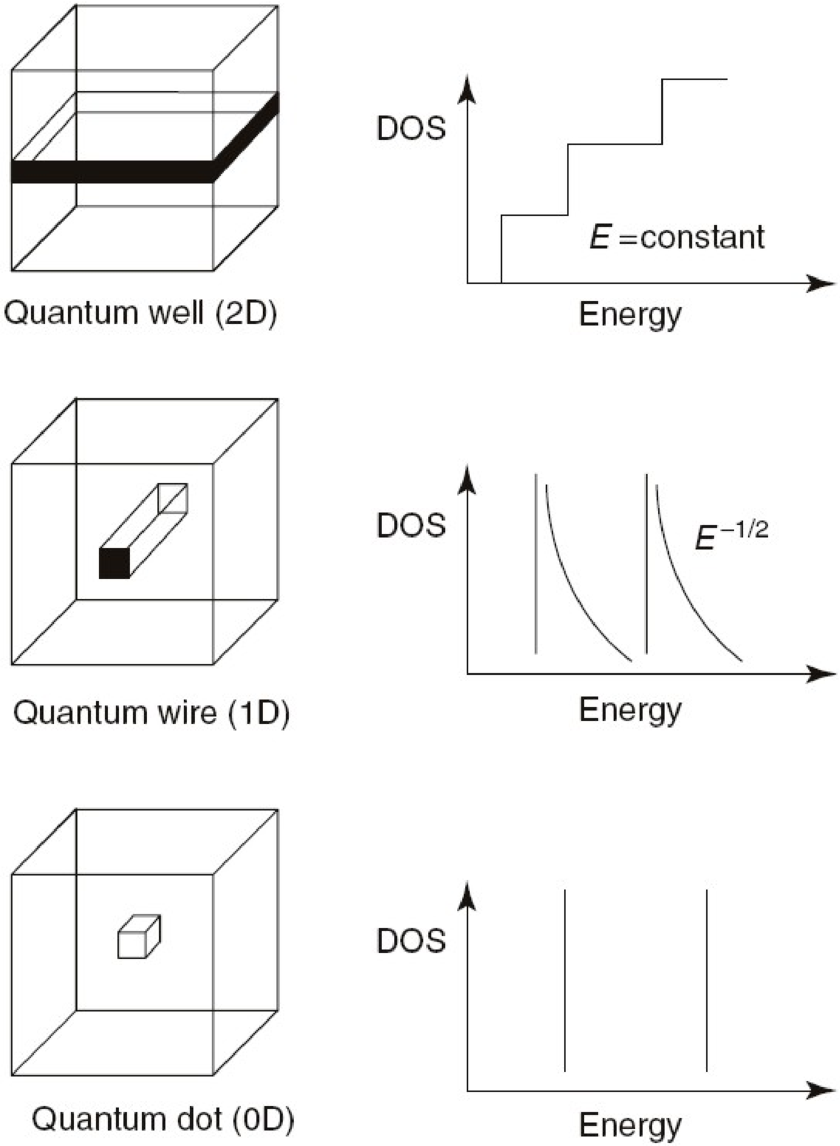
→ **Shrinking of parts of the transistor to atomic dimensions**

From solid state to molecules



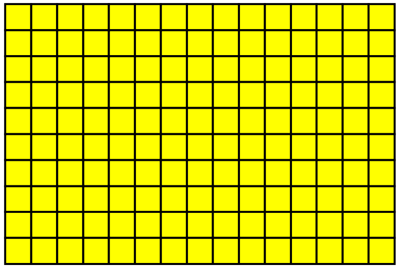
Electronic density of states - Overview

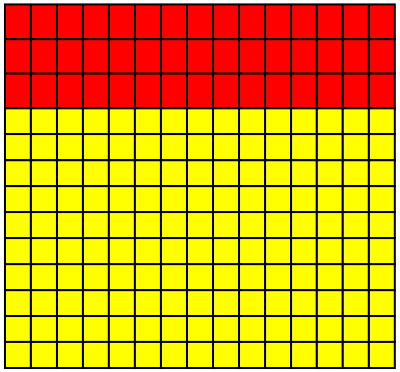


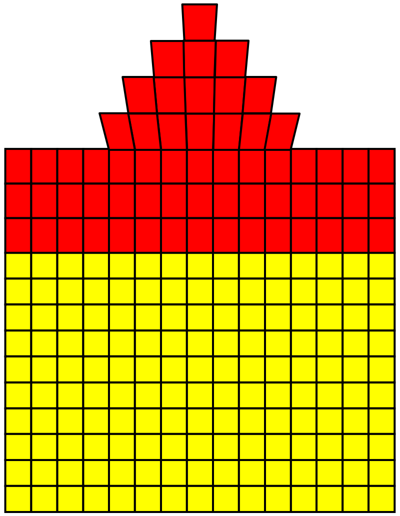


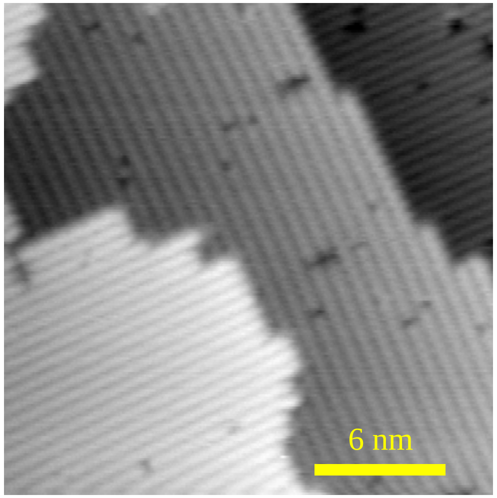
Stranski-Krastanov Growth of Semiconductor QDs

lattice constants: $a_A > a_B$

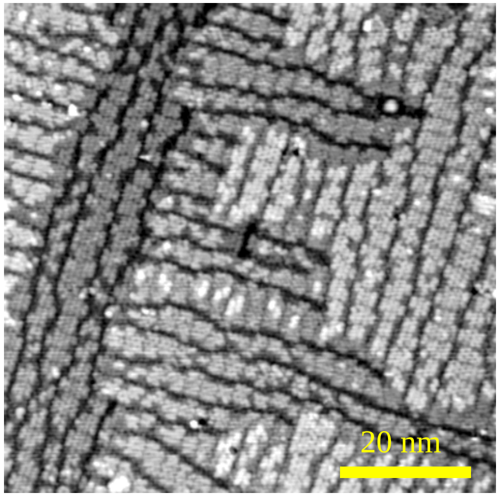

substrate


wetting layer

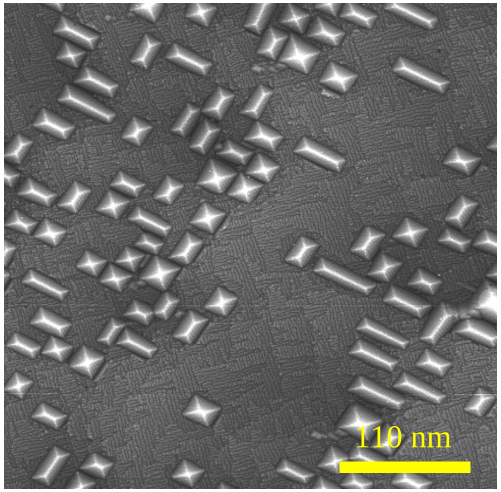

islands



Si(001)



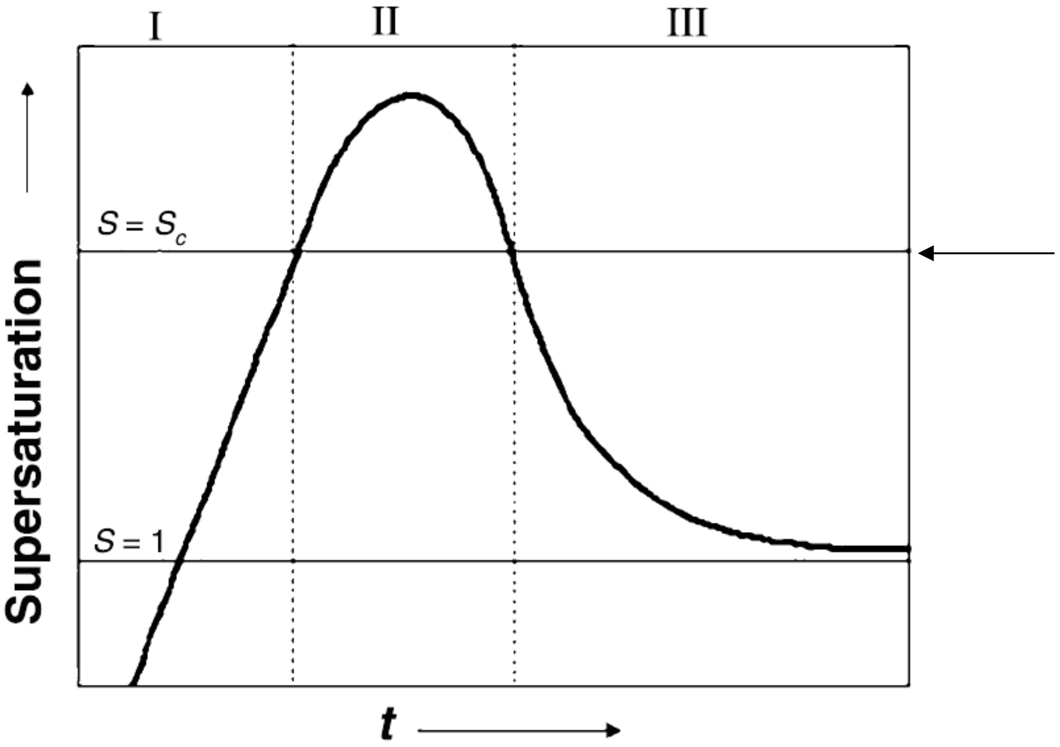
3ML Ge on Si(001)



6ML Ge on Si(001)

Solution-based nanocrystal synthesis

Degree of supersaturation vs. reaction time (LaMer plot):



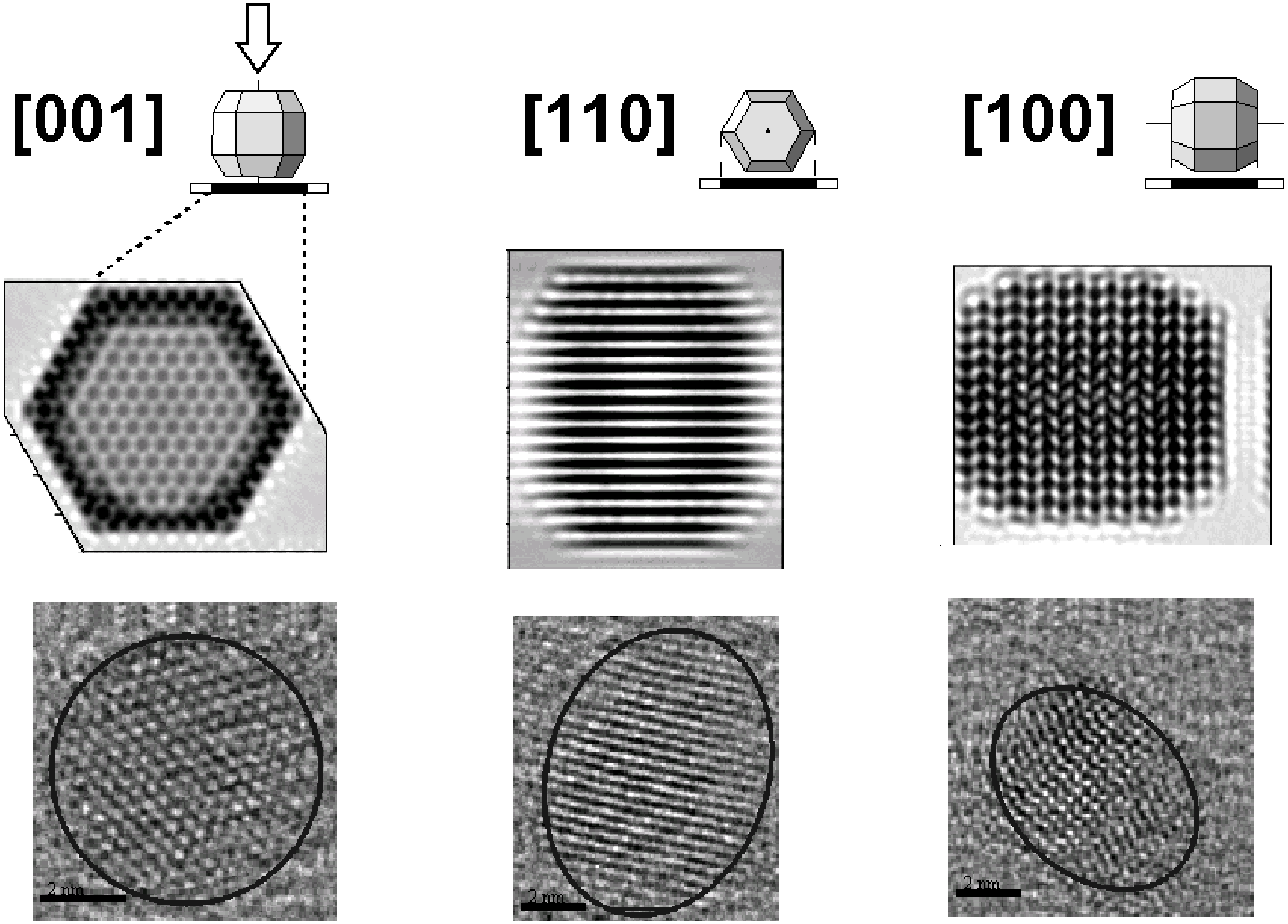
→ homogeneous nucleation requires temporal separation of nucleation and growth of the seeds.

hot injection method
(instantaneous nucleation)

heating-up method
(in situ formation of reactive species)

Microscopic structure

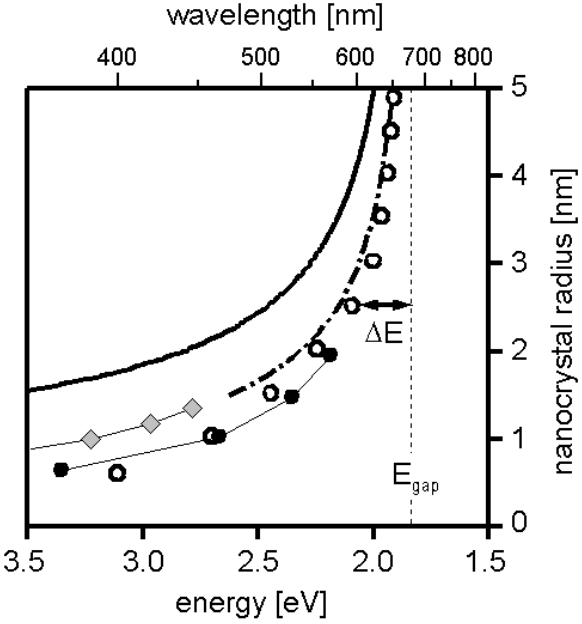
HRTEM images of hexagonal CdSe nanocrystals:

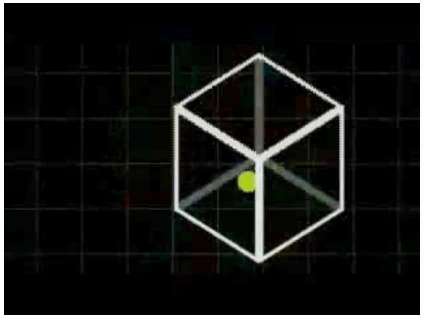


Size Dependence of Optical Absorption

2 nm → 7 nm

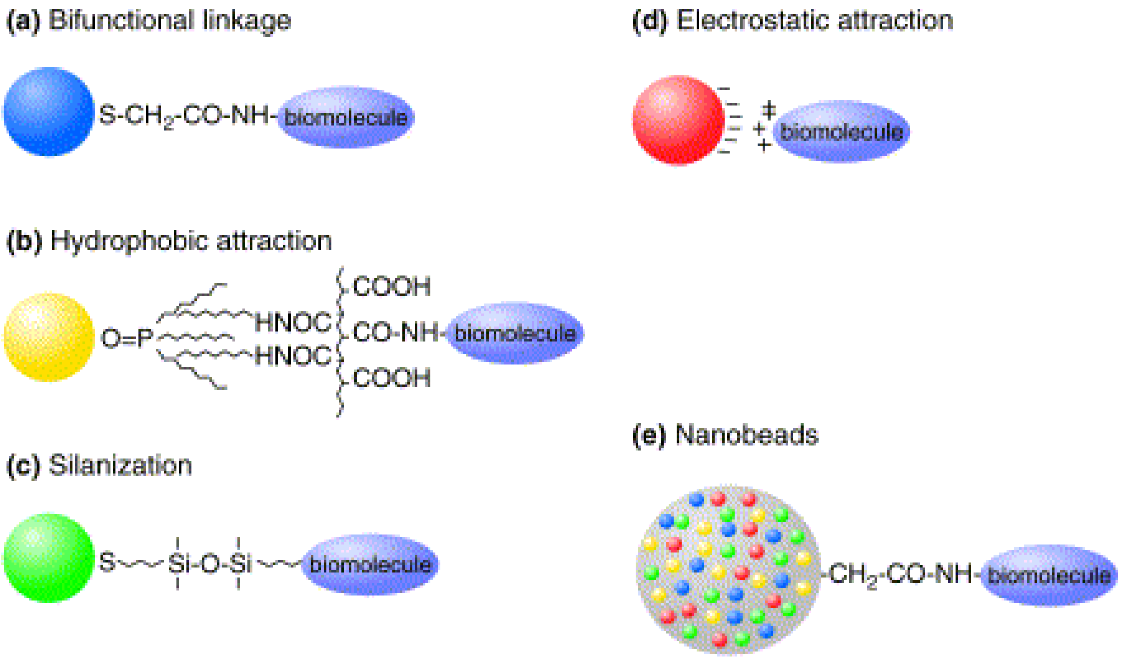
CdSe Nanocrystals



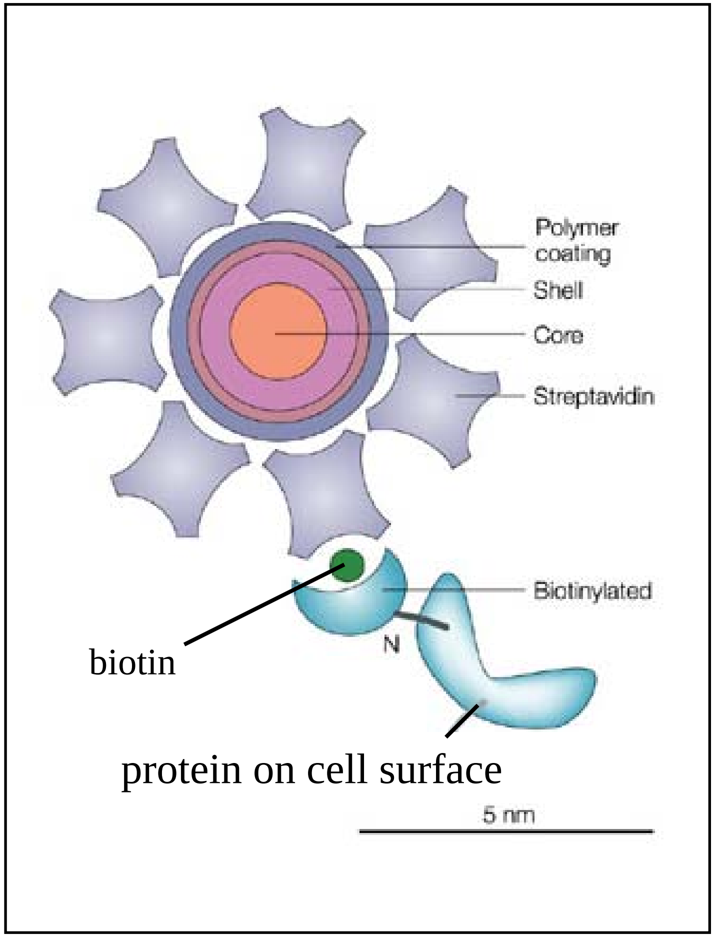


SC nanocrystals as bio-labels

bio-conjugation strategies:



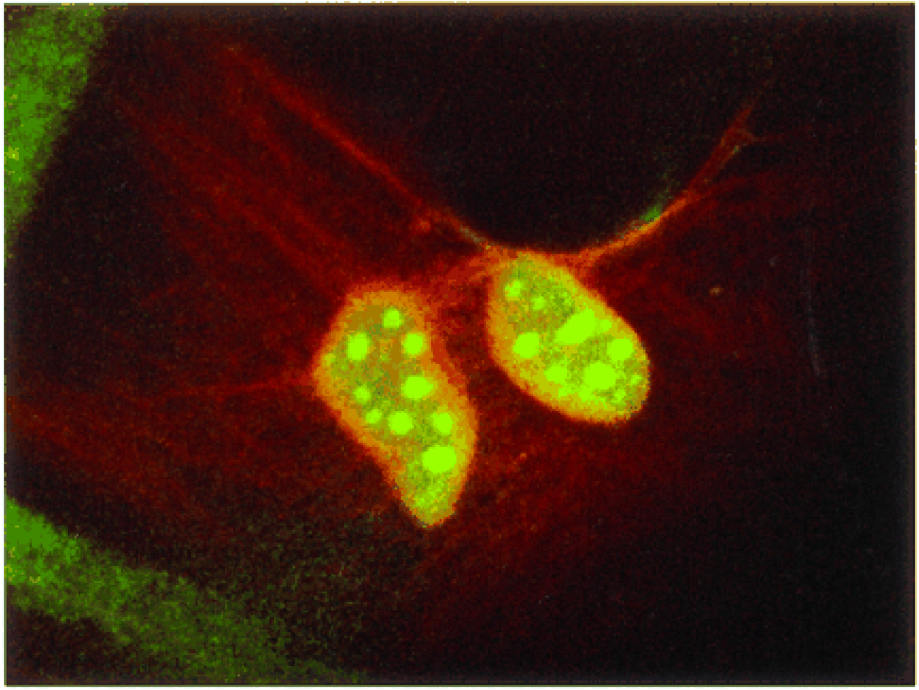
example:



core-shell nanocrystal conjugated to streptavidin

A. Miyawaki et al., Nature Cell Biology 5, S1-S7 (2003).

SC nanocrystals as bio-labels



mouse fibroplast cells stained with CdSe-CdS core-shell particles of different size (2 nm core particles: green emission;
4 nm core particles: red emission)

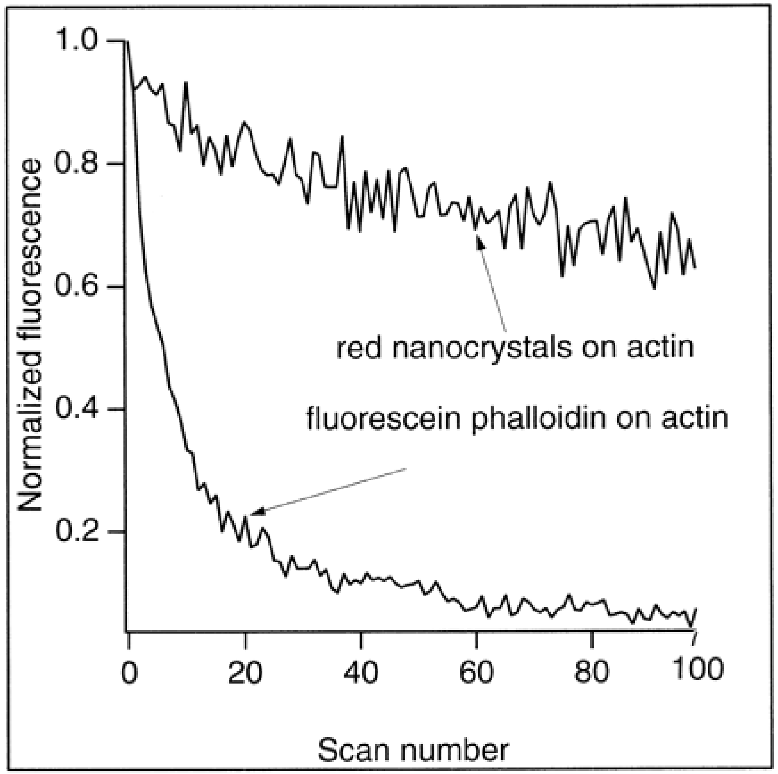
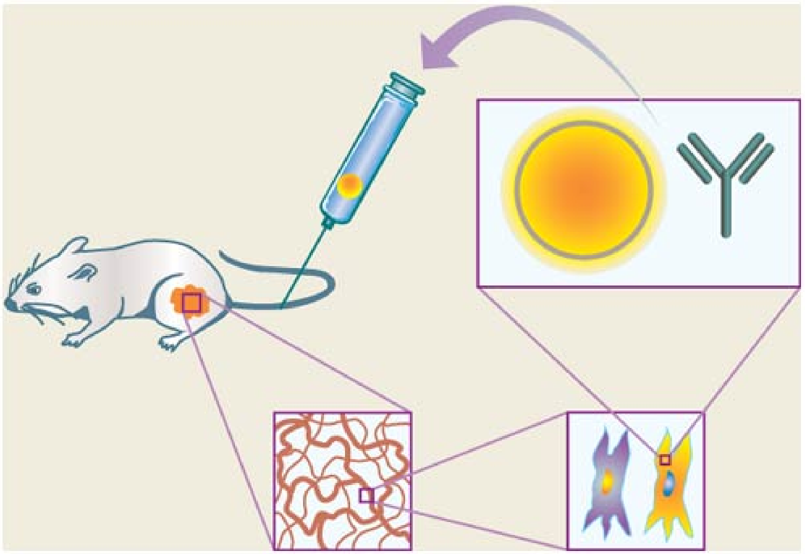
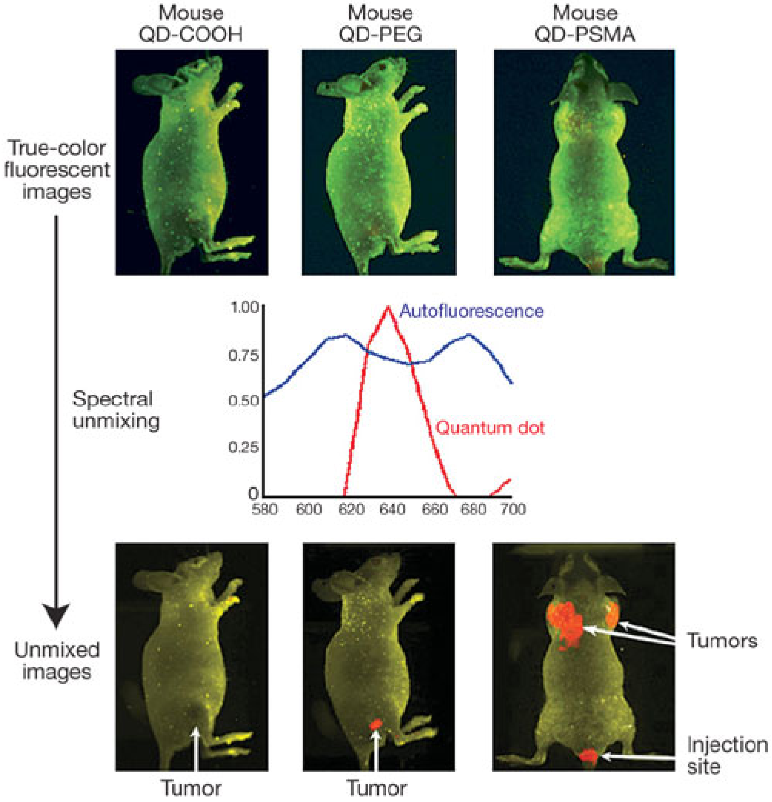


photo-stability comparison for actin fibres stained with fluorescent dyes or
core-shell nanocrystals

M. Bruchez et al., Science 281, 2013 (1998).

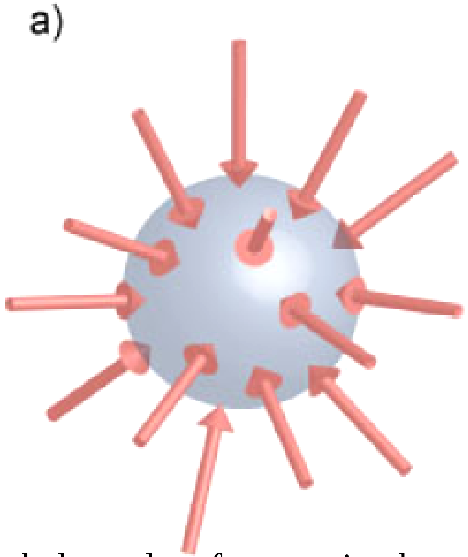
In-Vivo Cancer Imaging with QD´s



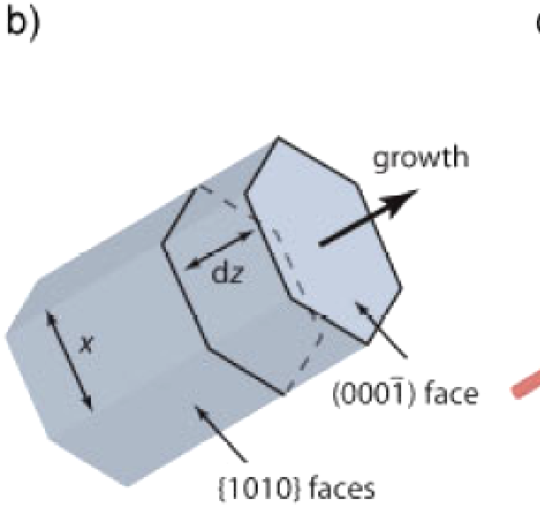


X. Gao et al.,Nature Biotechnology 22,969(2004)

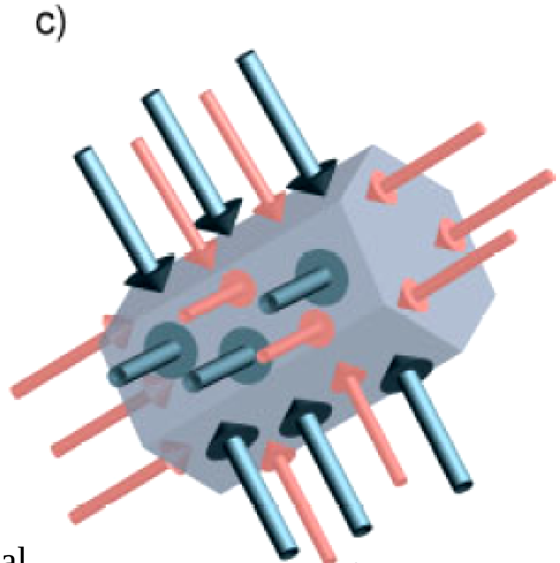
Shape control of NCs



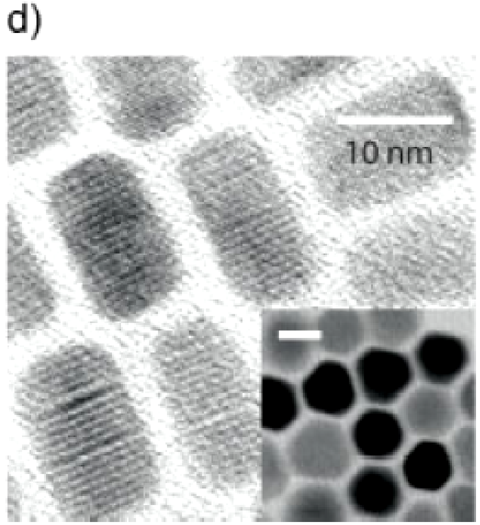
balanced surface tension by uniform ligand coverage



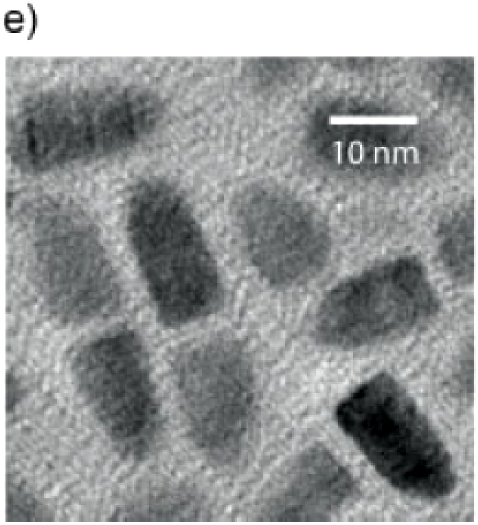
growth of a hexagonal nanocrystal along the c-axis



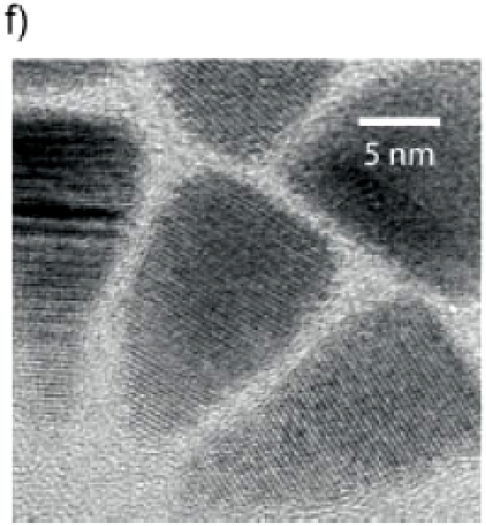
stabilization of rod-shape via non-uniform ligand coverage



CdSe nanorods

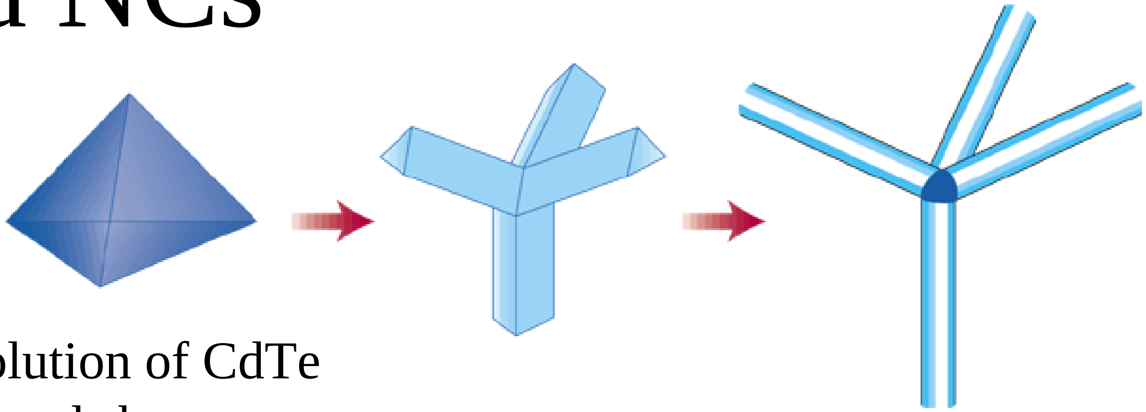


CdSe nanobullets

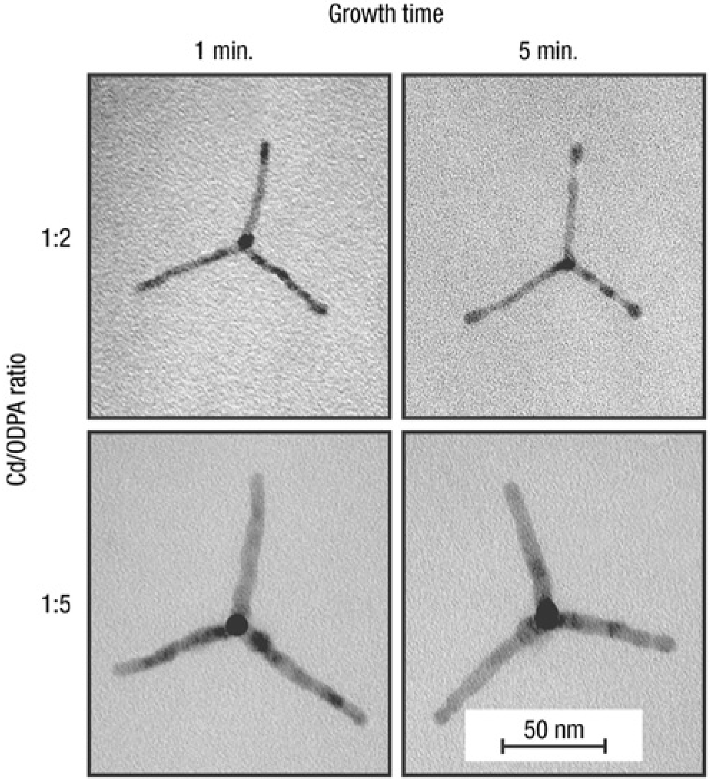


CdSe nanopyramids

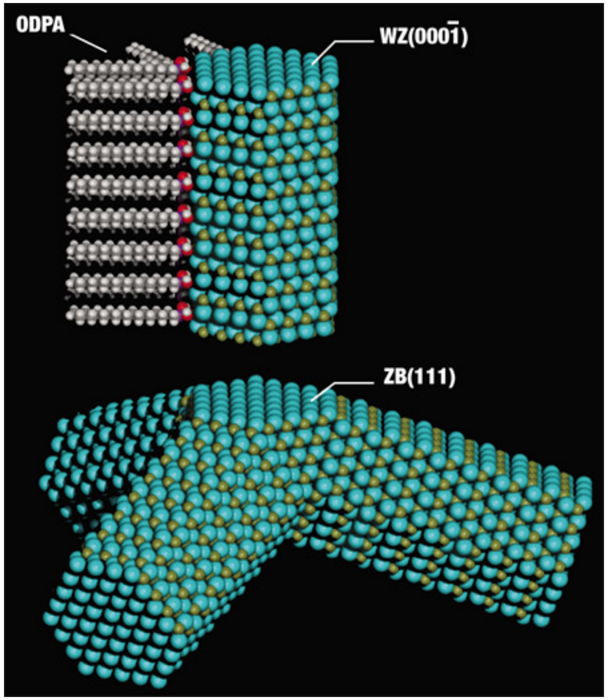
Branched NCs



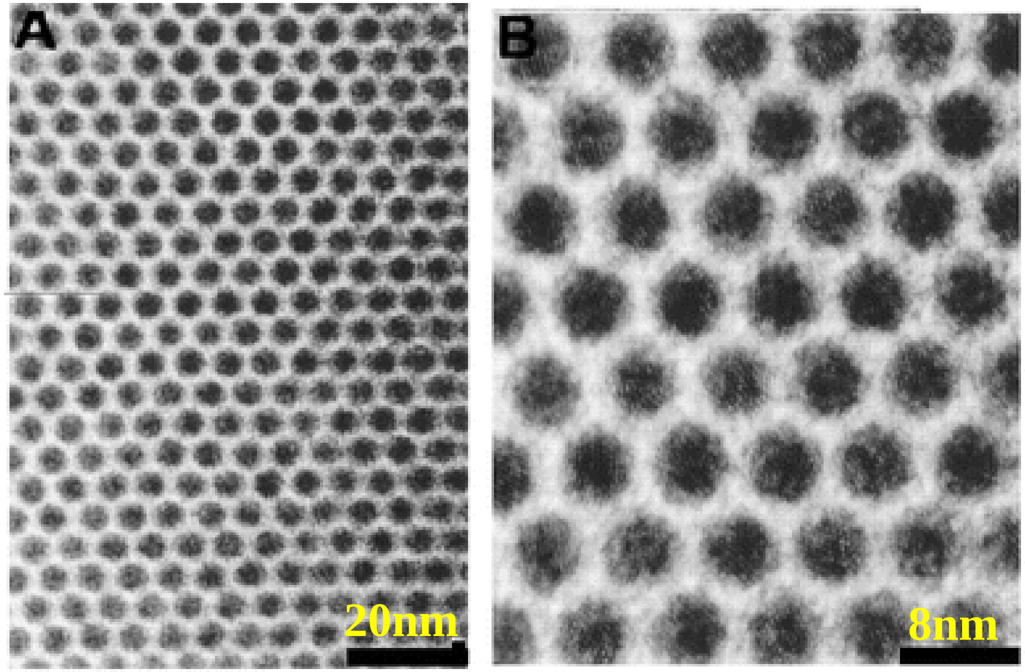
time evolution of CdTe tetrapod shape:



model of a CdTe tetrapod:



Ordered arrays of semiconductor NCs



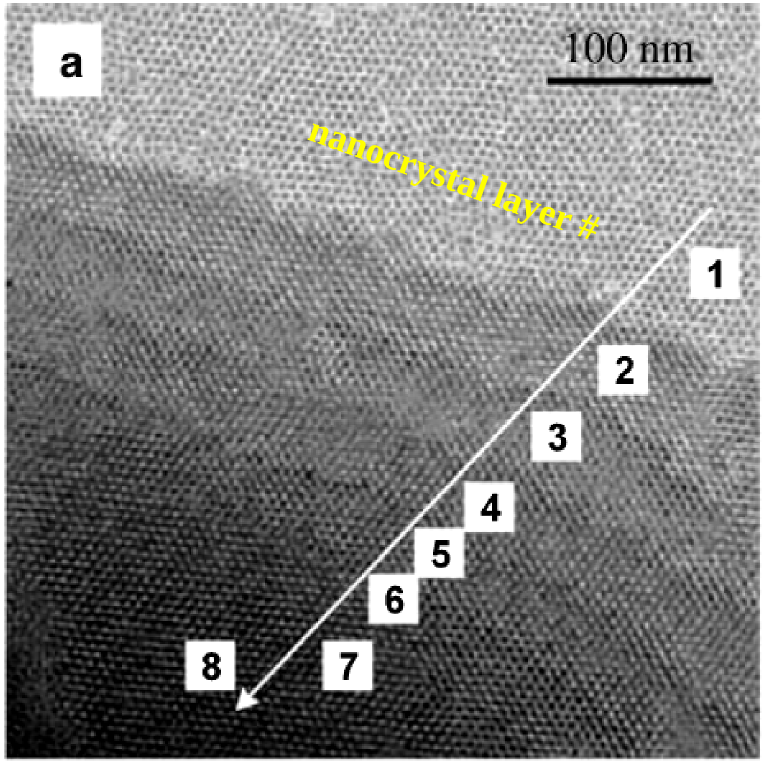
2D-array of CdSe nanocrystals
(d = 4.8nm) on carbon grid

C.B. Murray et al., Ann. Rev. Mater. Sci. 30, 545 (2000).

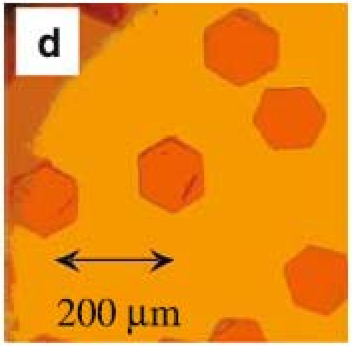
Semiconductor NC 3D-superstructures

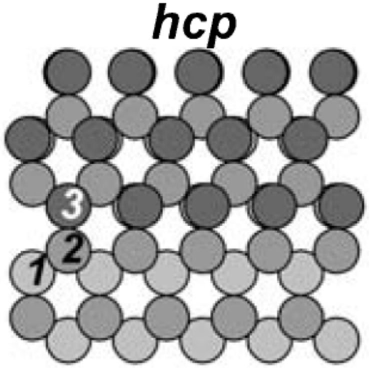
A *hcp* superlattice self-assembled from CdSe NCs:

[111] projection



optical micrograph

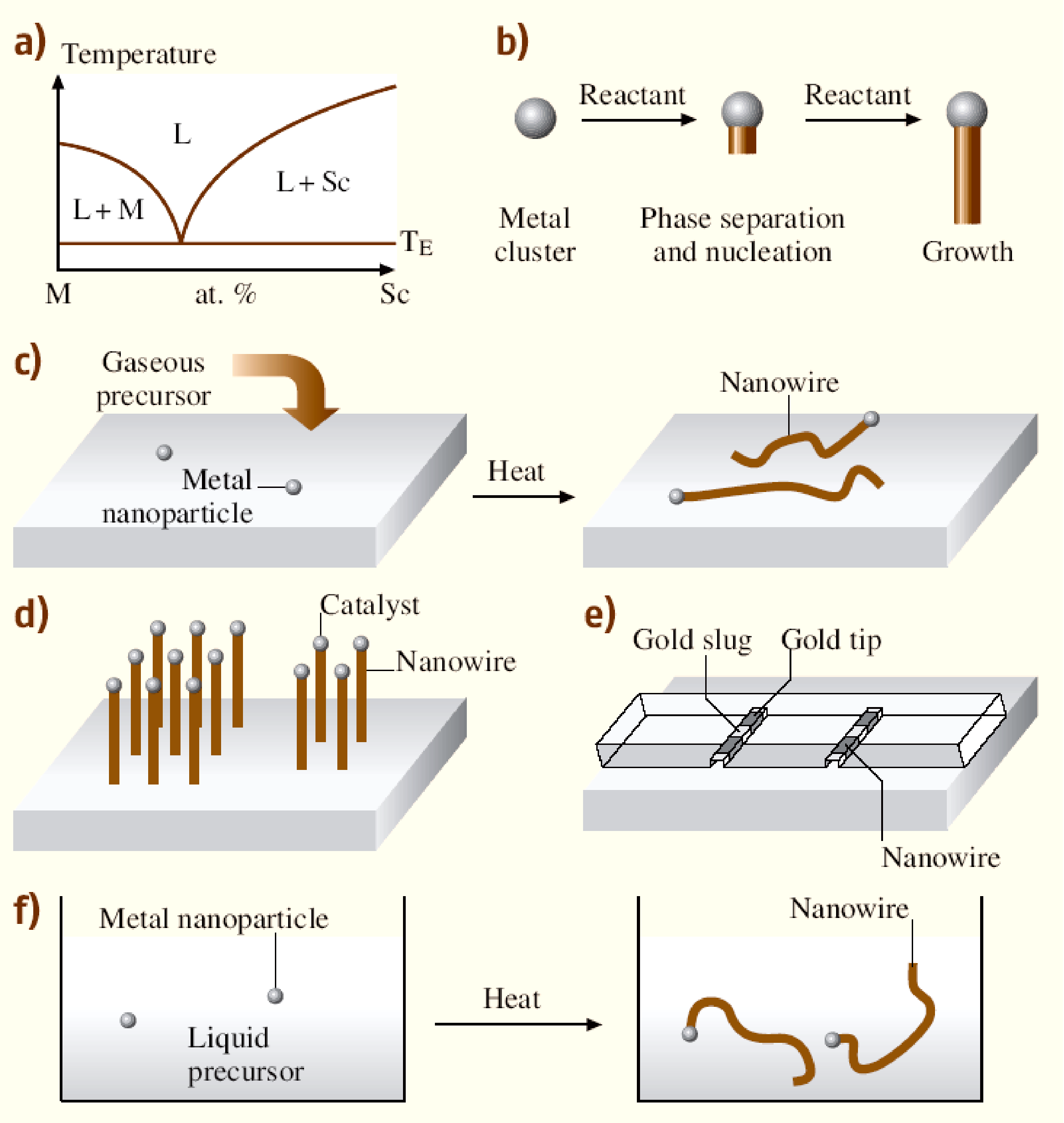




Semiconductor Nanostructures

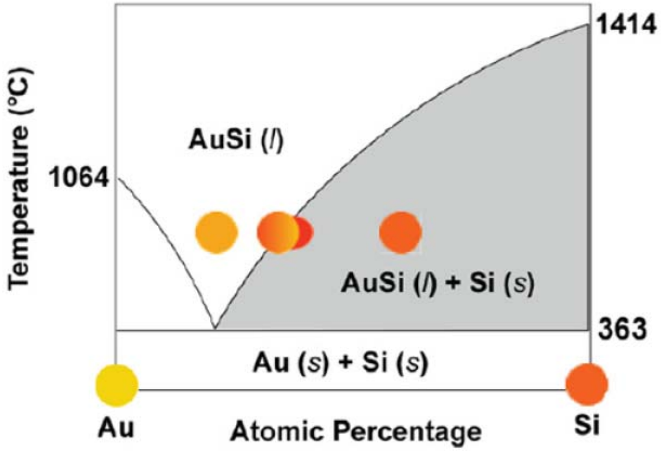
- basics
- MBE quantum dots
- nanocrystals
- nanowires

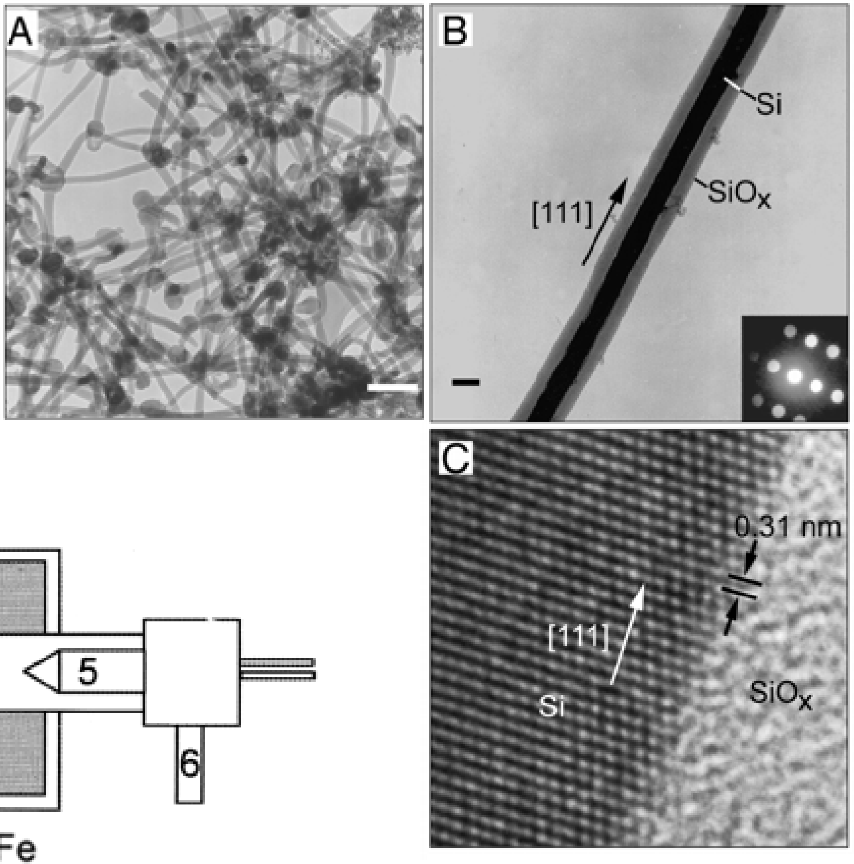
Synthesis methods - Overview

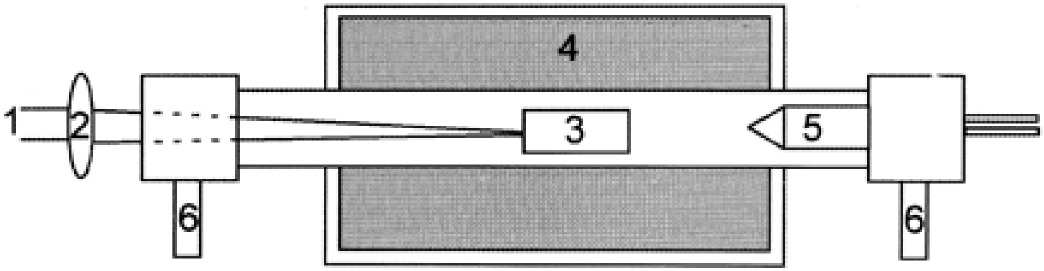


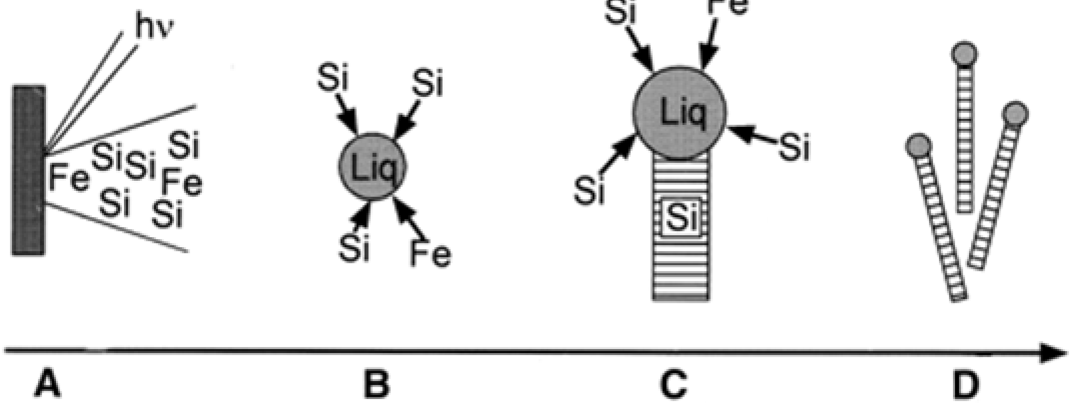
Vapor-liquid-solid growth mechanism

Si nanowire synthesis:

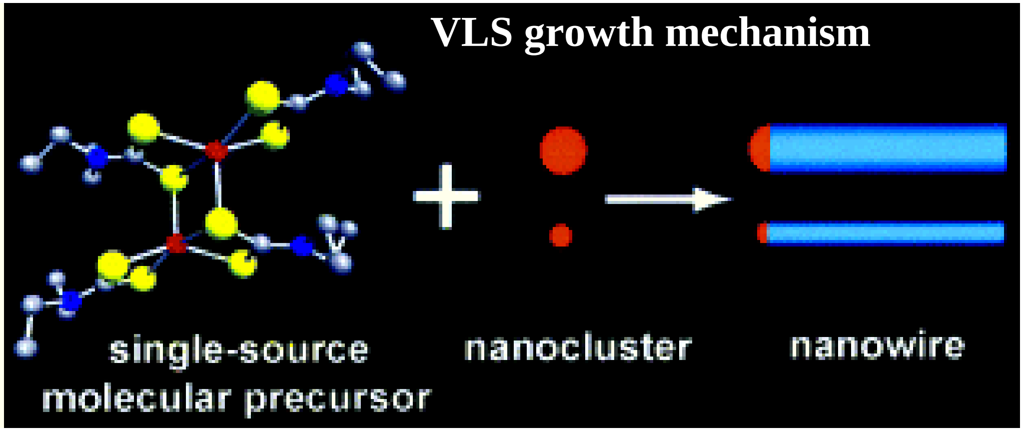




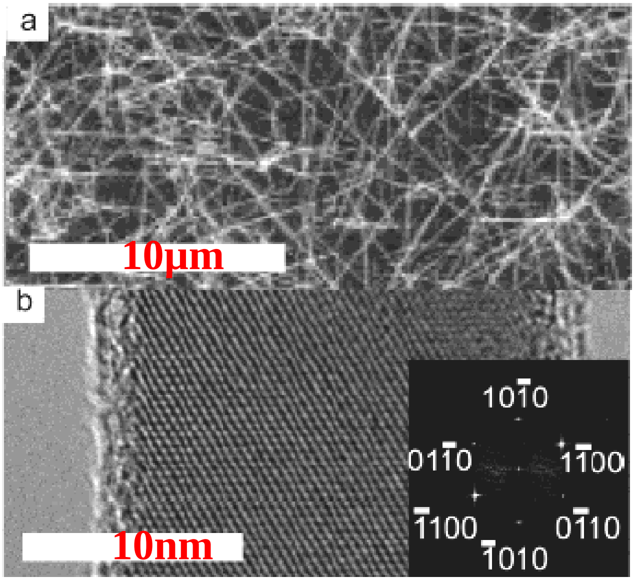




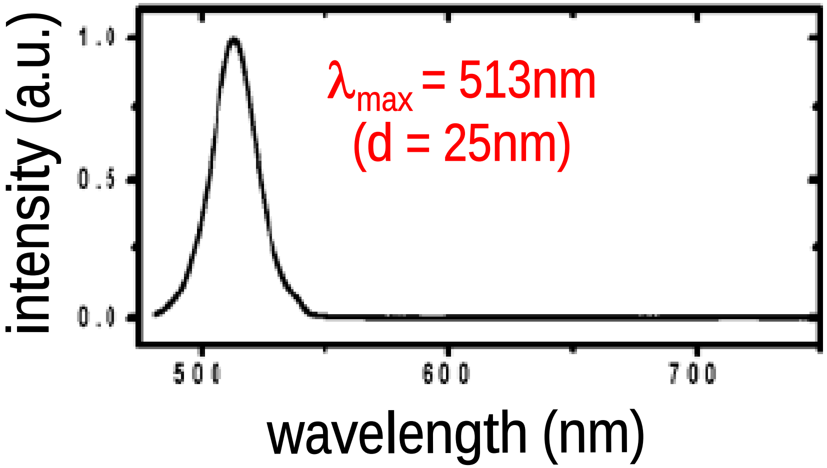
CdS NWs from molecular precursor



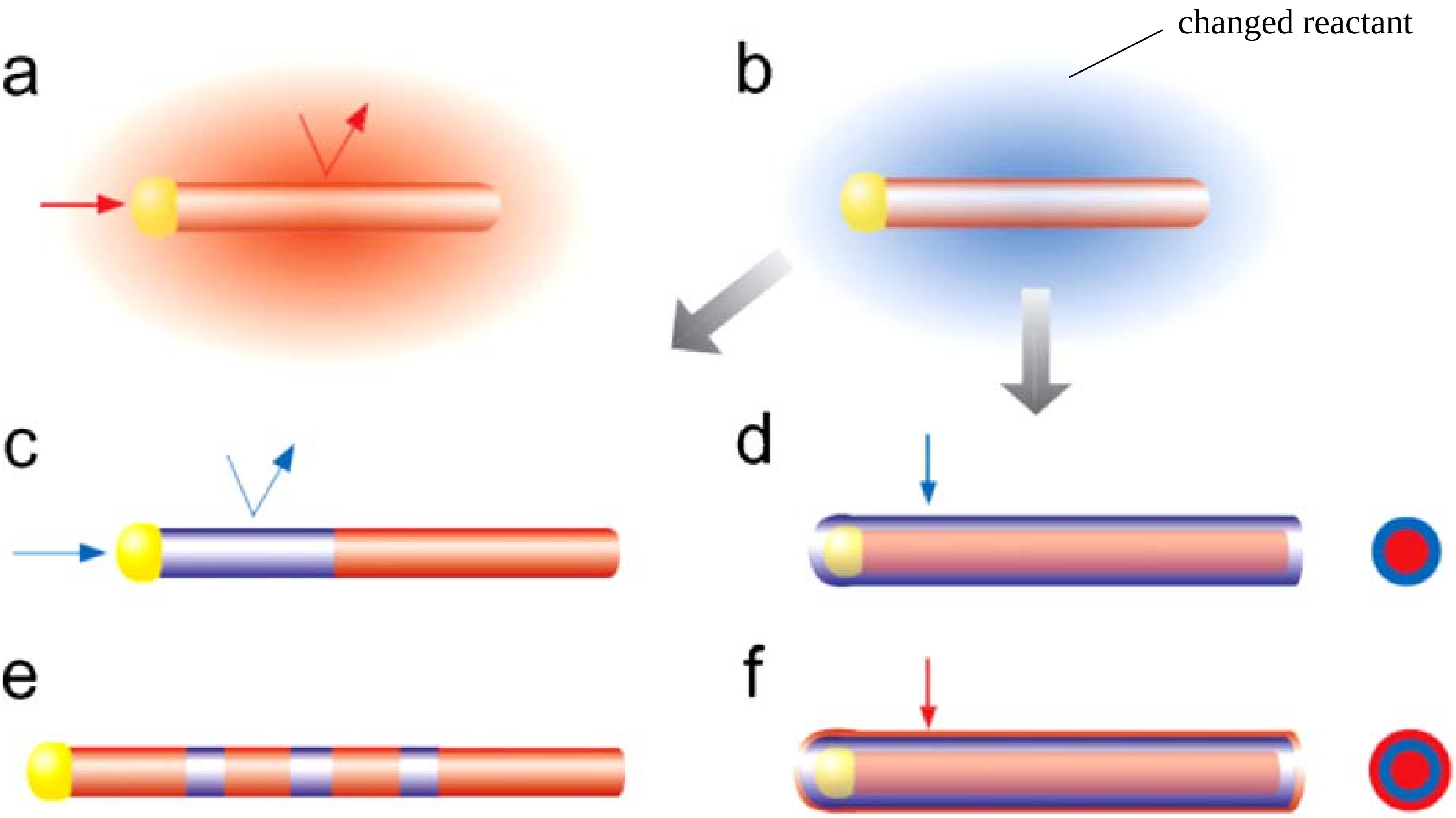
$Cd(S_2CNEt_2)$



photoluminescence (RT)



Nanowire heterostructure synthesis



changed reactant

axial heterostructure growth

radial heterostructure growth

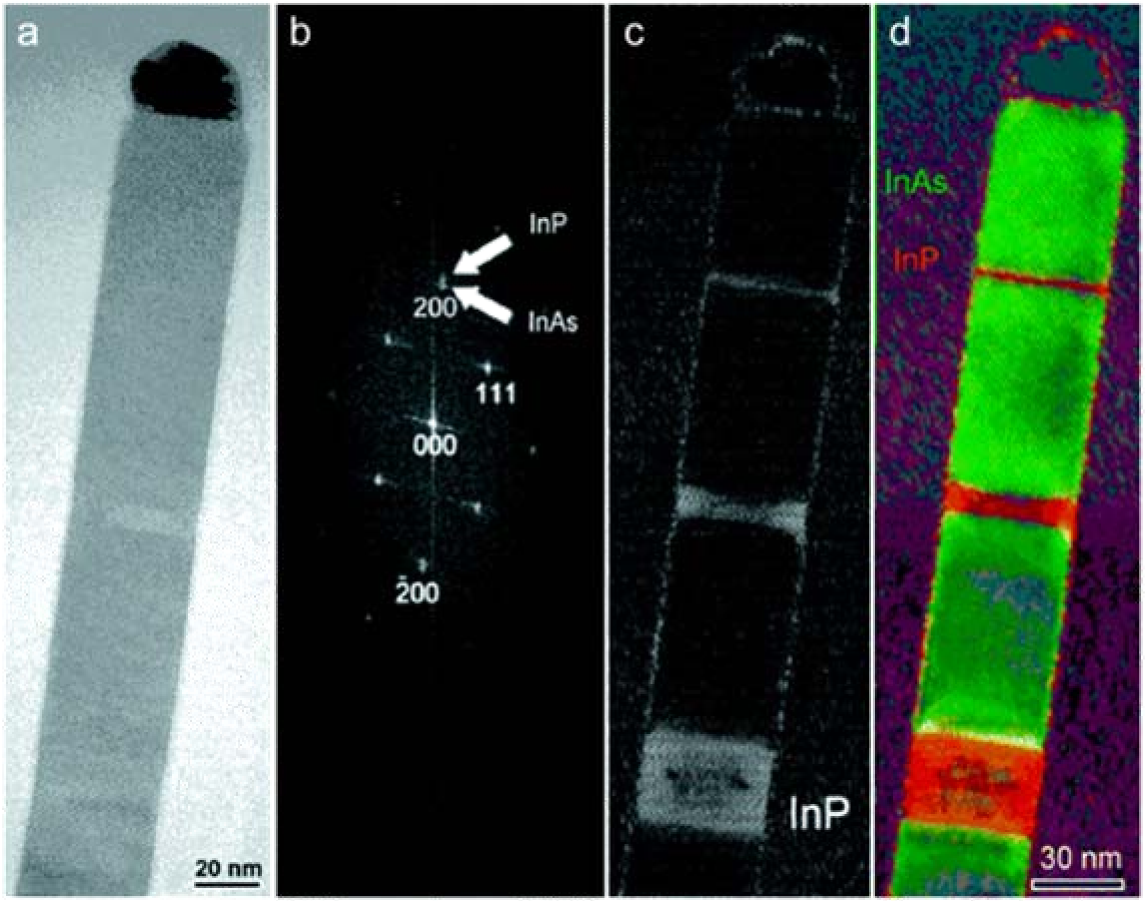
InAs/InP segmented nano-whiskers

TEM

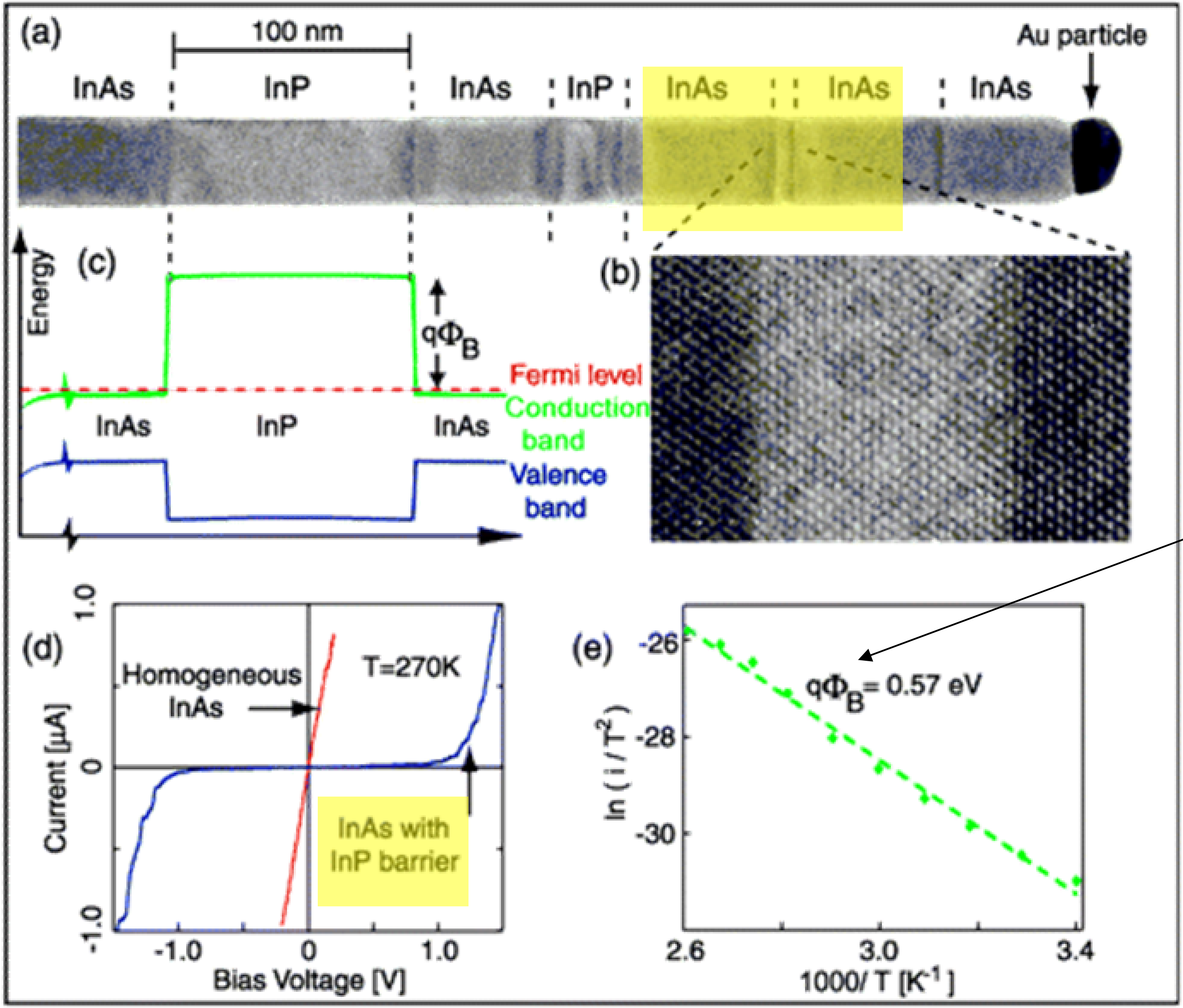
TED

Inverse FT

Superimposed, colour-coded images



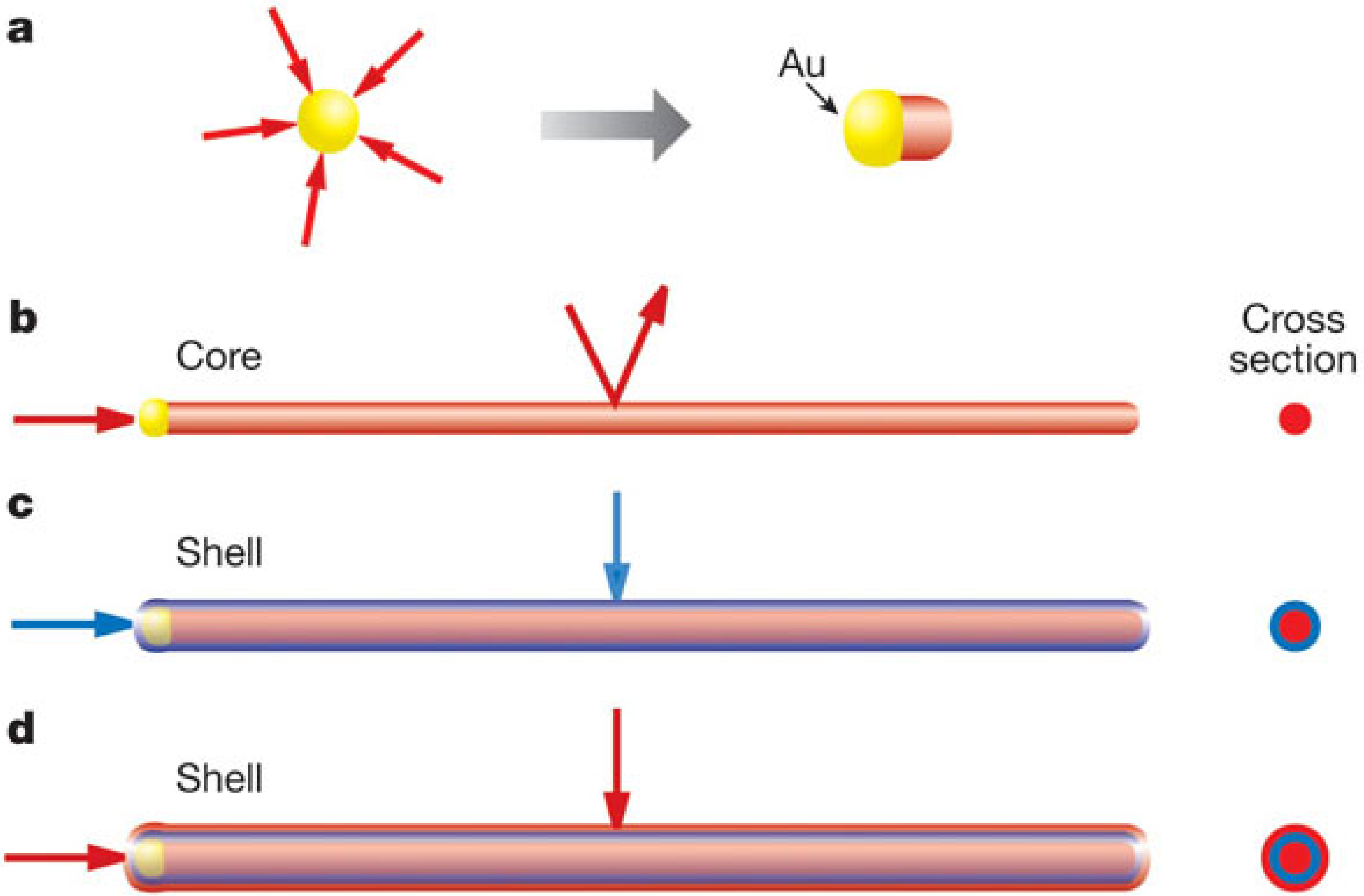
Charge transport over InP barrier



effective barrier height (thermionic excitation)

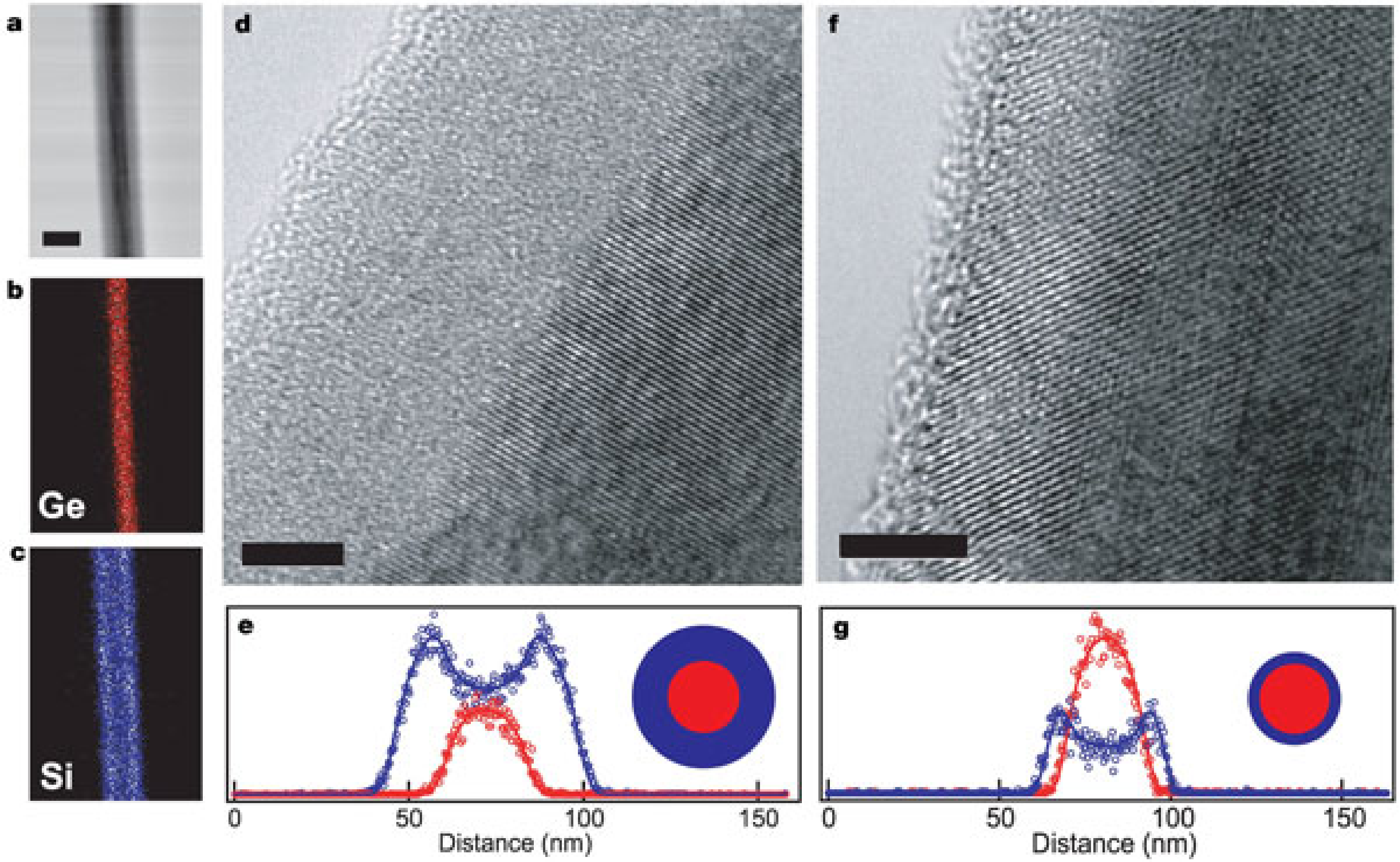
Core-shell semiconductor nanowires

synthesis by chemical vapour deposition:



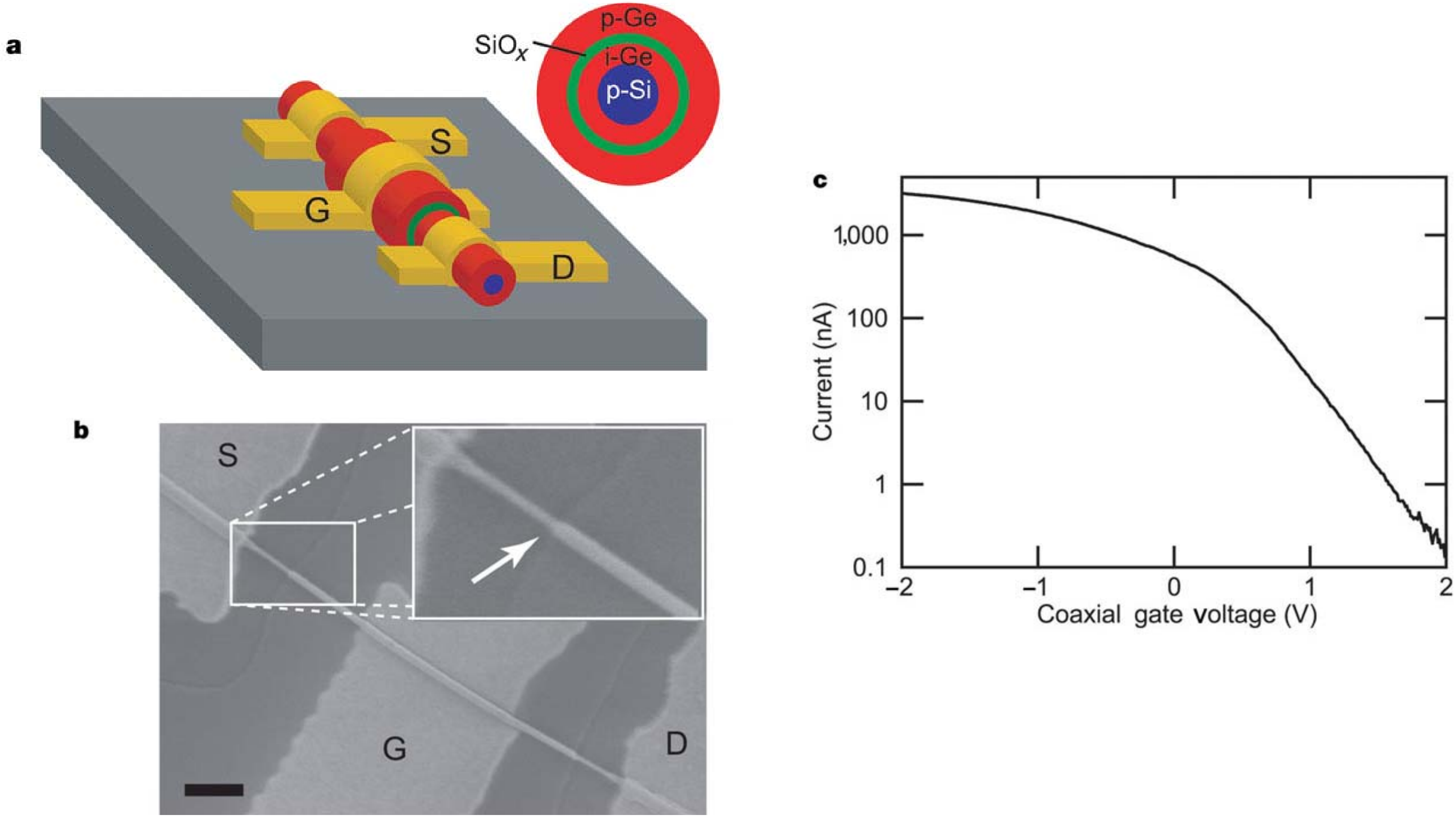
Core-shell semiconductor nanowires

TEM/EDX analysis of Ge-Si core-shell nanowires:



Core-shell semiconductor nanowires

coaxially gated nanowire field-effect transistor from
Ge-Si core-shell nanowires :



Electrons confined to nm dimensions

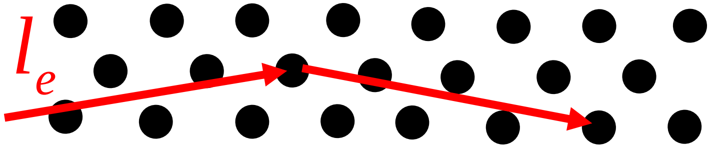
- can the electrons still be considered to be independent ?
- can the electrons still move in all three dimensions ?

Quantum Transport

- Transport through a 1D wire
- Coulomb blockade
- Single electron transistor
- Quantum Hall effect

Transport through a one dimensional wire (1)

Conductivity of a macroscopic sample



$$\sigma = \frac{ne^2}{m^*}\frac{l_e}{v_F}$$

Boltzman model

n : charge carrier density

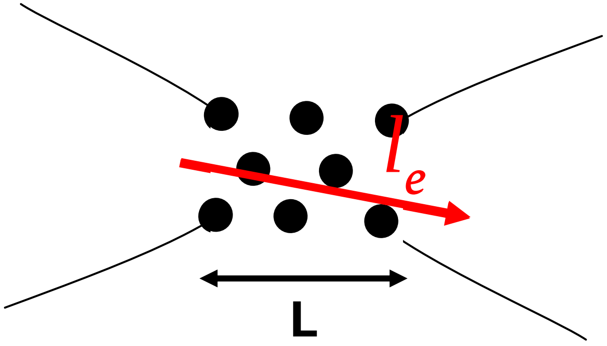
m^* : effective electron mass

l_e : mean free path

v_F : Fermi velocity

Example: for copper l_e is about 30 nm

Conductivity of a nanoscopic sample



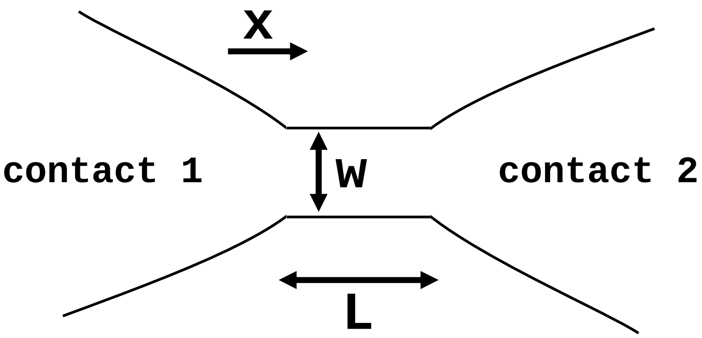
Drude model breaks down for

l_e > L

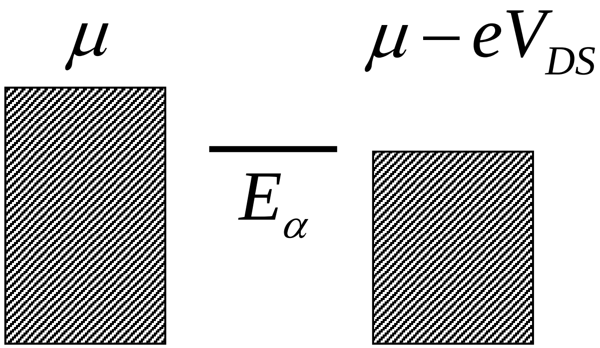
“ballistic conductance”

→ **infinite conductance ?**

Landauer formula (1)



L > l_e **and** λ_f **> w**



1D wire, one channel with the energy E_α

Energy of an electron

$$E = E_\alpha + \frac{\hbar k_x^2}{2m^*} \qquad \textbf{(1)}$$

its longitudinal velocity

$$v_\alpha(E) = \frac{\hbar k_x}{m^*} \qquad \textbf{(2)}$$

with **(1)** it follows

$$v_\alpha(E) = \sqrt{\frac{2}{m^*}(E - E_\alpha)} \qquad \textbf{(3)}$$

Landauer formula (2)

Current through one channel α

transmission probability

$$I_\alpha = e\cdot T_\alpha \int_{-\infty}^{\infty} D_\alpha(E)\cdot v_\alpha(E)[f(E-\mu)-f(E-(\mu-eV_{DS}))]dE$$

1D density of states

$$D_\alpha(E) = \frac{1}{2\pi}\frac{dk_\alpha}{dE}$$

From **(1)** it follows

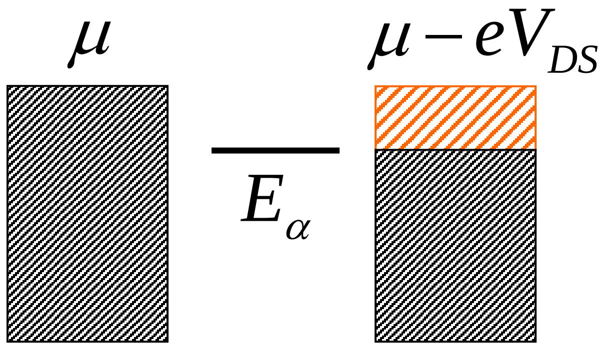
$$k_\alpha(E) = \frac{1}{\hbar}\sqrt{2m^*(E-E_\alpha)}$$

$$\longrightarrow D_\alpha(E) = \frac{1}{2\hbar\pi}\sqrt{\frac{m^*}{2(E-E_\alpha)}}$$

velocity of the electrons

$$v_\alpha(E) = \sqrt{\frac{2}{m^*}(E-E_\alpha)}$$

part of the Fermi distribution that participates in the transport



Landauer formula (3)

$$I_\alpha = e \cdot T_\alpha \int_{-\infty}^{\infty} \cancel{D_\alpha(E)} \cdot \cancel{v_\alpha(E)} [f(E-\mu) - f(E-\mu+eV_{DS})] dE$$

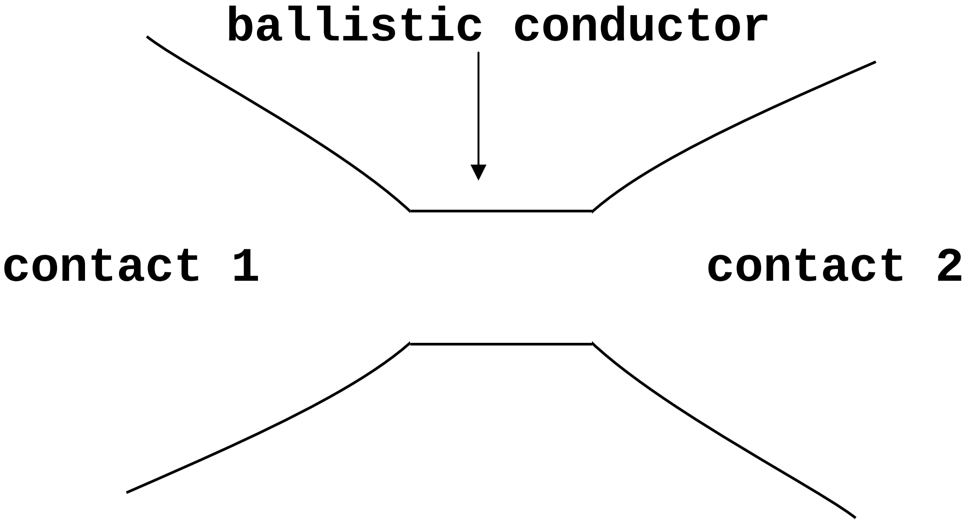
$$\sim \sqrt{\frac{1}{(E-E_\alpha)}} \qquad \sim \sqrt{(E-E_\alpha)}$$

with $\dfrac{df(E-\mu)}{d(-eV_{DS})} = \dfrac{f(E-\mu+eV_{DS}) - f(E-\mu)}{-eV_{DS}}$ **it follows**

$$\rightarrow \quad I_\alpha = \frac{e^2 \cdot V_{DS} \cdot T_\alpha}{h} \overbrace{\int_{-\infty}^{\infty} (-f'(E-\mu)) dE}^{=1} = \frac{e^2 V_{DS}}{h} T_\alpha$$

$$\boxed{\textbf{Conductance quantum} \quad G = \underbrace{2}_{\textbf{Spin}} \cdot \frac{e^2}{h} T_\alpha}$$

Interpretation of the conductance quantum



ballistic conductor

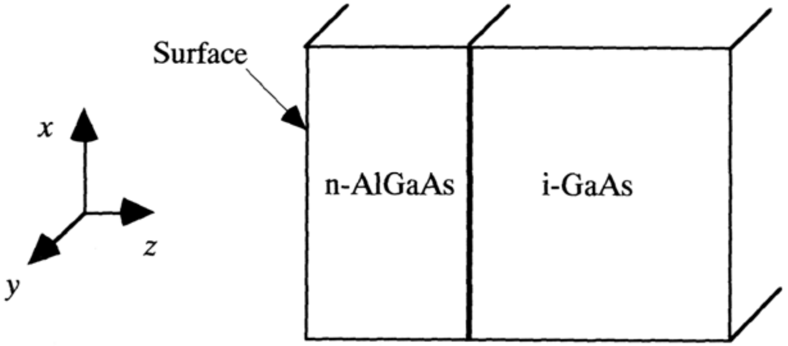
contact 1

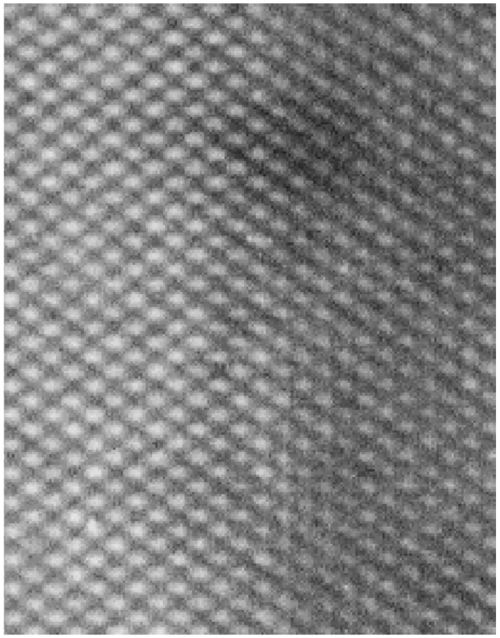
contact 2

$$G = \frac{e^2}{h}$$

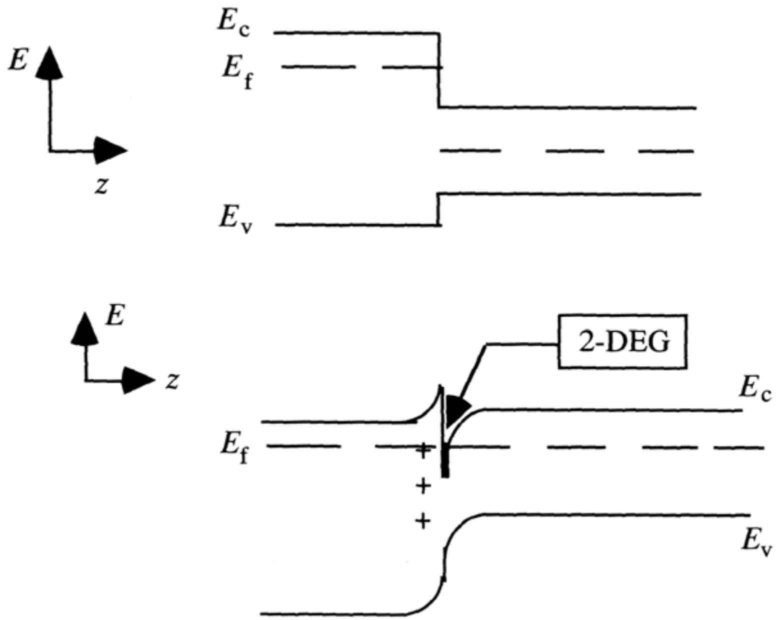
Conductance quantum can be rationalized as the **boundary resistance** between a perfect conductor and the contacts: The charge carriers have to scatter into the ballistic conductor from the contacts.

Experimental realization





TEM image

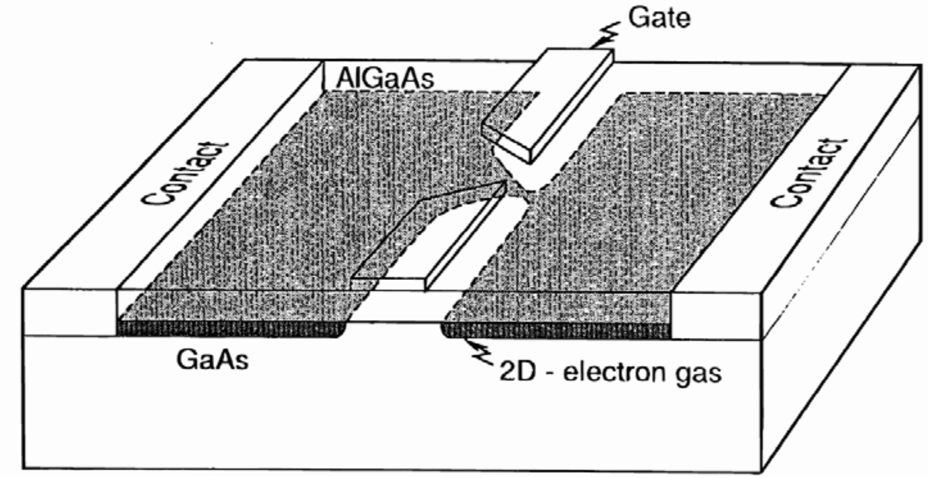


Band diagram before and after contact

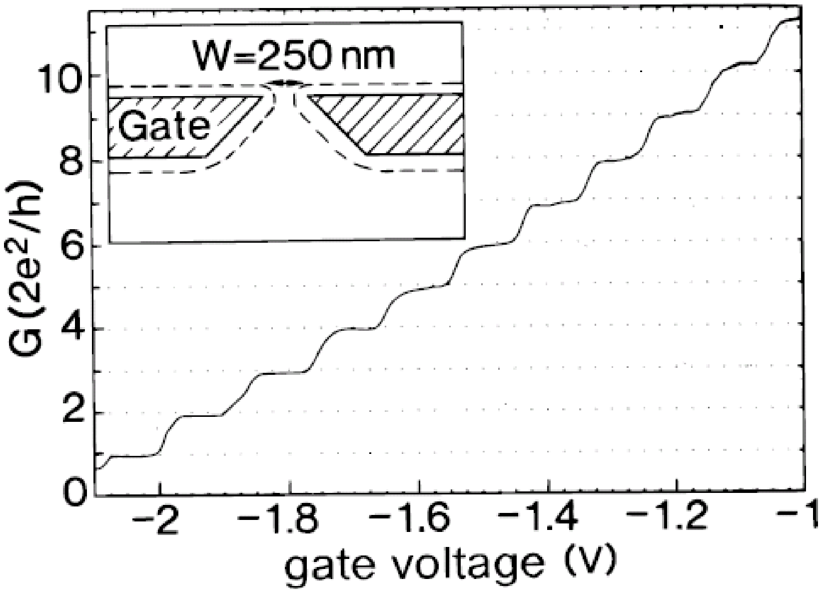
The 2-dimensional electron gas (2DEG) is formed at at the n-AlGaAs / GaAs interface

S. Datta *Electronic Transport in Mesoscopic Systems*

Experimental realization: Quantum point contact



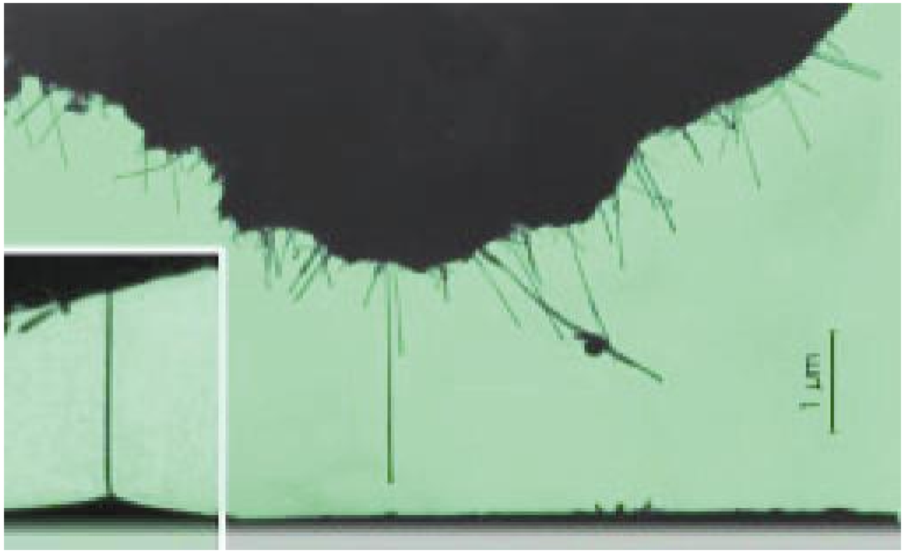
H. van Houten, *cond-mat/0512609*

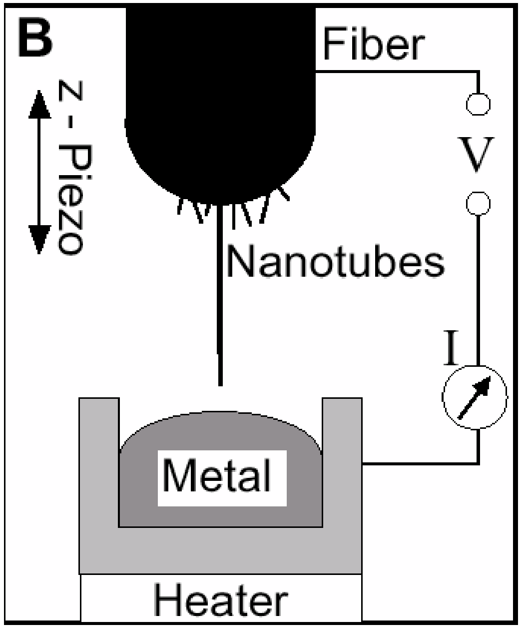


B.J. van Wees, *Phys. Rev. Lett.* **60** (1988) 848

By varying gate voltage the width of the constriction and with it the number of channels contributing to the conductance is changed

CNT – Quantum conductance



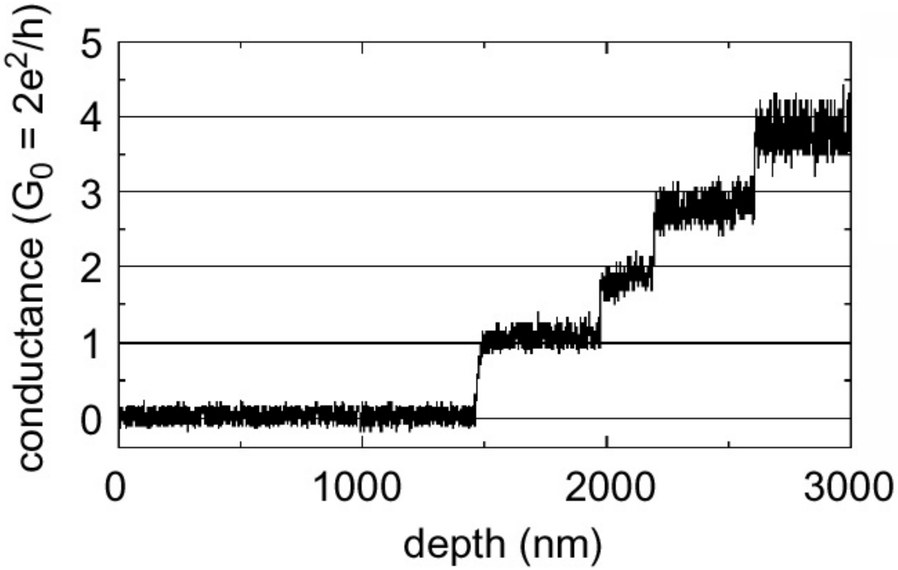


- ❑ MWCNT on a piezo-controlled tip

  → quantised conductance

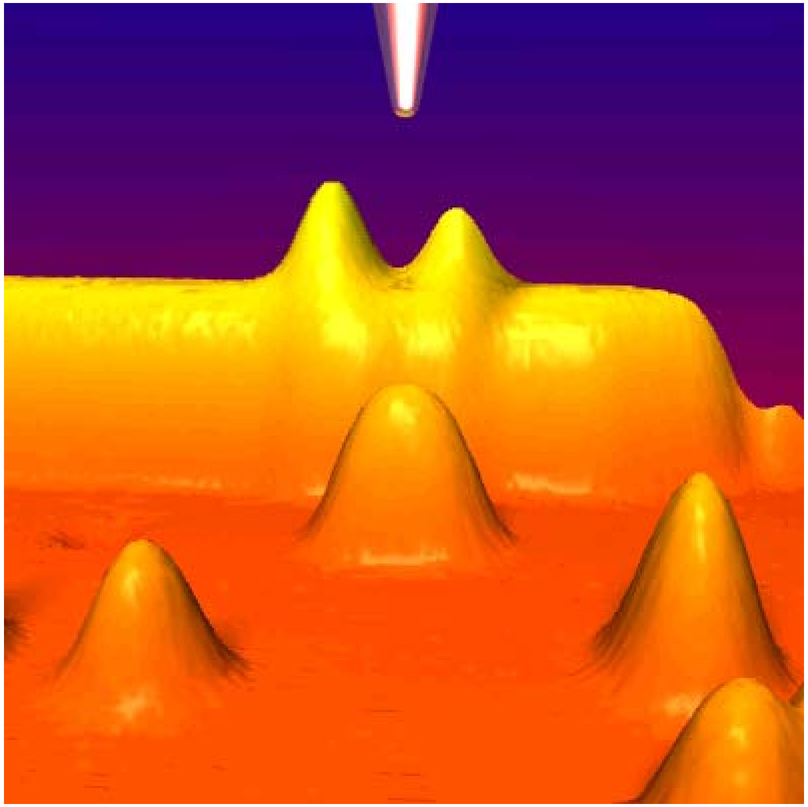
  $$nG_0 = n\,(2e^2/h) = n\,([12.9k\Omega]^{-1})$$

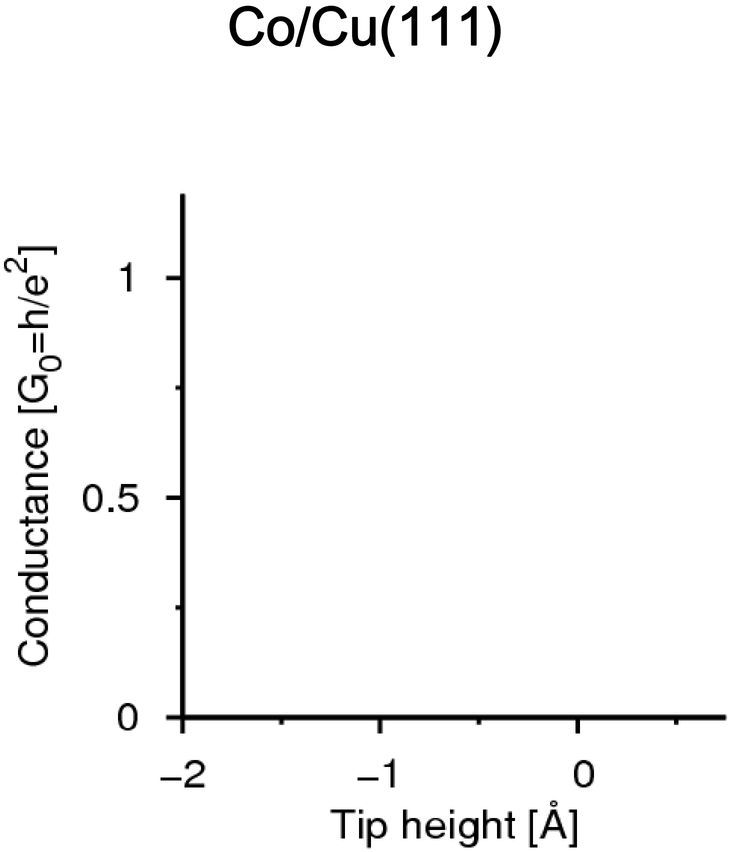
- ❑ Ballistic electron transport
  - resistance independent of tube length
  - upto 25mA per nanotube



Frank et al., Science 280, 1744, (1998)

STM Point Contacts



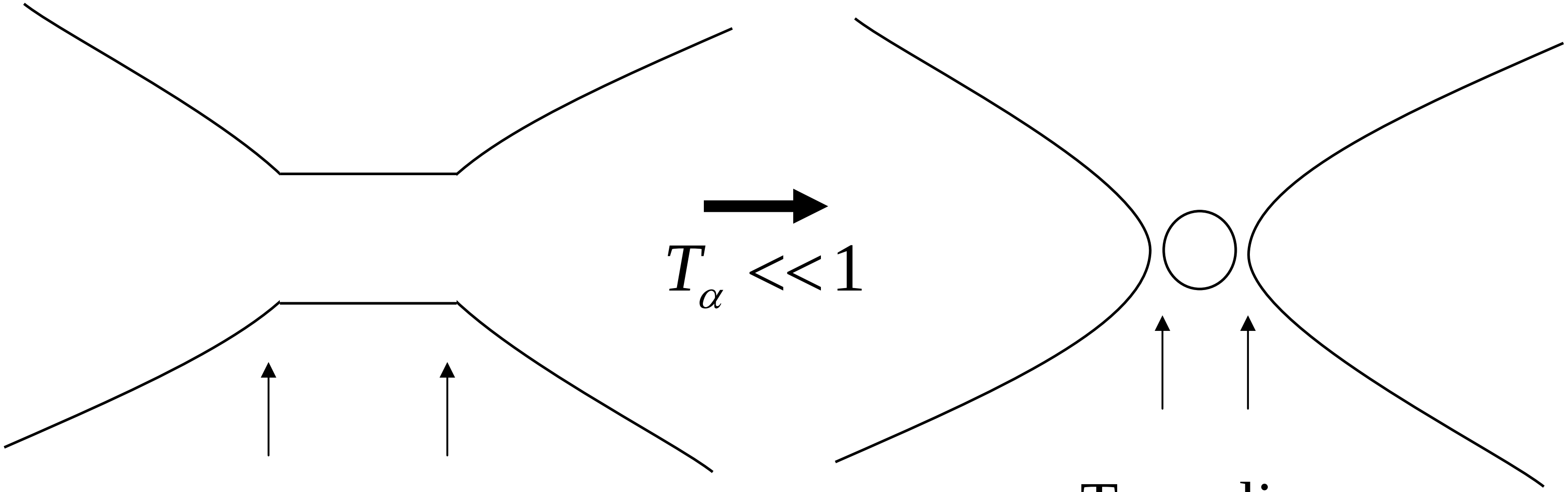


Up to now: non-interacting electrons ; highly transmissive contacts (T_α ~1)

Quantum Transport

- Introduction
- Transport through a 1D wire
- Coulomb blockade
- Single electron transistor
- Quantum Hall effect

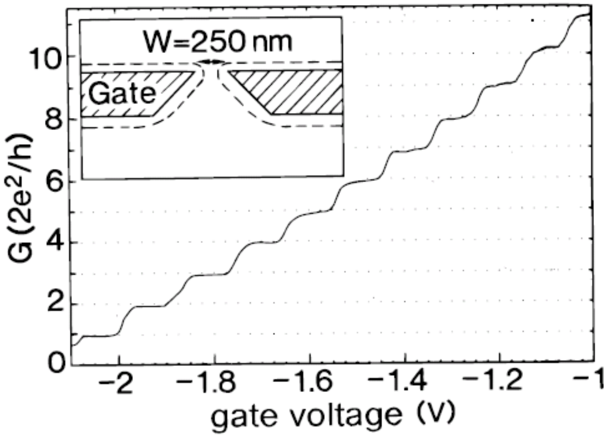
Confining electrons even more...

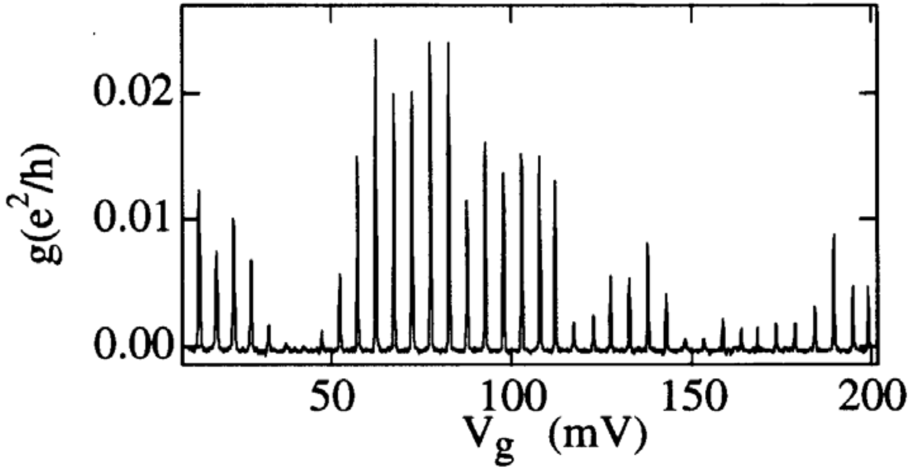


$T_\alpha << 1$

Up to now: $T_\alpha \approx 1$

Tunneling

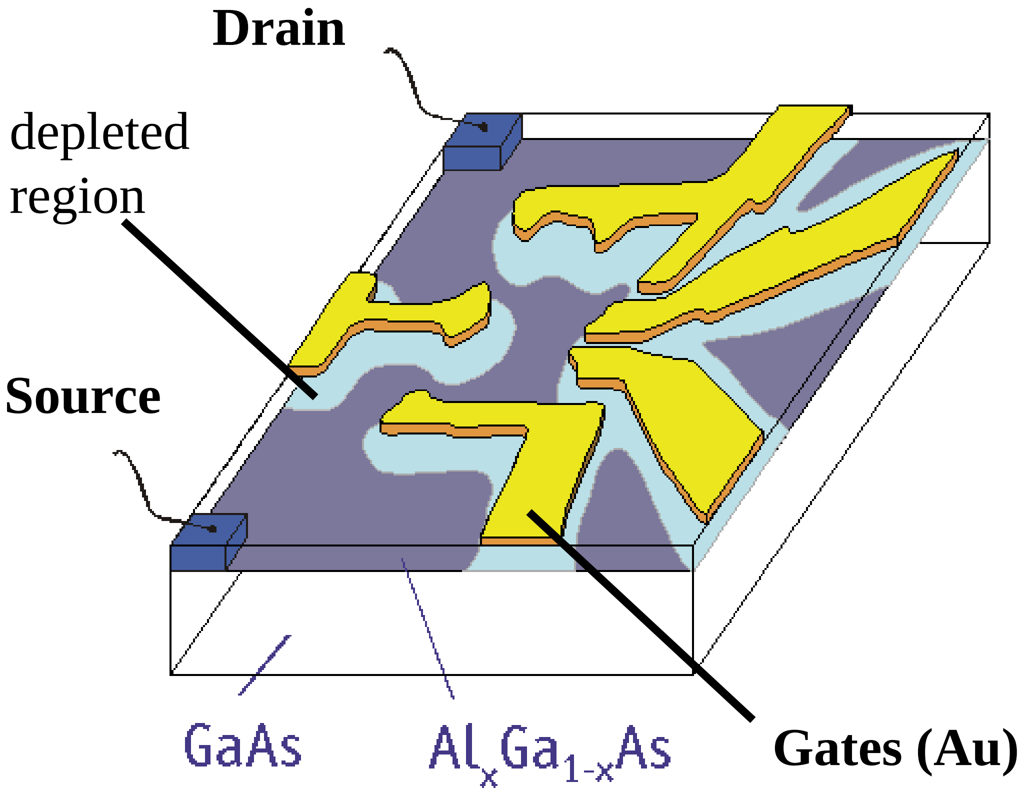


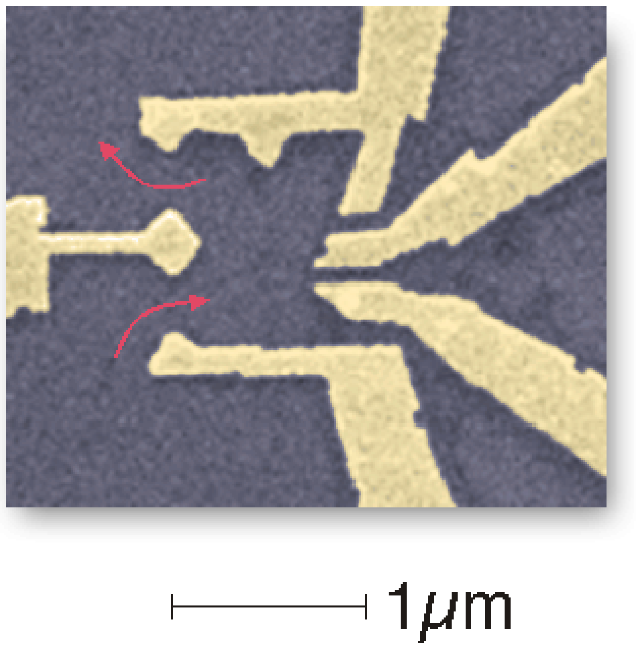


B.J. van Wees, *Phys. Rev. Lett.* **60** (1988) 848

J.A. Folk et.al., *Phys. Rev. Lett.* **76** (1996) 1699

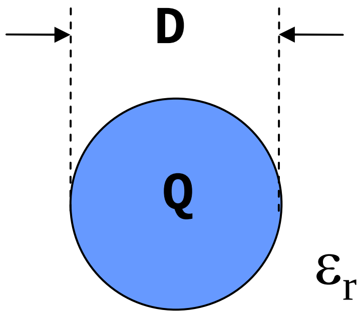
Experimental realization





L.P. Kouwenhoven et. al. Physics World, June 1998

Energy of an electron on a capacitor



$$C = 4\pi\varepsilon_0\varepsilon_r \frac{D}{2}$$

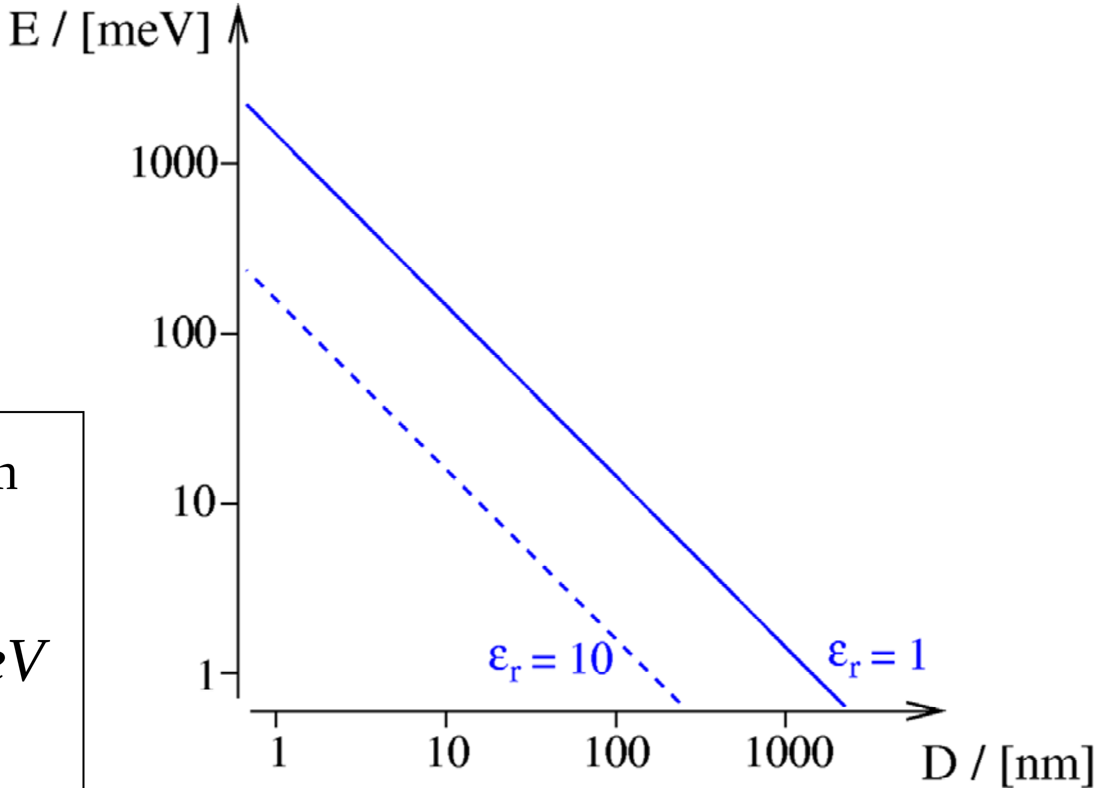
$$E_c = \frac{e^2}{4\pi\varepsilon_0\varepsilon_r D}$$

$$E_c = \frac{Q^2}{2C}$$

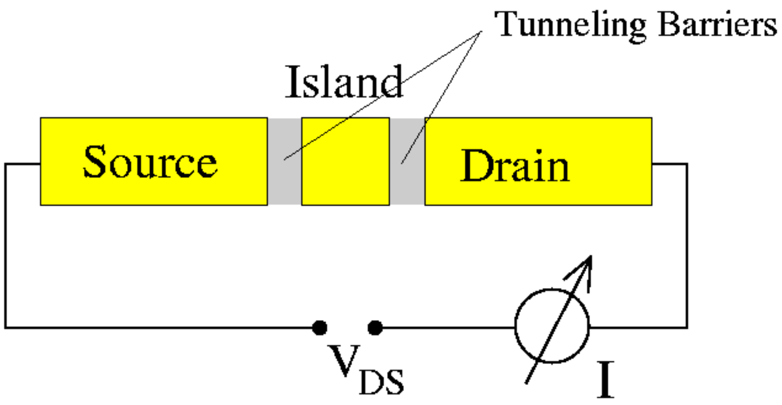
Energy of one additional electron on a 1µF capacitor :

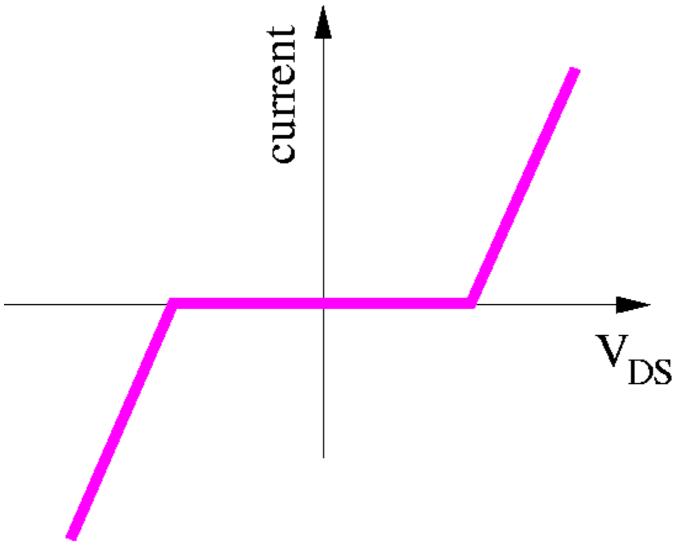
$$E_c = \frac{1.6\cdot 10^{-19} eV}{2\cdot 10^{-6}} = 0.8\cdot 10^{-12} eV$$

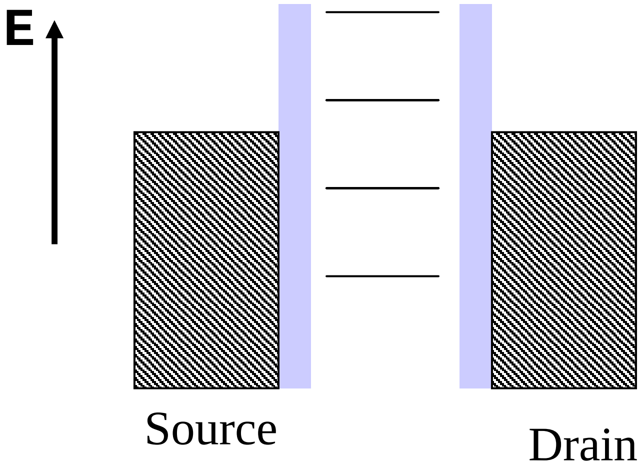
…too small to be measured



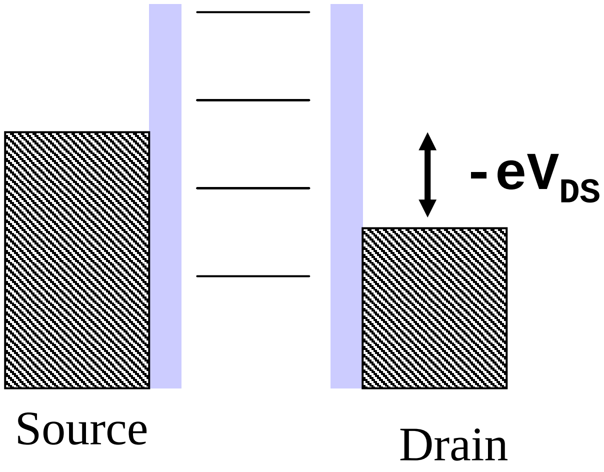
Coulomb blockade in transport





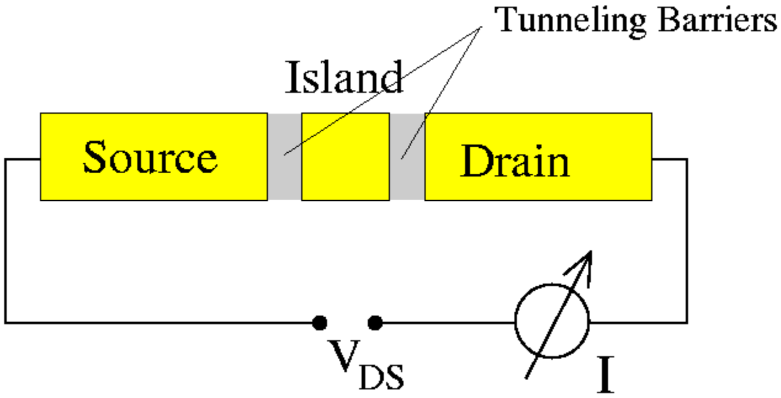


Blockade

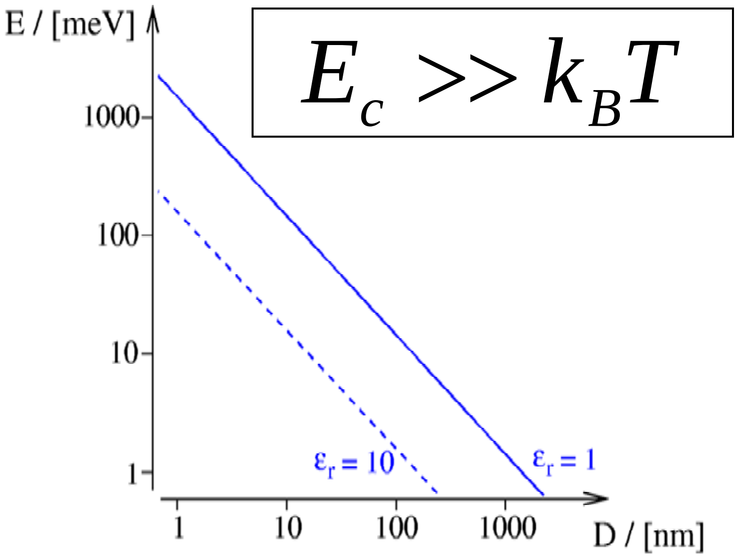


Current flow

When can the Coulomb blockade be seen ?



1. Thermal broadening



$$\boxed{E_c >> k_B T}$$

2. Uncertainty principle

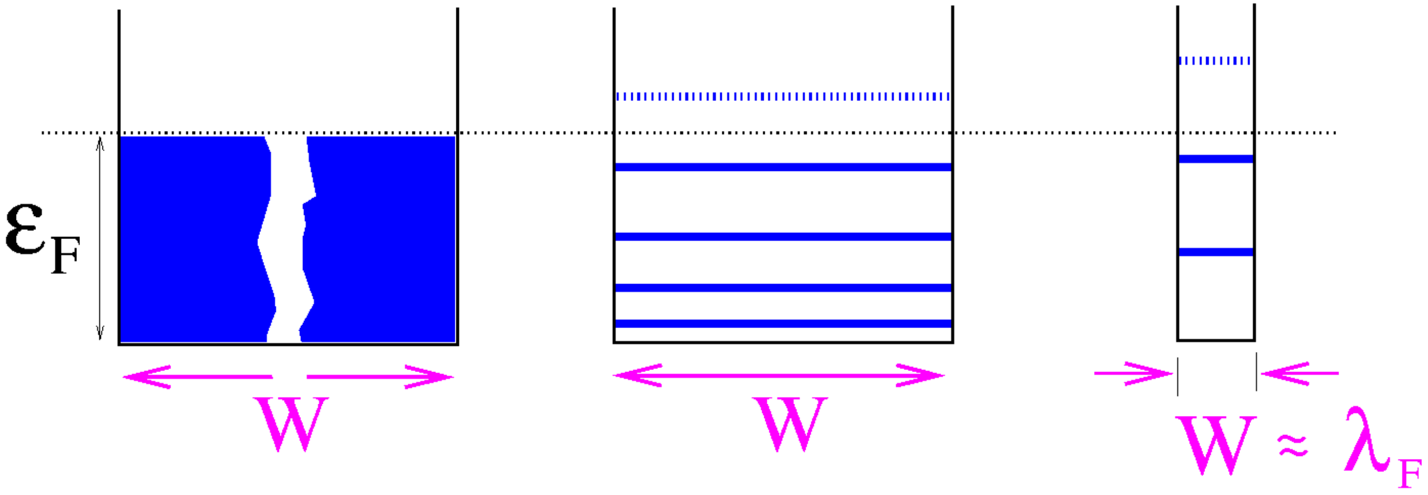
$$\Delta E \cdot \Delta t > h \qquad \frac{e^2}{C} \cdot R_t C > h$$

charging energy

lifetime of the charging

$$\boxed{R_t > \frac{h}{e^2}}$$

Level quantization (particle in a box) ?



$$\left.\begin{aligned} \varepsilon_F &= \frac{(\hbar k_F)^2}{2m} \\ k_F &= \frac{2\pi}{\lambda_F} \end{aligned}\right\} \lambda_F = \frac{h}{\sqrt{2m\varepsilon_F}}$$

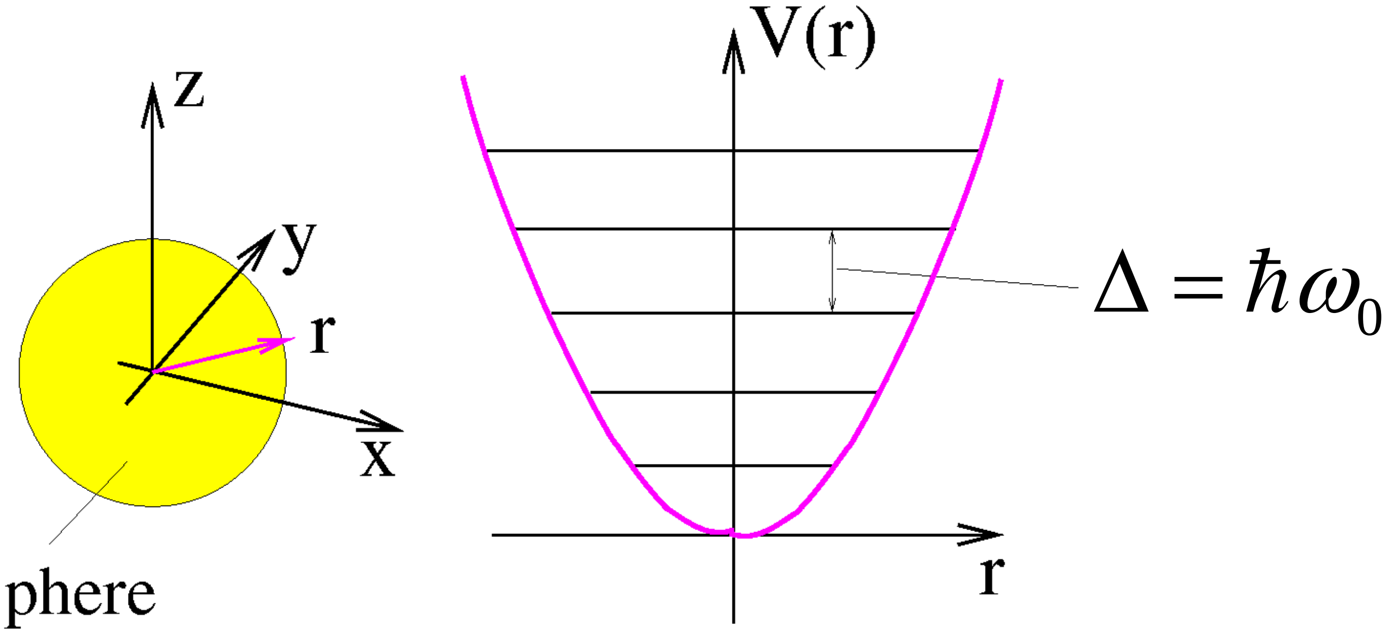
Si: $m = 0.98\ m_0$

GaAs: $m = 0.07\ m_0$

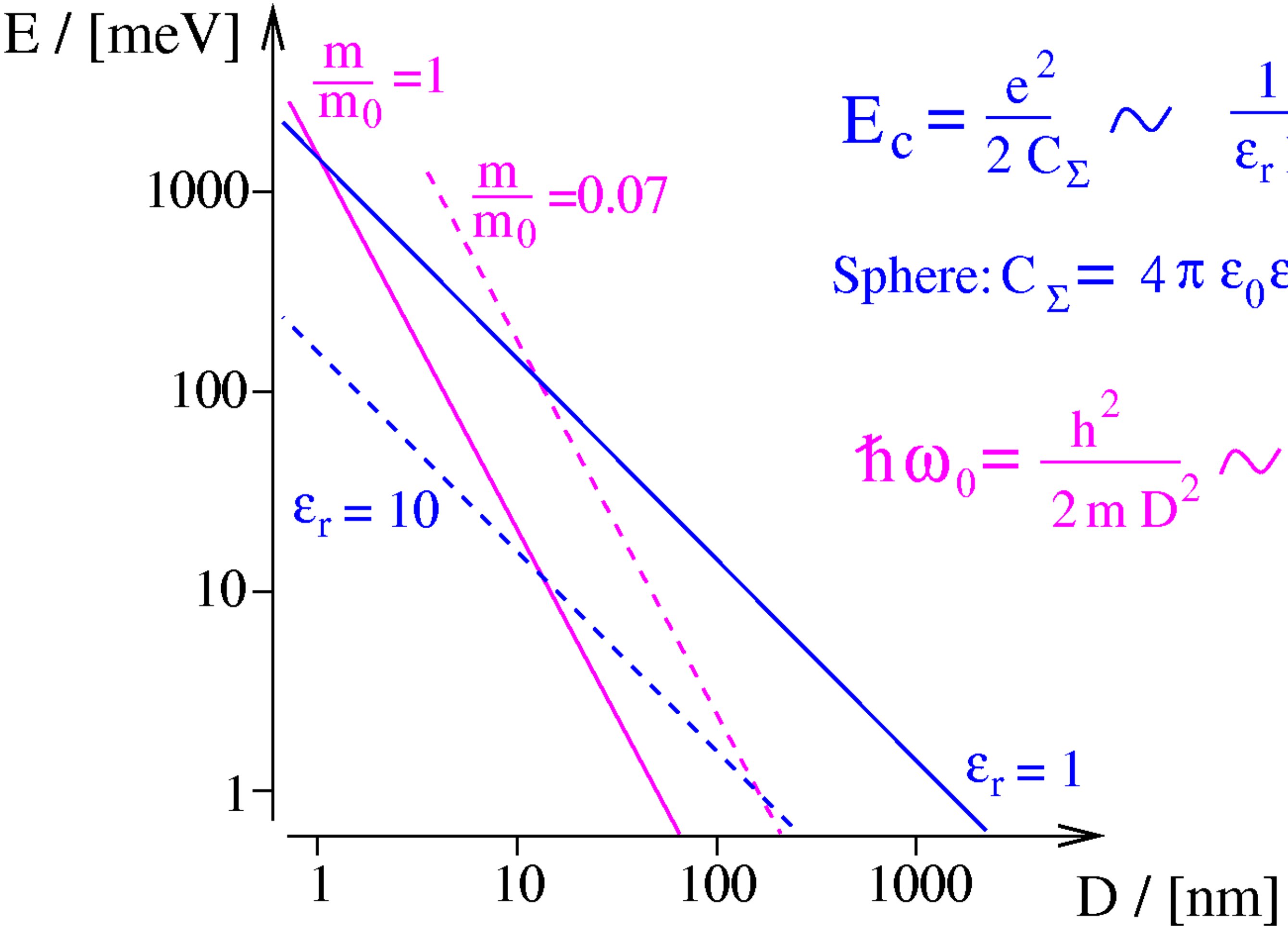
→ Electrons confined in all dimensions: QD can be considered to be 0D

Level quantization

Parabolic Confinement



E_c and Δ vs. Island Diameter D

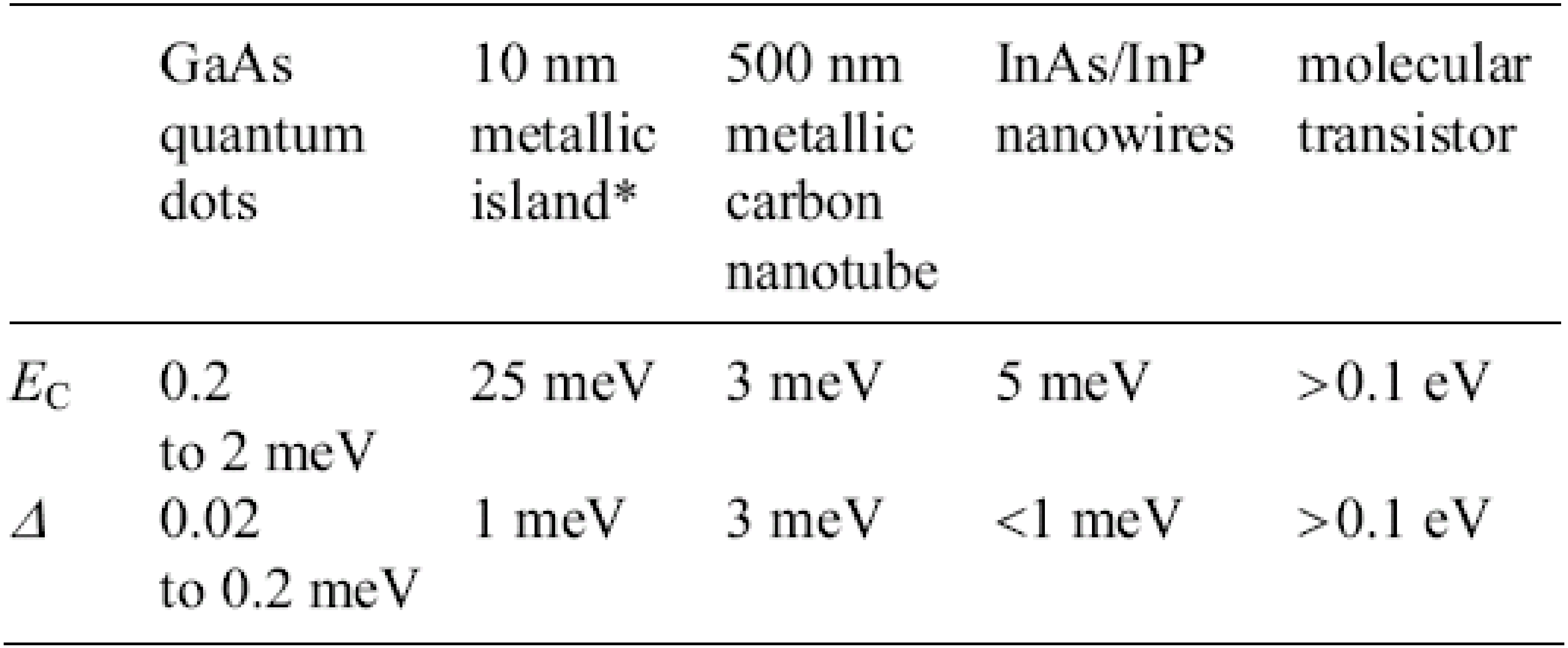


$$E_c = \frac{e^2}{2\,C_\Sigma} \sim \frac{1}{\varepsilon_r\,D}$$

$$\text{Sphere: } C_\Sigma = 4\pi\,\varepsilon_0\varepsilon_r\,\frac{D}{2}$$

$$\hbar\omega_0 = \frac{h^2}{2\,m\,D^2} \sim \frac{1}{m\,D^2}$$

E_c and Δ for different material systems



	GaAs quantum dots	10 nm metallic island*	500 nm metallic carbon nanotube	InAs/InP nanowires	molecular transistor
E_C	0.2 to 2 meV	25 meV	3 meV	5 meV	>0.1 eV
Δ	0.02 to 0.2 meV	1 meV	3 meV	<1 meV	>0.1 eV

J.M. Thijssen et. al. *phys. stat. sol. (b)* **245** (2008) 1455

Quantum Transport

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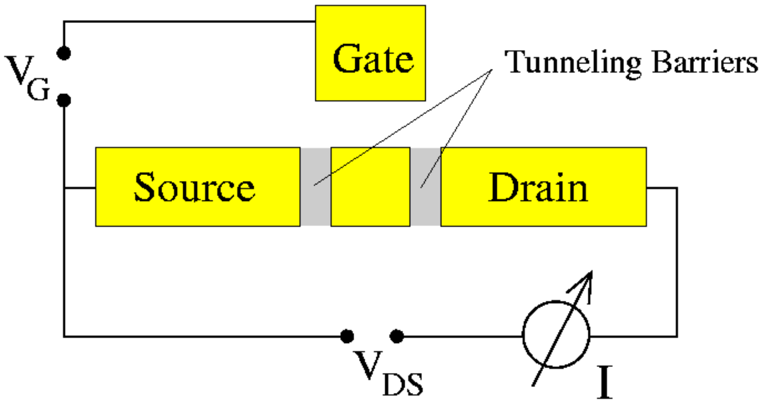
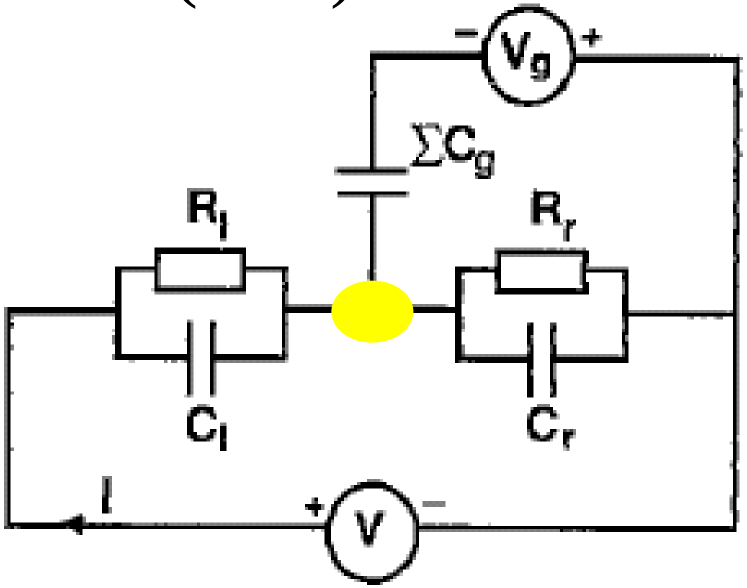
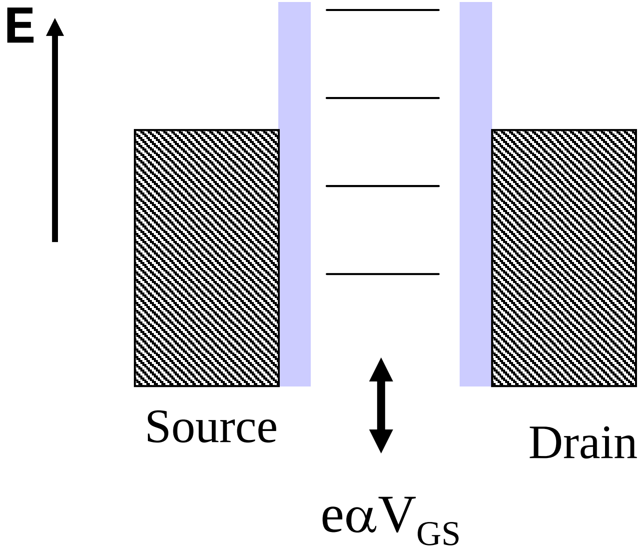
Single Electron Transistor (SET)







Total energy of the dot (CI-model):

$$U(N) = \boxed{\sum_{n=1}^{N} E_n} + \boxed{\frac{(-e(N-N_0)+Q_0)^2}{2C}}$$

Quantization energy

Electrostatic energy

$$Q_0 = C_l V_l + C_r V_r + C_G V_G$$

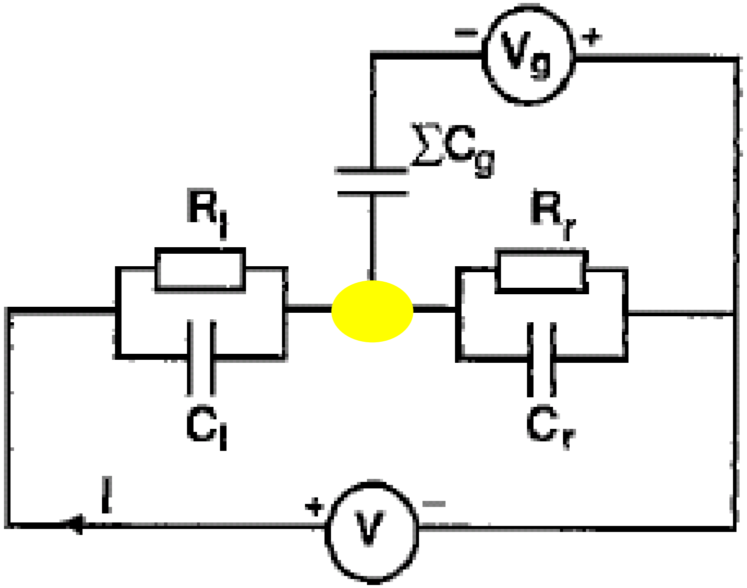
$$C = C_l + C_r + C_G$$

Proposed: D. Averin, K.K. Likharev, IEEE Trans. Magn. 23, 1142 (1987)

First Realisation: T.A. Fulton and G.J. Dolan, Phys. Rev. Lett. 59,109 (1987)

Energy needed to add one electron to the dot

Constant interaction model:
1.) single capacitance C between electrons on the dot and the environment
2.) single-particle energy-level spectrum independent of the number of electrons



Total energy of the dot:

$$U(N) = \sum_{n=1}^{N} E_n + \frac{(-e(N-N_0)+Q_0)^2}{2C}$$

$$Q_0 = C_l V_l + C_r V_r + C_G V_G$$

$$C = C_l + C_r + C_G$$

Electrochemical potential of N electrons on the dot

$$\mu(N) = U(N) - U(N-1)$$

$$= E_N + \frac{e^2(N-N_0-0.5)}{2C} - e\frac{C_g}{C}V_g$$

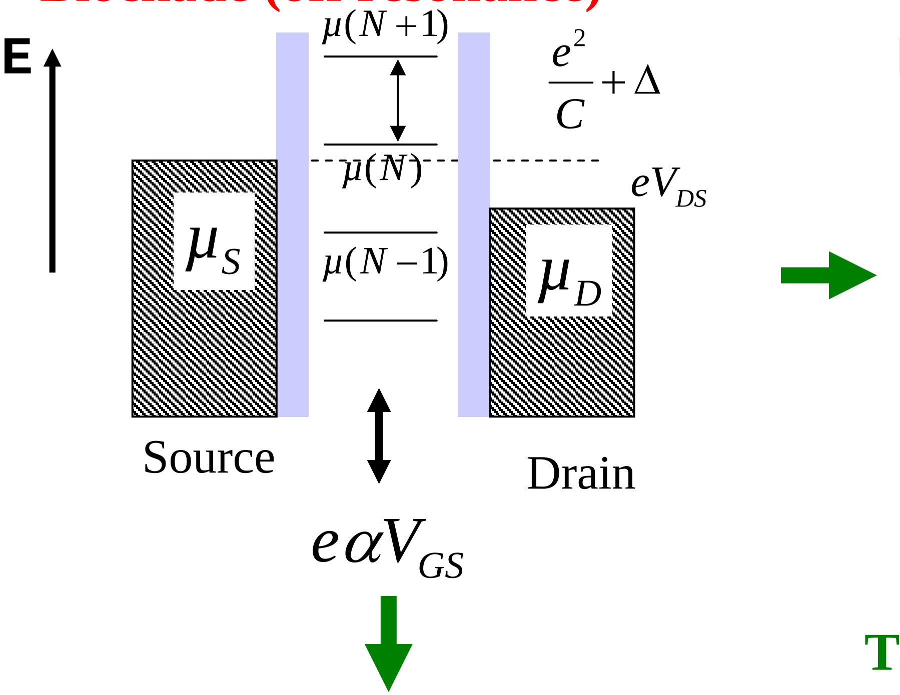
Energy to add one electron:

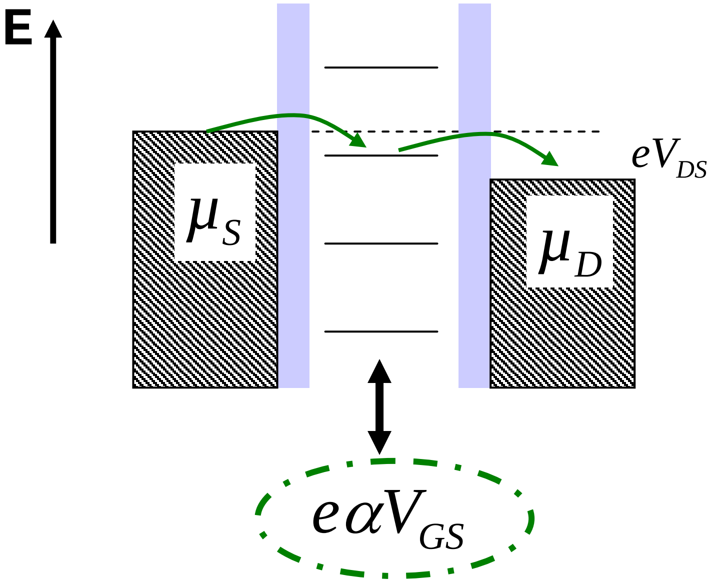
$$\mu(N+1) - \mu(N) = \frac{e^2}{C} + E_{N+1} - E_N$$

$$= \frac{e^2}{C} + \Delta$$

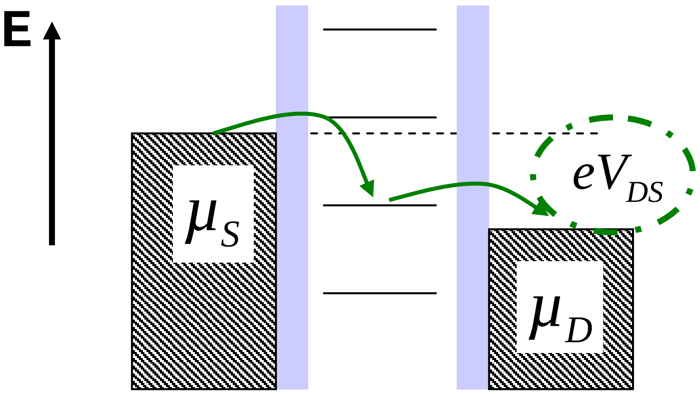
Single electron transport through the dot

Blockade (off resonance)

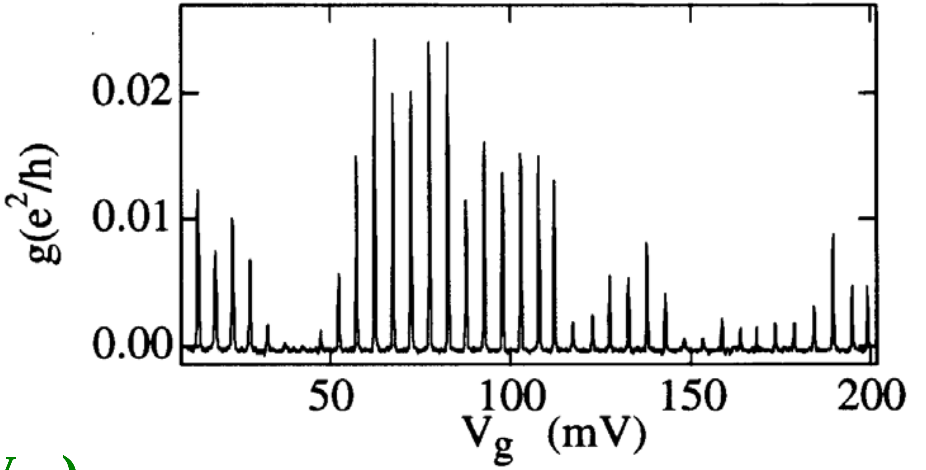




Transport (on resonance by changing V_{GS})



Transport (on resonance by changing V_{DS})

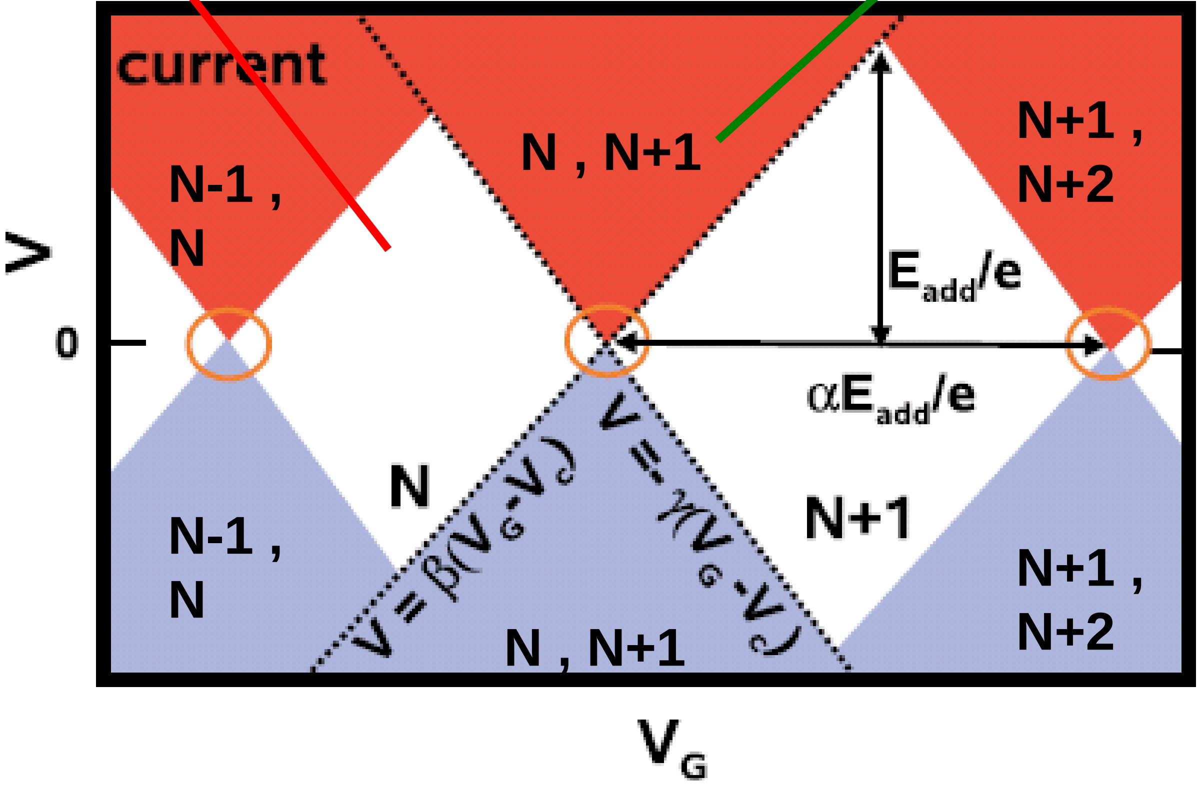


J.A. Folk et al., *PRL* **76** (1996) 1699

Stability diagram of a quantum dot (QD)

Blockade: Dot stable with N electrons

Transport: Dot stable with N or N+1 electrons



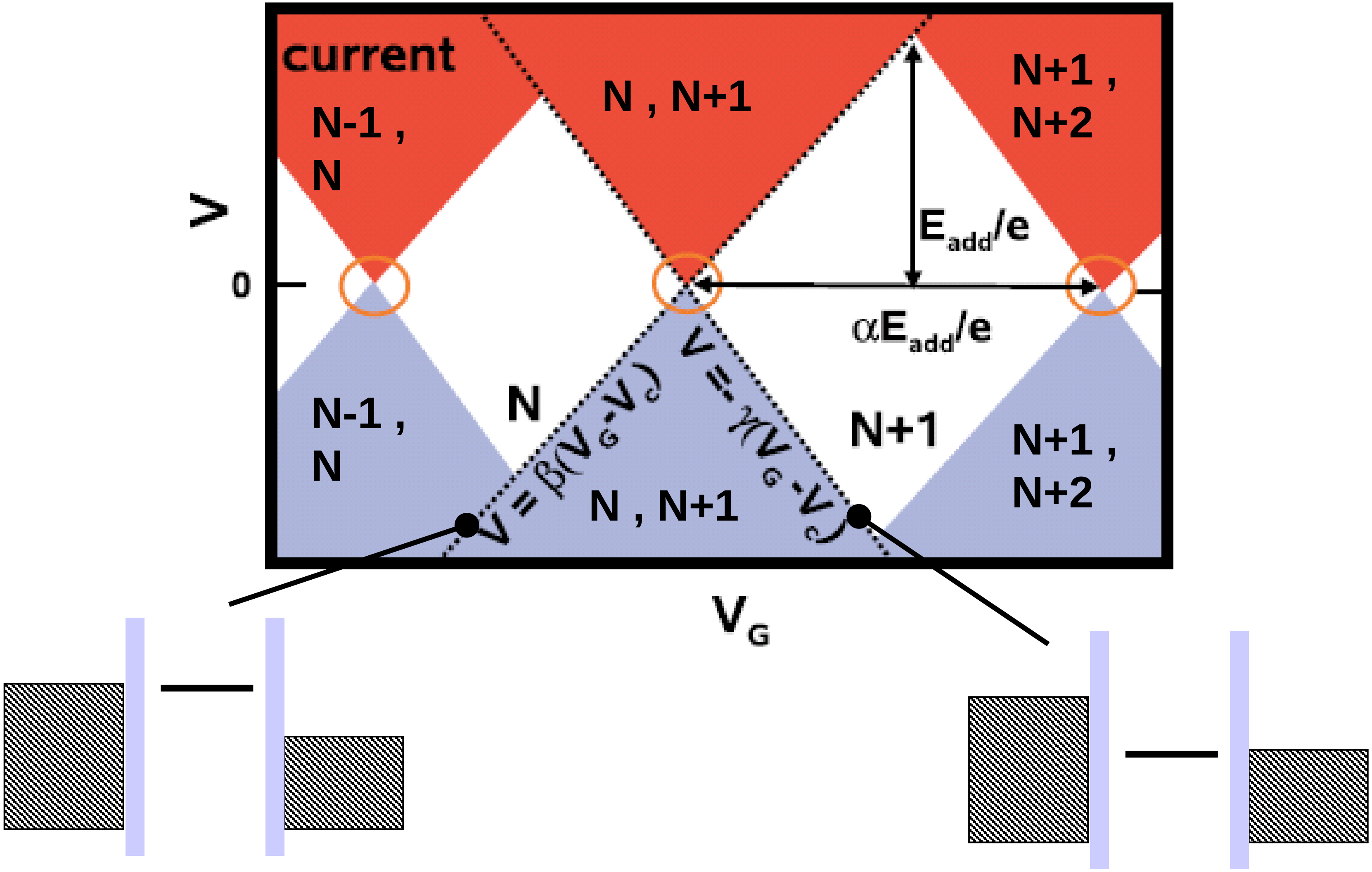
Periodicity of the oscillations with V_{GS}:

$$\mu(N, V_{GS}) = \mu(N+1, V_{GS} + \Delta V_{GS})$$

$$\Delta V_{GS} = \frac{C_G}{eC} E_{add} = \frac{\alpha E_{add}}{e}$$

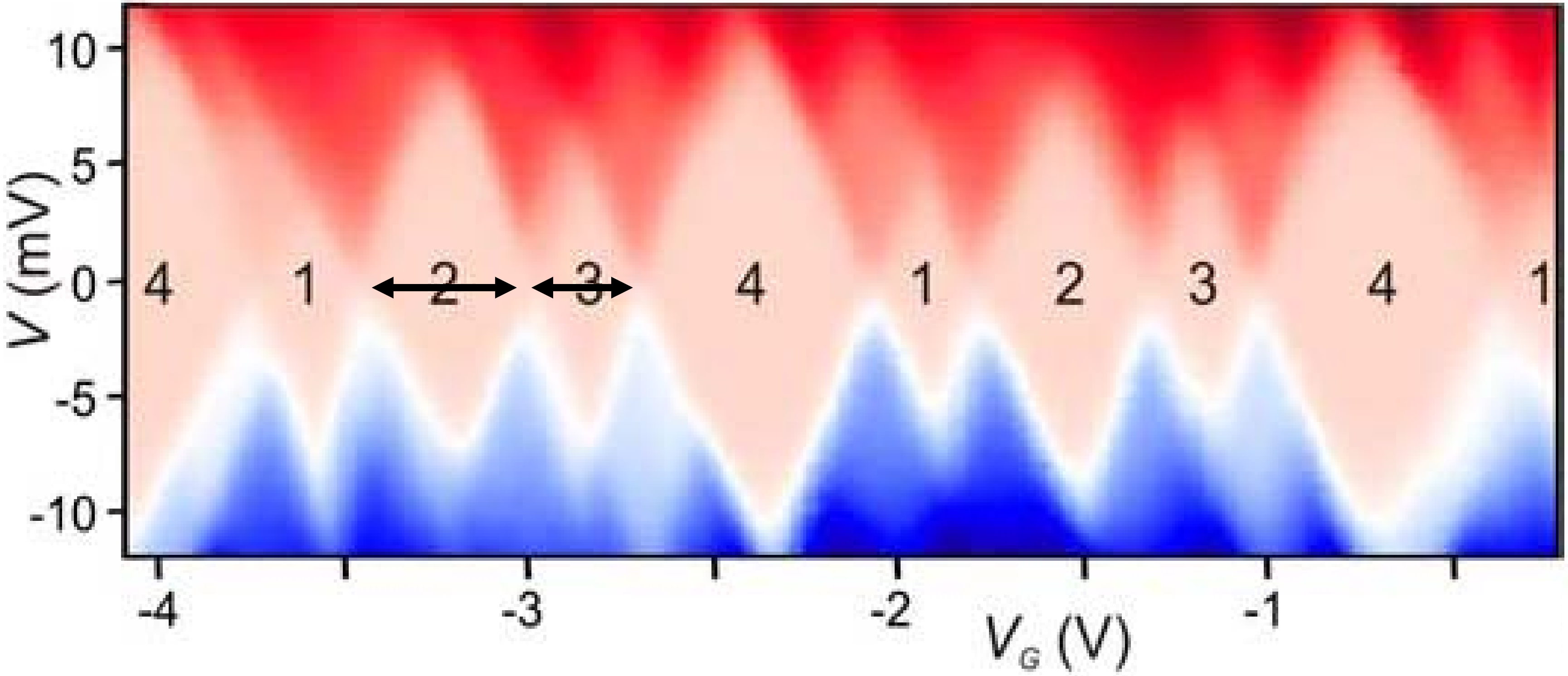
J.M. Thijssen et. al. *phys. stat. sol. (b)* **245** (2008) 1455

Stability diagram of a quantum dot



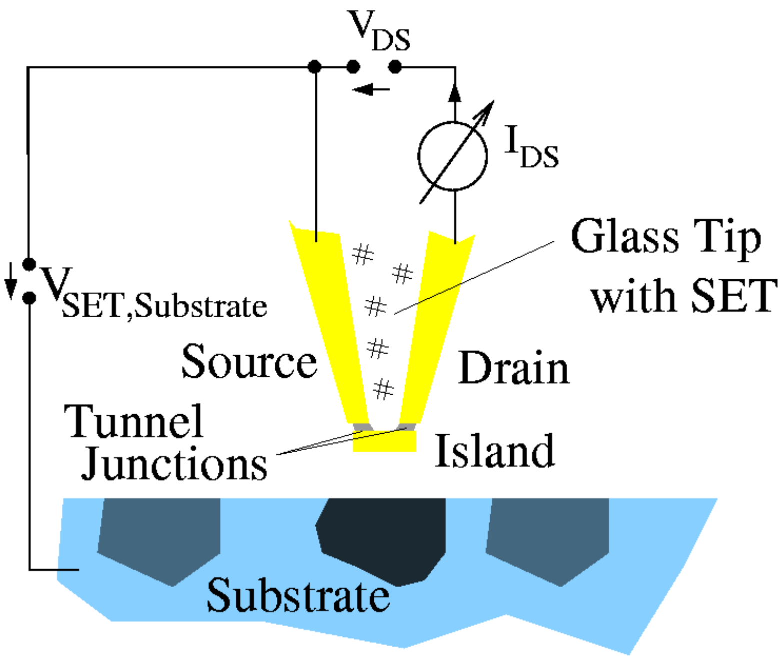
Experimental Stability Diagram: Carbon Nanotube QD

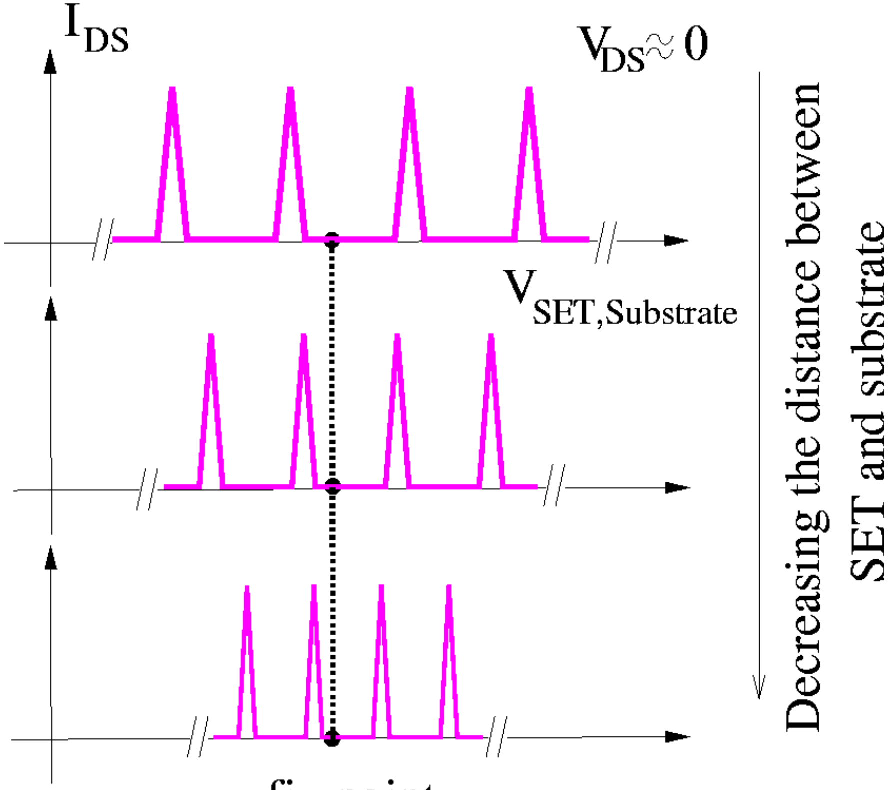
$$E_{add} \neq E_{add}^{'}$$



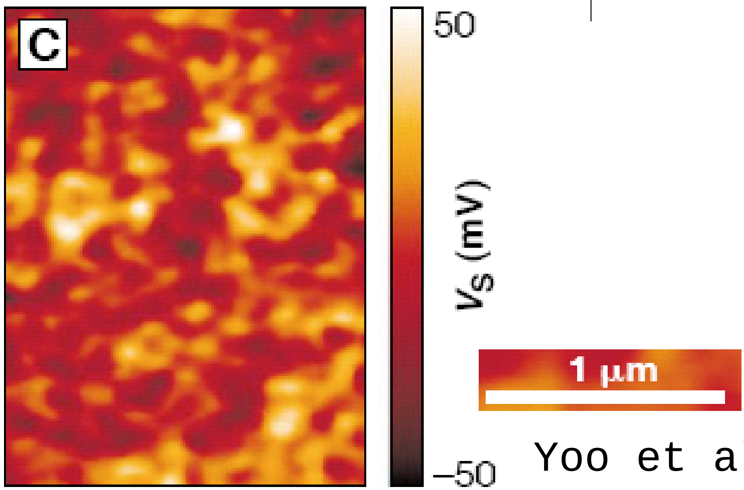
J.M. Thijssen et. al. *phys. stat. sol. (b)* **245** (2008) 1455

Applications of SETs: **Electrostatic Potential Probe**





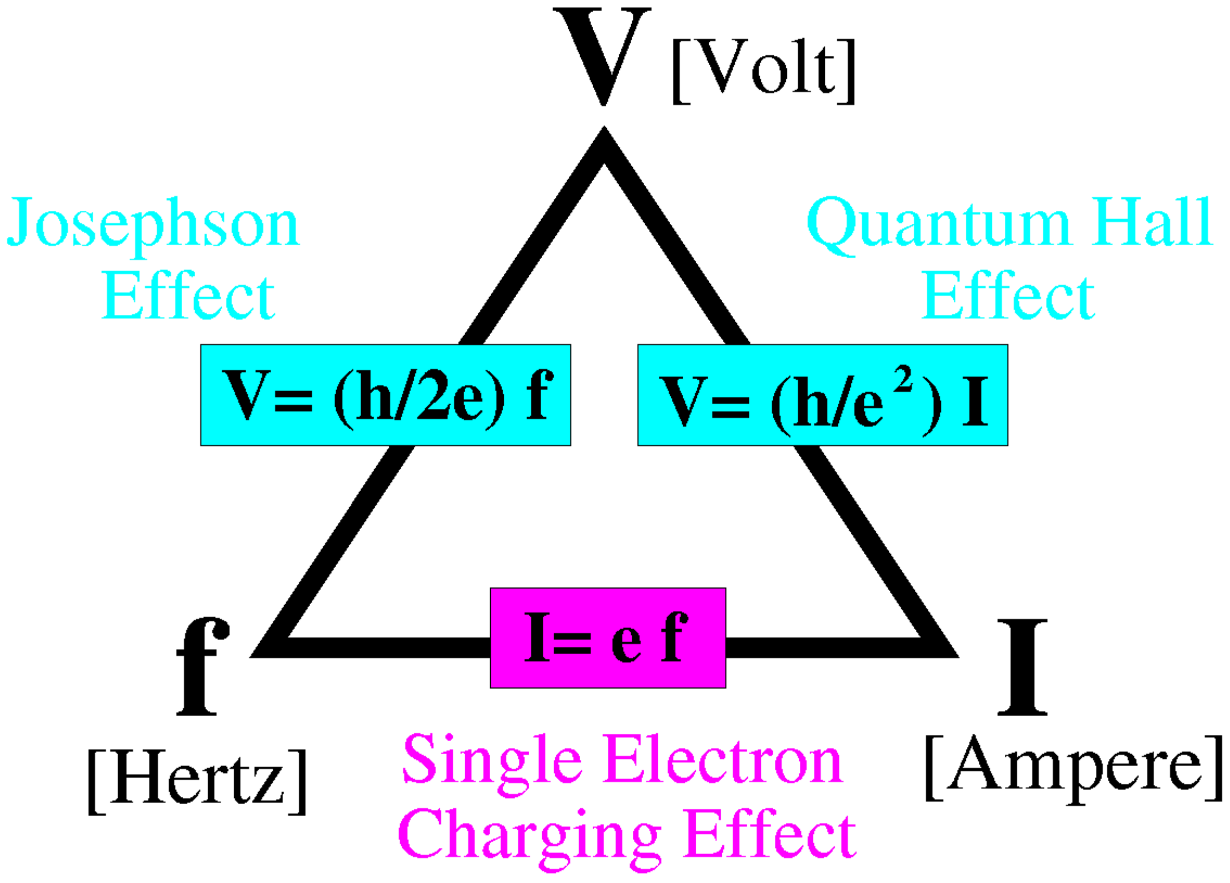
Map the potential landscape of a GaAs substrate with high sensitivity and high spatial resolution



Yoo et al. *Science* **276** (1997) 579

Applications of SETs: **Representation of physical units**

Quantum Metrological Triangle



V [Volt]

Josephson Effect

Quantum Hall Effect

$$V = (h/2e)\, f$$

$$V = (h/e^2)\, I$$

$$I = e\, f$$

f [Hertz]

Single Electron Charging Effect

I [Ampere]