Electronic transport in low dimensional systems

For example: 2D system l<<L; L = macroscopic extent

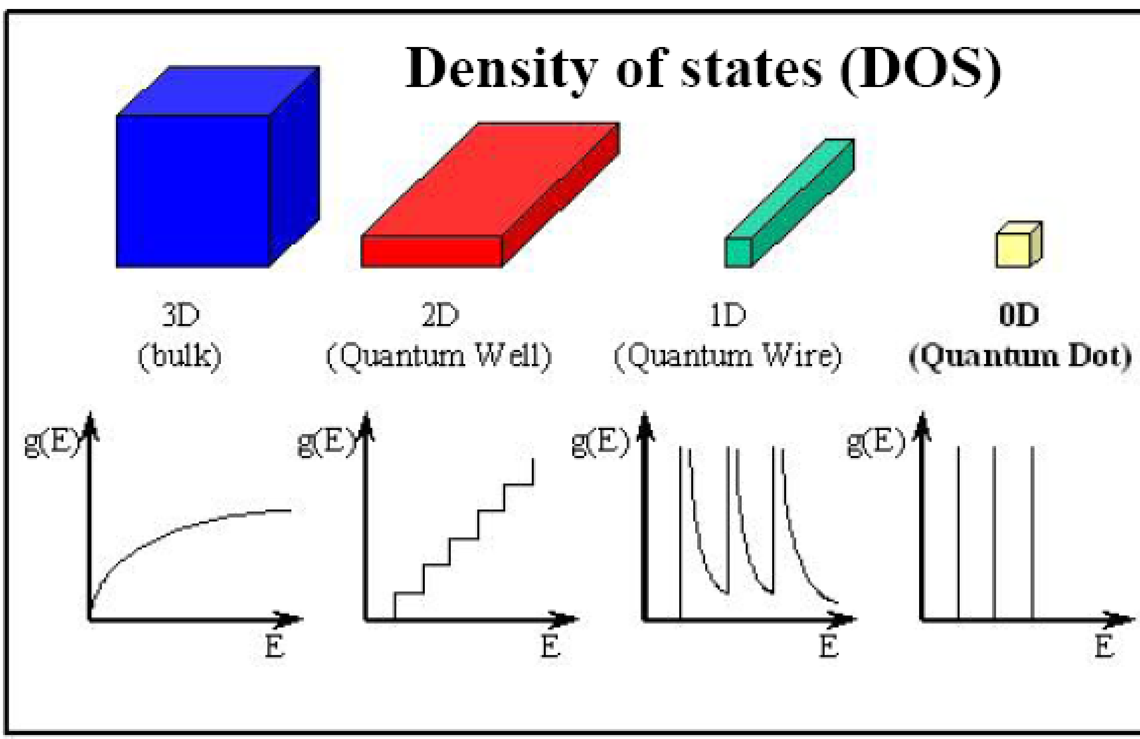
$$\Psi = \sqrt{\frac{2}{l}} \sin(\frac{n\pi}{l} x) \frac{1}{L} e^{i(k_y y + k_z z)}$$

$$E = \frac{\hbar^2}{2m_{//}} (\frac{n\pi}{l})^2 + \frac{\hbar^2}{2m_{\perp}} (k_y^2 + k_z^2)$$

$$g(E)dE = 2\frac{1}{(2\pi)^2} 2\pi k\, dk = \frac{m}{\pi\hbar^2} dE$$

For each sub-band defined by n

Electronic structure



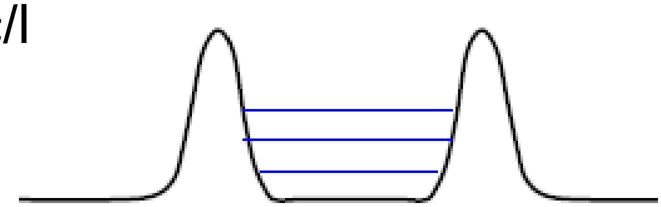
$$g(E)dE = 2\frac{1}{(2\pi)^p} \int \delta(E - \frac{\hbar^2 k^2}{2m}) d^p k$$

p = 0,1,2,3 is the system dimension

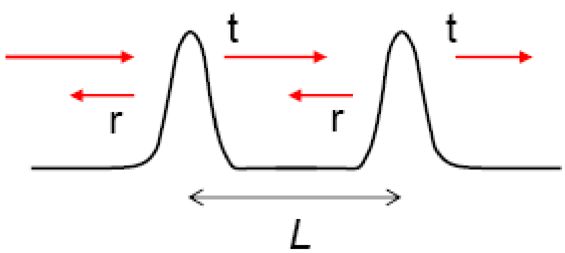
Quantum well (2D)

Confinement in x direction and free electrons in the y,z directions

$K_x = n\pi/l$



Two identical barriers in series:

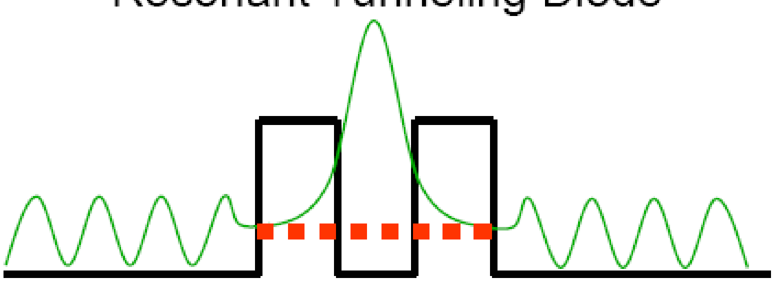


Coherent transport; complex transmission and reflection amplitudes:

$t = |t|e^{i\phi_t}$

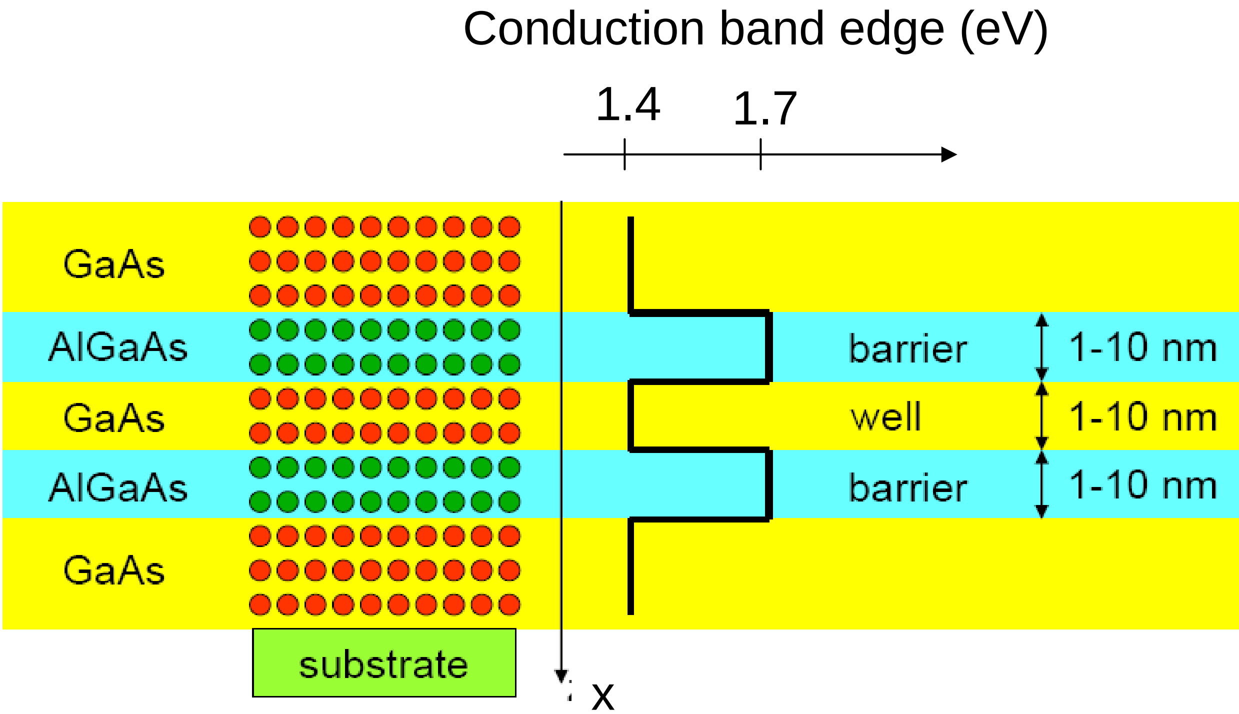
$r = |r|e^{i\phi_r}$

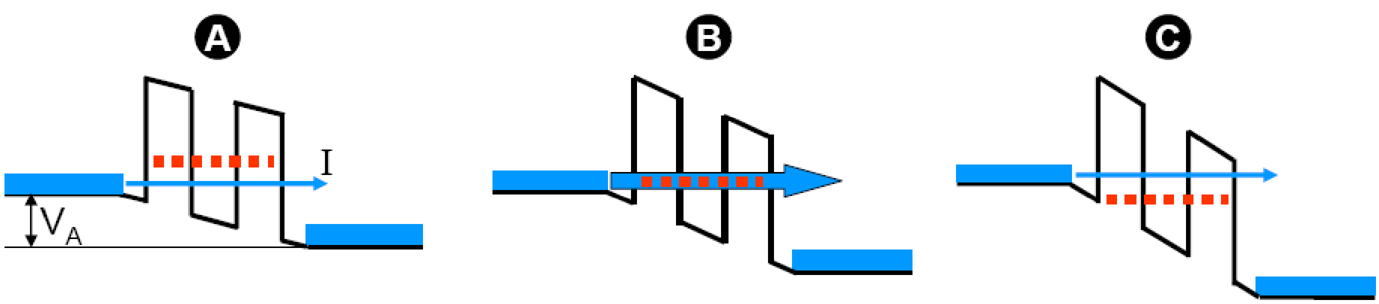
Resonant Tunneling Diode



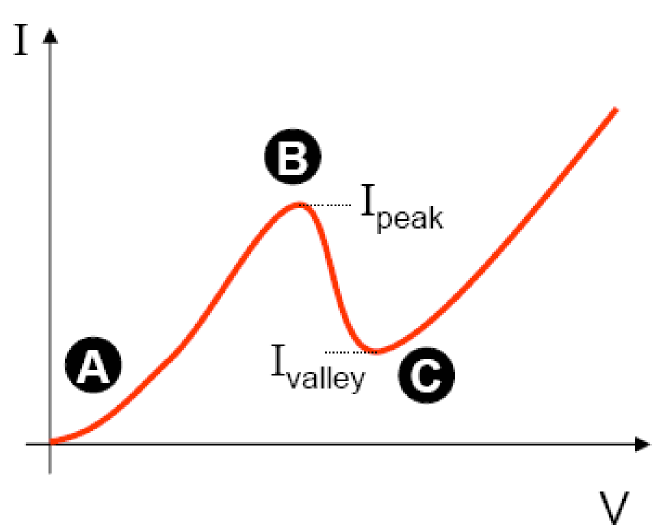
Quasi-localized state

Peak in the barrier transmission when the electron energy matches one of the well discrete energy level





Current peak when the electron energy matches one of the well discrete energy level



Resonant tunneling in semiconductor double barriers*

L. L. Chang, L. Esaki, and R. Tsu

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(Received 18 March 1974)

Appl. Phys. Lett., **24**, 593 (1974)

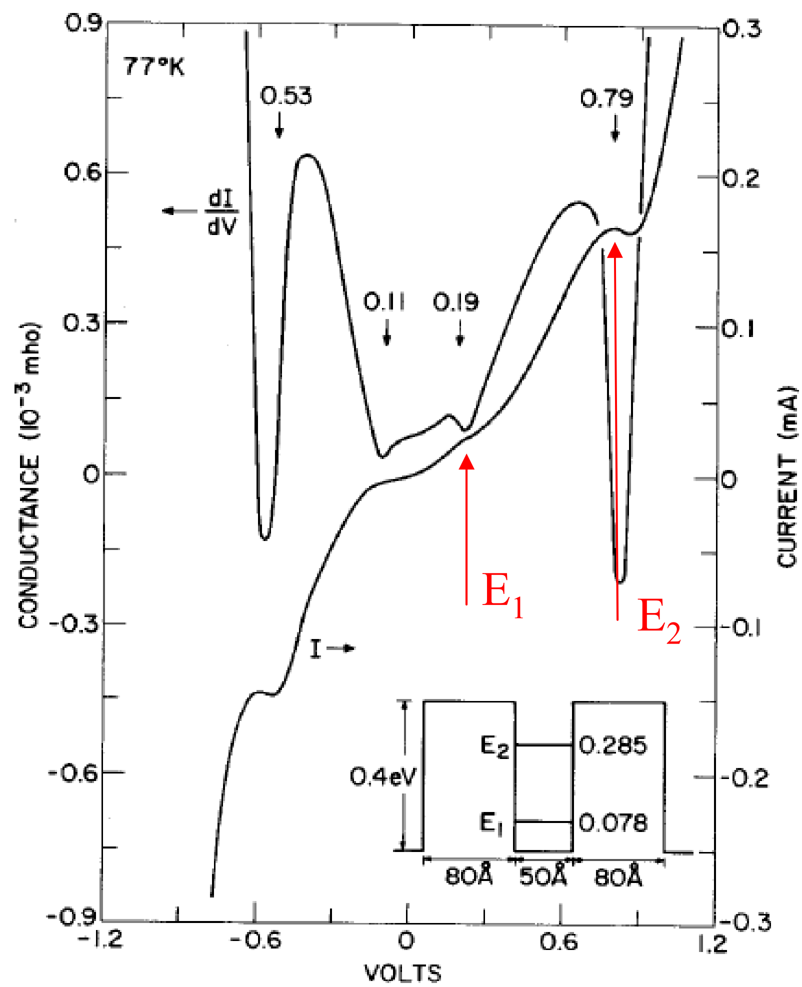


FIG. 1. Current and conductance characteristics of a double-barrier structure of GaAs between two $Ga_{0.3}Al_{0.7}As$, as shown in the energy diagram. Both the thicknesses and the calculated quasistationary states of the structure are indicated in the diagram. Arrows in the curves indicate the observed voltages of singularities corresponding to these resonant states.

In the simple model of square barriers the current peaks appear at about V = 2E

Observation of Discrete Electronic States in a Zero-Dimensional Semiconductor Nanostructure

M. A. Reed, J. N. Randall, R. J. Aggarwal,[a] R. J. Matyi, T. M. Moore, and A. E. Wetsel[b]
Central Research Laboratories, Texas Instruments Incorporated, Dallas, Texas 75265
(Received 2 October 1987)

Electronic transport through a three-dimensionally confined semiconductor quantum well ("quantum dot") has been investigated. Fine structure observed in resonant tunneling through the quantum dot corresponds to the discrete density of states of a zero-dimensional system.

Phys. Rev. Lett. **60**, 535 (1988)

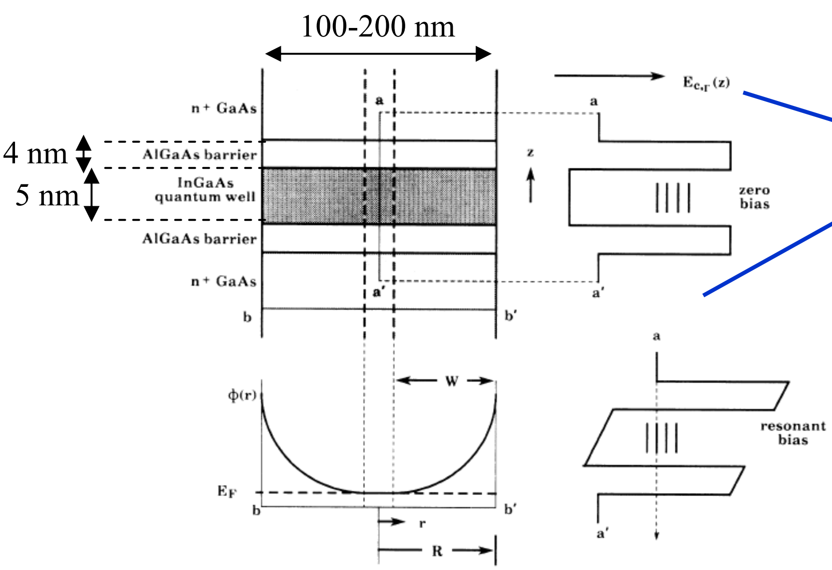


FIG. 2. Schematic illustration of the vertical (a-a') and lateral (b-b') potentials of a column containing a quantum dot, under zero and applied bias. $\Phi(r)$ is the (radial) potential, R is the physical radius of the column, r is the radial coordinate, W is the depletion depth, Φ_T is the height of the potential determined by the Fermi-level (E_F) pinning, and $E_{c,\Gamma}$ is the Γ-point conduction-band energy.

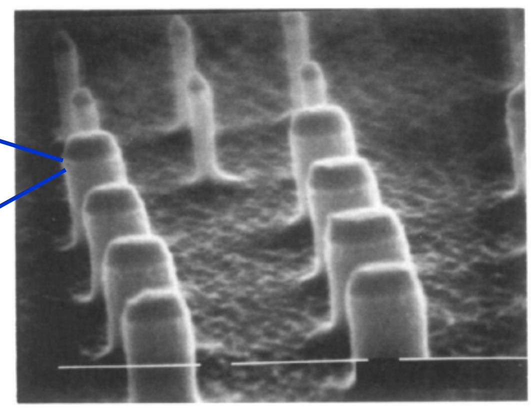


FIG. 1. A scanning electron micrograph of various size GaAs nanostructures containing quantum dots. The dark region on top of the column is the electron-beam defined Ohmic contact and etch mask. The horizontal bars are 0.5 μm.

The principal peak is due to the vertical confinement

Temperature smearing

The low temperature multi-peak structure reflect the lateral confinement

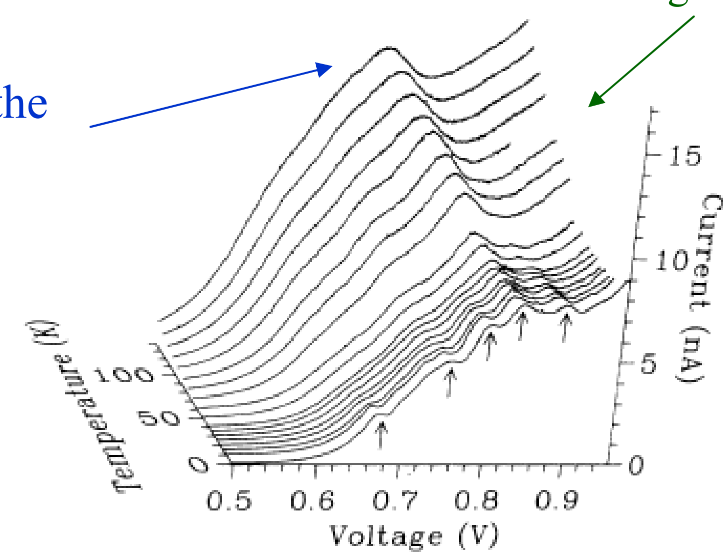
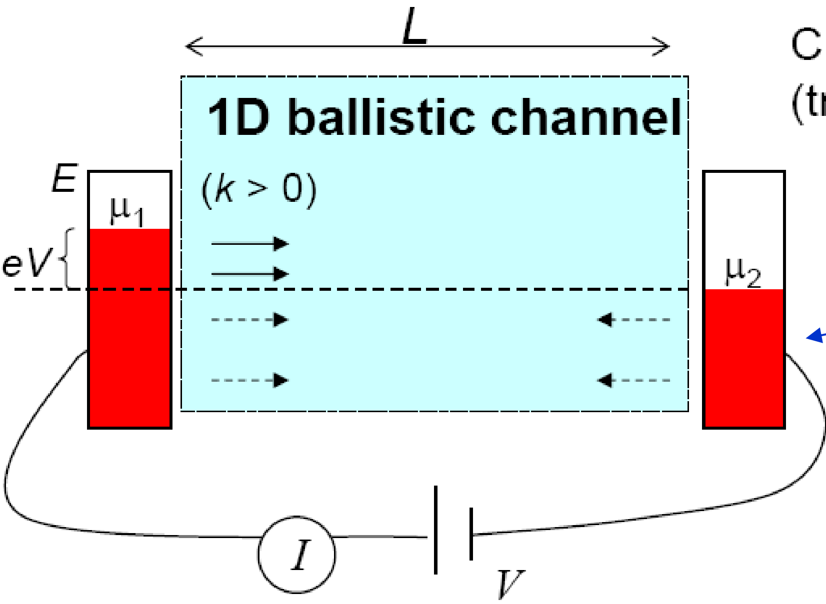


FIG. 3. Current-voltage characteristics of a single quantum-dot nanostructure as a function of temperature, showing resonant tunneling through the discrete states of the $n=2$ quantum well resonance. The arrows indicate the voltage positions of the discrete states for the $T=1.0$-K curve.

Quantum wire (1D)



Channel: 1D, ballistic
(transport without scattering)

N.B.: Ohmic contact
(in this case absence
of potential barrier)

$$\mu_2 - \mu_1 = eV$$

$$v(E) = \sqrt{\frac{2E}{m}}$$

$$g(E)\,dE = \frac{2}{2\pi}dk = \frac{m}{\pi\hbar}\frac{1}{\sqrt{2mE}}dE$$

velocity; 1D density; spin; $k > 0$

$$I = \int_{\mu_1}^{\mu_2} ev(E)\left(2\frac{1}{2}g_{1D}(E)\right)\mathrm{d}E = \int_{\mu_1}^{\mu_2} ev(E)\left(\frac{2}{hv(E)}\right)\mathrm{d}E$$

$$= \frac{2e}{h}(\mu_2 - \mu_1) = \frac{2e}{h}(eV)$$

$$G = I/V = \frac{2e^2}{h}$$

Conductance independent on the length L

Quantized conductance through individual rows of suspended gold atoms

Nature **395**, 781 (1998)

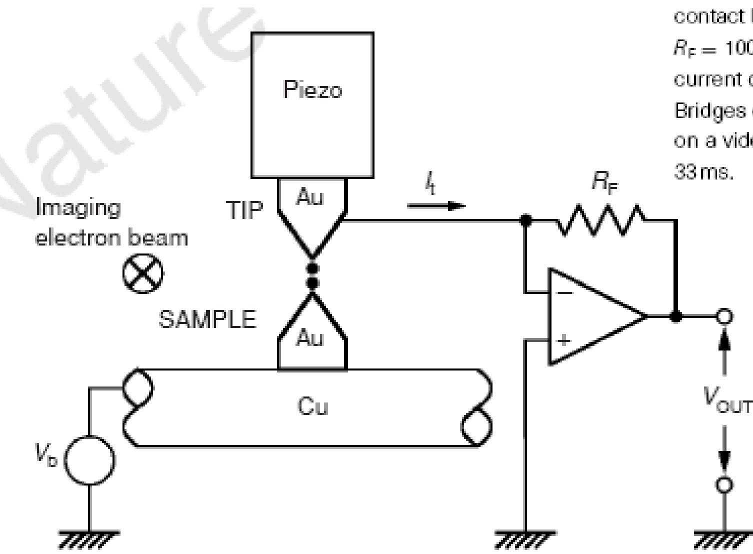


Figure 1 Scanning tunnelling microscope (STM) configuration built at the specimen stage of a UHV electron microscope. The conductance, $G = I_t/V_b$, was obtained by measuring $V_{out} = -R_F I_t$ for a constant bias voltage V_b, where R_F is the feedback resistor for current sensing, and I_t is the current passing through the contact between the tip and the sample. For most experiments, $V_b = 13$ mV and $R_F = 100$ kΩ. The imaging electron beam entered from top and typically had a current density of 0.45 fA nm^{-2} at the contact between the tip and the substrate. Bridges of gold atoms formed at the contact were imaged at $\times 10^7$ magnification on a video monitor and simultaneously recorded with the current at intervals of 33 ms.

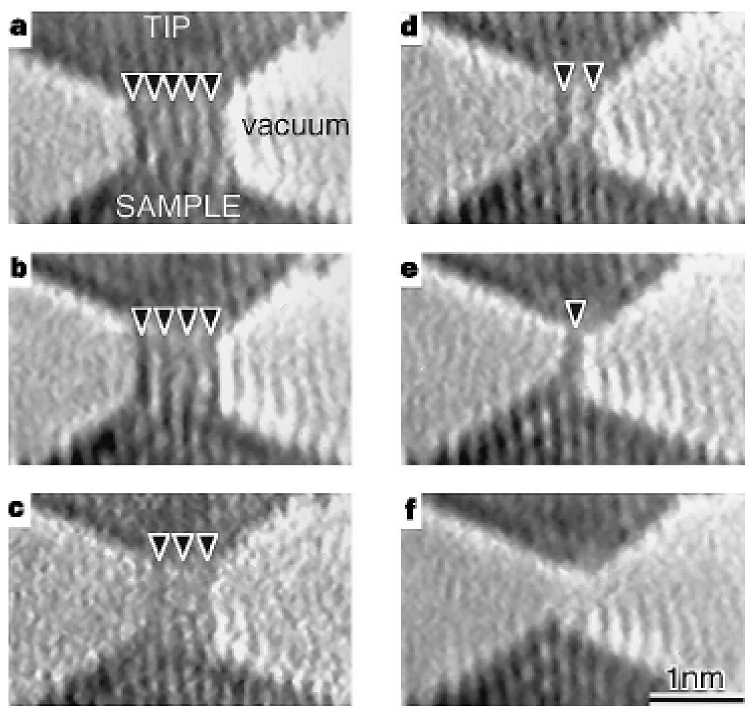
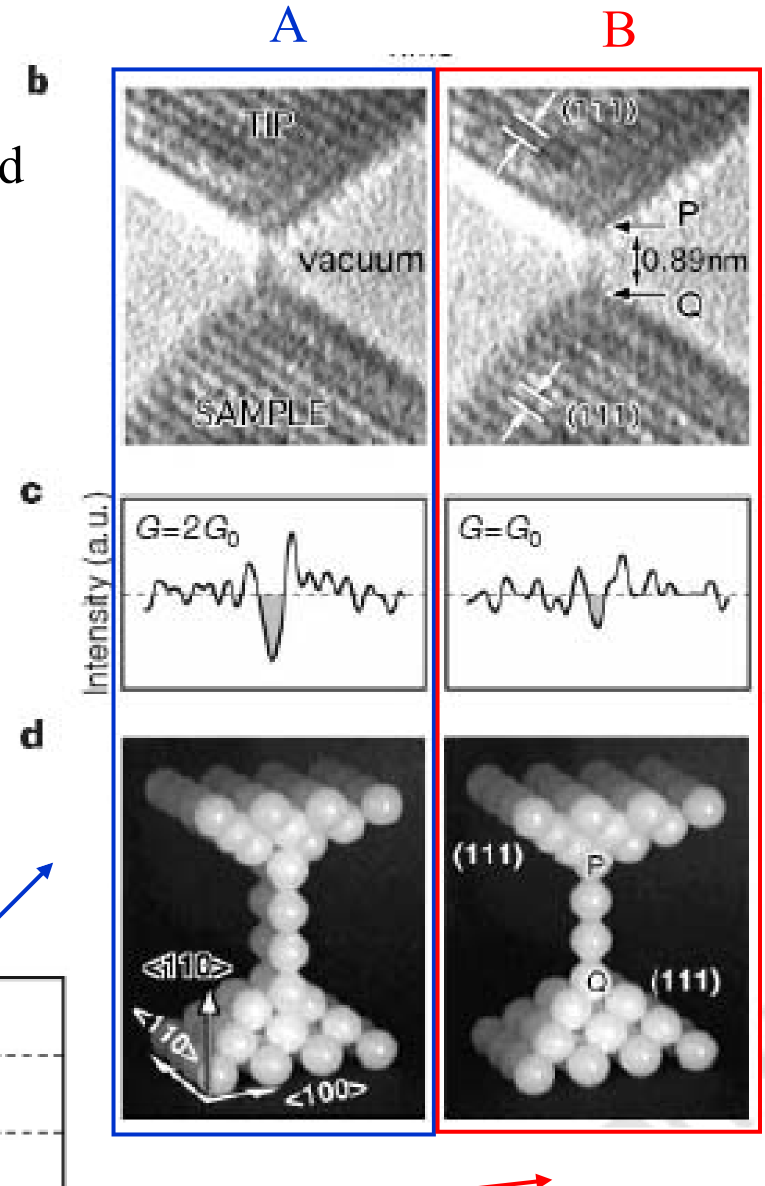
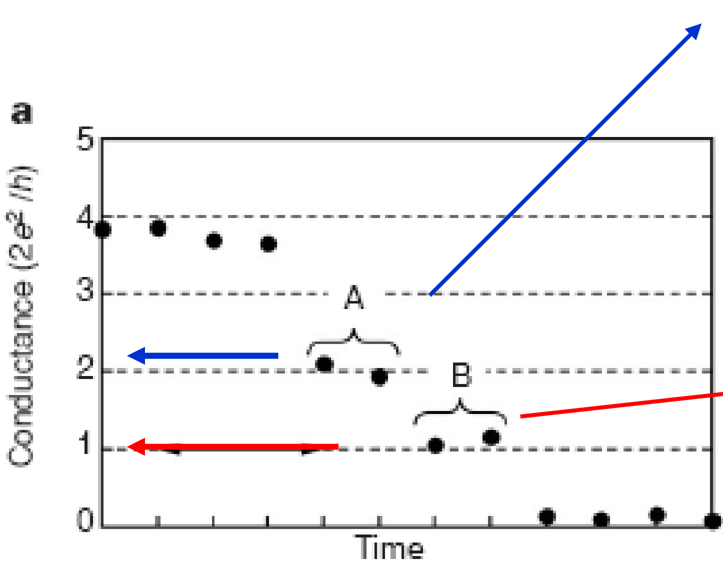


Figure 2 Electron microscope images of a contact while withdrawing the tip. A gold bridge formed between the gold tip (top) and gold substrate (bottom), thinned from **a** to **e** and ruptured at **f**, with observation times of 0, 0.47, 1.23, 1.33, 1.80 and 2.17 s, respectively. Dark lines indicated by arrowheads are rows of gold atoms. The faint fringe outside each bridge and remaining in **f** is a ghost due to interference of the imaging electrons. The conductance of the contact is 0 at **f** and ~$2 \times (13\ \text{k}\Omega)^{-1}$ at **e**. $V_b = -10$ mV and $R_F = 10$ kΩ.

The conductance is quantized

The conductance of a mono-atomic wire is $2e^2/h$ and that of a bi-atomic wire is twice as large





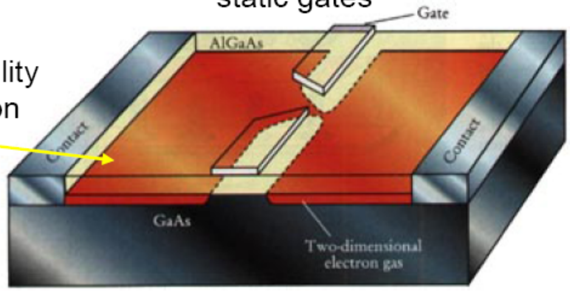
Quantized conductance of a single and a double strand of gold atoms. a) Conductance change of a contact while withdrawing the tip. Conductance is shown in units of $G_0 = 2e^2{=}h$. b) Electron microscope images of gold bridges obtained simultaneously with the conductance measurements in a). Left, bridge at step A; right, bridge at step B. c) Intensity profiles of the left and right bridges shown in b). The shaded area is the intensity from the bridge after subtraction of the background noise. d) Models of the left and right bridges. The bridge at step A has two rows of atoms; the bridge at step B has only one row of atoms.

Quasi 1D channel in 2D electron system

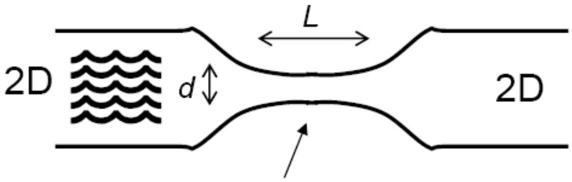
Phys. Rev. Lett. **60**, 848 (1988)

Depletion by electro-static gates

High mobility 2D electron gas ('ballistic')

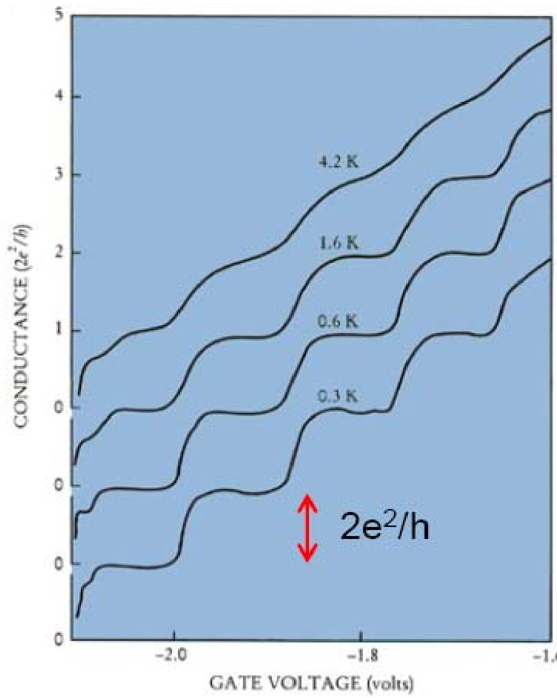


'Quantum Point Contact'



Narrow constriction; quasi-1D (width d ~ Fermi wavelength λ_F)

d=250nm; L=1000 nm



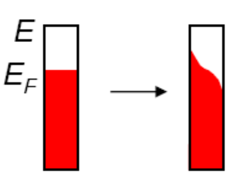
With *N* parallel 1D channels (subbands):

$$G(E_F) = \frac{2e^2}{h} \sum_n T_n(E_F) \ (T = 0)$$

Limited conductance $2e^2/h$ even without scattering, regardless of length *L*: "contact resistance"

Temperature effect

- Electrons populate leads according to Fermi-Dirac distribution:



$$f(E, E_F) = \frac{1}{\exp\left(\frac{E - E_F}{k_B T}\right) + 1}$$

- Conductance at finite temp. *T*:

$$G(E_F, T) = \frac{2e^2}{h} \sum_n \int T_n(E) \left(-\frac{df}{dE}\right) dE \approx \frac{2e^2}{h} \sum_n f(E_n - E_F)$$

eg. thermal smearing of conductance staircase

- Higher *T*: incoherent transport
  (dephasing due to inelastic scattering, phonons etc)

Coulomb blockade in quantum dots (0D)

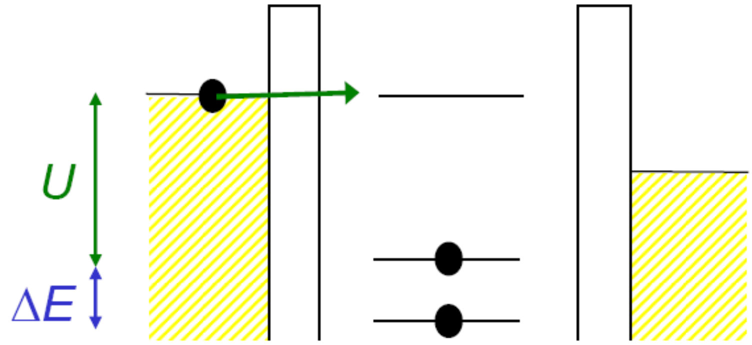
Rev. Mod. Phys. **74**, 1283 (2002)

Quantum dots
(artificial atoms)

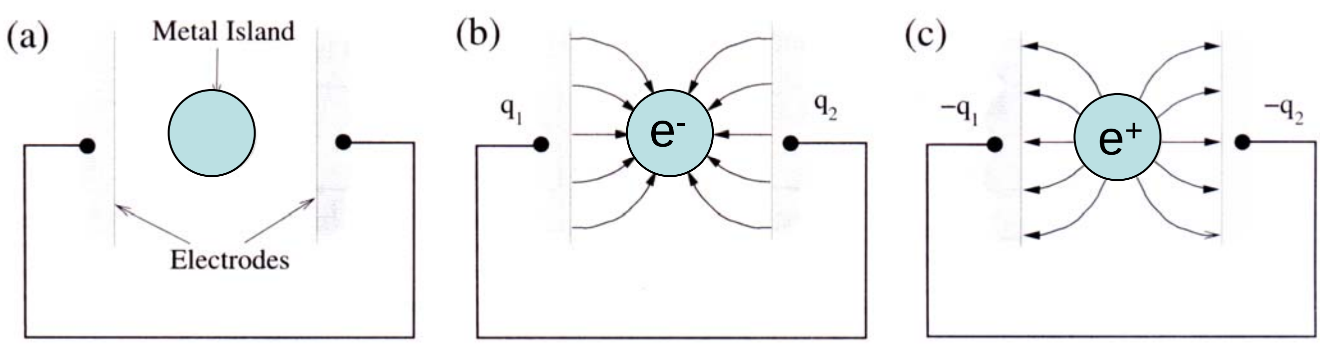
Two energy parameters:

U – 'charging energy' e^2/C
(e-e interaction strength)

ΔE – single-particle level spacing



Tunnel resistance >>h/e^2 -> dot fully decoupled from the electrode



a) A metal island embedded between electrodes which are electrically connected. Transferring an electron onto the island (b) or taking off the electron from the island (c) charges the capacitor formed by the island and the electrodes

Classical charging energy for a capacitor $E=q^2/2C=e^2N^2/2C$ for N electrons (C is the island capacitance)

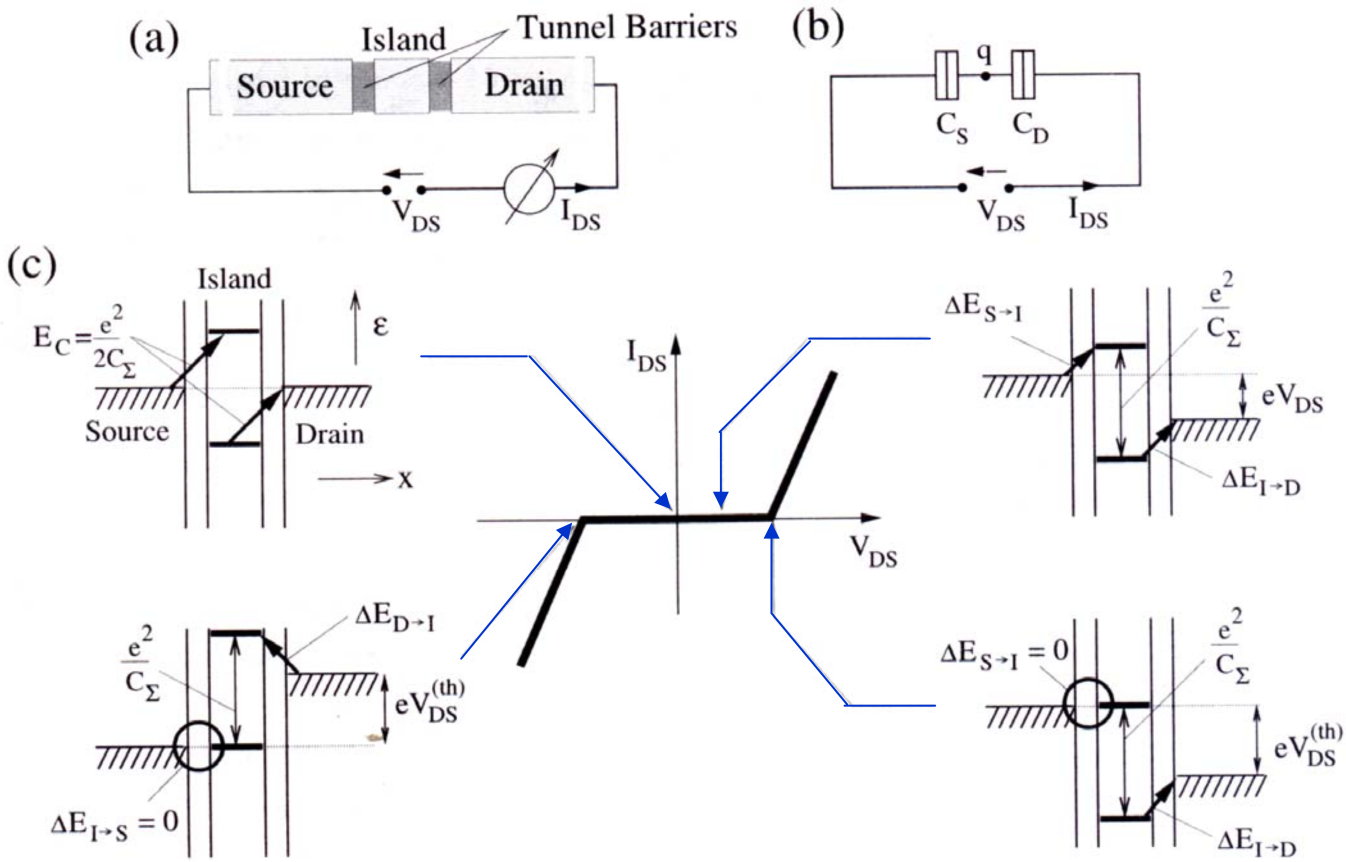
$$\mu(N+1) = E(N+1) - E(N) = \frac{e^2}{C}(N+\frac{1}{2})$$

$$\mu(N) = E(N) - E(N-1) = \frac{e^2}{C}(N-\frac{1}{2})$$

$$\boxed{\mu(N+1) - \mu(N) = \frac{e^2}{C}}$$

Additional energy to spend for adding one electron

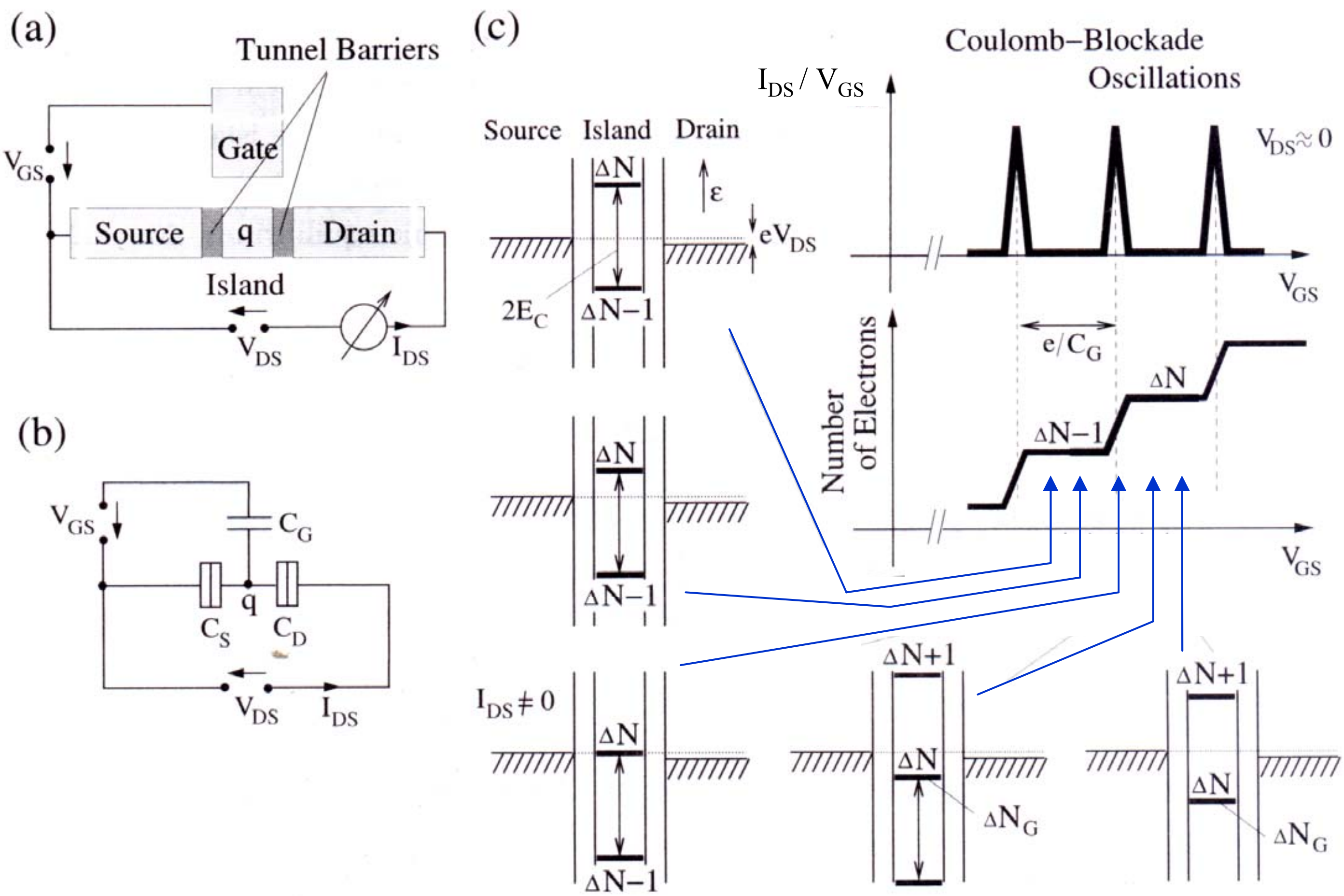
Two terminal arrangement



Since a current flow through the island requires the electron number to fluctuate at least by one electron, the energy barrier due to the charging energy inhibits transport for

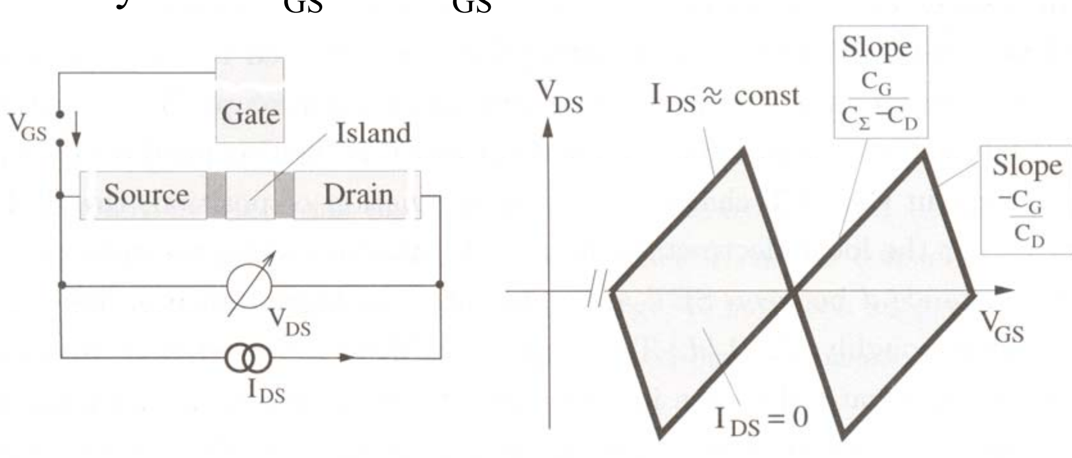
$E = eV_{DS} < e^2/C$ (Coulomb blockade)

Single electron transistor



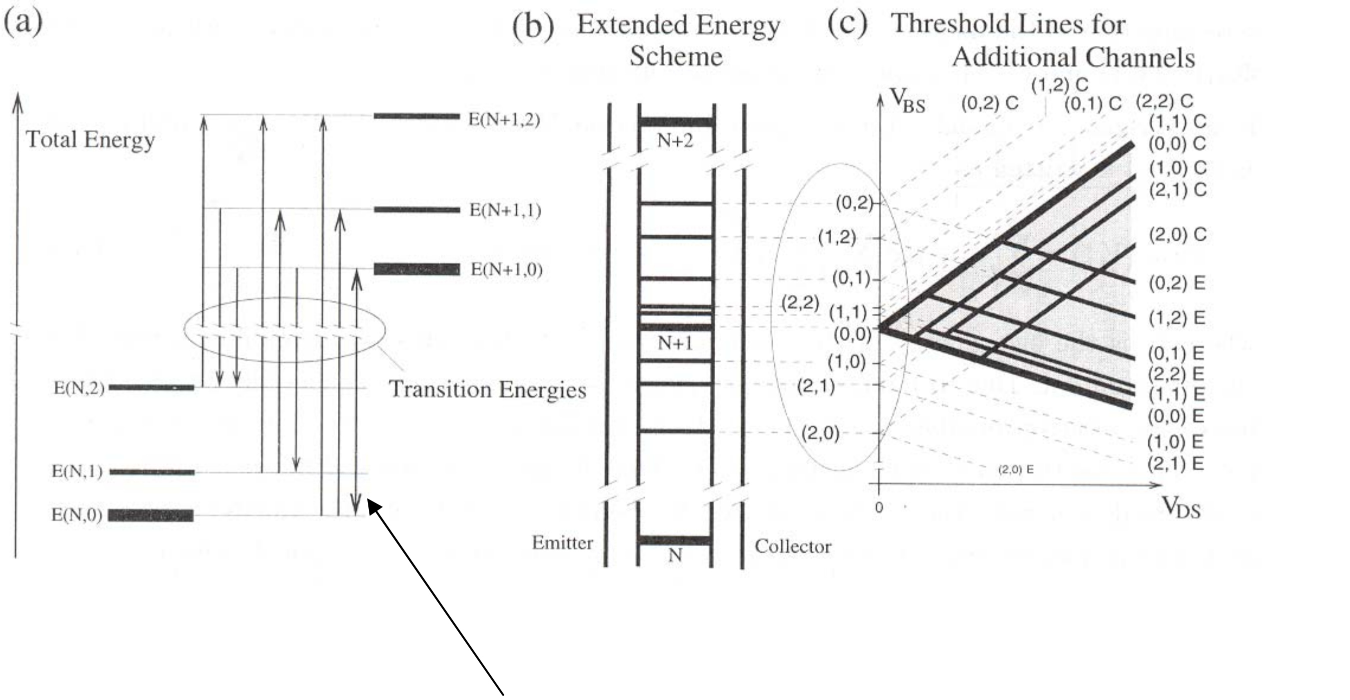
With increasing gate-source voltage eV_{GS} electrons are accumulated on the island. Whenever the charge state fluctuates by e^-, current I_{DS} flows for small applied voltage V_{DS} through the island, leading to a periodically modulated $I_{DS} - V_{GS}$ characteristic.

Actually both V_{GS} and V_{GS} can be used to control the current



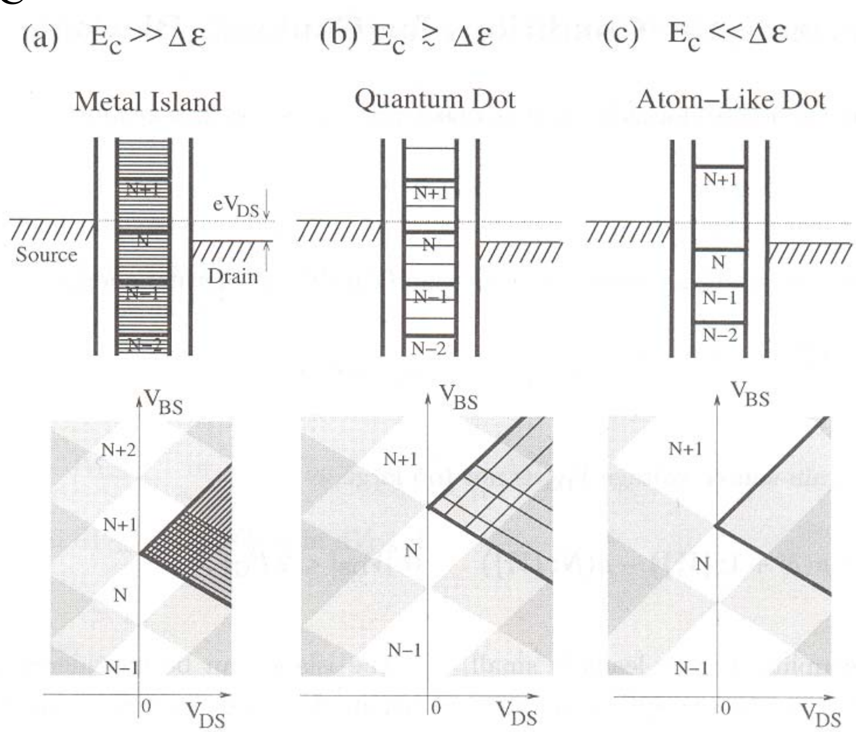
Competing channel in single electron tunneling through quantum dot

Fictitious total energy spectra of N and N+1 electrons confined in a quantum dot



Energy levels due to the confinement $\Delta\varepsilon$

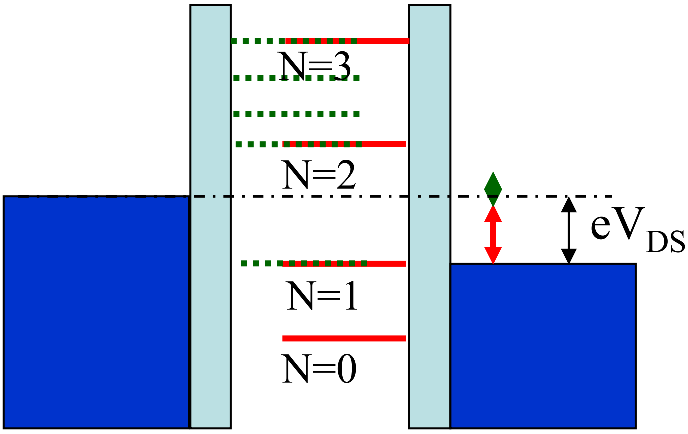
$$E(N+1,0)-E(N,0) = E_C = e^2/C$$



The measured spectrum depends on the dot size

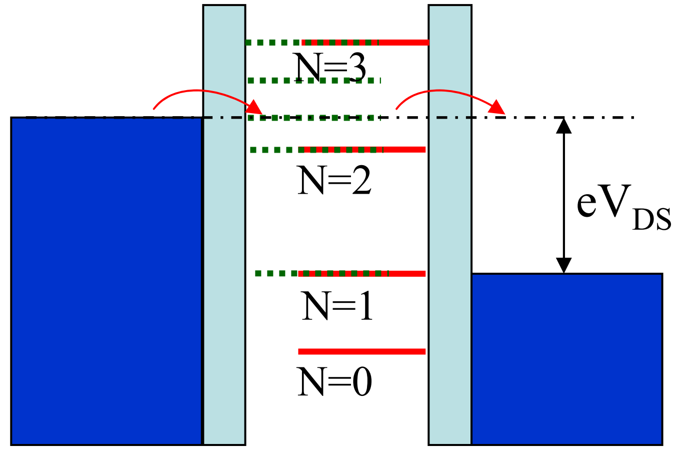
Coulomb diamonds

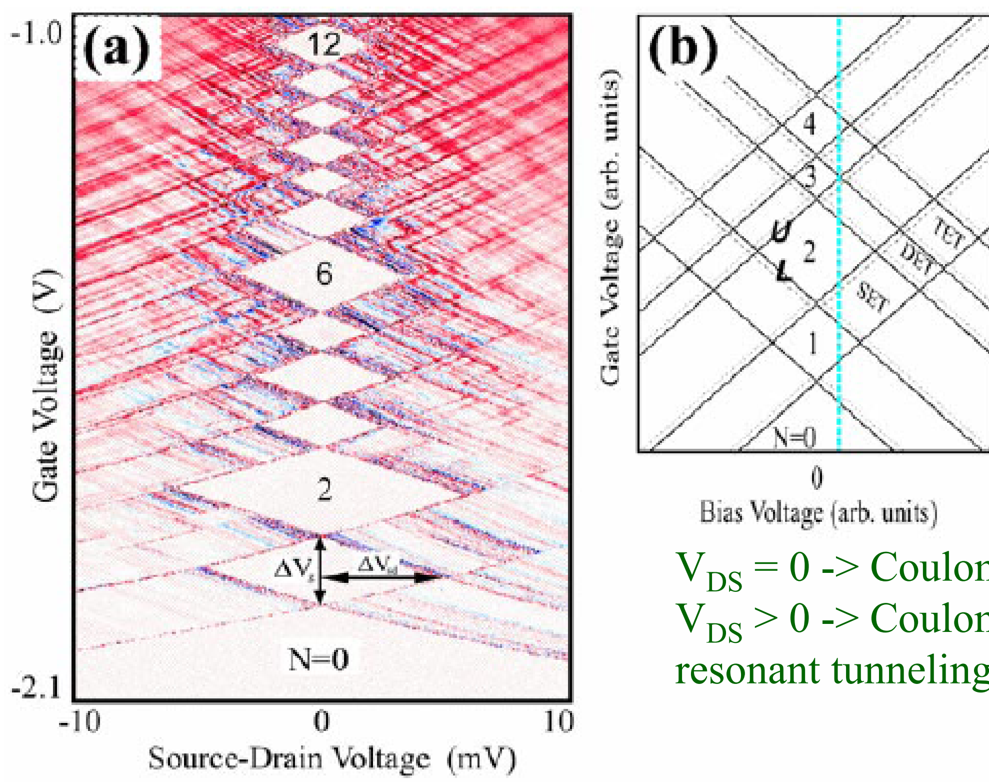
Coulomb blockade



$$\mu(N+1)-\mu(N)=\frac{e^2}{C}+\Delta E$$

Coulomb blockade + resonant tunneling





V_{DS} = 0 -> Coulomb blockade
V_{DS} > 0 -> Coulomb blockade + resonant tunneling

(*a*) Differential conductance, $\partial I/\partial V_{sd}$, plotted in a color scale in the plane of (V_g, V_{sd}) for $N = 0$–12 at $B = 0$. The white regions (i.e. the Coulomb diamonds) correspond to $\partial I/\partial V_{sd} \approx 0$, red indicates a positive $\partial I/\partial V_{sd}$, while blue indicates some regions of negative $\partial I/\partial V_{sd}$. (*b*) Schematic stability diagram. In the diamonds at non-zero bias voltages transport can take place via **single-electron resonant tunneling (SET),** double-electron resonant tunneling (DET), etc.

Competing Channels in Single-Electron Tunneling through a Quantum Dot

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(Received 3 August 1993)

Coulomb blockade effects are investigated in lateral transport through a quantum dot defined in a two-dimensional electron gas. Tunneling through excited states of the quantum dot is observed for various tunneling barriers. It is shown that transport occurring via transitions between ground states with different numbers of electrons can be suppressed by the occupation of excited states. Measurements in a magnetic field parallel to the current give evidence for tunneling processes involving states with different spin.

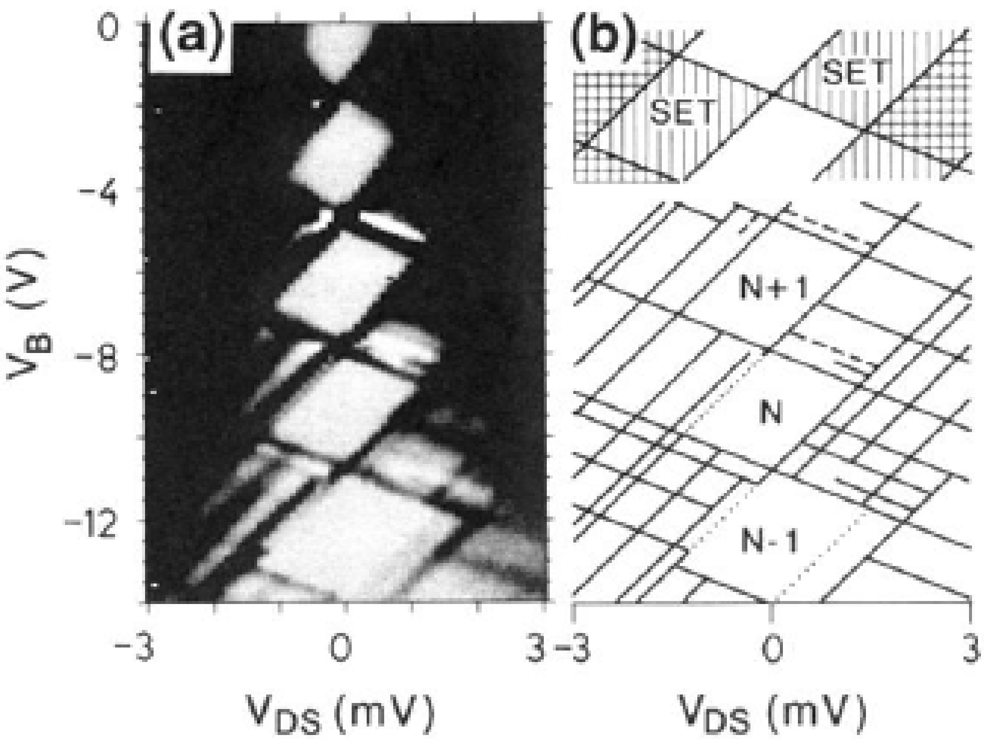
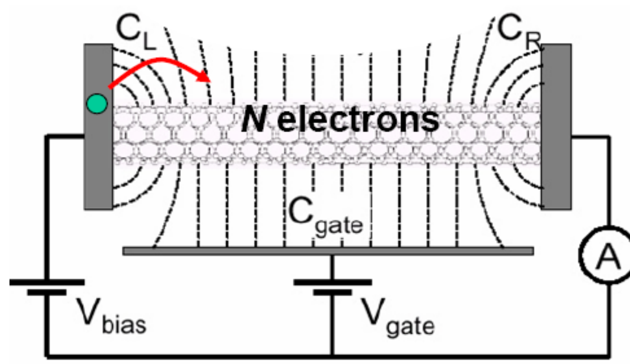


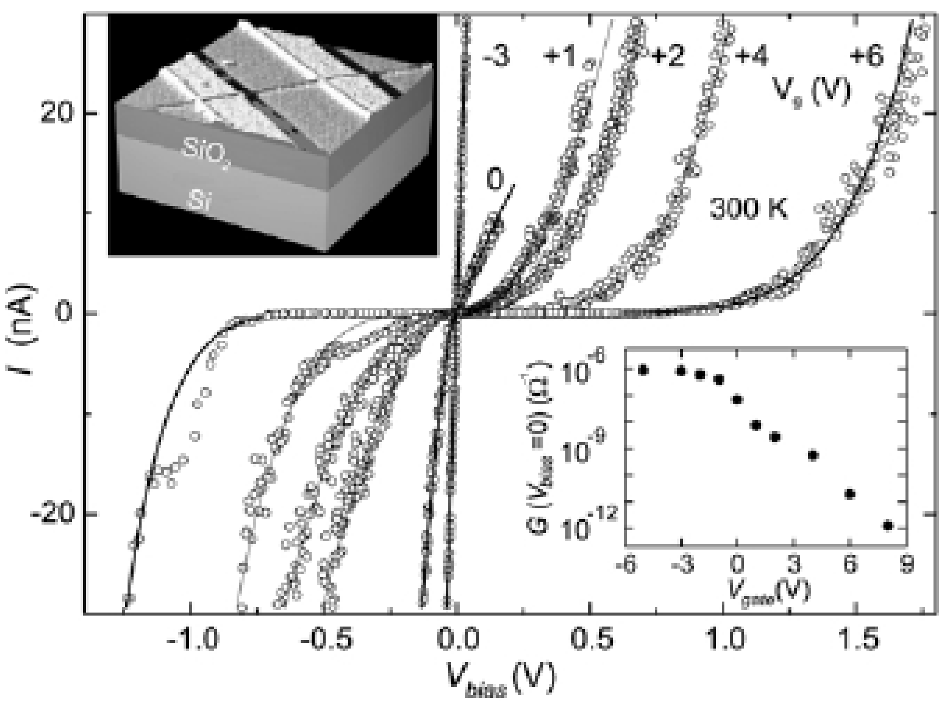
FIG. 2. (a) Differential conductance dI/dV_{DS} given in linear grey scale (white $\leq -0.1\ \mu$S, black $\geq 2\ \mu$S) as a function of back-gate voltage V_B for different bias voltages V_{DS}. (b) At the top: regions of SET are hatched, regions where the number of electrons can change by two at a time are cross-hatched. Lower part: The main structures visible in (a) are sketched. Dashed lines show regime of negative differential conductance; dotted lines show suppressed conductance.

Room-temperature transistor based on a single carbon nanotube

Sander J. Tans, Alwin R. M. Verschueren & Cees Dekker

Nature, **393**,49 (1998)





The nanotube transistor (‘TUBEFET’). Main: room-temperature $I-V$ traces measured in vacuum for a nanotube device at a series of gate voltages. *Left inset:* AFM image showing the nanotube lying across platinum electrodes. The gate voltage is applied to the *n*+ doped Si underneath the 300-nm-thick SiO2 layer. *Right inset:* Small-signal conductance *G* vs. *V*g

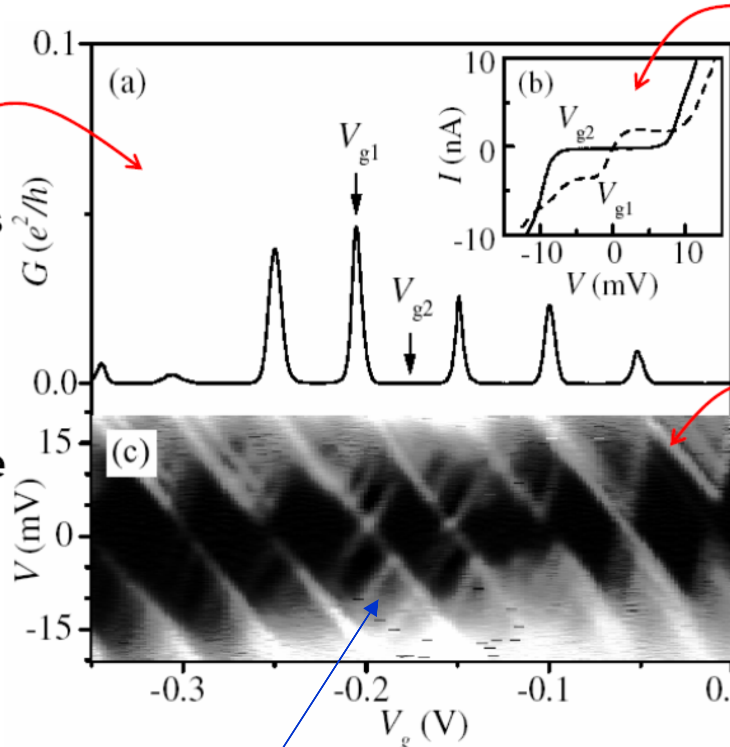
Coulomb diamonds

Actually both source-gate and source-drain voltages affect the conductance

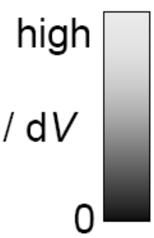
For a fixed gate voltage V_g, the source-drain voltage V can be used to open or close the conducting channel

Coulomb blockade peaks (zero bias)

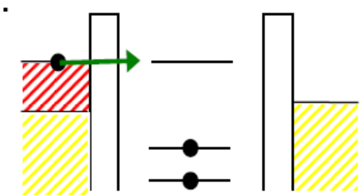
Nonlinear *I-V* curve (fixed gate volt.)



Differential conductance



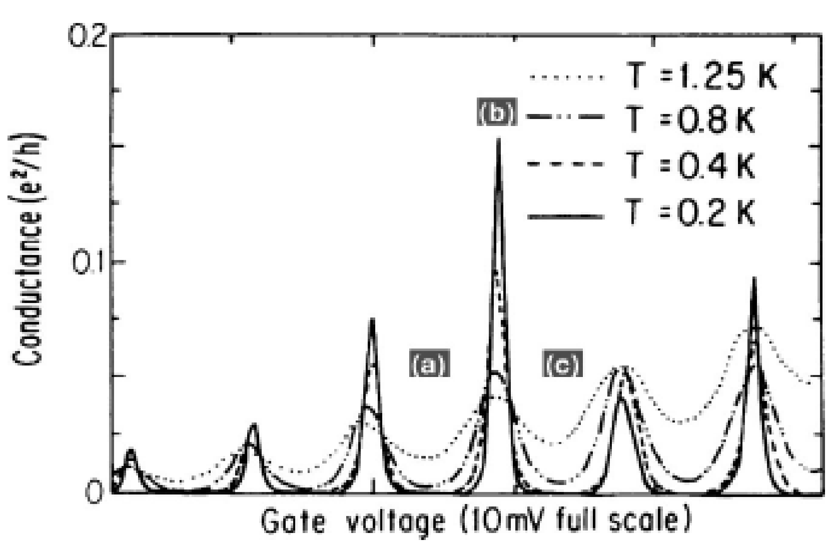
”Bias spectroscopy”, ”Coulomb diamonds”, …



Multi structure spectrum due to charging and confinement energy levels

To observe the Coulomb blockade $kT < e^2/C$

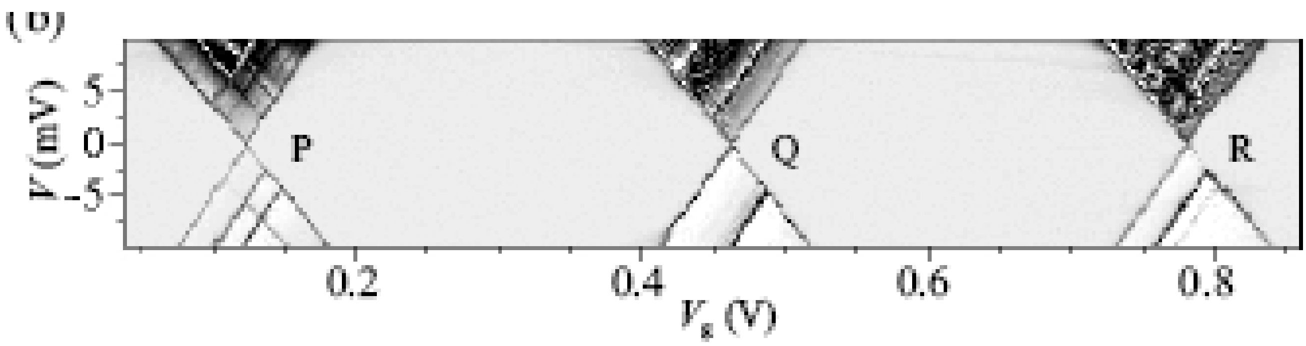
e^2/C = 1-10 meV



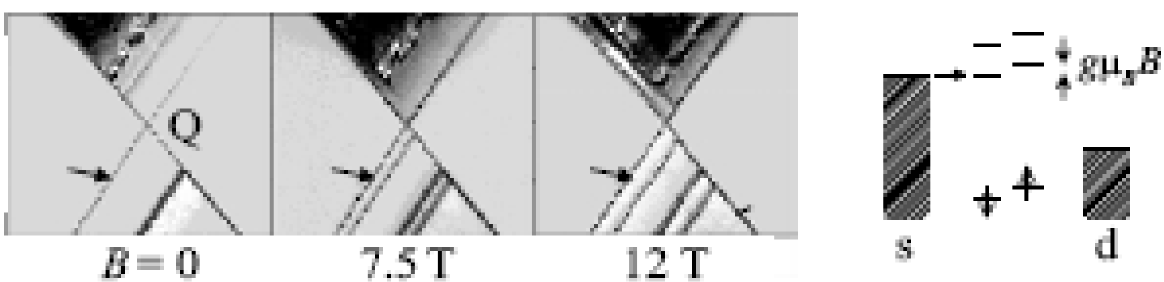
Coulomb blockade in a magnetic field

Appl. Phys. A **69**, 297 (1999)

T = 100 mK



(c)



Zeeman splitting: 1/2gμB-(-1/2gμB)=gμB

Two peaks in the conductance separated by gμB

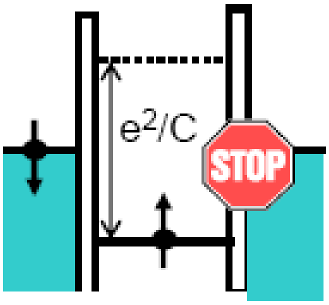
gμ = 0.11 meV/T ⇨ For B = 7.5 T
gμ B= 0.8 meV

T<< gμ B/k = 9 K

Kondo effect

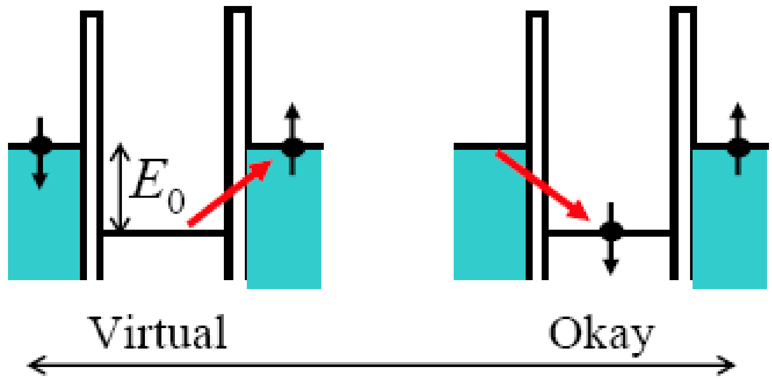
Physics World, 33 January 2001

Normal ground state:



Coulomb blockade

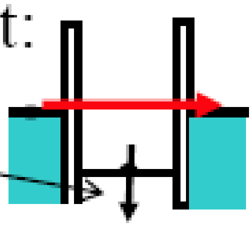
But, imagine ….



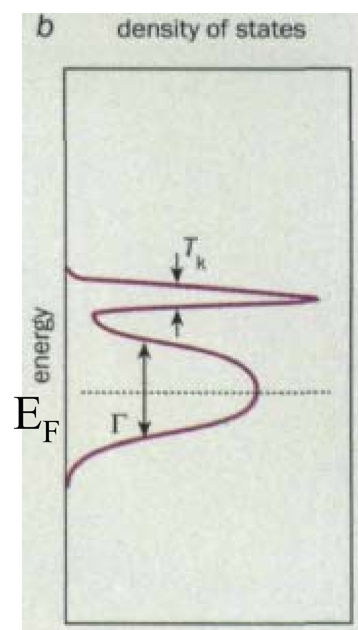
Heisenberg: $\Delta t \sim h/E_0$

Net result:

- transport→
- spin flip



Many such events combine to produce the Kondo effect, which leads to the appearance of an extra resonance very close to the Fermi level



Kondo temperature

$$k_B T_K = \frac{\sqrt{\Gamma U}}{2} \exp\left[-\frac{\pi (E_F - E_0)(U + E_0 - E_F)}{\Gamma U} \right]$$

Γ is the width of the energy level affected by the tunneling process; $U = e^2/C$

The conductance depends on the electron number

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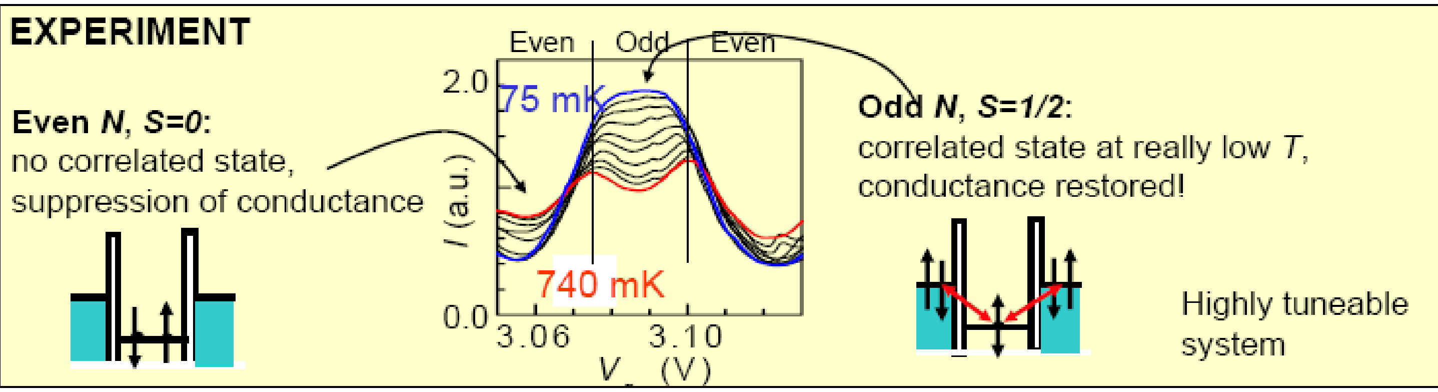
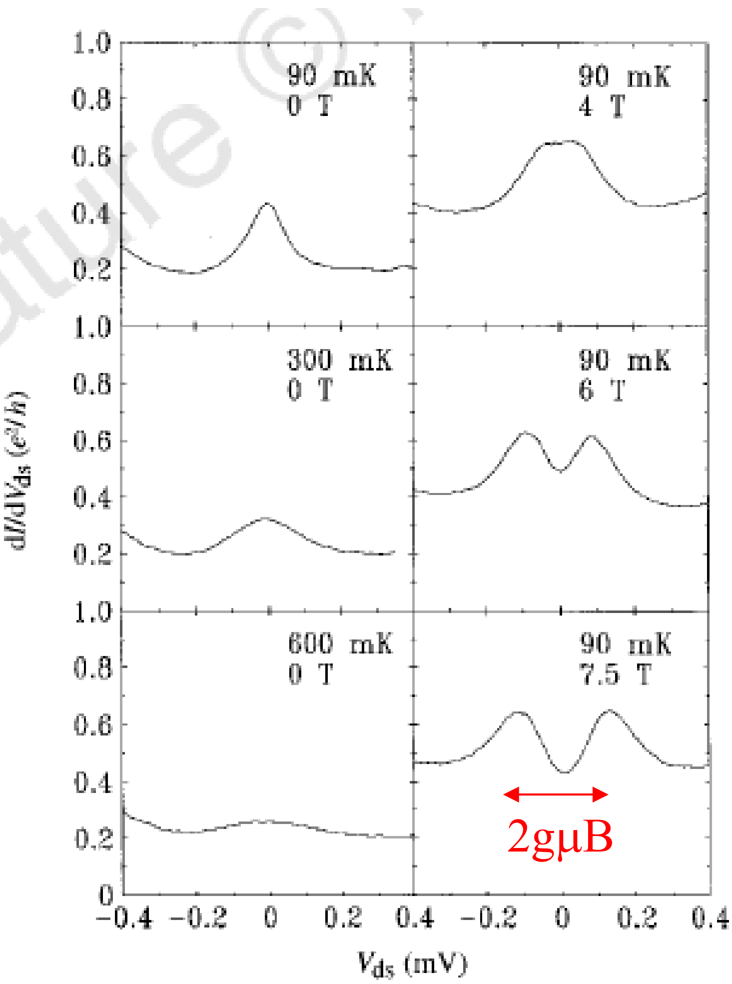
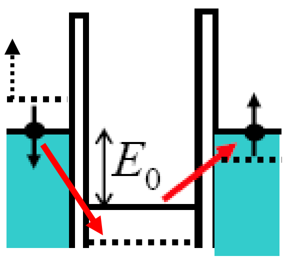
Kondo resonance makes it easier for states belonging to the two opposite electrodes to mix. This mixing increases the conductance (i.e. decreases the resistance).





Temperature and magnetic field dependence of the zero-bias Kondo resonance measured in differential conductance. Increasing temperature suppresses the resonance, whereas increasing magnetic field causes it to split into a pair of resonances at finite bias.



$$eV_{DS} = g\mu B$$

Nature **391**, 156 (1998)