

# Solar Photovoltaics & Energy Systems

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Lecture 1 – The nature of Solar irradiation,  
thermodynamics of solar energy conversion, solar  
thermal limitations

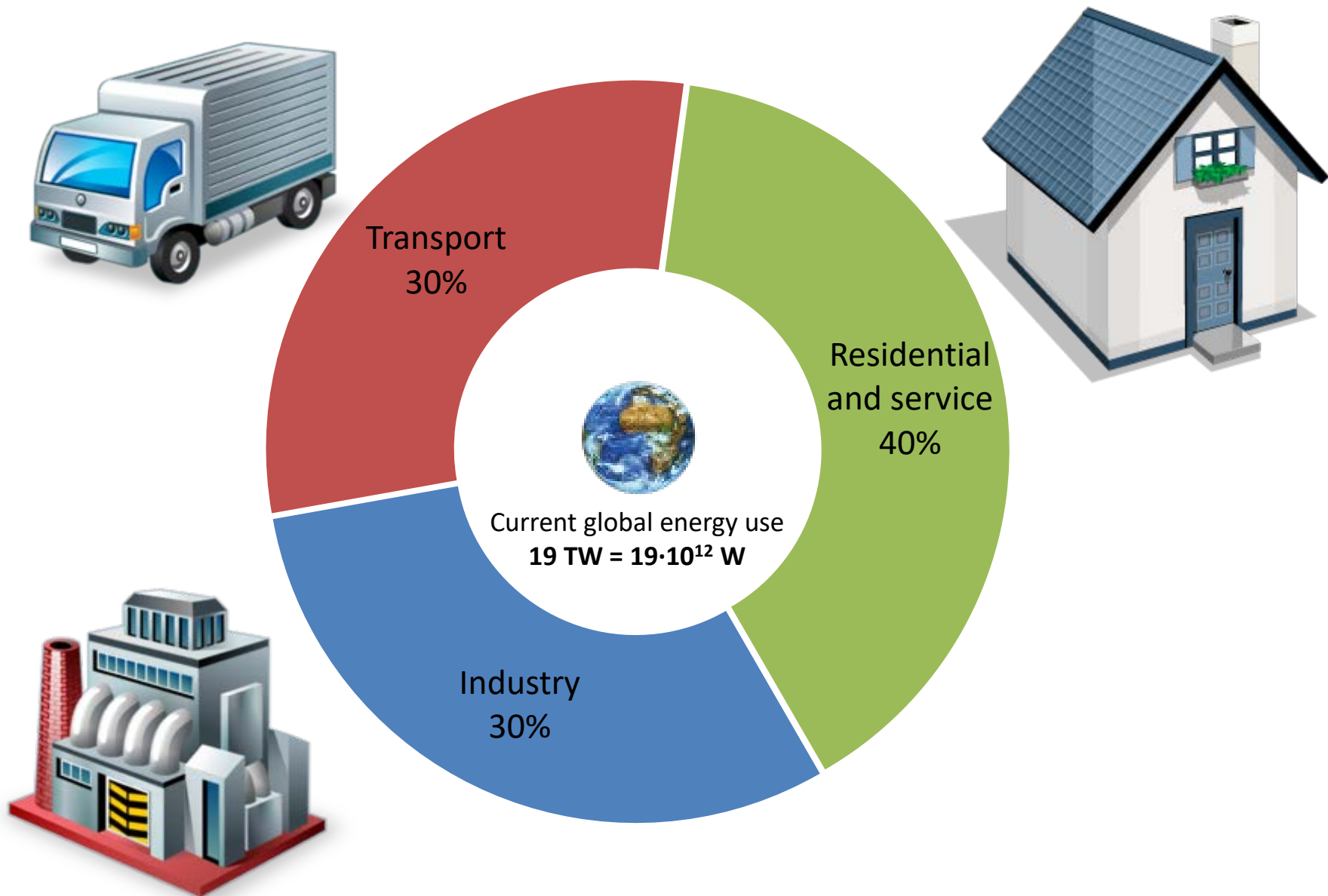
ChE-600

Kevin Sivula, Spring 2018

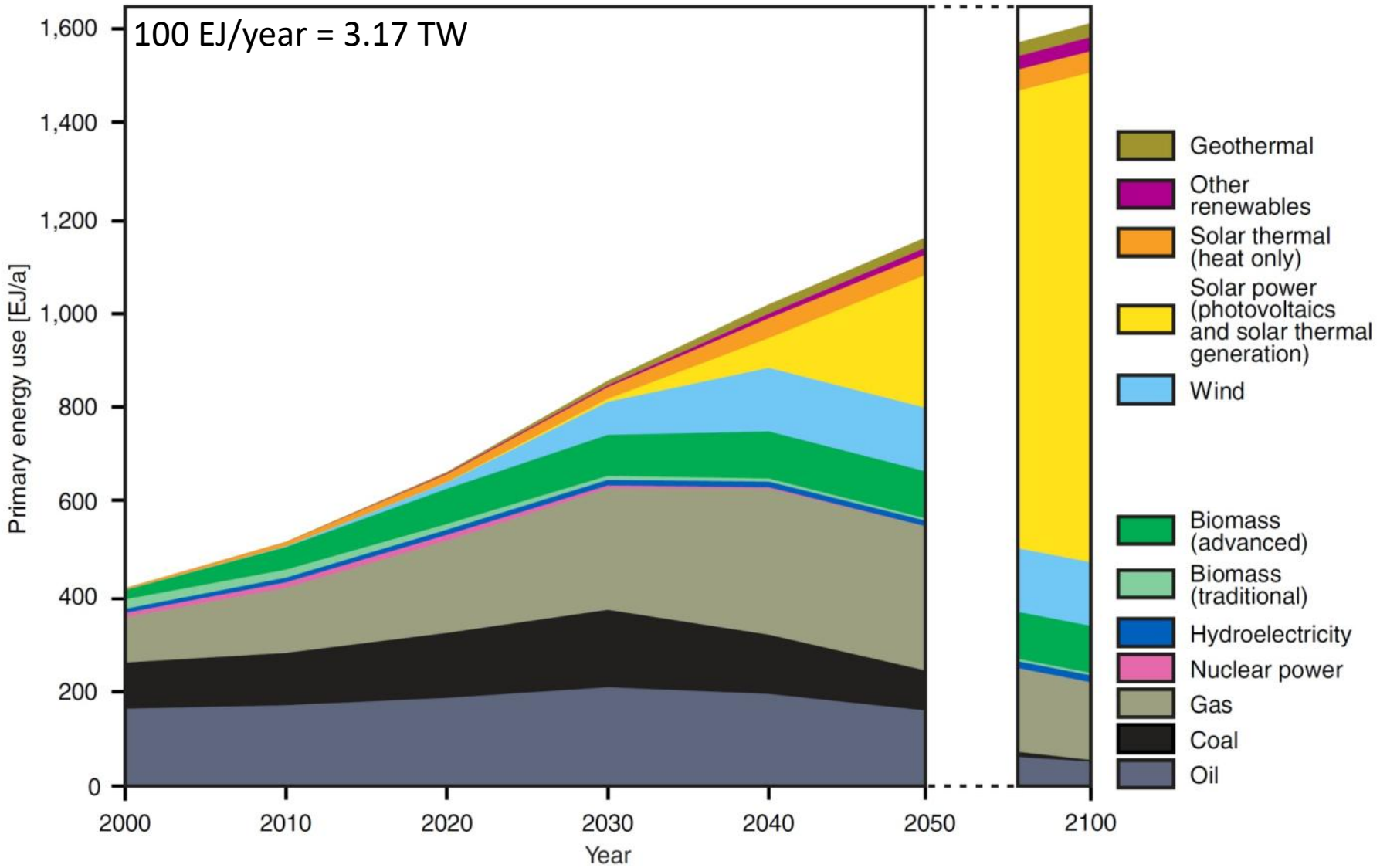


ÉCOLE POLYTECHNIQUE  
FÉDÉRALE DE LAUSANNE

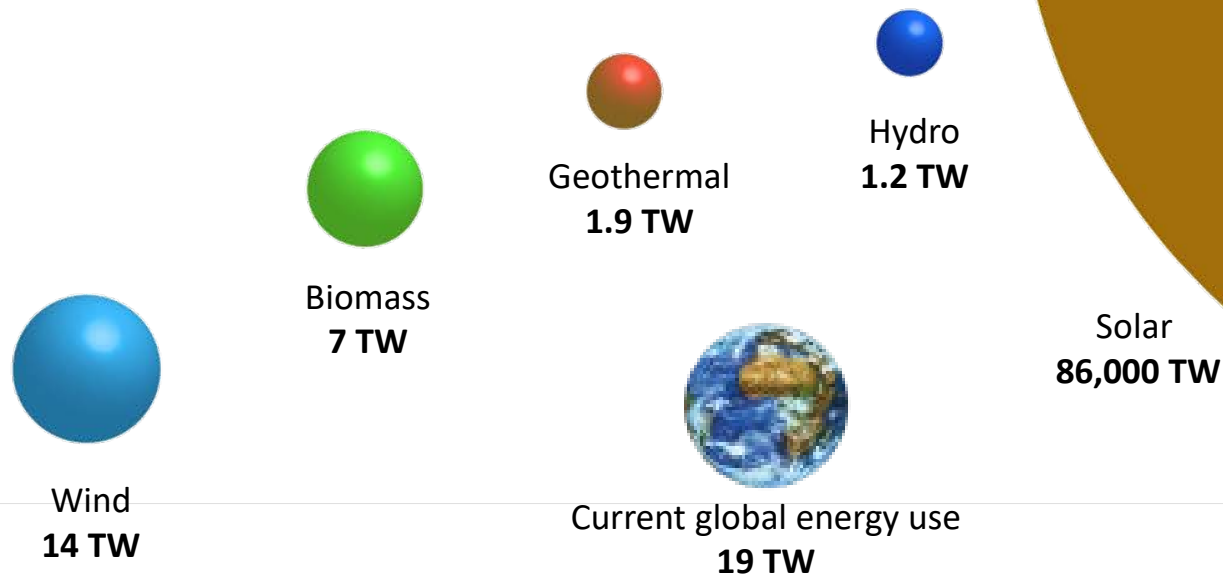
# Energy use today and in the future



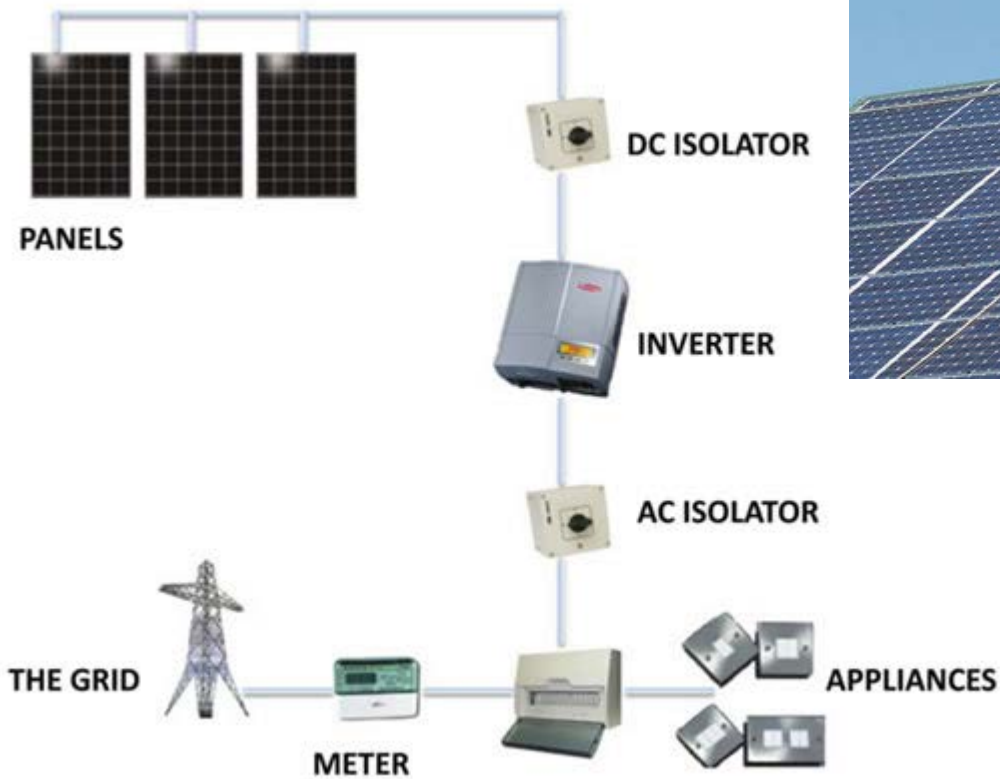
# Energy use today and in the future



# Energy use today and in the future

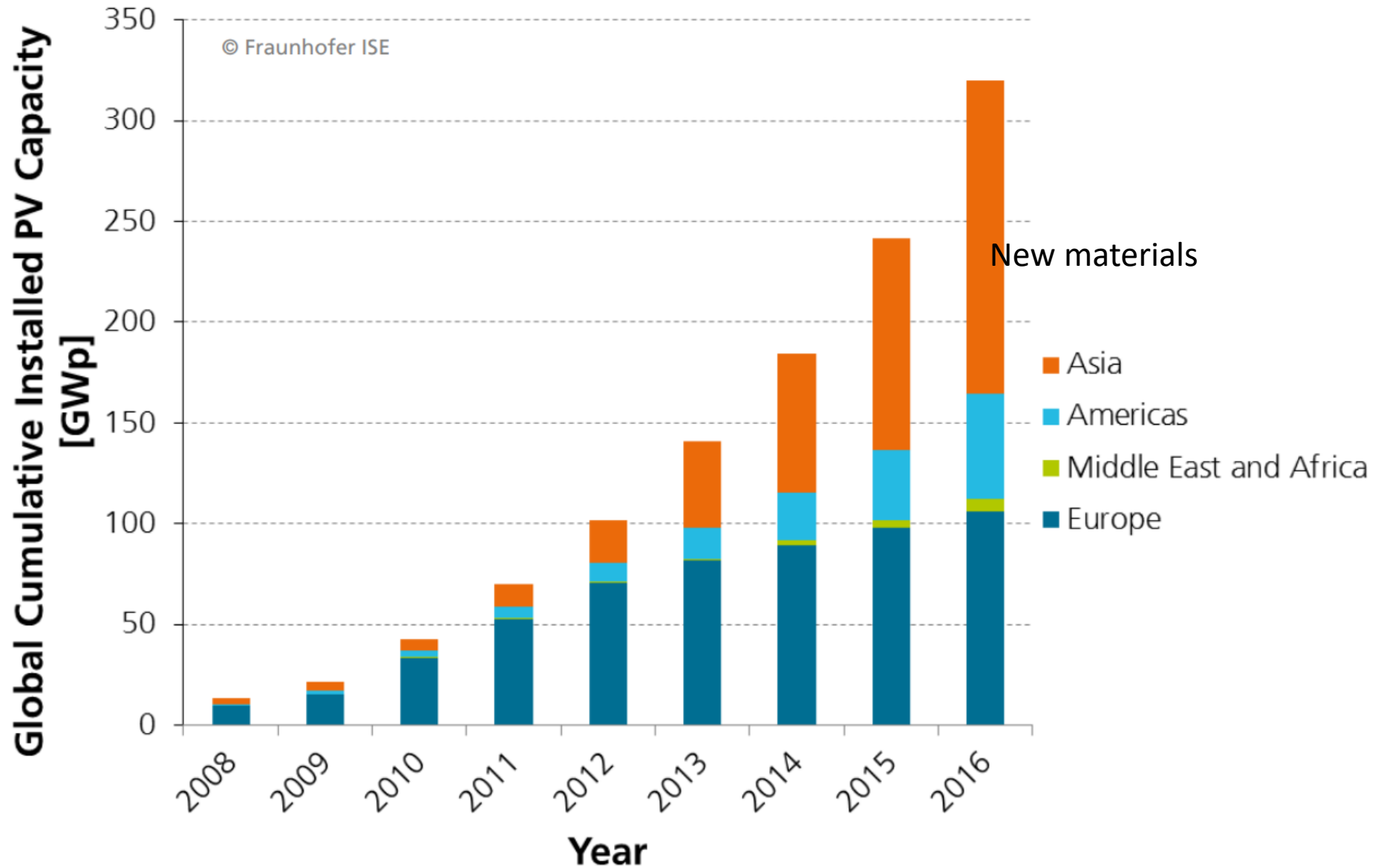


# Modern Si photovoltaic installations



**7.7MW in Rion-des-Landes**

# Growth of photovoltaic installations

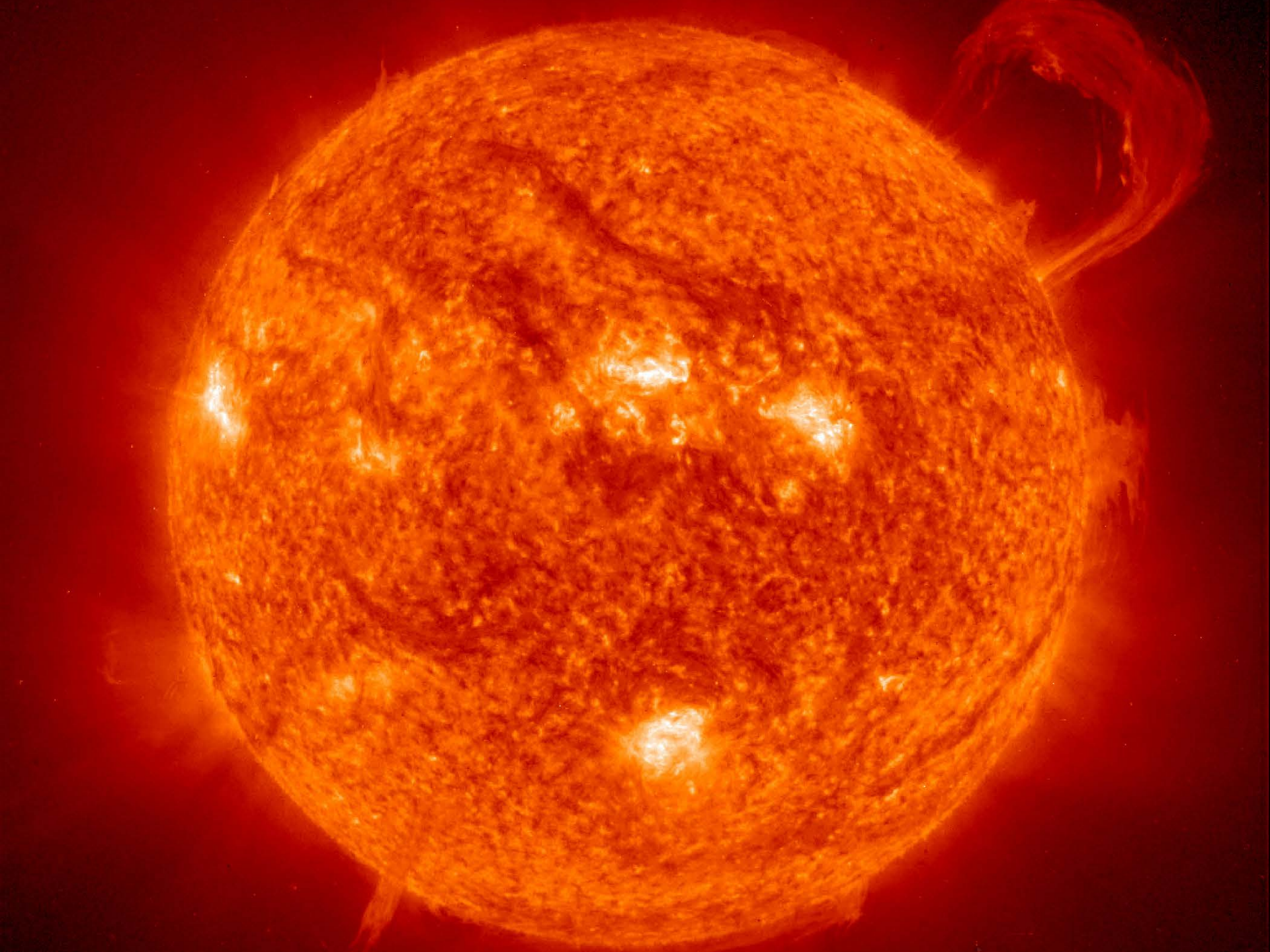


# Course Schedule

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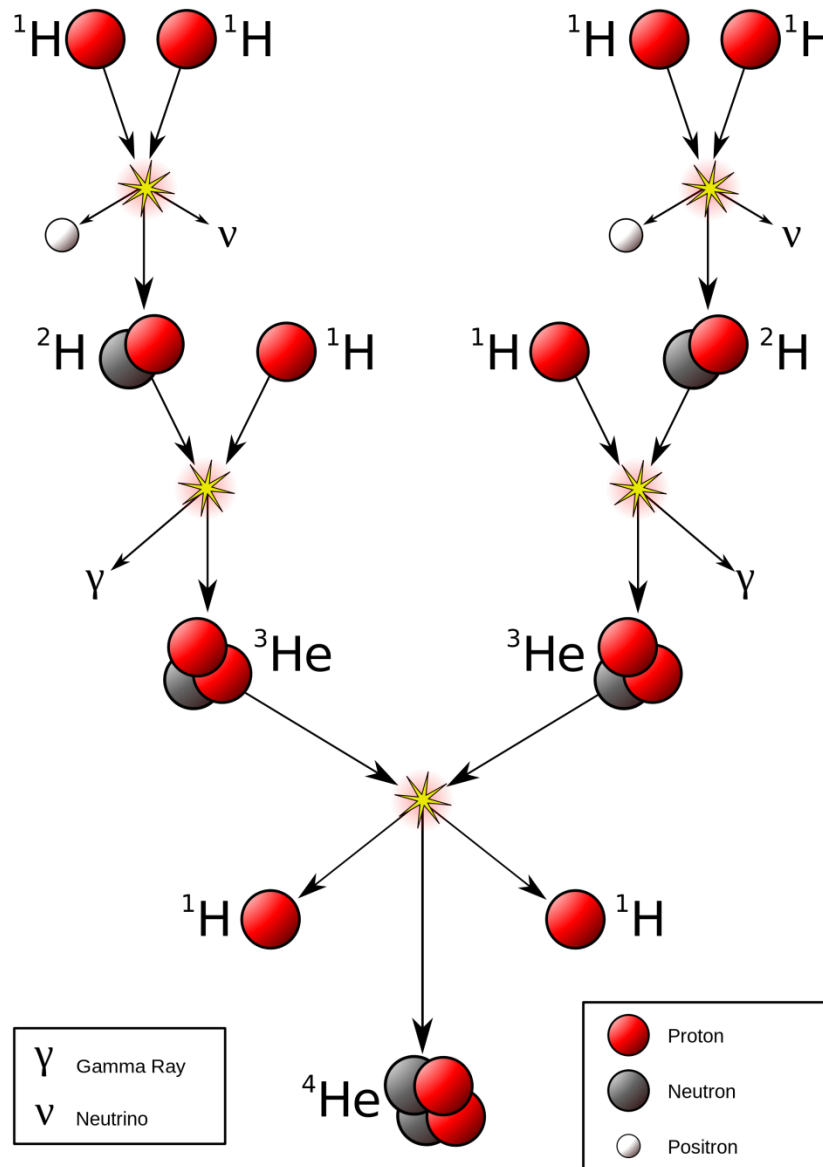
<b>Date</b>	<b>Lecturer</b>	<b>Topic</b>
27-02-18	Kevin Sivula	The nature of Solar irradiation, thermodynamics of solar energy conversion, solar thermal approaches
01-03-18	Kevin Sivula	Thermodynamics of band-gap based solar energy conversion, the SQ limit
06-03-18	Wolfgang Tress	pn Junctions, Silicon Solar Cells, and III-V materials
08-03-18	Wolfgang Tress	Heterojunction, Organic and Dye Sensitized Solar Cells
13-03-18	Wolfgang Tress	Perovskite and Third Generation Solar Cells
15-03-18	Néstor Guijarro	Solar-to-fuel conversion, motivation and approaches
20-03-18	Néstor Guijarro	Direct solar to fuel with photoelectrochemical and photocatalytic systems
22-03-18	Student presentations	Advanced strategies and materials (Literature Project)
27-03-18	Student presentations	Advanced strategies and materials (Literature Project)
29-03-18	Student presentations	Advanced strategies and materials (Literature Project)

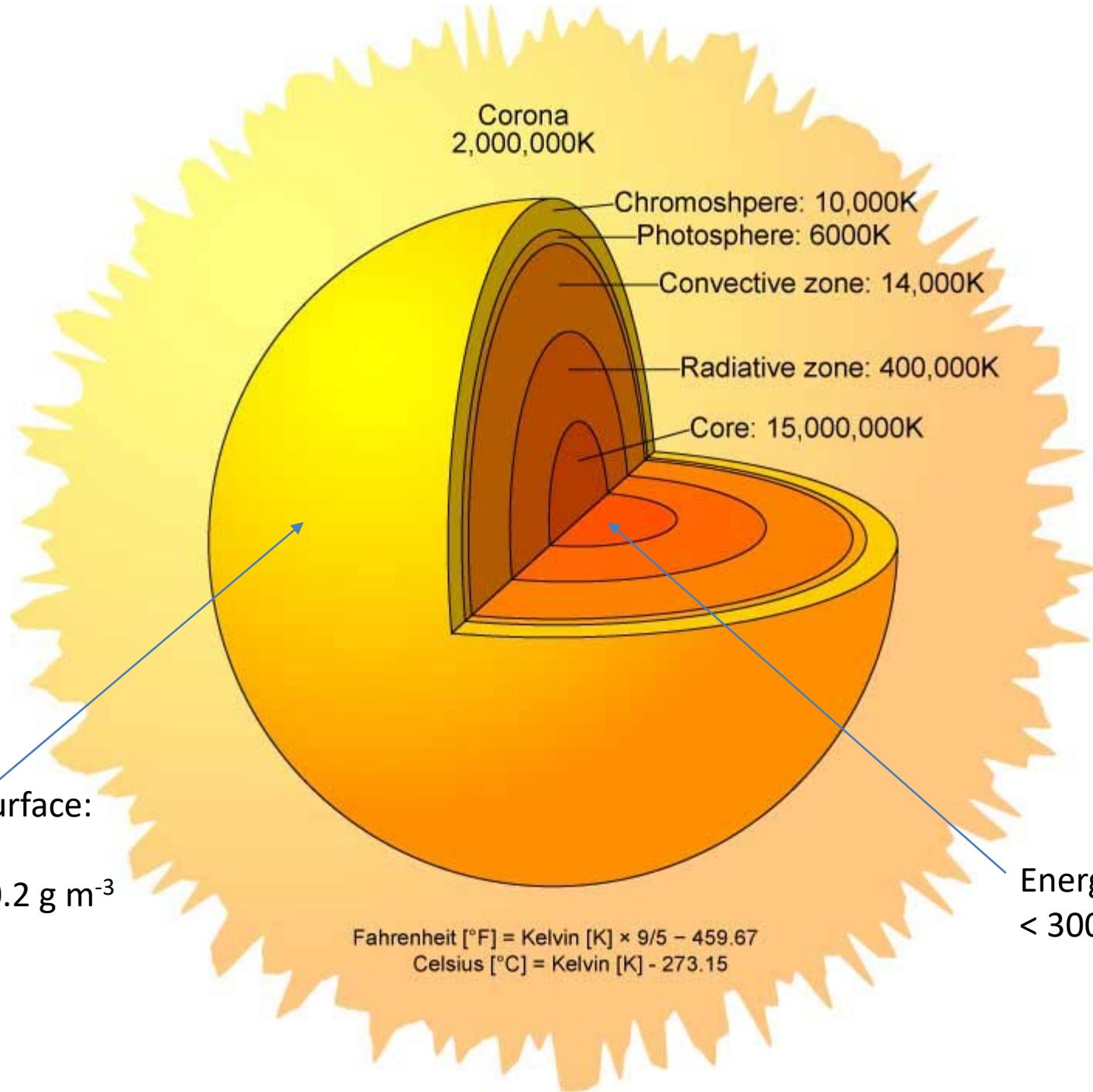
URL: [https://go.epfl.ch/ChE\\_600\\_2018](https://go.epfl.ch/ChE_600_2018)





# Why do stars “shine”?





Corona  
2,000,000K

Chromosphere: 10,000K

Photosphere: 6000K

Convective zone: 14,000K

Radiative zone: 400,000K

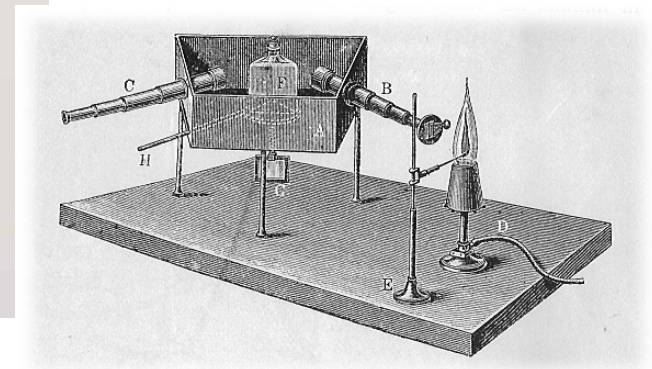
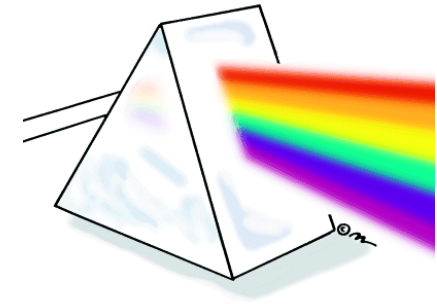
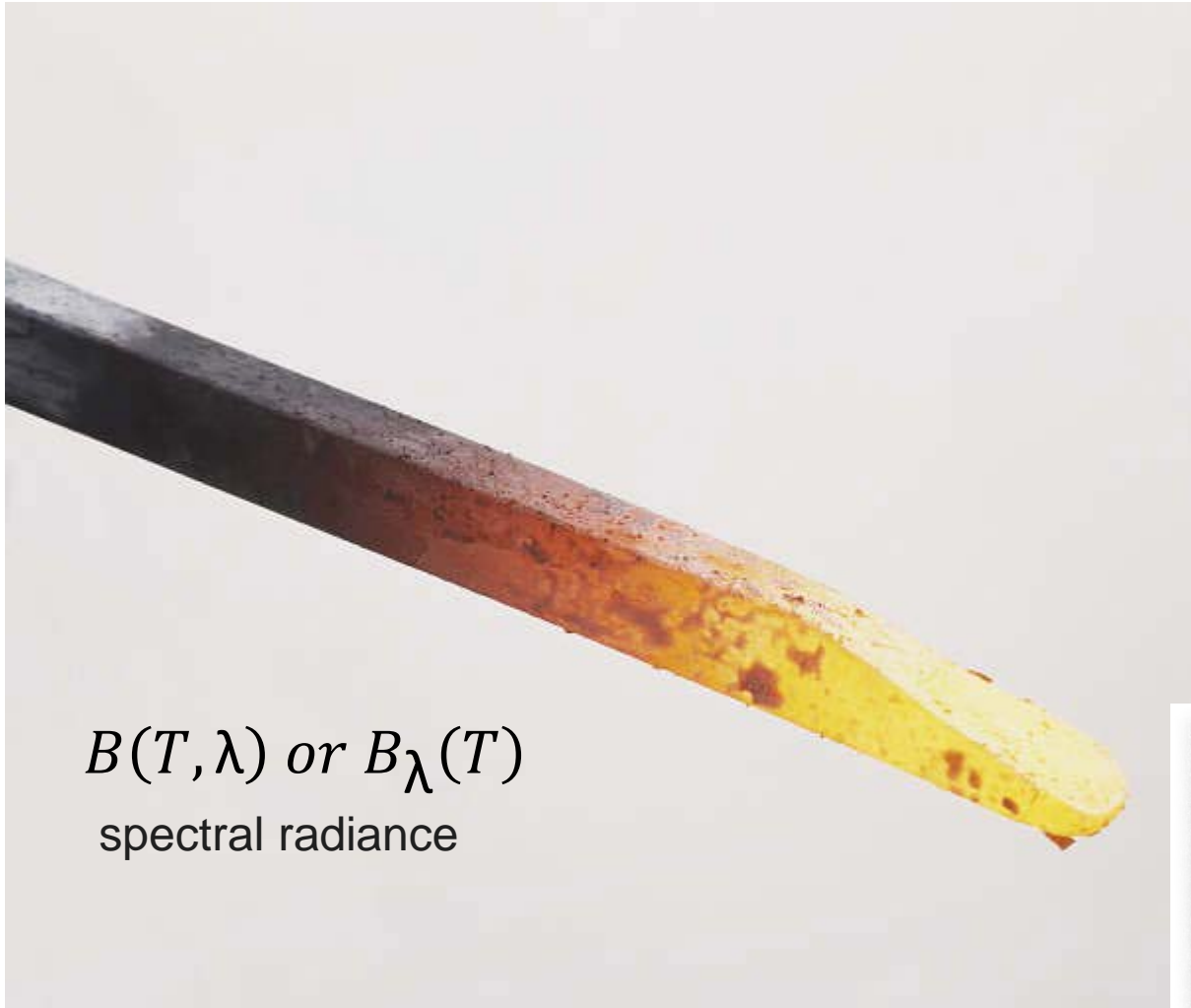
Core: 15,000,000K

Visible surface:  
5700 K  
density  $0.2 \text{ g m}^{-3}$

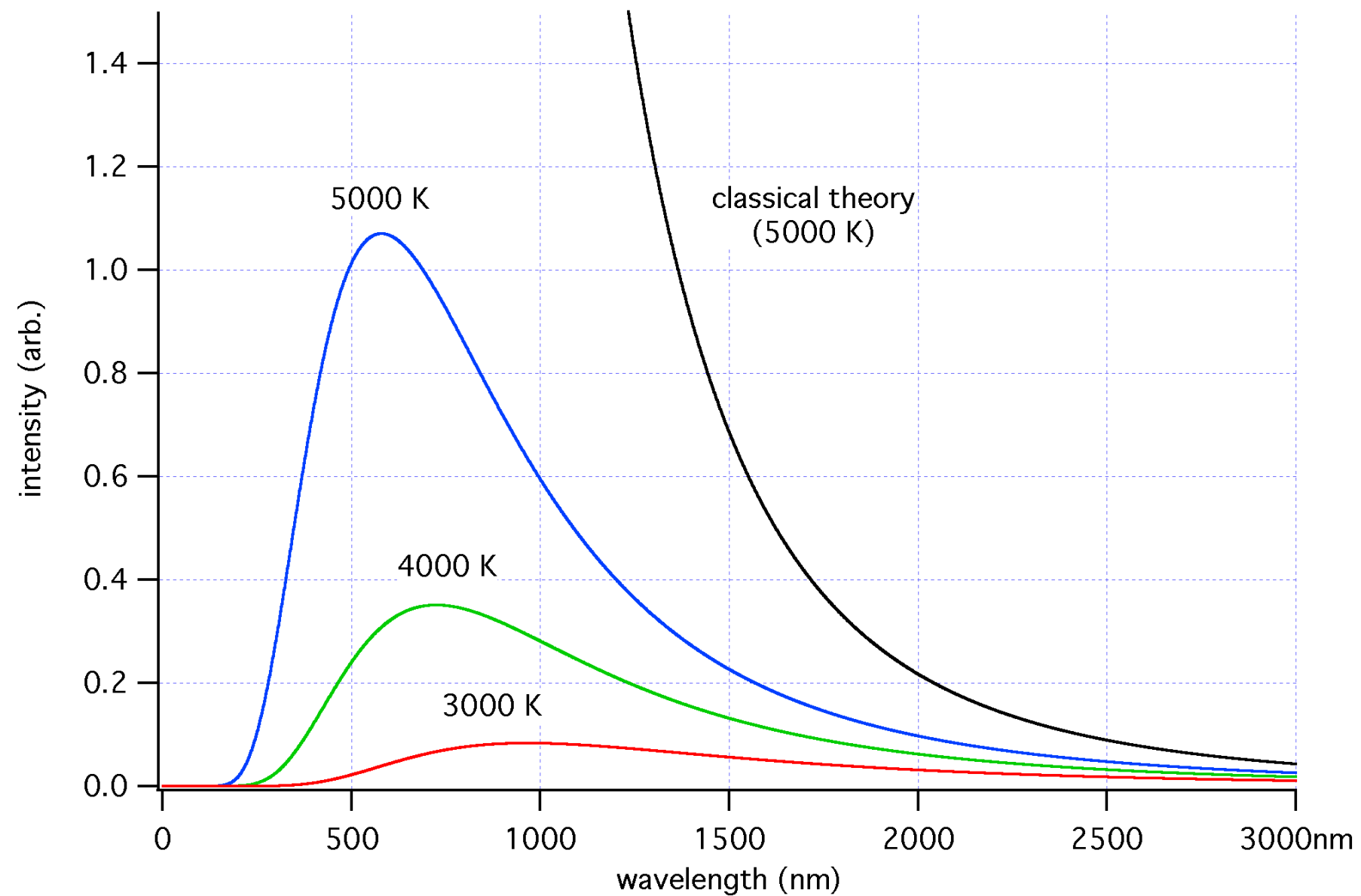
Energy production  
 $< 300 \text{ W m}^{-3}$

Fahrenheit [°F] = Kelvin [K]  $\times 9/5 - 459.67$   
Celsius [°C] = Kelvin [K] - 273.15

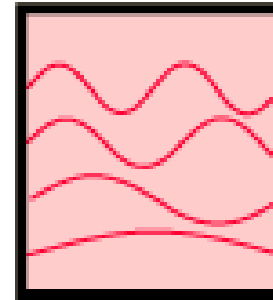
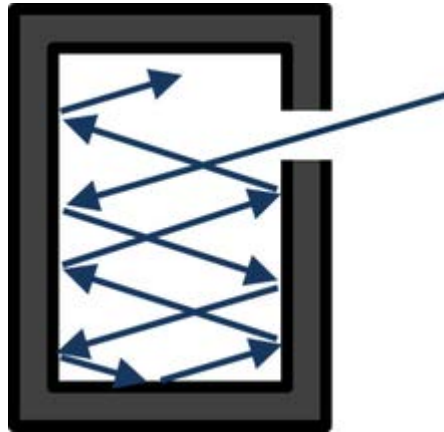
# Blackbody radiation



# Classical theory vs. experimental results

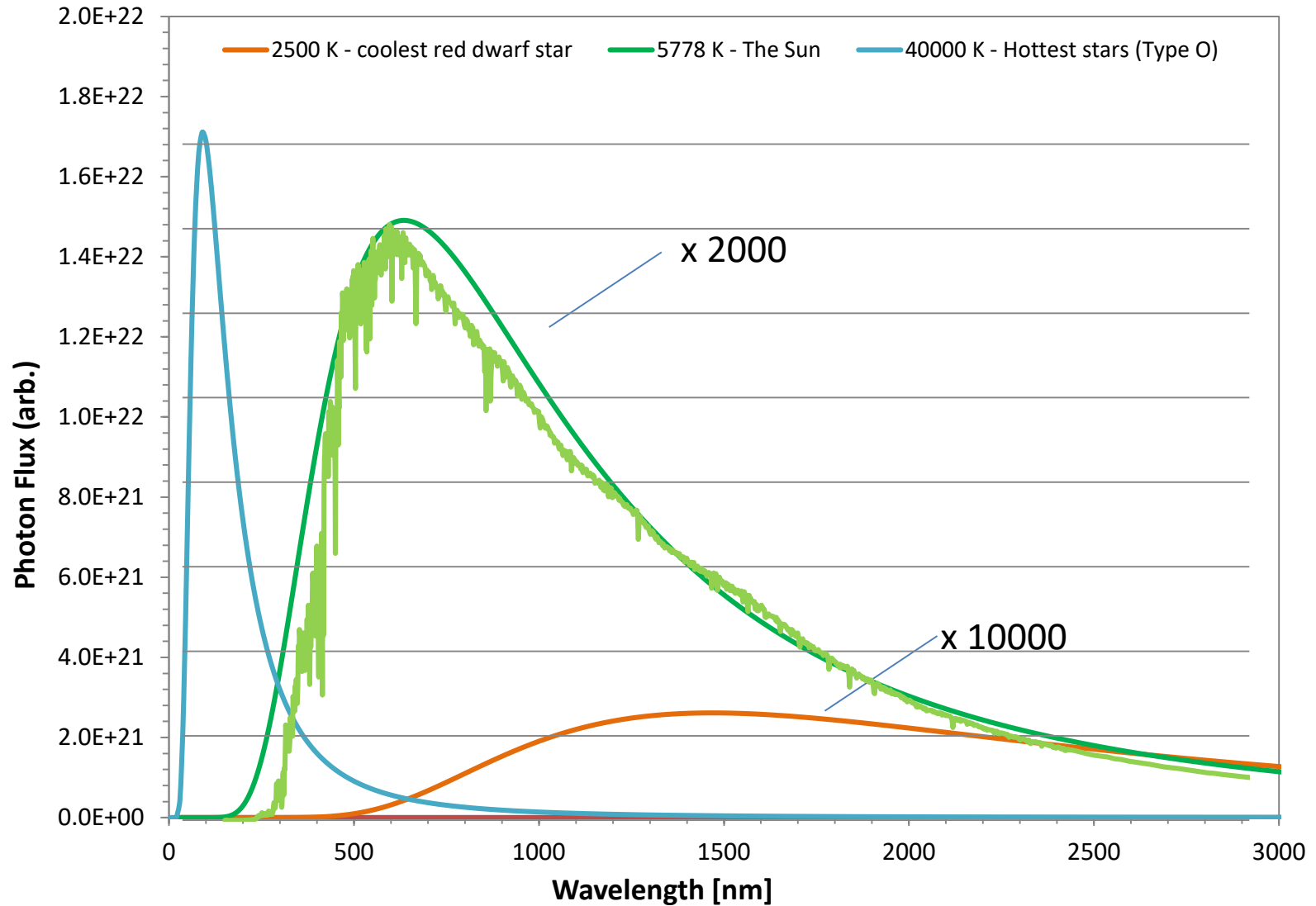


# Blackbody radiation



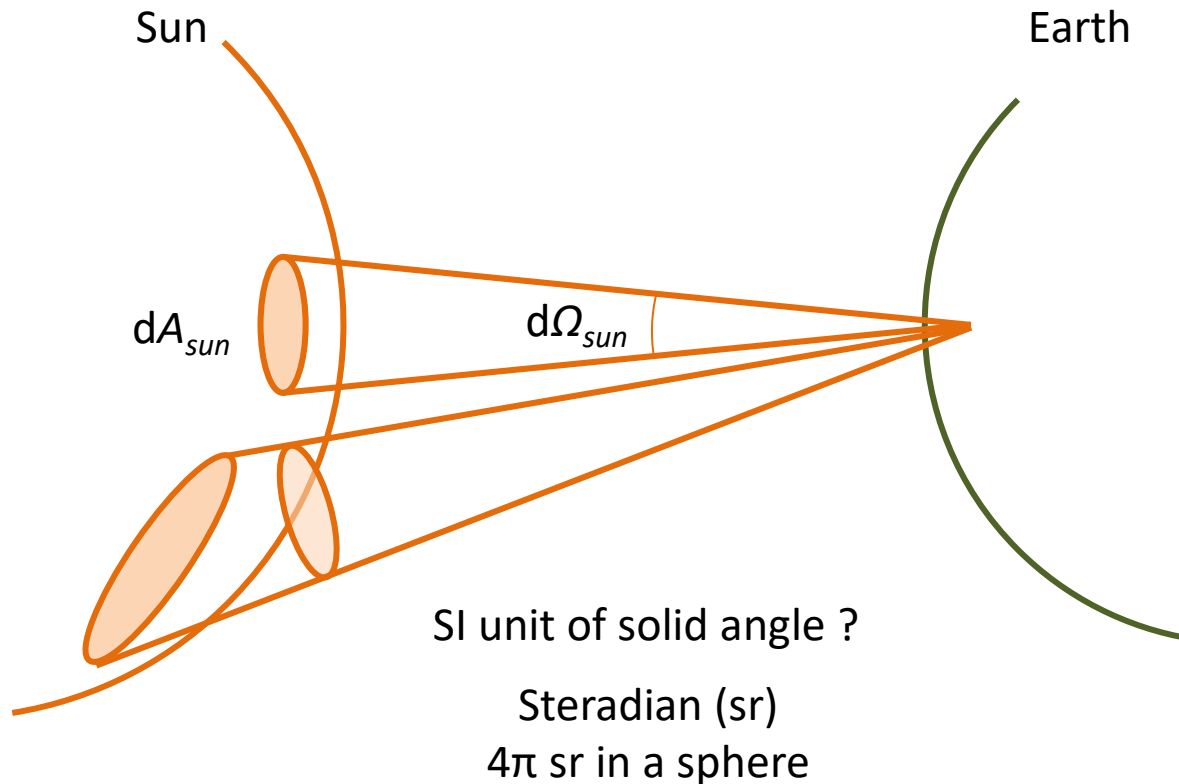
	<u>N° of modes</u> unit volume	Probability of occupying modes	Average energy per mode	Prediction
Classical	$\frac{8\pi}{c\lambda^2}$	Equal for all modes	$\frac{1}{2}k_B T$	$B_\lambda(T) = \frac{2ck_B T}{\lambda^4}$
Quantum	$\frac{8\pi}{c\lambda^2}$	Quantized modes: need $h\nu$ to excite, higher modes less probable	$\frac{\frac{hc}{\lambda}}{e^{\frac{hc}{\lambda k_B T}} - 1}$	$B_\lambda(T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$

# Blackbody radiation



# Flux of photons from a blackbody

$$B_{\lambda}(T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} [=] \frac{\text{Energy carried by photons}}{\text{time} \cdot \text{projected surface area} \cdot \text{wavelength} \cdot \text{solid angle}}$$



Total solid angle of the sun from the viewpoint of the earth is  $\Omega_{se} = 6.8 \times 10^{-5}$  sr

# Flux of photons from the sun

$$B_{600\text{nm}}(5778\text{K})\Omega_{se} = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} \Omega_{se} = 153169 \frac{\text{mW}}{\text{cm}^2 \text{ sr nm}} \frac{1}{e^{4.15} - 1} \Omega_{se}$$

$$= 2453 \frac{\text{mW}}{\text{cm}^2 \text{ sr nm}} \left( \frac{6.2415 \times 10^{15} \text{ eV}}{\text{mW} \cdot \text{s}} \right) \left( \frac{\text{photon}}{2.067 \text{ eV}} \right) \Omega_{se}$$

$$= 7.405 \times 10^{18} \frac{\text{photons}}{\text{cm}^2 \text{ s sr nm}} (6.8 \times 10^{-5} \text{ sr})$$

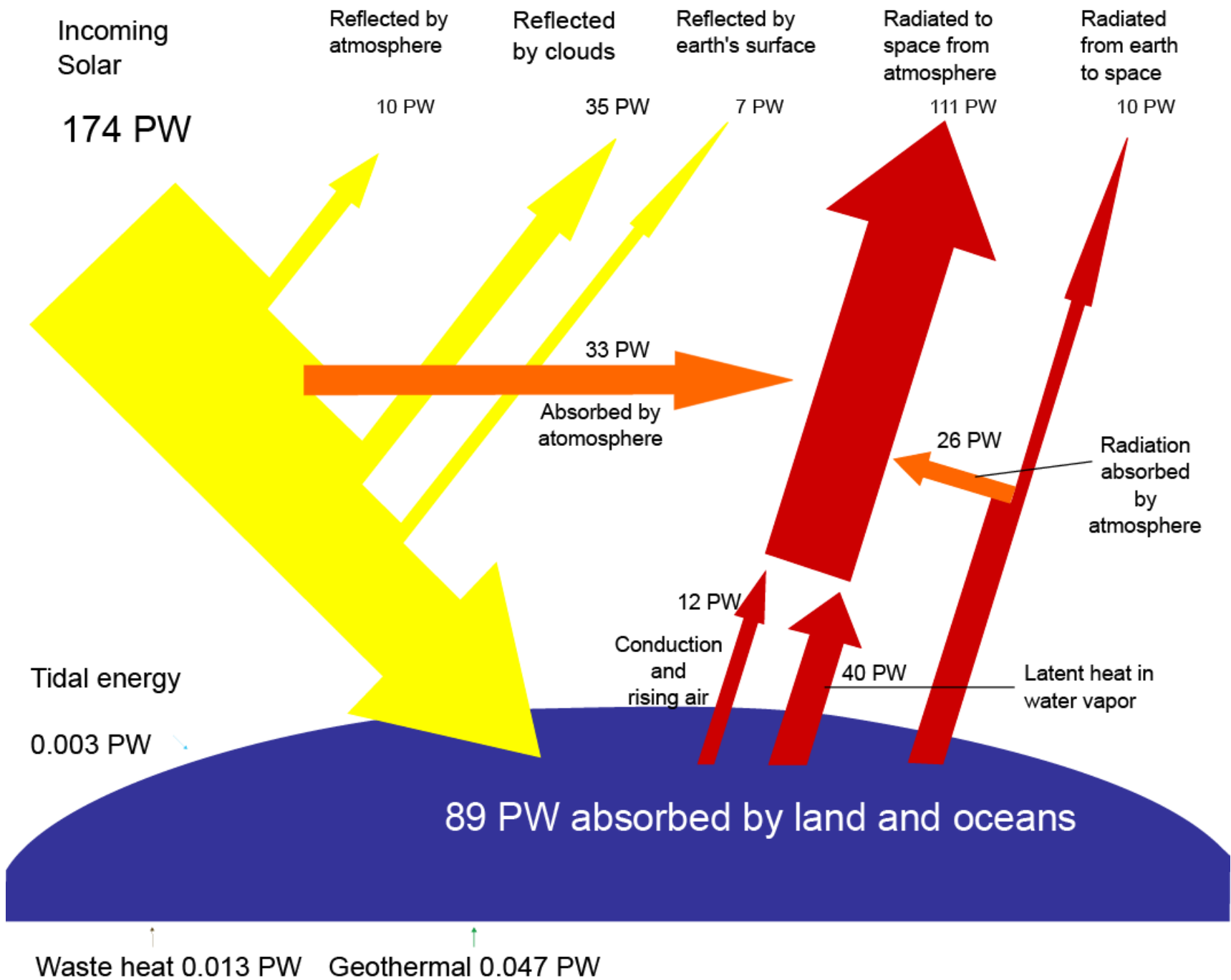
$$= 5.03 \times 10^{14} \frac{\text{photons}}{\text{cm}^2 \text{ s nm}} \quad \text{measured value} = 5.34 \times 10^{14} \frac{\text{photons}}{\text{cm}^2 \text{ s nm}}$$

The solar constant:

$$\int_{\lambda=0}^{\infty} B_{\lambda}(5778\text{K})\Omega_{se} d\lambda = 136.8 \frac{\text{mW}}{\text{cm}^2} \quad \text{measured value} = 136.1 - 136.2 \frac{\text{mW}}{\text{cm}^2}$$

$$136.8 \frac{\text{mW}}{\text{cm}^2} (1.27 \times 10^8 \text{ km}^2) = 174 \text{ PW}$$





Incoming Solar  
174 PW

Reflected by atmosphere  
10 PW

Reflected by clouds  
35 PW

Reflected by earth's surface  
7 PW

Radiated to space from atmosphere  
111 PW

Radiated from earth to space  
10 PW

33 PW

Absorbed by atmosphere

26 PW

Radiation absorbed by atmosphere

12 PW

Conduction and rising air

40 PW

Latent heat in water vapor

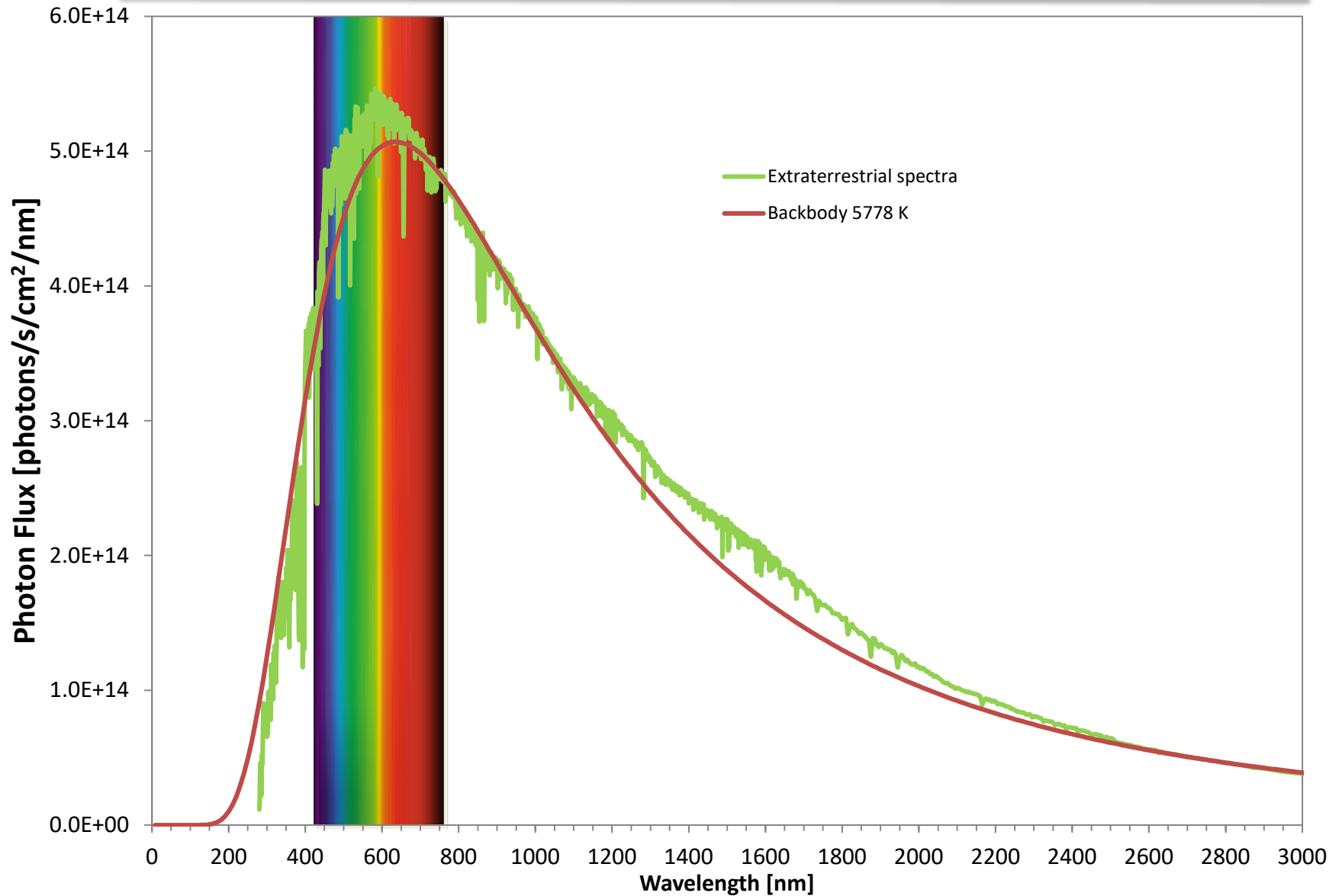
Tidal energy  
0.003 PW

89 PW absorbed by land and oceans

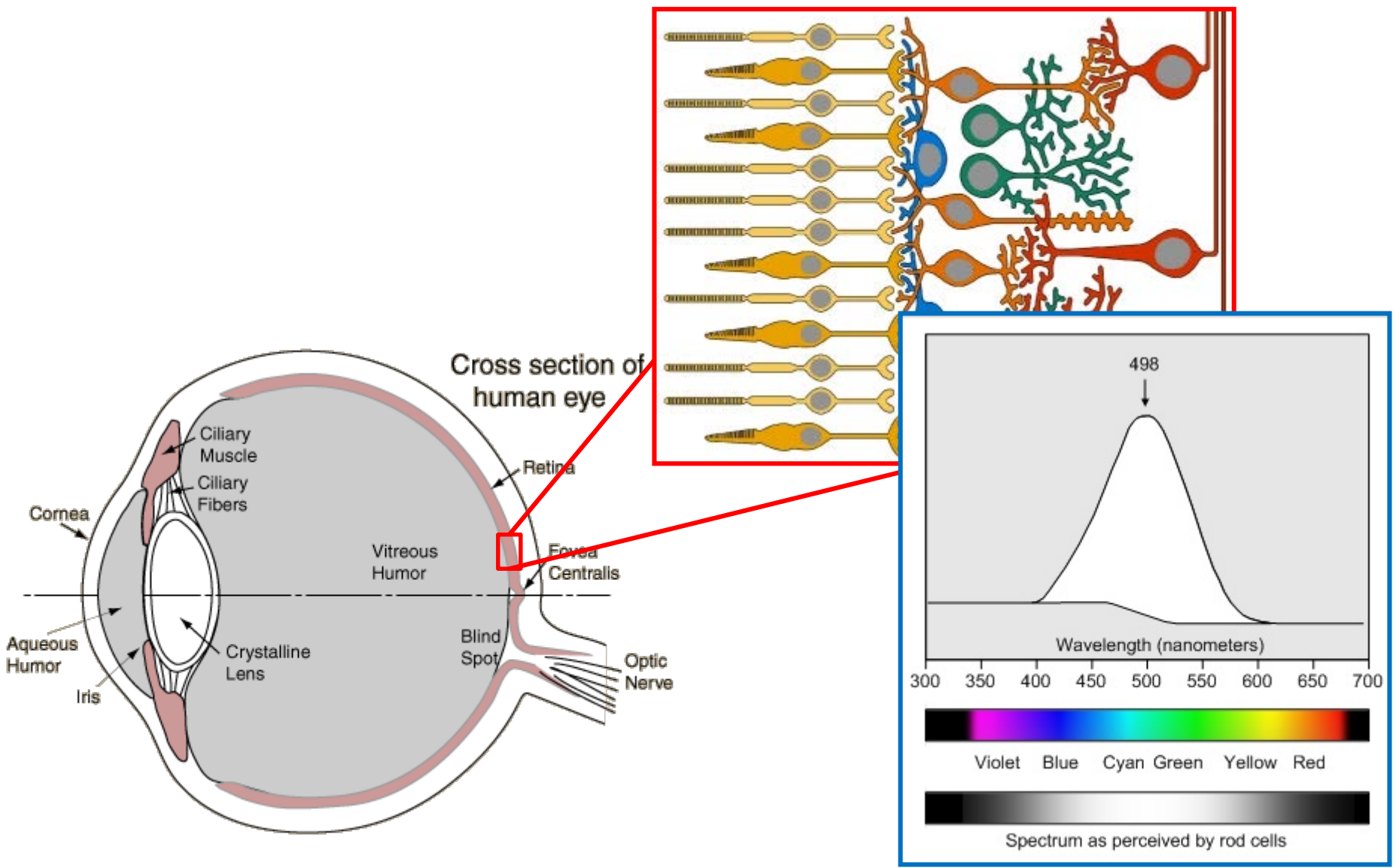
Waste heat 0.013 PW

Geothermal 0.047 PW

# Blackbody radiation

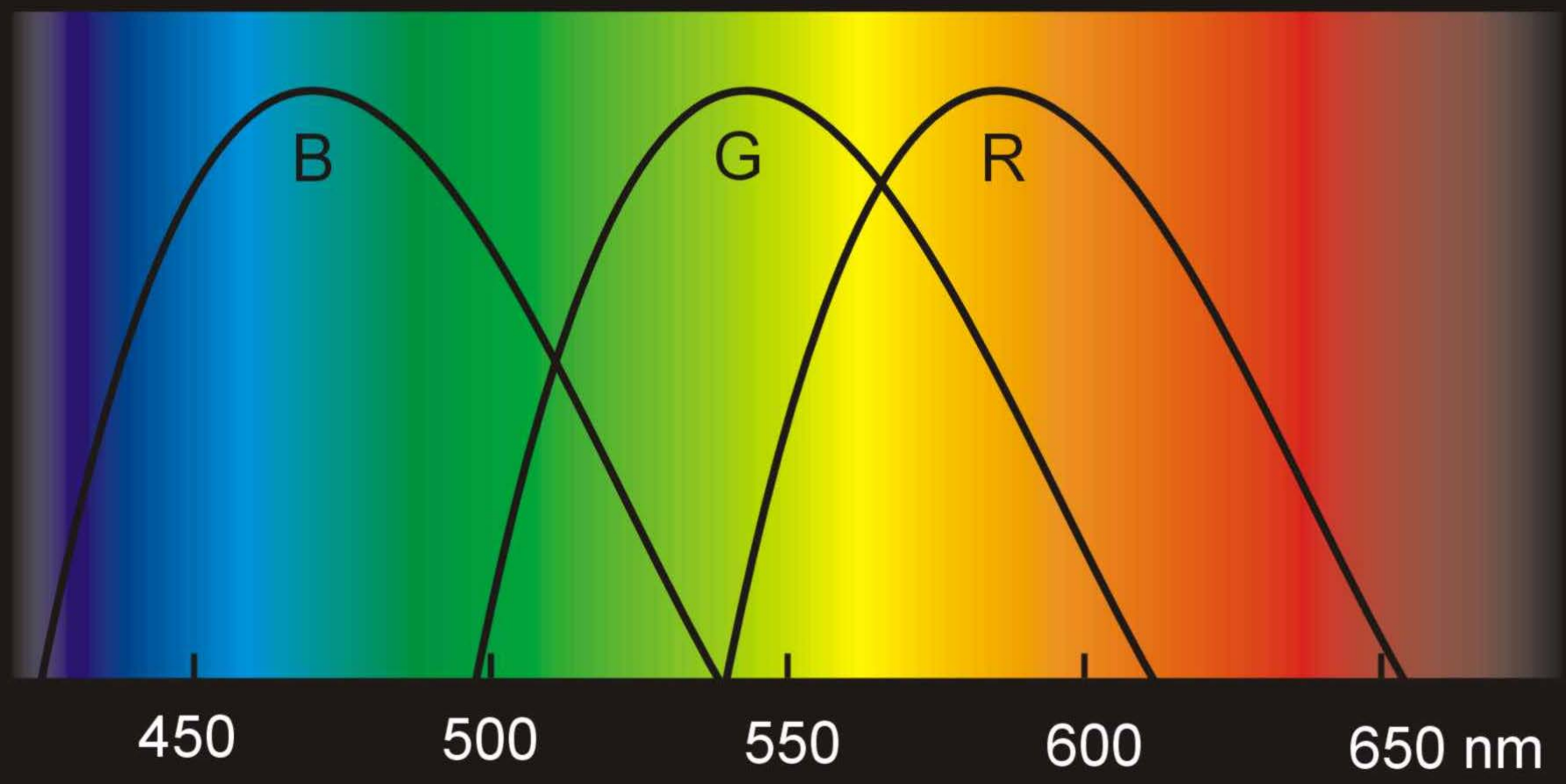


# Solar energy conversion by the human eye



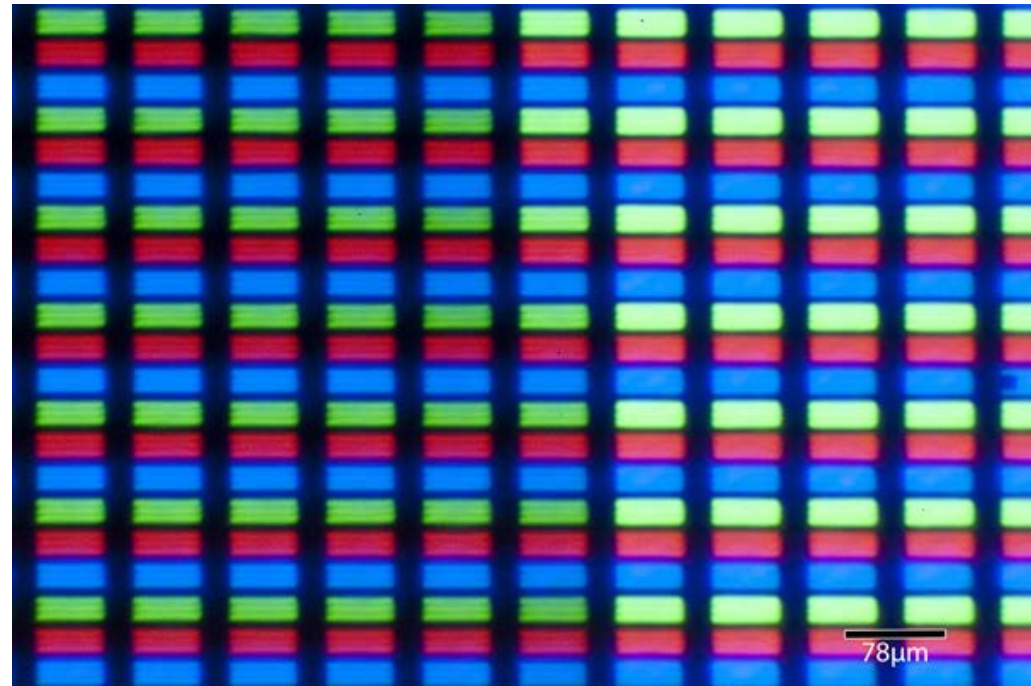
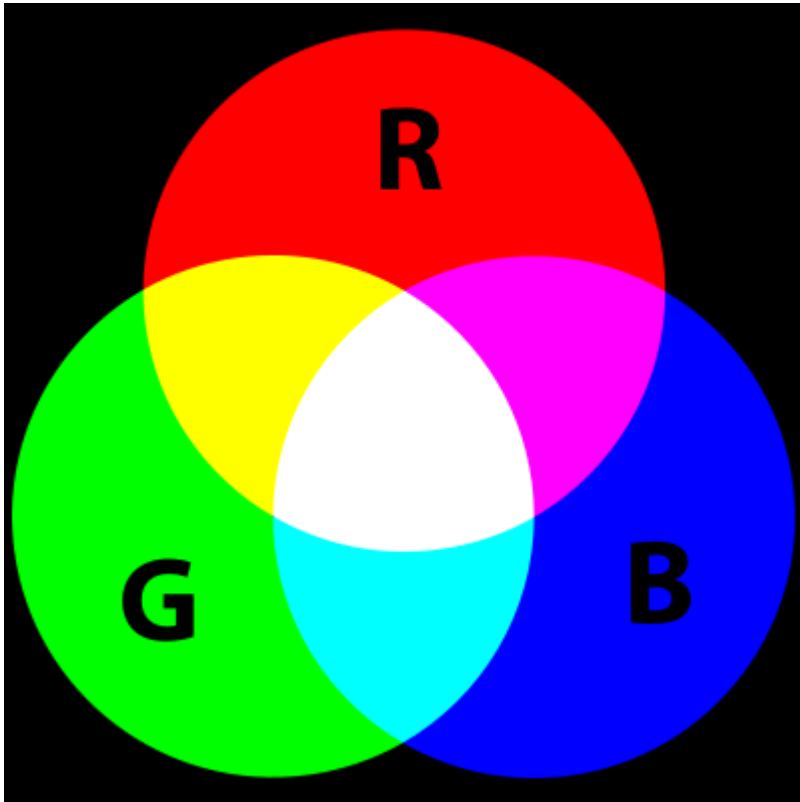
# Spectral sensitivities of human cone cells

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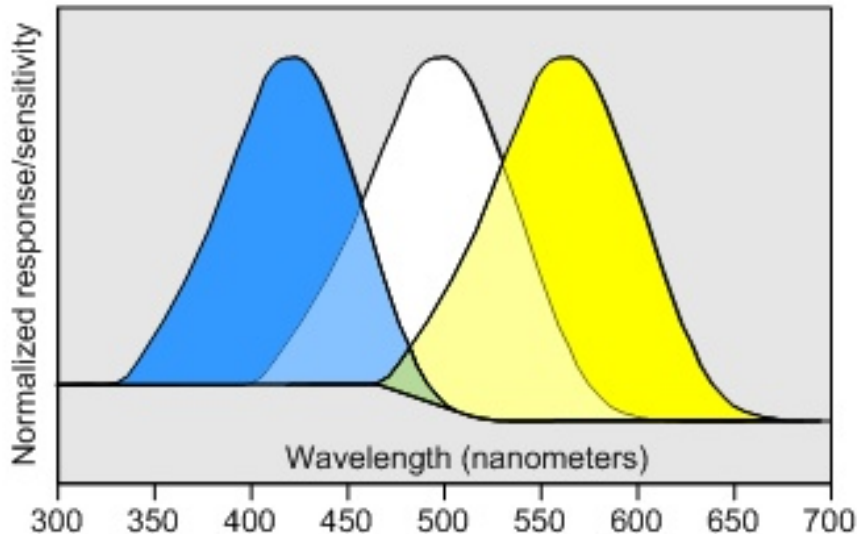
# Trichromatic vision

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iphone 4 display

# Dichromatic vision



Violet Blue Cyan Green Yellow Red



Spectrum as perceived by dogs

**Dogs are dichromats  
(two color cone/pigment types – blue and yellow)**

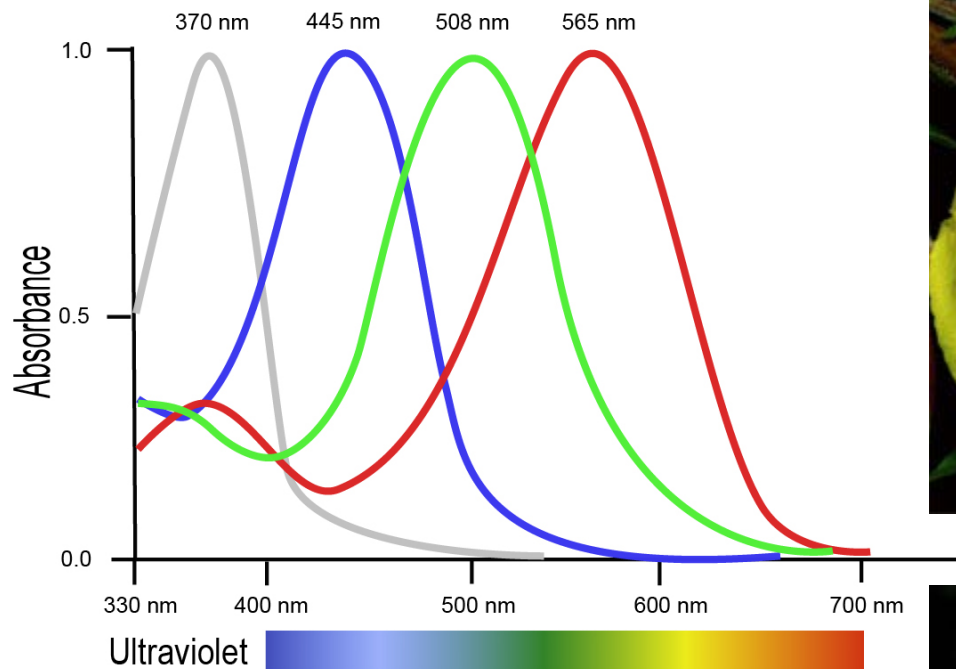


The way a mammalian trichromat (three cones) would see a scene

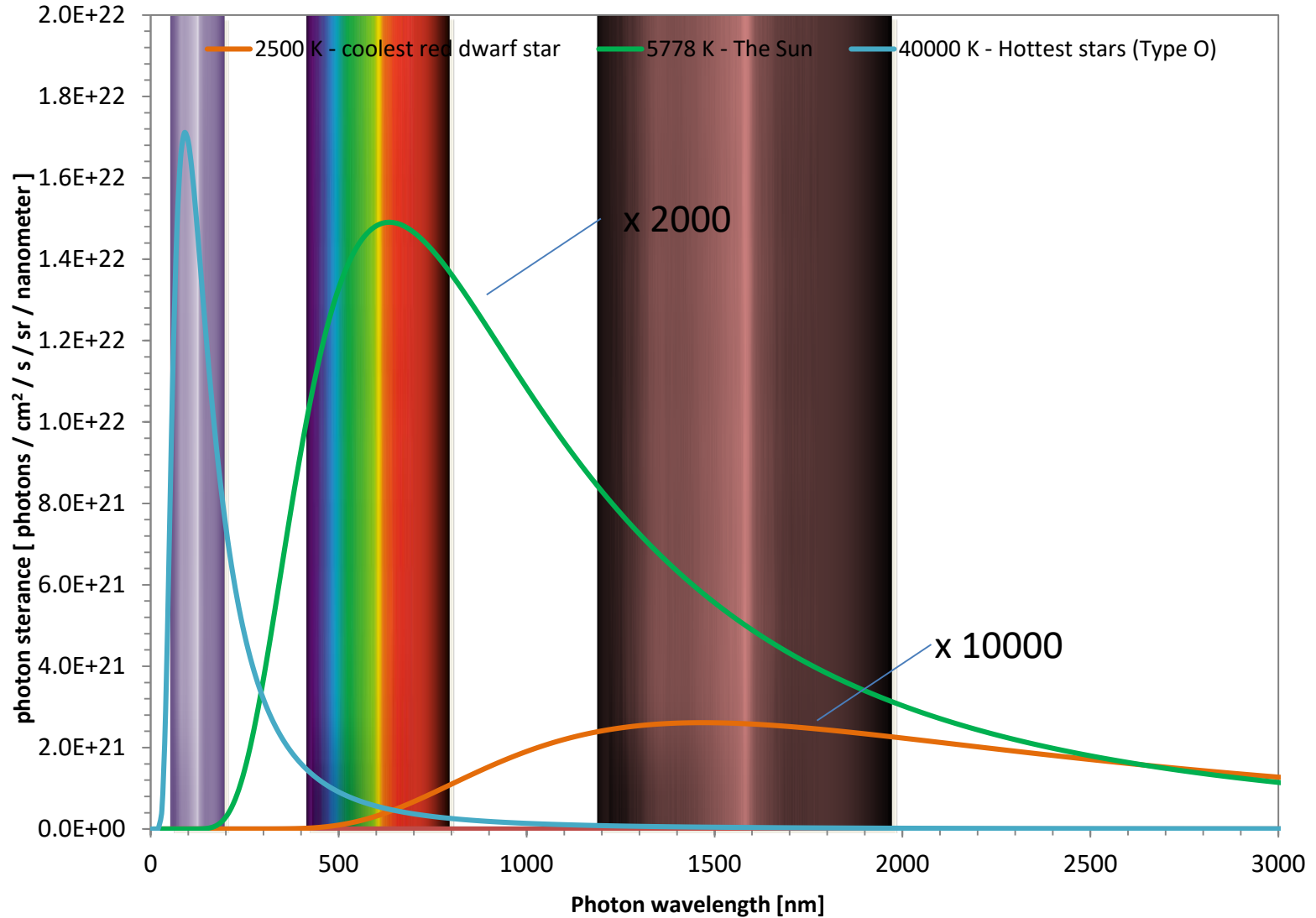


The way a mammalian dichromat (two cones) would see the same scene

# Tetrachromatic vision

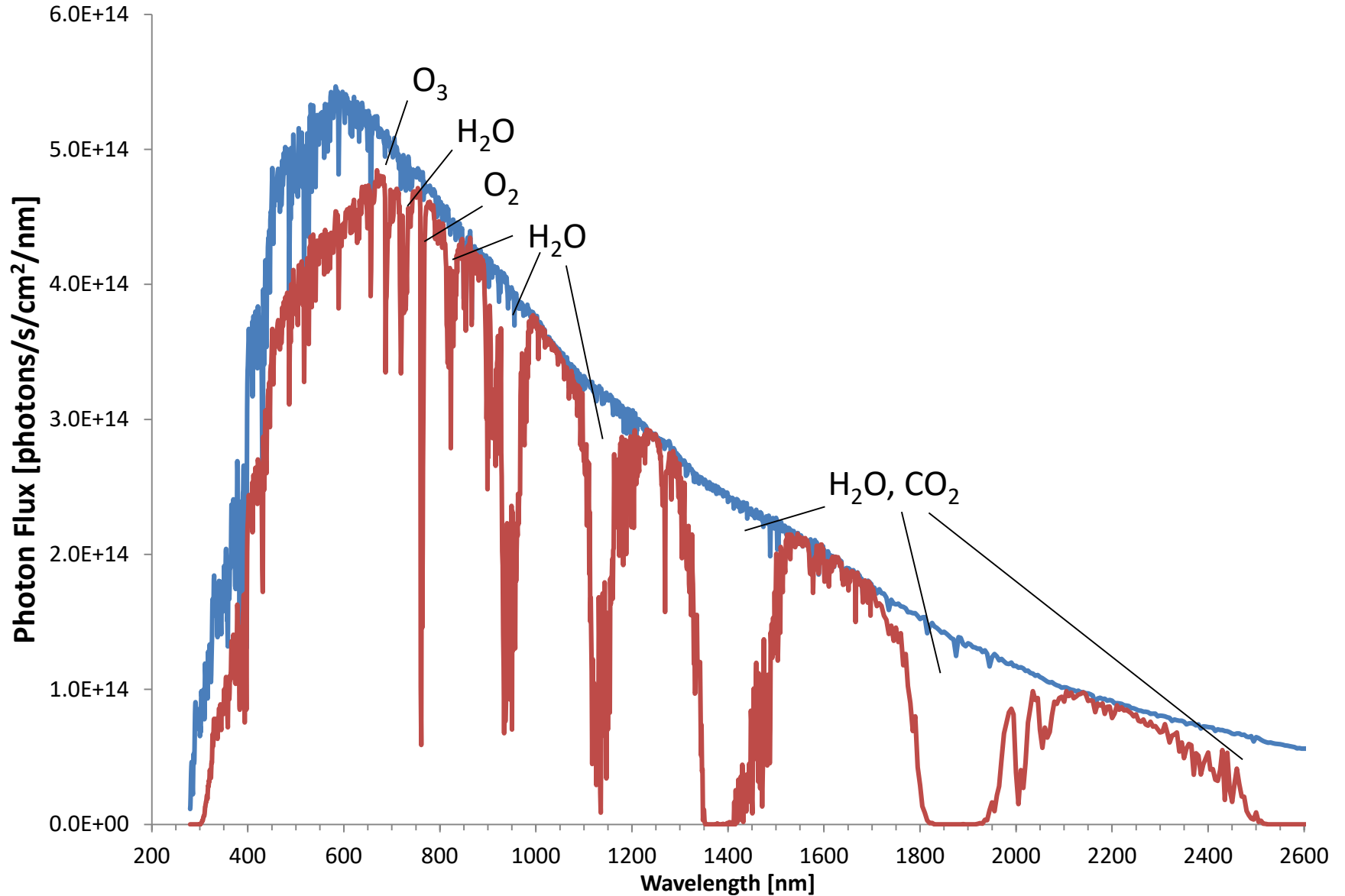


# Another digression

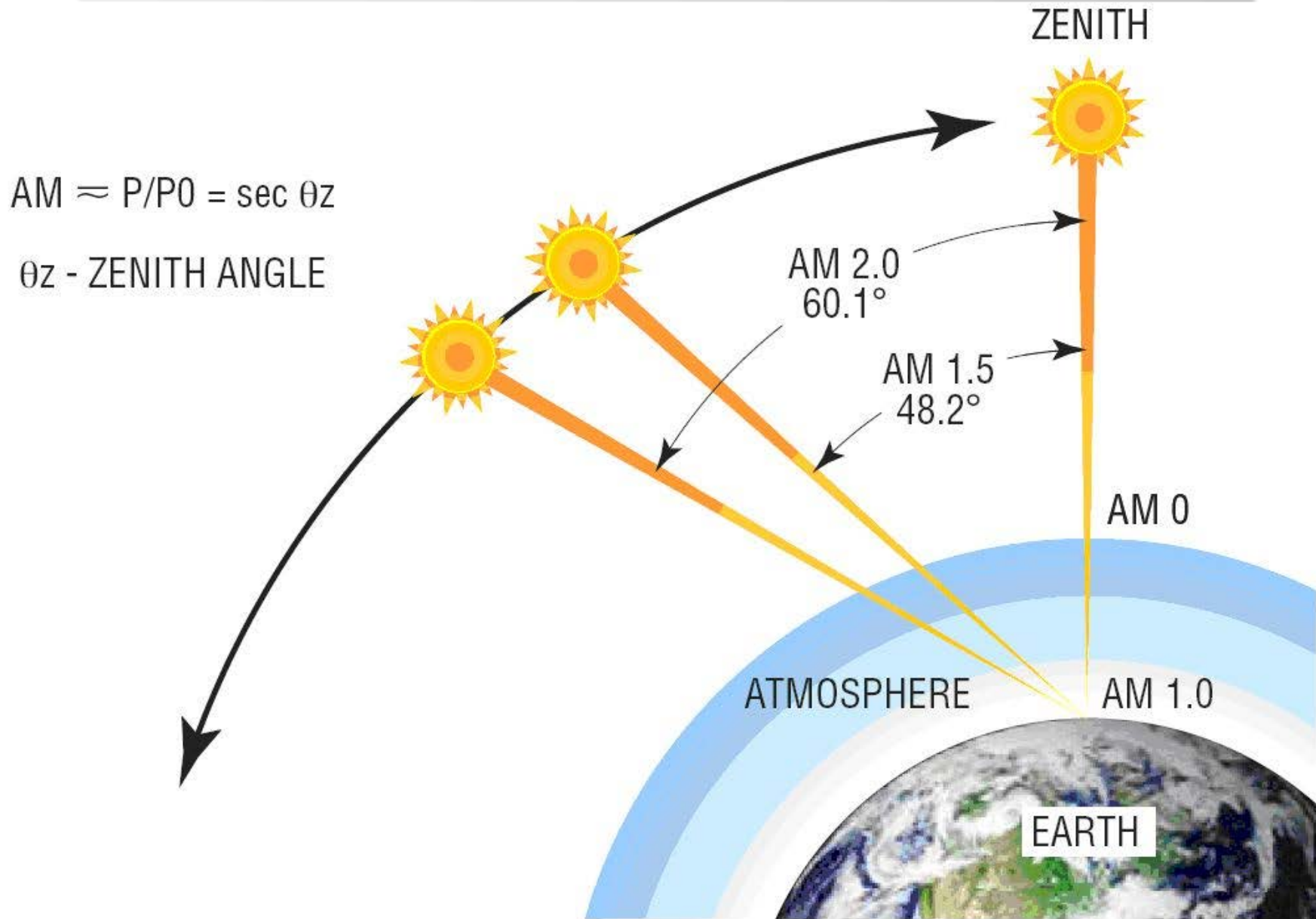




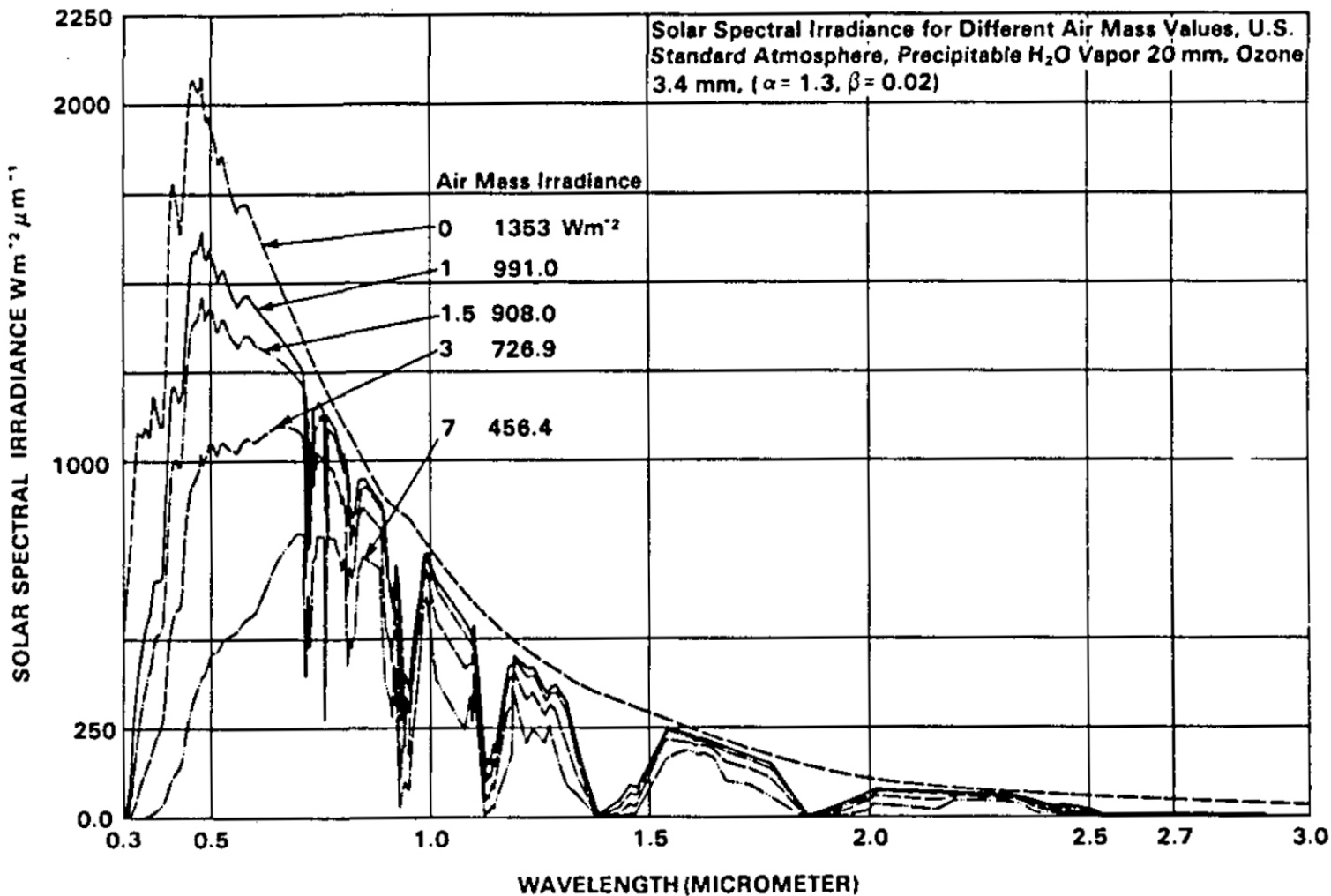
# Solar Spectra



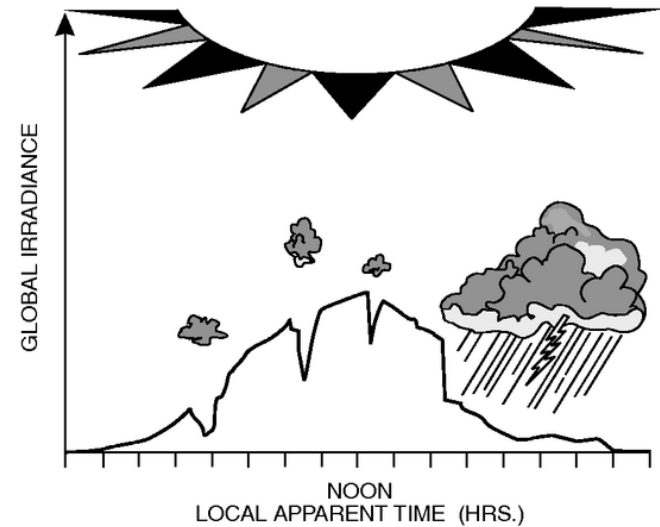
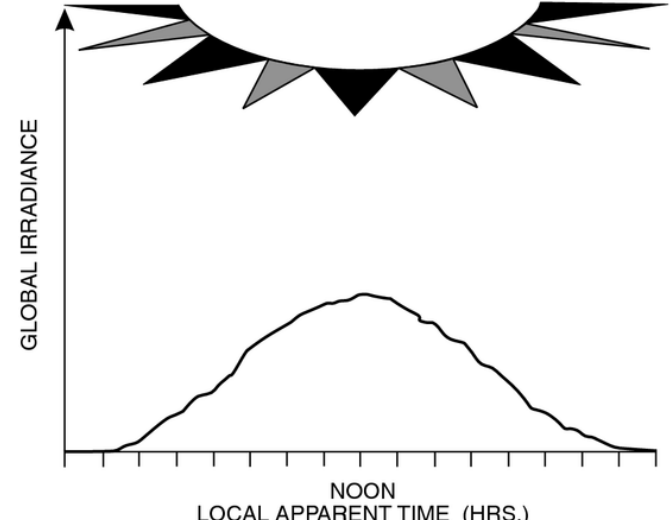
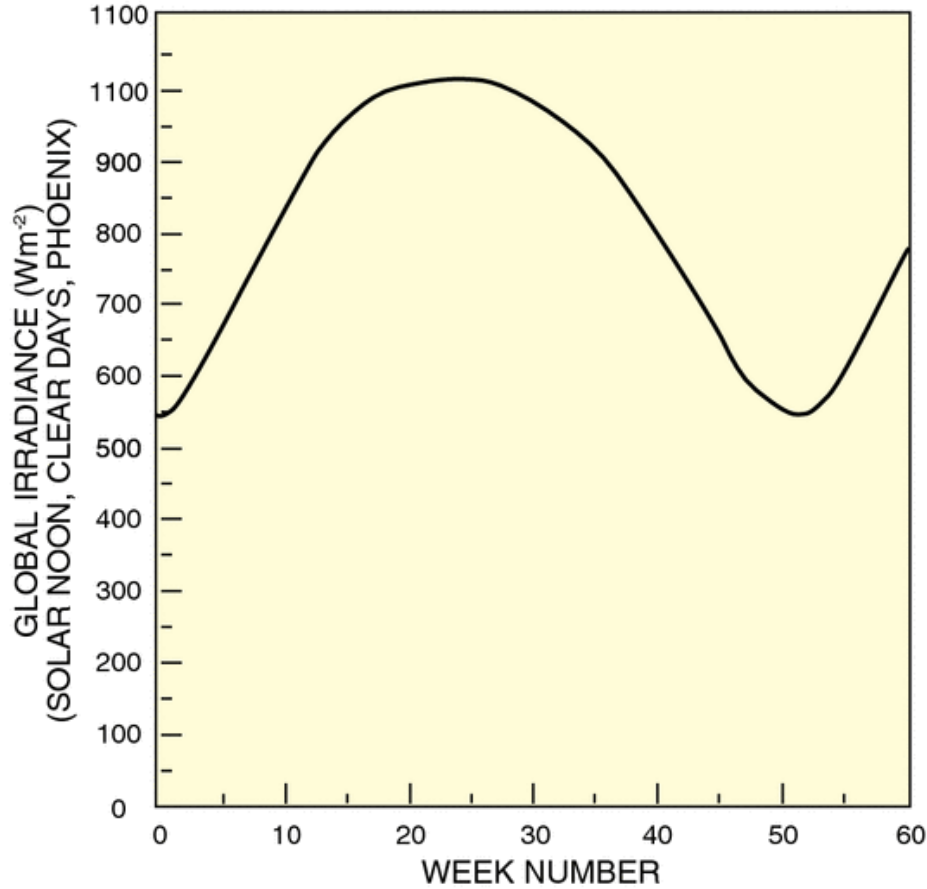
# Atmospheric effects



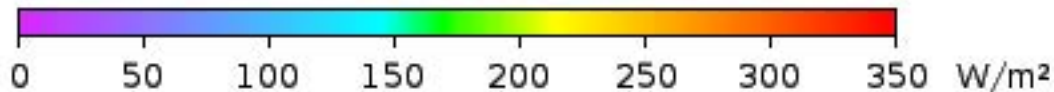
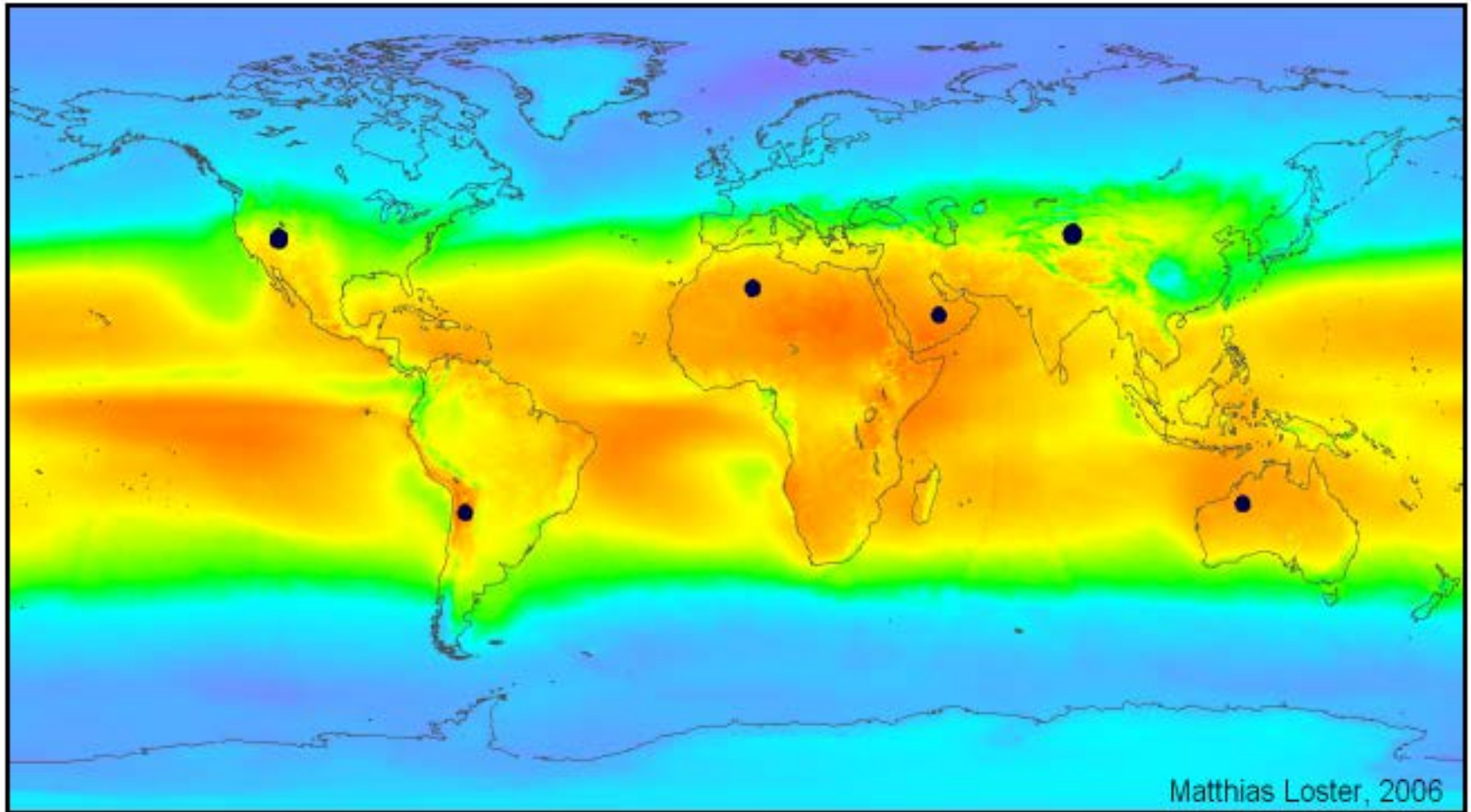
# Solar Spectra



# Variations in solar irradiance



# Averaged Solar Radiation 1990-2004

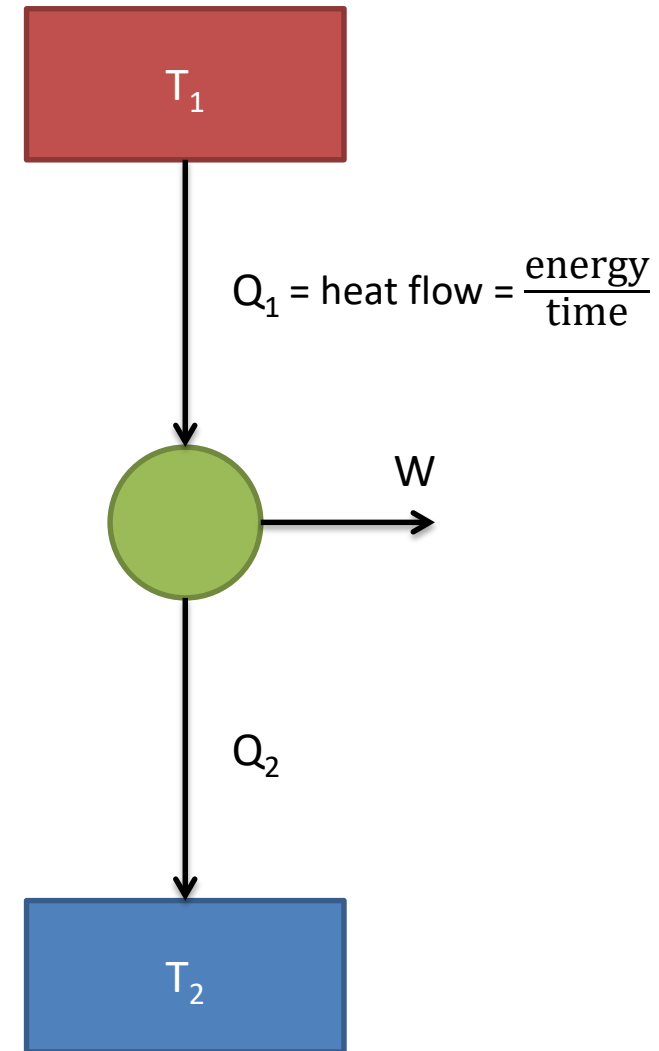


$\Sigma \bullet = 18 \text{ TWe}$

Assuming 8% energy conversion efficiency

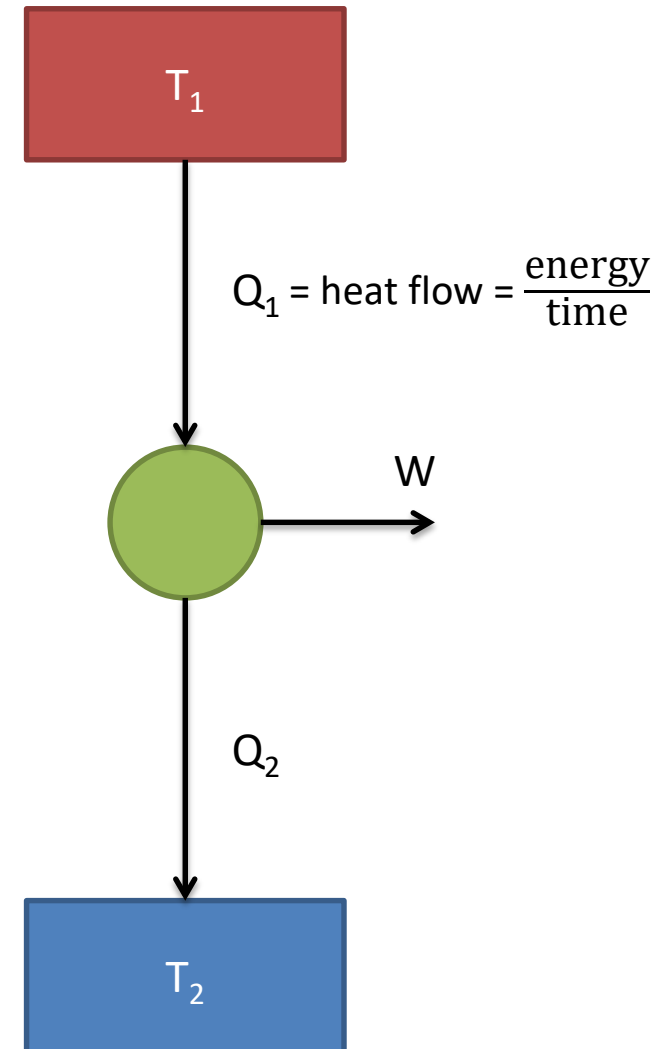
# Energy conversion thermodynamics

- How can we understand the limits of solar energy conversion?
  - Let's start with a simple thermodynamic model (Carnot Engine)
  - Add linear (conduction) heat transfer (Curzon-Ahlborn Engine)
  - Add radiative heat transfer (The Stefan-Boltzmann Engine)
  - Include a Sun-planet model (The Müser Engine)
  - Include a “band gap” to derive the Shockley–Queisser limit



# Thermodynamics: The Carnot engine

- The Carnot Engine
  - Normally  $T_1 > T_2$
- Axiom 1 (first law)
  - $\Sigma Q + \Sigma W = dU = 0$
- Axiom 2 (second law)
  - $\Sigma S = 0$  (for reversible processes)
- Postulate: heat flow is accompanied by an entropy flow  $Q/T$ , and work flow is accompanied by an entropy flow of 0, then
  - $\Sigma(Q/T) = 0$
- It follows:
  - $Q_1 = W + Q_2$  and  $Q_1/T_1 = Q_2/T_2$
- Introduce “efficiency”
  - $W = \eta_c Q_1$
- $\eta_c = 1 - T_2/T_1$



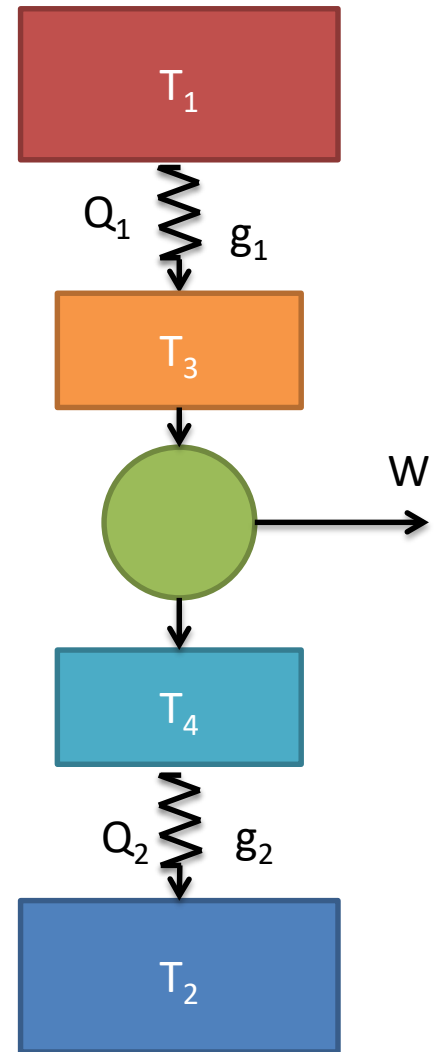
If  $W > 0$ : positive work flow, “heat engine”

If  $W < 0$ : negative work flow, two subcases:

- 1)  $T_1 > T_2$  this is a “heat pump” with  $\eta_c > 0$
- 2)  $T_2 > T_1$  this is a “refrigerator” with  $\eta_c < 0$

# The Carnot engine with linear conduction

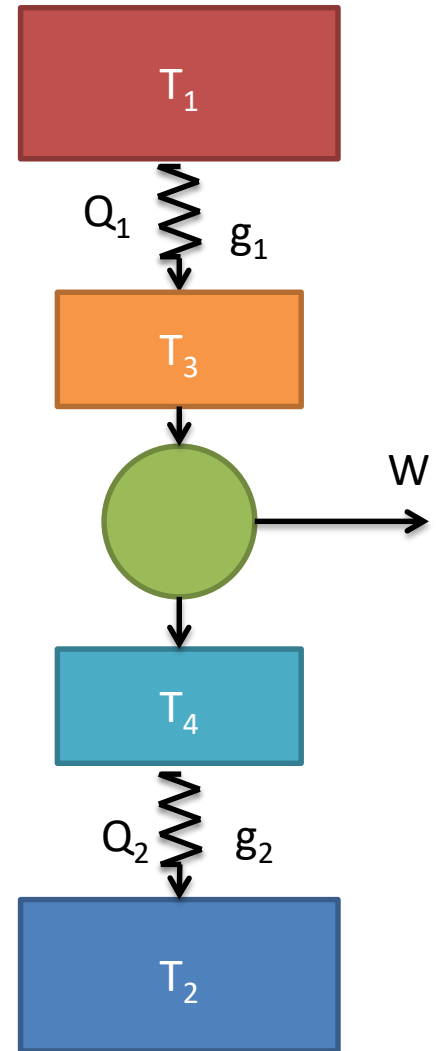
- The Curzon-Ahlborn Engine  
Normally  $T_1 > T_2$  as in Carnot  
Two irreversible components  
Reversible Carnot engine between  $T_3$  and  $T_4$
- Axiom 1 (first law)  
 $\Sigma Q + \Sigma W = 0 \Rightarrow Q_1 = W + Q_2$
- Axiom 2 (second law)  
 $\Sigma(Q/T) = 0 \Rightarrow Q_1/T_3 = Q_2/T_4$
- $\eta_{CA} = 1 - T_4/T_3$
- Fourier's law:
- $Q = -\lambda_{th} \nabla T$  (assume linear conduction,  $\lambda_{th}$  thermal conductivity)
  - $Q = g \Delta T$  ( $g = \lambda_{th}/L$ )
    - $Q_1 = g_1(T_1 - T_3)$
    - $Q_2 = g_2(T_4 - T_2)$



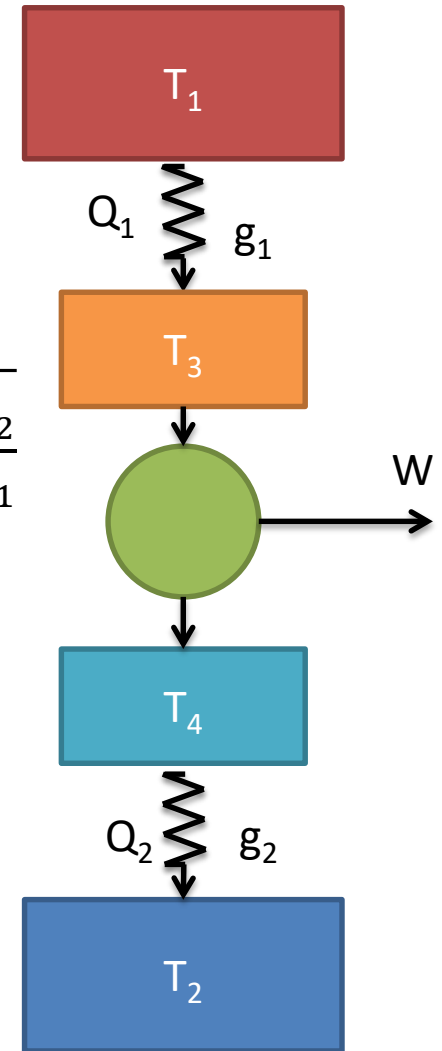
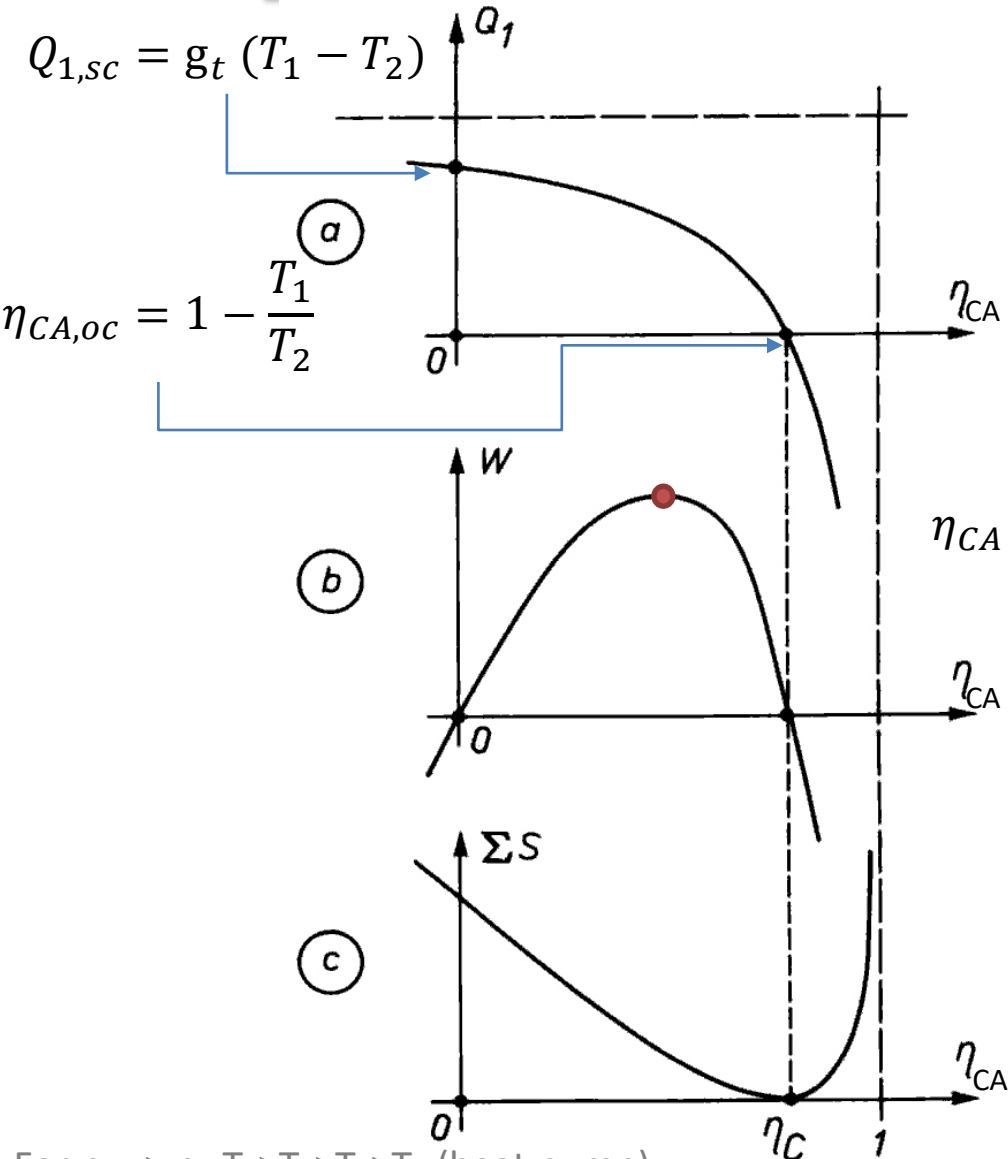


# Curzon-Ahlborn Engine

- $Q_1 = g_t \frac{T_1 - T_2 - T_1 \eta_{CA}}{1 - \eta_{CA}}, \quad g_t = \frac{g_1 g_2}{g_1 + g_2}$
- $W = g_t \frac{\eta_{CA}(T_1 - T_2 - T_1 \eta_{CA})}{1 - \eta_{CA}}$
- $\Sigma S = g_t \frac{(T_1 - T_2 - T_1 \eta_{CA})^2}{T_1 T_2 (1 - \eta_{CA})}$

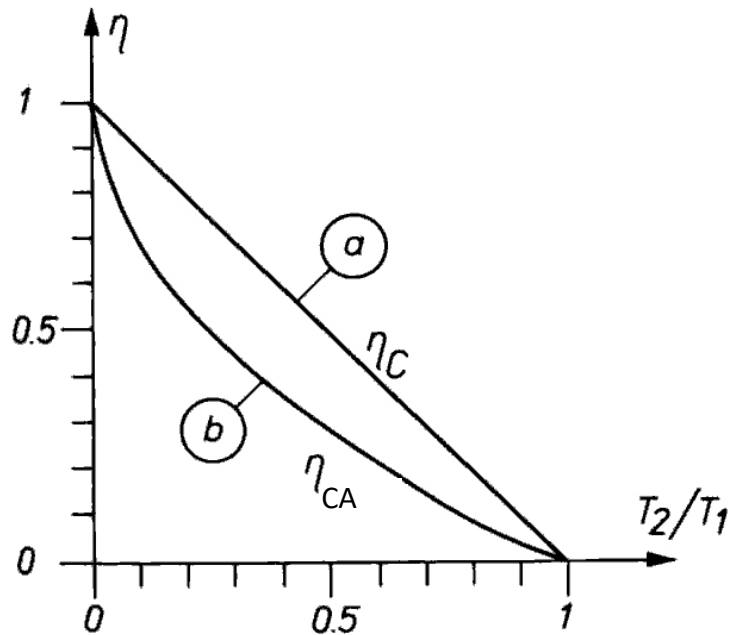


# Curzon-Ahlborn Engine

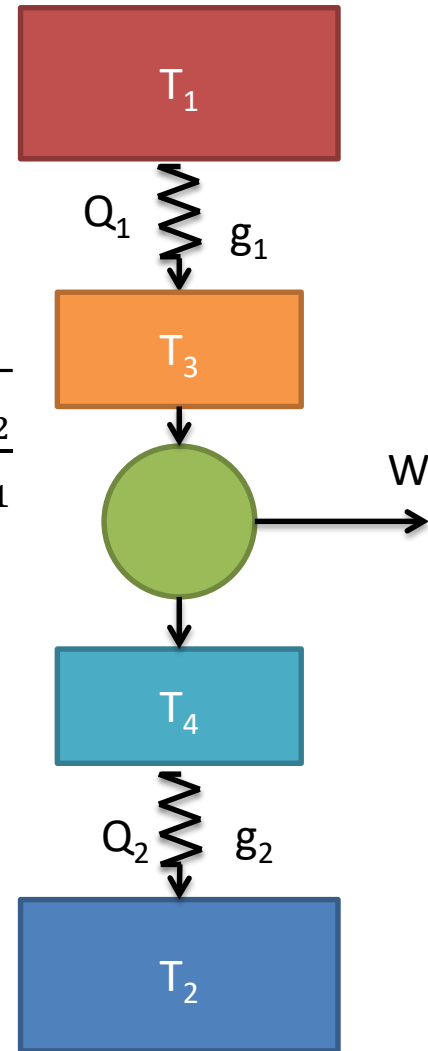


For  $\eta_{CA} > \eta_C$   $T_3 > T_1 > T_2 > T_4$  (heat pump)  
 For  $-\infty < \eta_{CA} < 0$ ,  $T_1 > T_3$ ,  $T_4 > T_3$ ,  $T_4 > T_2$  (refrigerator)

# Curzon-Ahlborn Engine



$$\eta_{CA} = 1 - \sqrt{\frac{T_2}{T_1}}$$



$$Q_1 = g\sqrt{T_1} \left( \sqrt{T_1} - \sqrt{T_2} \right), \quad g = \frac{g_1 g_2}{g_1 + g_2}$$

$$W = g \left( \sqrt{T_1} - \sqrt{T_2} \right)^2$$

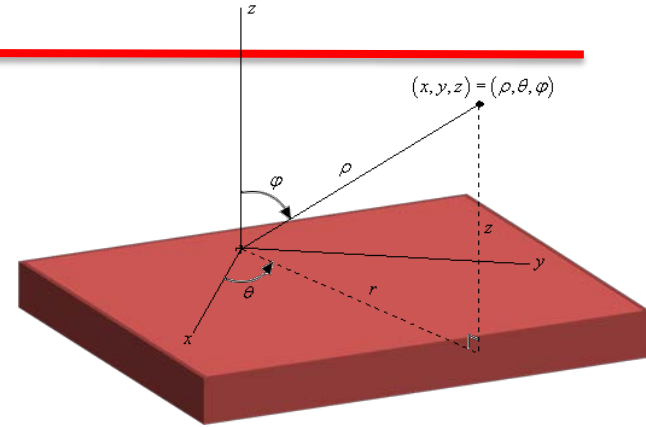
$$\Sigma S = g \frac{\left( \sqrt{T_1} - \sqrt{T_2} \right)^2}{\sqrt{T_1 T_2}}$$

# Heat transfer by Radiation

The maximum heat transfer from the surface of a blackbody can be calculated from the spectral distribution:

$$\frac{Q}{A} = q = \int_{\lambda=0}^{\infty} B_{\lambda}(T) d\lambda \int_{\theta=0}^{2\pi} d\theta \int_{\varphi=0}^{\pi/2} \cos \varphi \sin \varphi d\varphi$$

$$= \pi \int_{\lambda=0}^{\infty} B_{\lambda}(T) d\lambda = \int_{\lambda=0}^{\infty} \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1} d\lambda = \sigma T^4$$



$$\zeta(s) = \frac{1}{\Gamma(s)} \int_0^{\infty} \frac{x^{s-1}}{e^x - 1} dx$$

Stefan-Boltzmann law:

$$\sigma = \frac{2\pi^5 k_B^4}{15c^2 h^3}$$

$$Q/A = \varepsilon \sigma T^4$$

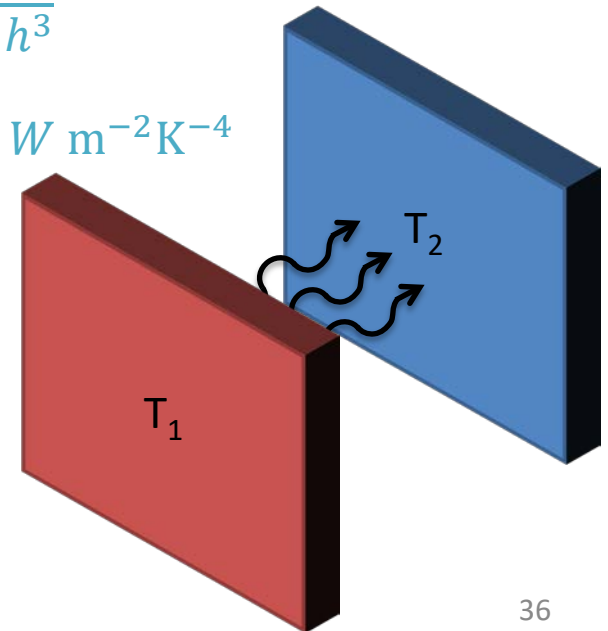
$\varepsilon$  = emissivity of the material ( $\varepsilon=1$  for a blackbody)

(grey body,  $\alpha(\lambda) = \varepsilon(\lambda)$ )

$$\sigma = 5.670 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$$

For two (infinite) parallel plates of the same area and same material :

$$Q = \varepsilon \sigma (T_1^4 - T_2^4)$$



# 5.1 Heat transfer by Radiation

Material	Average Emissivity
Aluminum foil	0.03
Aluminum, anodized	0.9
Asphalt	0.88
Brick	0.90
Concrete, rough	0.91
Copper, polished	0.04
Copper, oxidized	0.87
Glass, smooth (uncoated)	0.95
Ice	0.97
Limestone	0.92
Marble (polished)	0.89 to 0.92
Paint (including white)	0.9
Paper, roofing or white	0.88 to 0.86
Plaster, rough	0.89
Silver, polished	0.02
Silver, oxidized	0.04
Snow	0.8 to 0.9
Water, pure	0.96

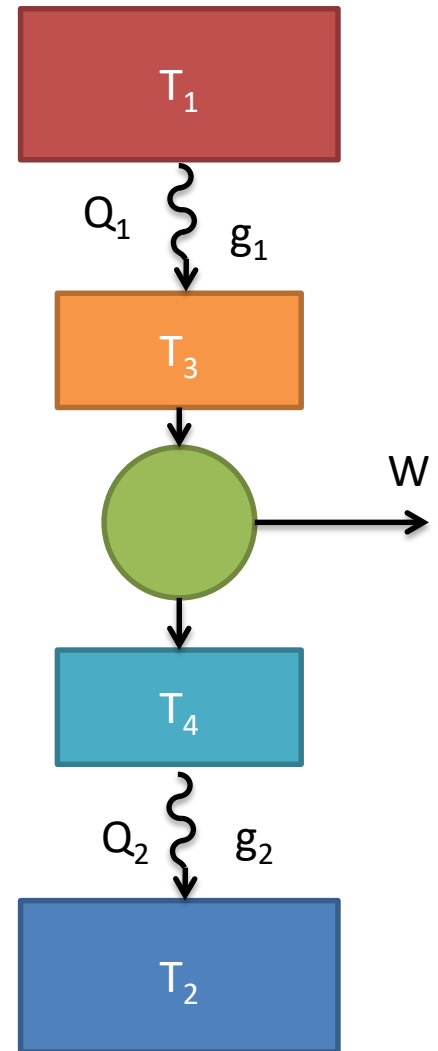
# The Stefan-Boltzmann Engine

- $Q = \sigma \varepsilon A T^4$  (assume radiative heat transfer)
  - $Q_1 = g_1 (T_1^4 - T_3^4)$
  - $Q_2 = g_2 (T_4^4 - T_2^4)$
  - $\eta_{SB} = 1 - T_4/T_3$

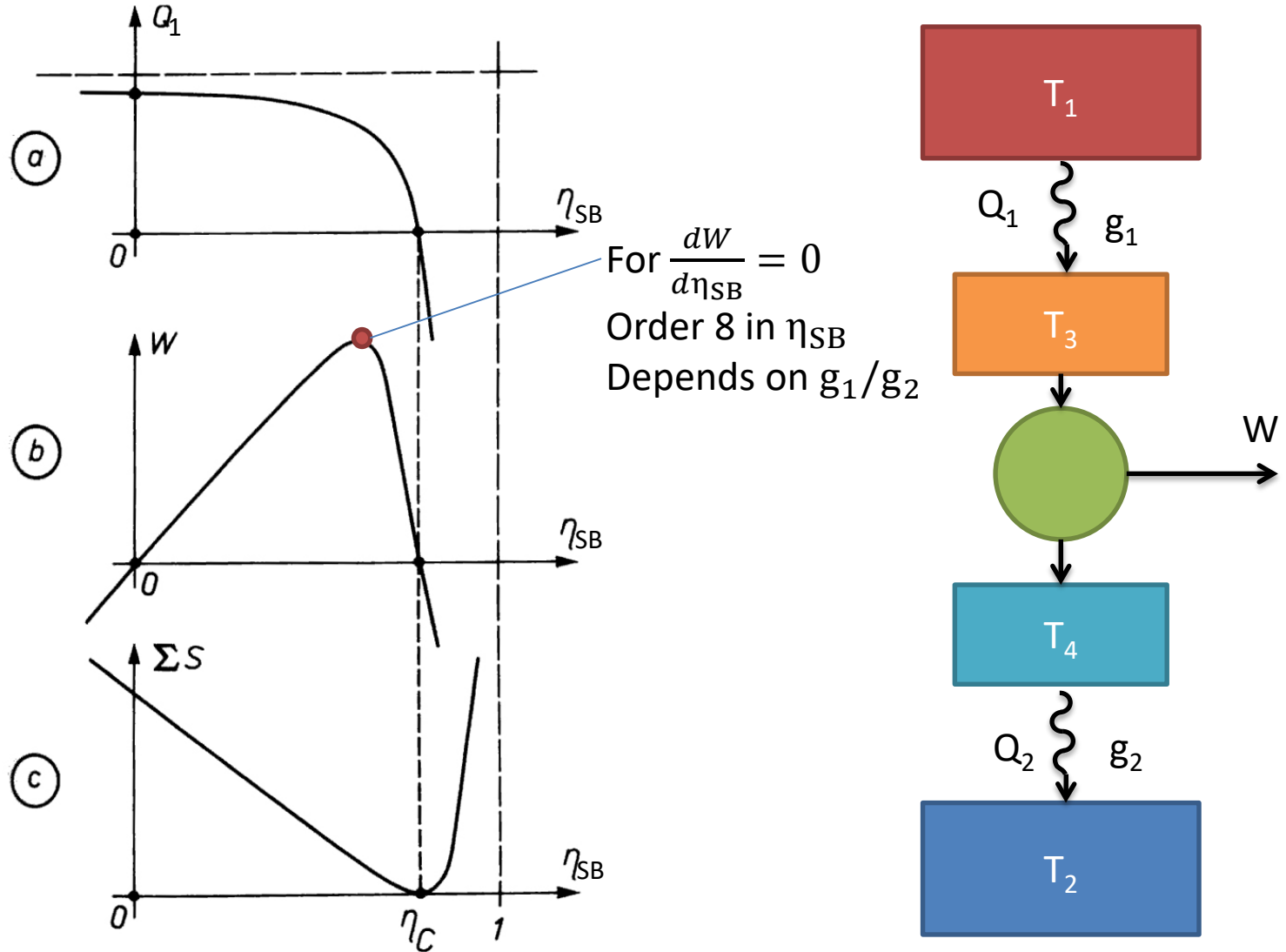
- $Q_1 = g_1 g_2 \frac{(1-\eta_{SB})^4 T_1^4 - T_2^4}{g_1(1-\eta_{SB}) + g_2(1-\eta_{SB})^4}$

- $W = g_1 g_2 \eta_{SB} \frac{(1-\eta_{SB})^4 T_1^4 - T_2^4}{g_1(1-\eta_{SB}) + g_2(1-\eta_{SB})^4}$

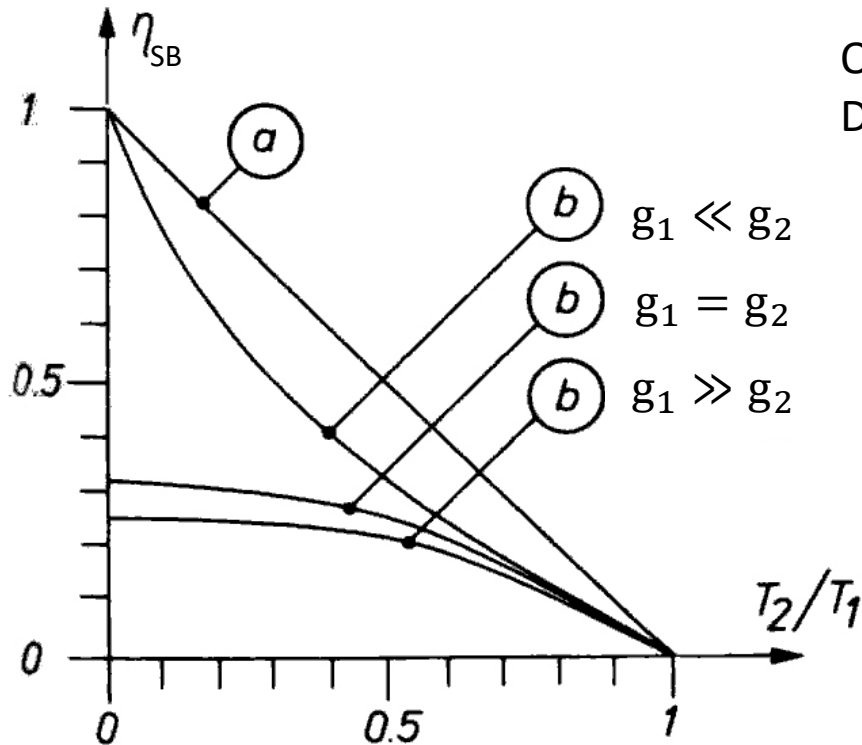
- $\Sigma S = \frac{g_1 g_2}{T_1 T_2} \frac{((1-\eta_{SB})T_1 - T_2)((1-\eta_{SB})^4 T_1^4 - T_2^4)}{g_1(1-\eta_{SB}) + g_2(1-\eta_{SB})^4}$



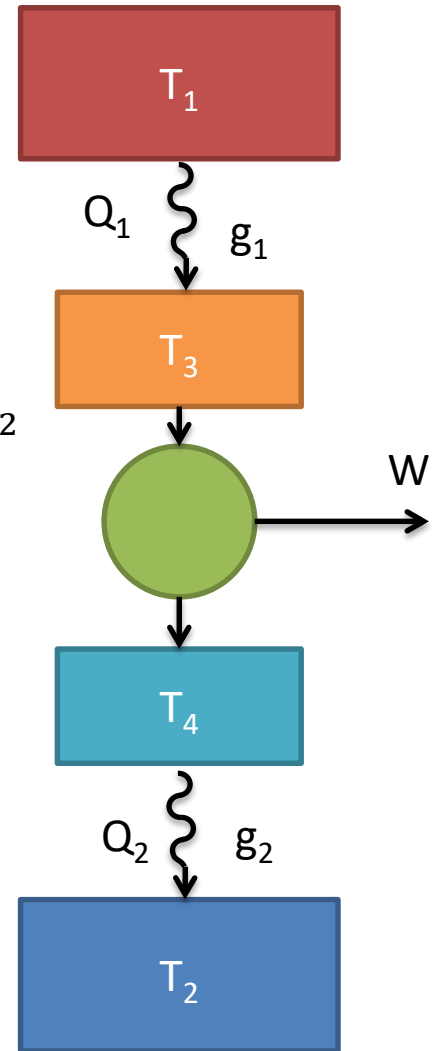
# The Stefan-Boltzmann Engine



# The Stefan-Boltzmann Engine

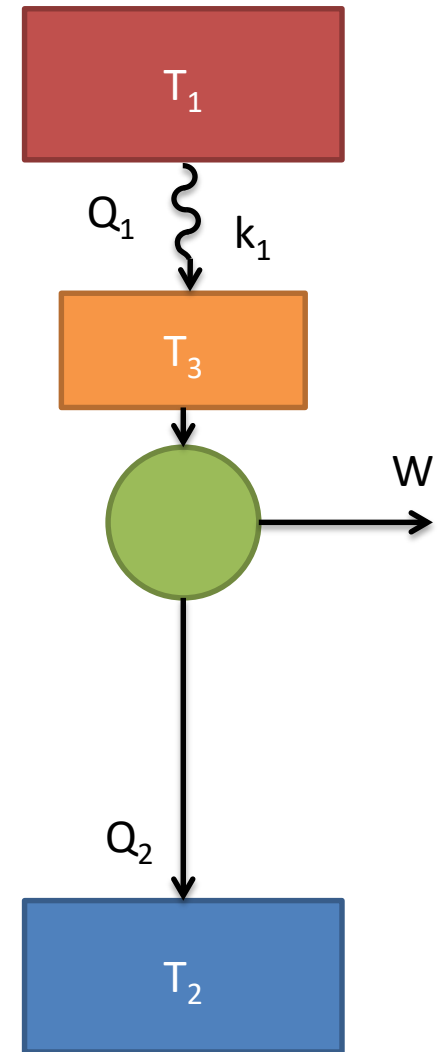
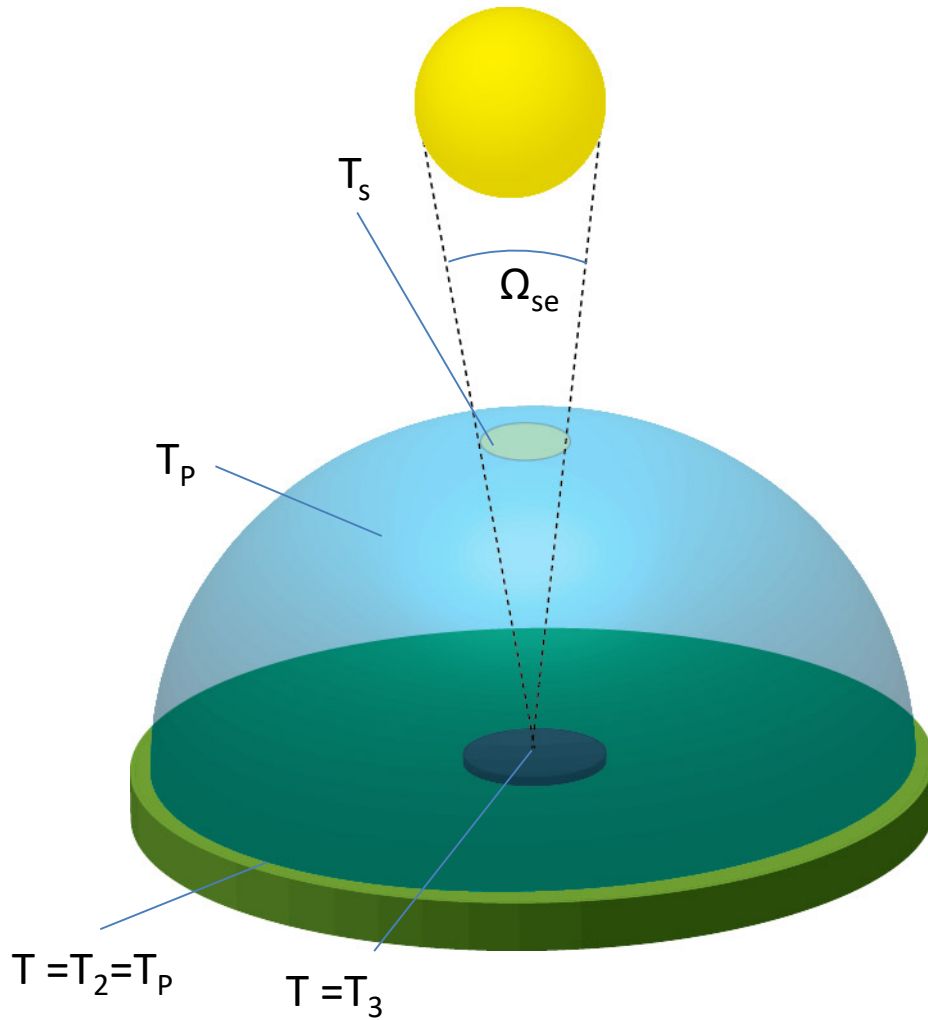


For  $\frac{dW}{d\eta_{SB}} = 0$   
 Order 8 in  $\eta_{SB}$   
 Depends on  $g_1/g_2$

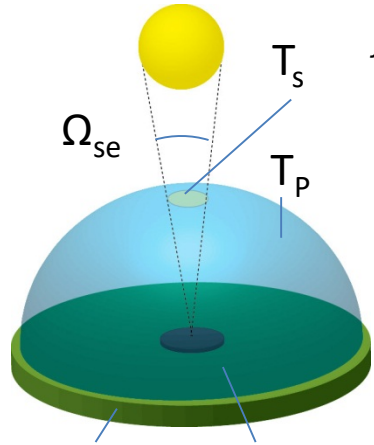




# The Müser Engine



# The Müser Engine



$$f = \frac{\text{apparent area subtended by sun}}{\text{apparent area of the hemisphere}}$$

$$= \frac{\iint_{\Omega} \cos \theta \, d\Omega}{\iint_{2\pi} \cos \theta \, d\Omega} \approx \frac{\Omega_{se}}{\pi}$$

$$= \frac{6.8 \times 10^{-5}}{\pi} = 2.16 \times 10^{-5}$$

$$T = T_2 = T_p \quad T = T_3$$

$$Q_{\text{sun-earth}} = f \sigma T_s^4$$

$$Q_1 = f \sigma T_s^4 + (1 - f) \sigma T_p^4 - \sigma T_3^4 \quad T_1 = [f T_s^4 + (1 - f) T_p^4]^{1/4}$$

$$= \sigma [f T_s^4 + (1 - f) T_p^4 - T_3^4] \quad g_1 = \sigma$$

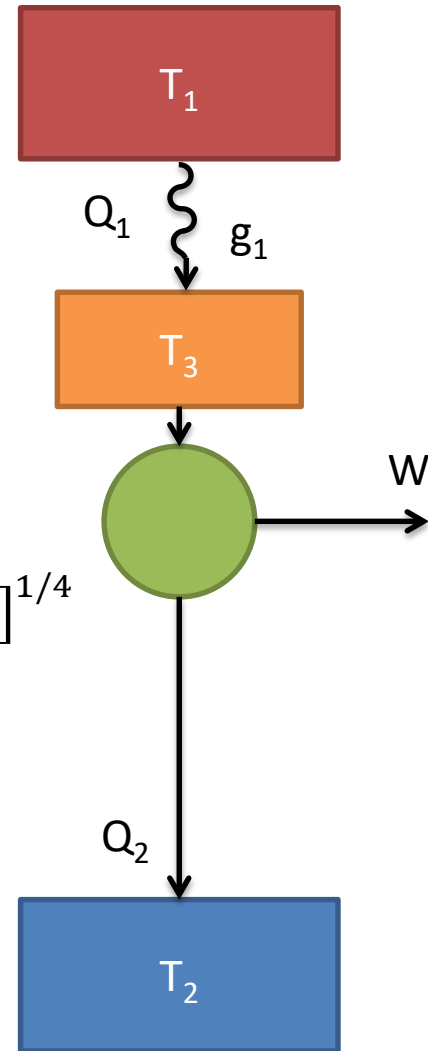
$$Q_1 = g_1 (T_1^4 - T_3^4)$$

The Müser engine efficiency:

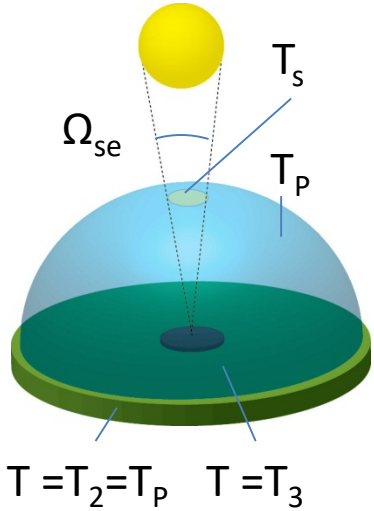
$$W = \eta_M Q_1$$

Solar energy efficiency:

$$W = \eta_{\text{solar}} f \sigma T_s^4$$



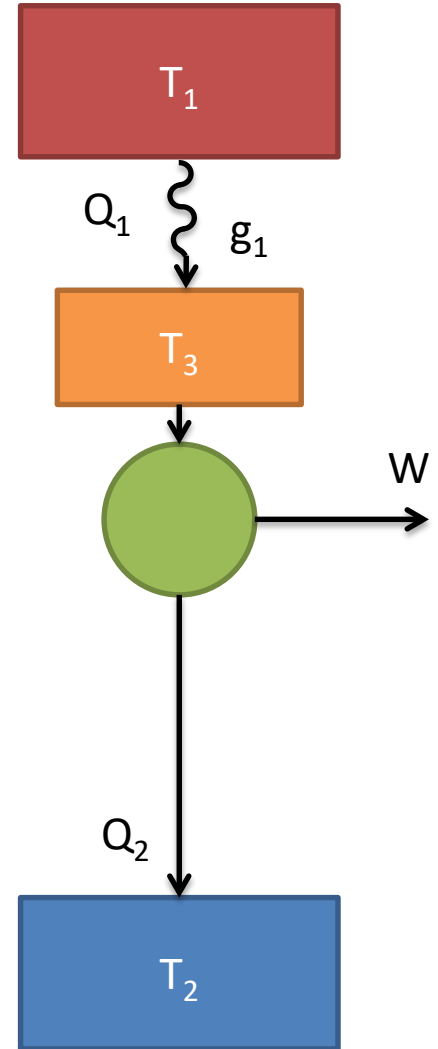
# The Müser Engine



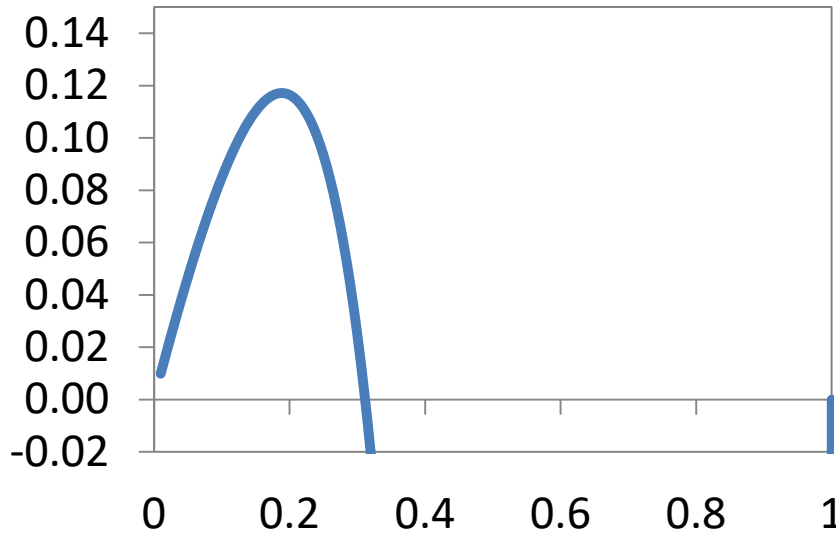
$$T_3 = \frac{T_p}{1 - \eta_M}$$

$$W = \eta_M \left[ f \sigma T_s^4 + \left( 1 - f - \frac{1}{(1 - \eta_M)^4} \right) \sigma T_p^4 \right]$$

$$\eta_{solar} = \eta_M \left[ 1 + \frac{(1 - f)(1 - \eta_M)^4 - 1}{(1 - \eta_M)^4} \frac{T_p^4}{f T_s^4} \right]$$



Solar energy conversion  
efficiency



Müser engine efficiency

# The Müser Engine on other planets

$$\eta_{solar} = \eta_M \left[ 1 + \frac{(1-f)(1-\eta_M)^4 - 1}{(1-\eta_M)^4} \frac{T_p^4}{fT_s^4} \right]$$

A first approximation:

$$T_p^4 = \frac{f}{4} T_s^4$$

Planet	Avg. distance (Gm)	$f$	$S$ (W m <sup>-2</sup> )	$T_p$ (K)	Max. $\eta_{solar}$ (%)
Mercury	57	$1.48 \times 10^{-4}$	9228	440	13.5
Venus	108	$4.14 \times 10^{-5}$	2586	733	0.95
Earth	150	$2.16 \times 10^{-5}$	1353	288	11.8
Mars	227	$9.37 \times 10^{-6}$	586	220	13.6
Jupiter	778	$8.01 \times 10^{-7}$	50	124	12.3
Saturn	1426	$2.38 \times 10^{-7}$	15	95	11.3
Uranus	2868	$5.89 \times 10^{-8}$	4	59	15.2
Neptune	4497	$2.40 \times 10^{-8}$	2	59	8.75

# Concentrating solar irradiation

Concentration factor:

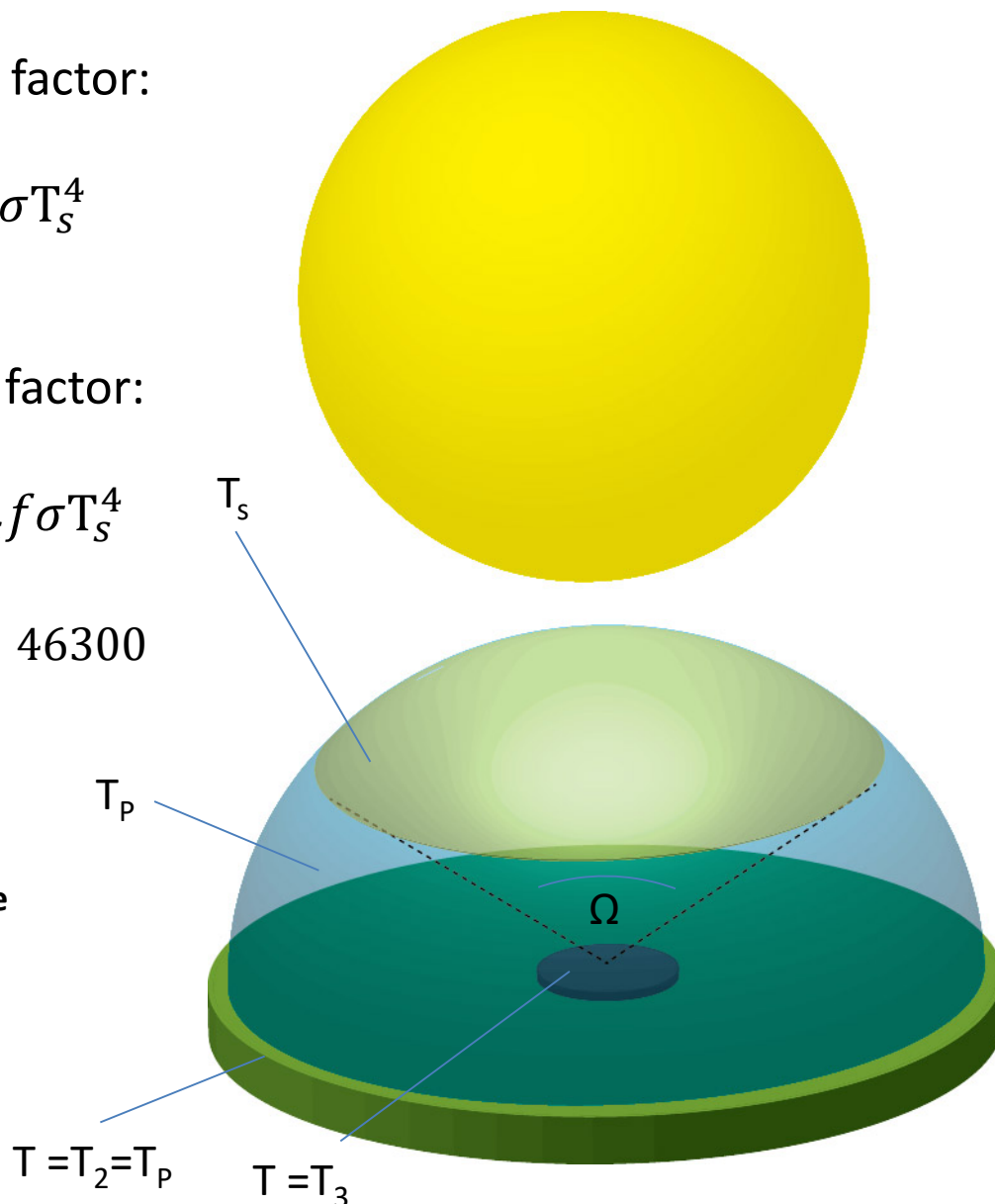
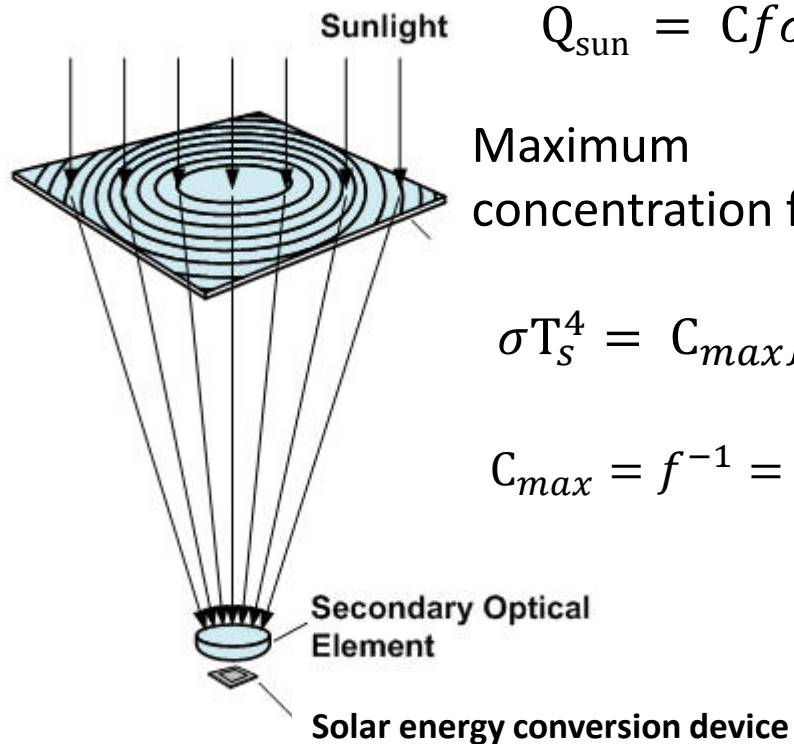
$$C > 1$$

$$Q_{\text{sun}} = C f \sigma T_s^4$$

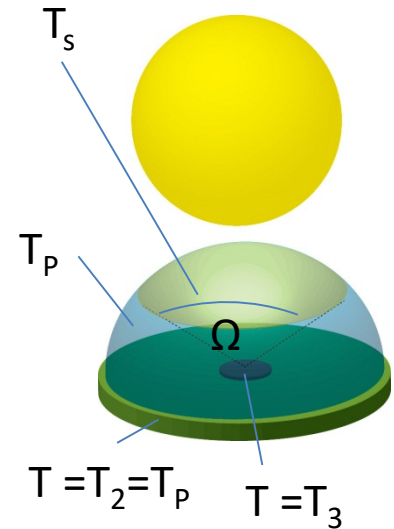
Maximum  
concentration factor:

$$\sigma T_s^4 = C_{\text{max}} f \sigma T_s^4$$

$$C_{\text{max}} = f^{-1} = 46300$$



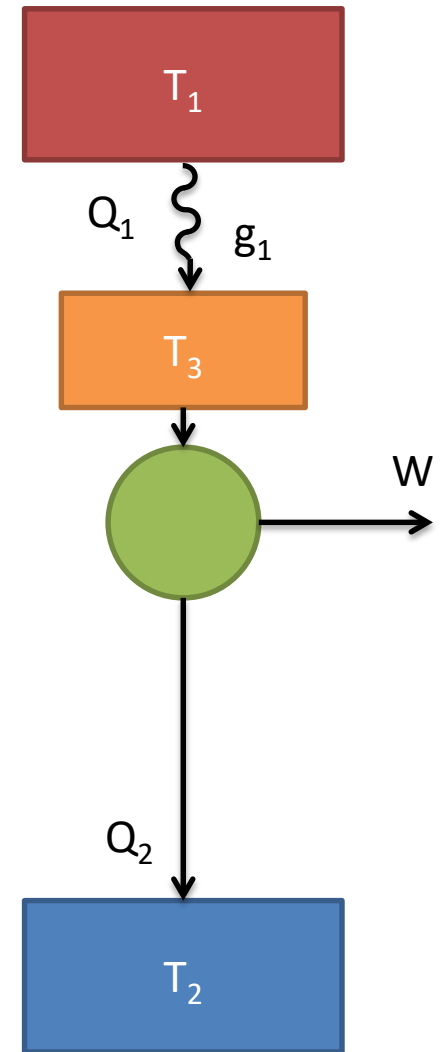
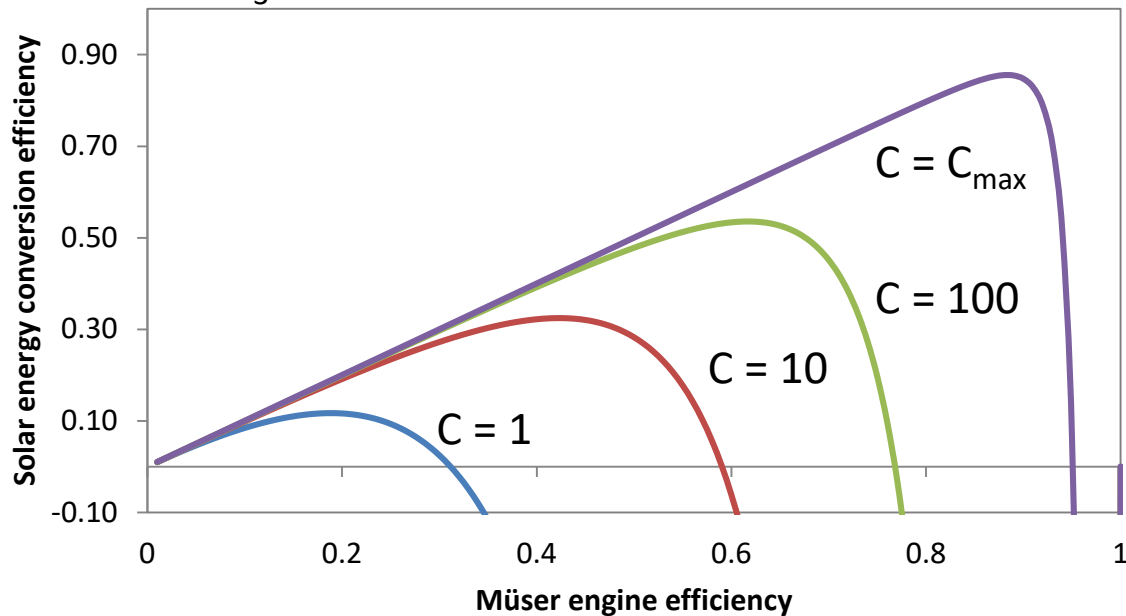
# The Müser Engine with a concentrator



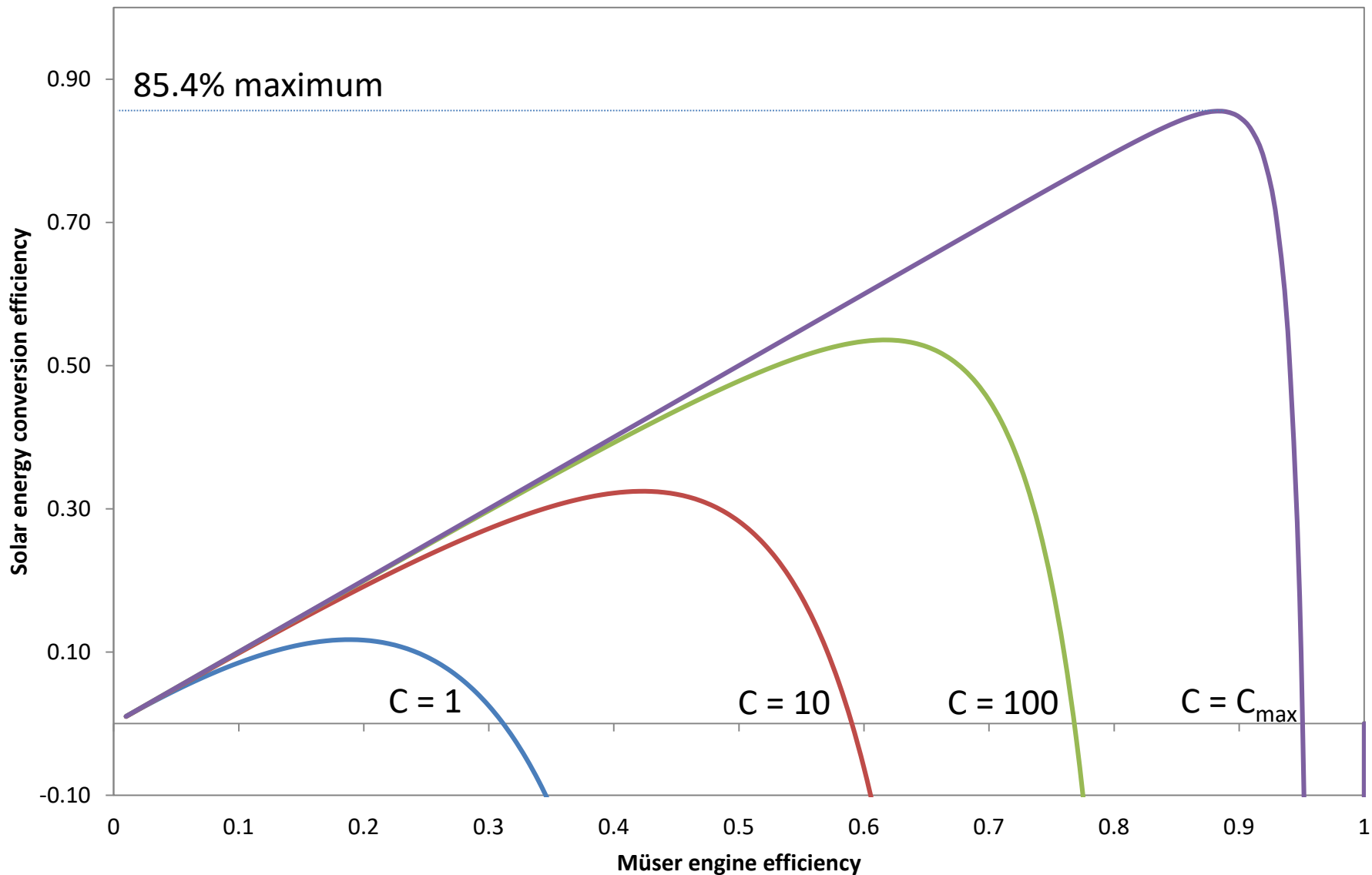
$$Q_1 = \sigma [CfT_s^4 + (1 - Cf)T_p^4 - T_3^4]$$

Solar energy efficiency:

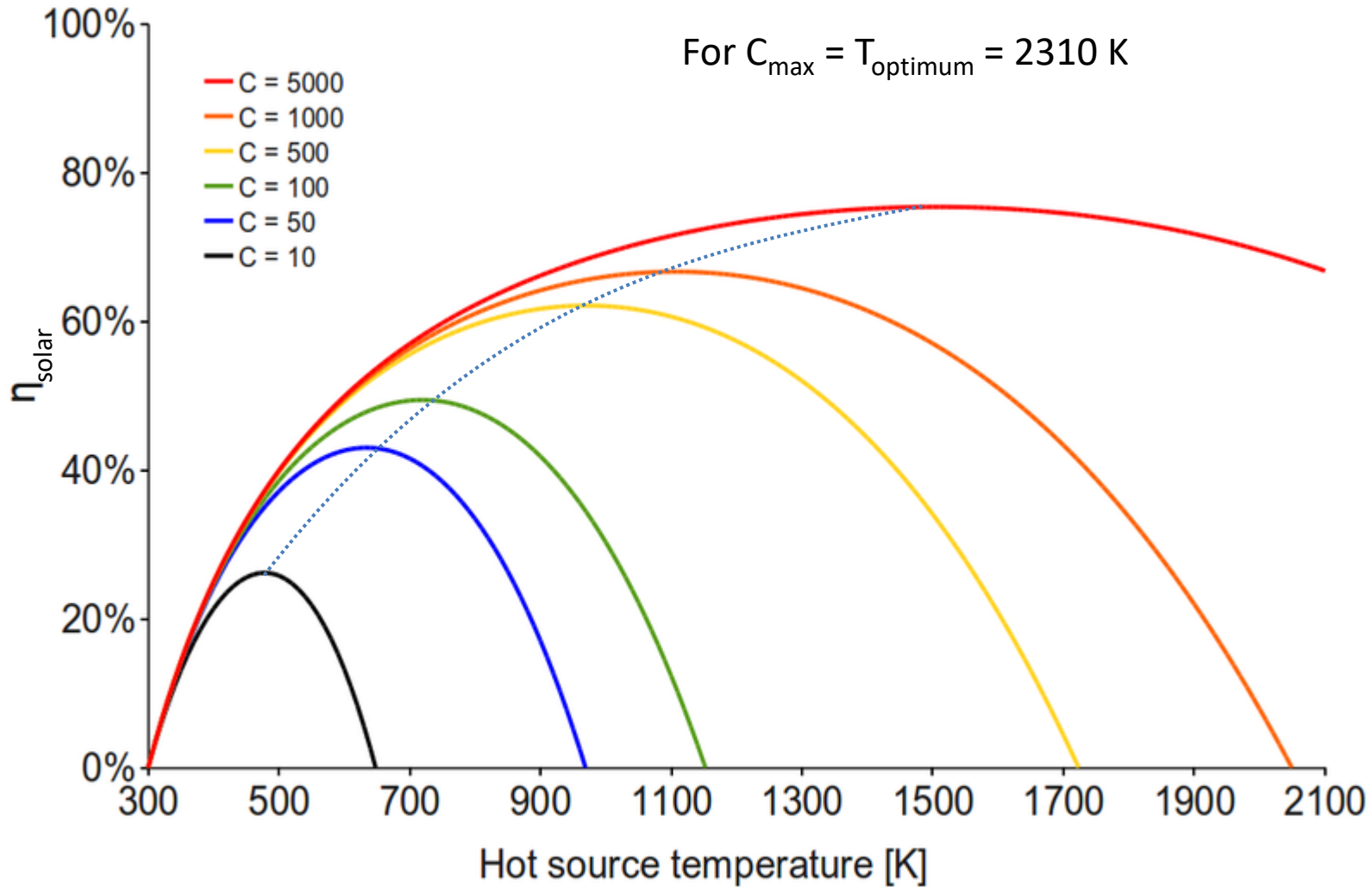
$$W = \eta_{solar} Cf \sigma T_s^4$$



# The Müser Engine with a concentrator



# $\eta_{\text{solar}}(T_3)$ for different C





# The Müser Engine with concentrator on other planets

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Planet	Avg. distance (Gm)	f	S (W m <sup>-2</sup> )	T <sub>p</sub> (K)	Max. $\eta_{solar}$ (%) With conc.
Mercury	57	$1.48 \times 10^{-4}$	9228	440	79.7
Venus	108	$4.14 \times 10^{-5}$	2586	733	75.0
Earth	150	$2.16 \times 10^{-5}$	1353	288	85.4
Mars	227	$9.37 \times 10^{-6}$	586	220	88.1
Jupiter	778	$8.01 \times 10^{-7}$	50	124	92.5
Saturn	1426	$2.38 \times 10^{-7}$	15	95	93.9
Uranus	2868	$5.89 \times 10^{-8}$	4	59	95.6
Neptune	4497	$2.40 \times 10^{-8}$	2	59	95.7
Pluto	5874	$1.18 \times 10^{-8}$	2	44	96.6

# Summary

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- The Curzon-Ahlborn engine provides the framework for energy conversion with heat transfer
- The Stefan-Boltzmann engine uses radiative heat transfer and shows that the highest efficiency can be gained if the thermal transfer to the cold sink is infinitely good
- The Müser engine approximates this and reveals that only about 12% solar energy conversion efficiency is possible even with a Carnot cycle.
- Concentration allows the maximum efficiency of 85.4%