11: Echo formation and spatial encoding

1. What makes the magnetic resonance signal spatially dependent ?
2. How is the position of an MR signal identified ?
   - Slice selection
3. What is echo formation and how is it achieved ?
   - Echo formation
   - Gradient echo sequence
4. How is a two-dimensional MR image encoded ?

After this course you
1. Understand the principle of slice selection
2. Are familiar with dephasing and rephasing of transverse magnetization and how it leads to echo formation
3. Understand the principle of spatial encoding in MRI
4. Can describe the basic imaging sequence and the three necessary elements
5. Understand the principle of image formation in MRI and how it impacts spatial resolution

11-1. What do we know about magnetic resonance so far ?

Adding a 3rd magnetic field

So far
1) Excite spins using RF field at ω_L
2) Record time signal (Known as FID)
3) M_{xy} decays, M_z grows (T_2 and T_1 relaxation)

RF coils measure signal from **entire** body (no spatial information)

Precessional Frequency $\omega_L = \gamma B_0$ Static Magnetic Field

How to encode spatial position ?

$B(x) = B_0 + G_x x$ → ω_L=f(x)

Magnetic field B along z varies spatially with x, y, and/or z:

$$\vec{G} \equiv \frac{dB_z}{d\vec{r}}$$

B_0: Static Magnetic Field
- Creates equilibrium magnetization
- 0.1 T to 12 T
  - » Earth's field is 0.5 10^{-4} T

B_1: Radiofrequency Field (RF)
- 0.05mT, on resonance
- Detection of MR signal (RF coils)

$$B_z(\vec{r}) = B_0 + \vec{G} \cdot \vec{r}$$

e.g. **G**=(G_x,*0*,*0*)

How is the gradient field created ?

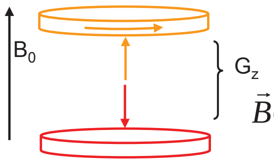
One coil for each spatial dimension: G_x, G_y, G_z

G: Gradient Field $\vec{G} \equiv \frac{dB_z}{d\vec{r}}$

10-50 mT/m in ~100µs

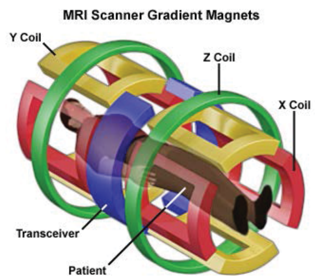
Used to determine spatial position of signal (frequency)

Example: z-gradient coil principle (Helmholtz pair)



$$\vec{B}(\vec{r}) \propto \int \frac{\mu_0}{4\pi} I \frac{d\vec{l} \times \vec{r}}{r^3}$$

Created by a set of 3 additional coils (gradient coil)



NB. Why are MRI scans so loud ?

Lorentz-force of B_z (3T) on rapidly switched current in gradient coil (wire)

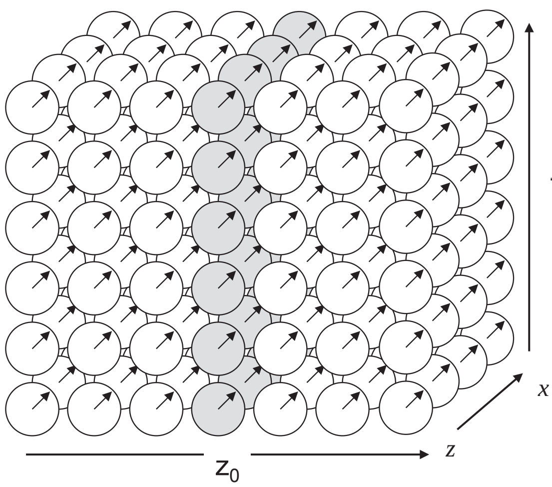
(~100A in ~100µs)

How is slice-selection achieved ?

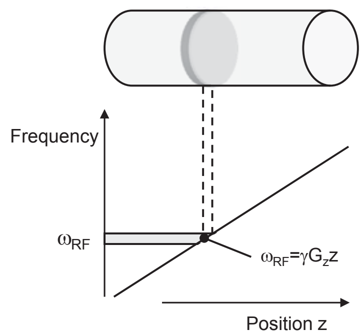
Only magnetization on-resonance is excited

On-resonance:
Frequency ω_{RF} of RF field B_1 matches the precession frequency of magnetization

Moving Frequency ω_{RF} alters position of slice :



$\omega_{RF}=\gamma B_0+\gamma G_z z_0$



NB. Not to confuse:

(x,y) refers to spatial dimensions

M_{xy} M or $M_\perp$ refers to transverse magnetization (in magnetization space)

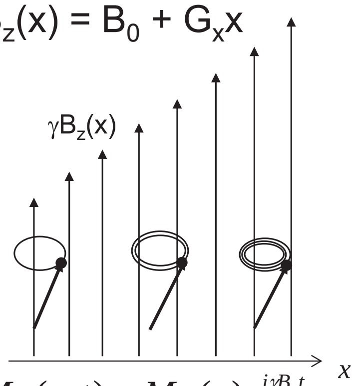
(coordinate systems are different, but share z)

11-2. What is the basic principle of encoding spatial information ?

frequency encoding - 1D example

Spatial-varying resonance frequency $\gamma B(x)$ during detection

$B_z(x) = B_0 + G_x x$



$$M_\perp(x,t) = M_\perp(x) e^{i\gamma B_z t}$$

$$M_\perp(x) e^{i\gamma B_0 t + i\gamma G_x x t}$$

Rotating frame: $B(x) = G_x x$

$$M_\perp(x) e^{i\gamma G_x x t}$$

Detected signal = sum of all precessing magnetization:

$$S(t) \propto \int_{object} M_\perp(x,0) e^{i\gamma G_x x t} dx$$

What does this resemble ?

$$S(t) \propto \int_{object} M_\perp(\omega,0) e^{i\omega t} d\omega$$

= Inverse Fourier Transformation !

$$S(k) \propto \int_{object} M_\perp(x,0) e^{ikx} dx$$

FT of S(t) (or S(k)) → M(x)

Reconstruction as in CT (in principle)

For 2D object:

$$M(x) = \int M(x,y) dy$$

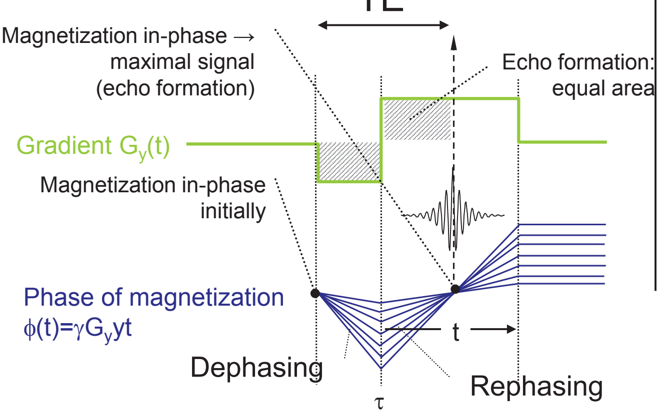
=Radon Transform

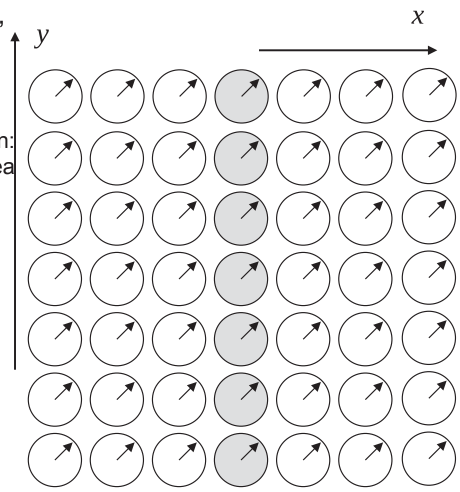
11-3. When is the signal maximal in the presence of G ?

Echo formation: Dephasing and rephasing

$$S(t) \propto \int_{object} M(y,0) e^{i\gamma G_y y t} dy$$

S(t)<S(0) → "**dephasing**"





τ=TE/2:

$$S(\tau) = \int_{object} M(y,0) e^{-i\gamma G_y y \tau} dy$$

$$S(t) = \int_{object} M(y,\tau) e^{i\gamma G_y y t} dy$$

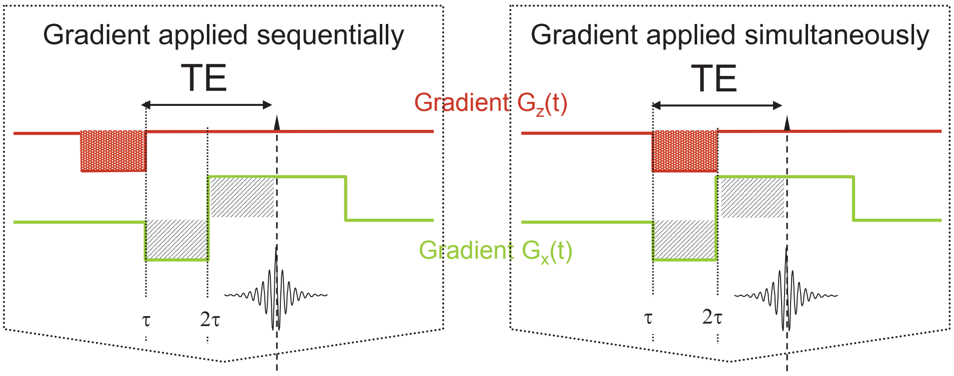
$$S(t) = \int_{object} M(y,0) e^{-i\gamma G_y y \tau} e^{i\gamma G_y y t} dy$$

$$e^{i\gamma G_y y (t-\tau)}$$

S(t)=maximal (constant G_y) when $t=\tau$: echo formation

Is it important when a gradient is applied ?

gradient applied at different time has the same effect on magnetization phase



Question: Is there a difference in effect on echo ?

$$M(\tau) = M(0)e^{-i\gamma G_z z\tau}$$

$$M(2\tau) = M(\tau)e^{-i\gamma G_x x\tau} = M(0)e^{-i\gamma G_x x\tau}e^{-i\gamma G_z z\tau}$$

Application of two orthogonal gradients simultaneously or sequentially generates the same phase for M_{xy}

$$\gamma(\vec{G}\cdot\vec{r})\tau$$

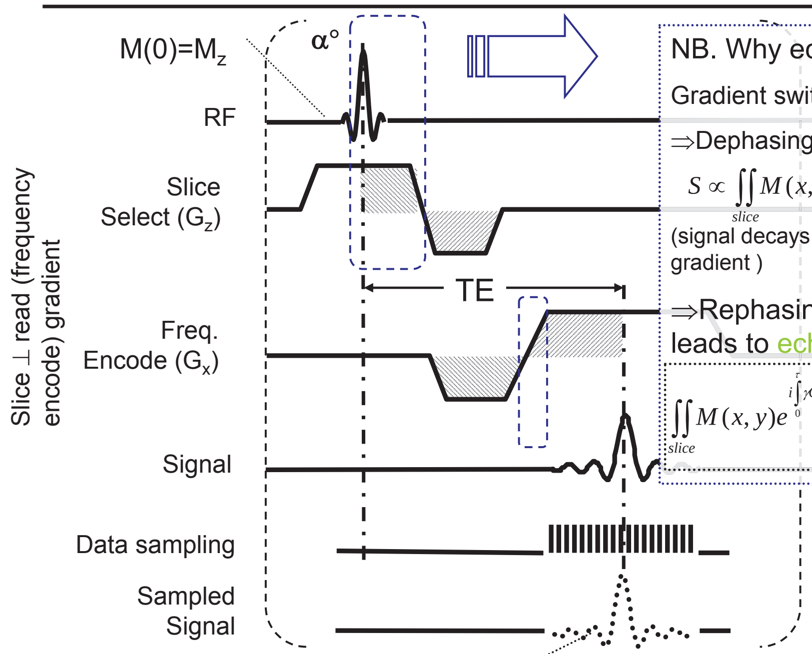
$$M(\tau) = M(0)$$

$$M(2\tau) = M(\tau)e^{-i\gamma(G_x x + G_z z)\tau} = M(0)e^{-i\gamma G_x x\tau}e^{-i\gamma G_z z\tau}$$

$$M(TE) = M(2\tau)e^{i\gamma G_x xTE/2}$$

$$M(TE) = M(0)e^{-i\gamma(G_x x+G_z z)\tau}e^{i\gamma G_x xTE/2} = M(0)e^{-i\gamma G_z z\tau}$$

What are the basic elements of the Gradient echo sequence ?



First half of echo also measured (more signal)

NB. Why echo formation ?

Gradient switching → Finite rise time

⇒Dephasing of magnetization

$$S \propto \iint_{slice} M(x,y)e^{i\gamma G_z z\tau}dz \xrightarrow[time]{} 0$$

(signal decays like FID in presence of gradient)

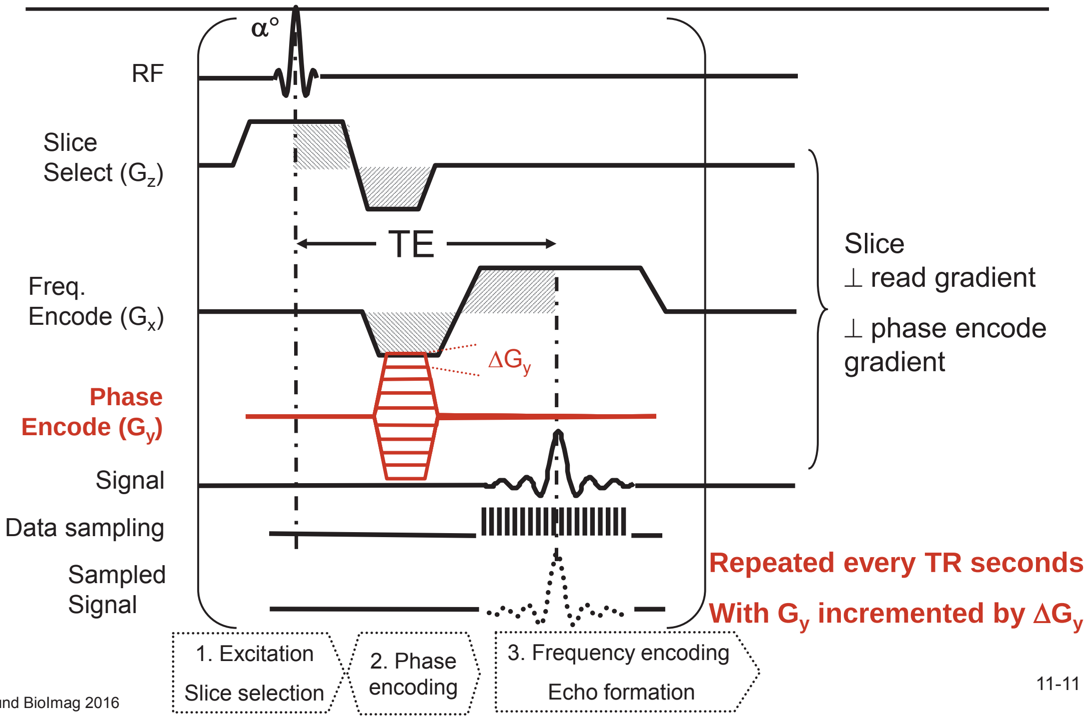
⇒Rephasing (negative) gradient leads to echo formation

$$\iint_{slice} M(x,y)e^{i\int_0^\tau \gamma G_z(t)zdt}dz = \max \quad \int_0^\tau G_z(t)dt = 0$$

(at t=τ)

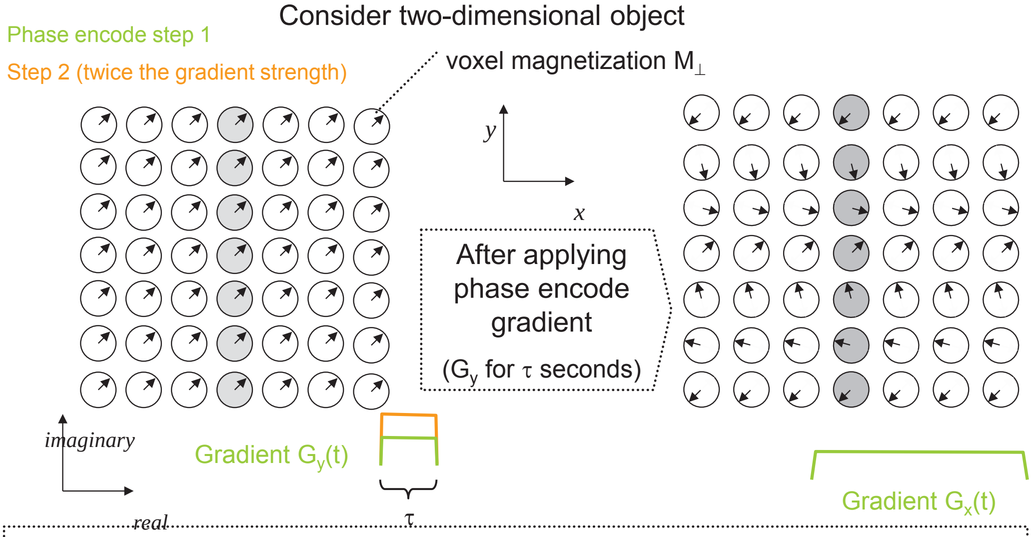
11-4. How is the 2nd dimension encoded ?

gradient echo imaging sequence



How does the phase encoding gradient encode the 2nd spatial dimension ?

Consider two-dimensional object

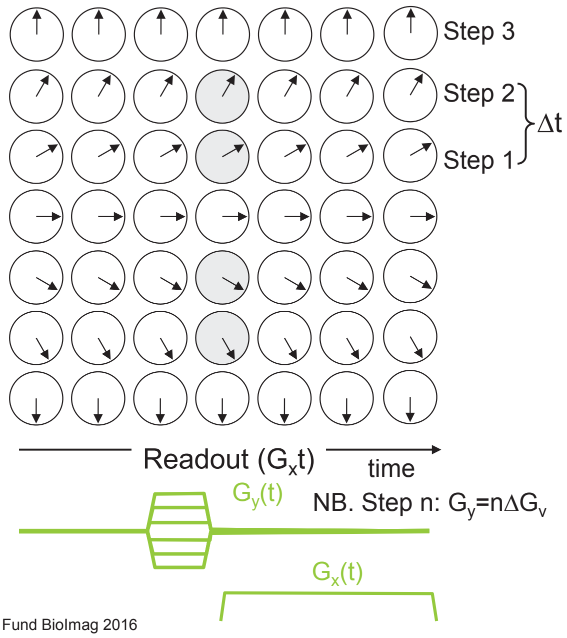


Phase of voxel magnetization $e^{i\phi}$: $e^{i\gamma G_y y \tau}$

$$\underbrace{e^{i\gamma G_y y \tau} \quad e^{i\gamma G_x x t}}_{e^{ik_x x} e^{ik_y y} = e^{i(k_x x + k_y y)}}$$

How is incrementing the phase step-by-step (phase encoding) equivalent to frequency encoding ?

Phase ϕ of a single pixel in x,y plane:



$$\omega \equiv \frac{d\phi}{dt}$$

Signal of the single voxel:

$$S(n,\Delta G_y,t) \propto M_\perp(x,y)e^{i\gamma n\Delta G_y y\tau}e^{i\gamma G_x xt}$$

$k_x \equiv \gamma G_x t$, $k_y \equiv \gamma G_y \tau$

$$S(k_x,k_y) \propto M_\perp(x,y)e^{i(k_x x+k_y y)}$$

Signal of the entire object :

$$S(k_x,k_y) \propto \iint_{object} M_\perp(x,y)e^{i(k_x x+k_y y)}dxdy$$

What does this resemble ?

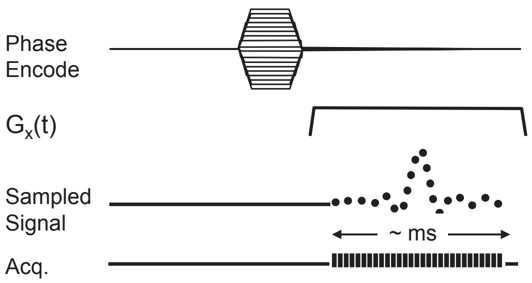
$$M_\perp(x,y) \propto \iint_{object} S(k_x,k_y)e^{-i(k_x x+k_y y)}dk_x dk_y$$

MR image generation:

FT of the signal

11-5. How is the spatial information encoded in MRI ?

scanning k-space (Fourier or reciprocal space) sequentially

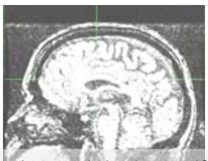


For k-space line every TR=1s:
256^2 image matrix > 4 min

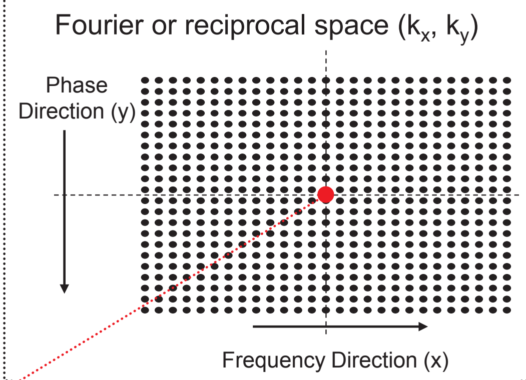
One line of k-space acquired per TR

MR scans are long and motion-sensitive

Subject moved head during acquisition



Ghosting and ringing artifacts



Maximum k_x (or k_y) ⇔ Resolution (Nyquist)

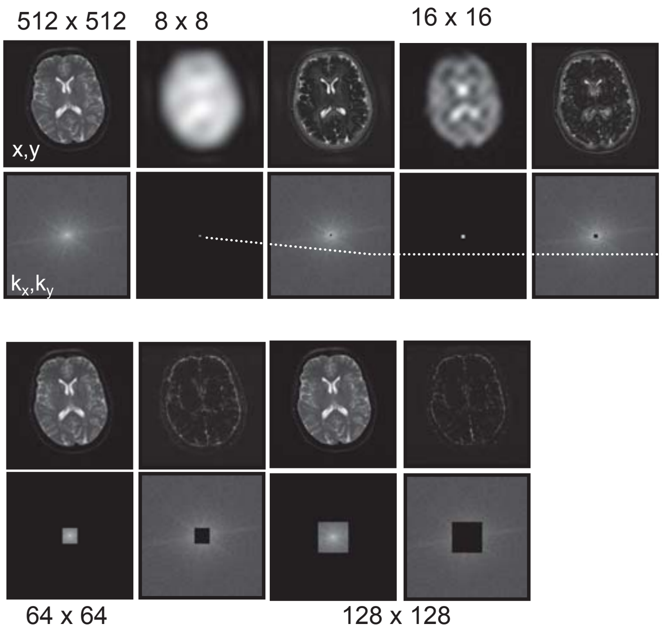
Increment Δk ⇔ Field-of-view

Uniform resolution and sensitivity (Limited by voxel magnetization)

center of k-space (k_x,k_y=0)

What are some effects of incomplete sampling ?

of Fourier space (k-space)

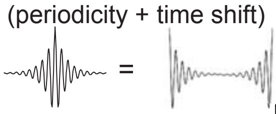


Time of acquisition of center of k-space point (k_x,k_y=0) determines contrast of image:

$$(0,0) = \iint_{object} M_{\perp}(x,y)\,dxdy \Big|_{k_x=0,\ k_y=0}$$

Discrete FFT

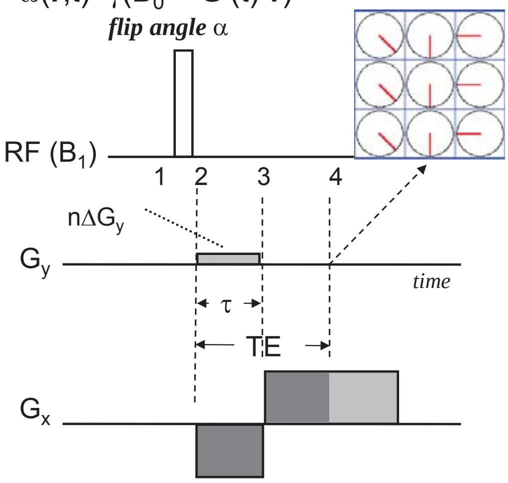
(periodicity + time shift)



Summary: Spatial encoding with gradients

Phase encoding, echo formation + 2DFT

$\omega(\mathbf{r},t)=\gamma(B_0 + \mathbf{G}(t)\cdot\mathbf{r})$



Magnetization at time points specified:

1: $(0,0,M_z)$

rotated by RF pulse by α^0 about x:

2: $(0,M_z\sin\alpha,M_z\cos\alpha)\equiv(0,M_y,\ldots)$

[now only consider M_{xy}]

Precesses with B = $-\gamma G_y y$ $-\gamma G_x x$

3: $M_y(\sin[-\gamma(n\Delta G_y y+G_x x)\tau], \cos[-\gamma(n\Delta G_y y+G_x x)\tau]$

$\equiv M_y\ [\sin(-\phi_x), \cos((-\phi_x)]$,

with $\phi_x = \gamma G_x x\tau$ (rotation by angle $-\phi_x$)

inverting gradient, i.e. B = $+\gamma G_x x$:

after another τ, **rotates by angle $+\phi_x$**

$\Rightarrow$ maximal signal at TE=2τ (ΔGy=0)

4: $M_y\ (0,1,\ldots) = (0,M_z\sin\alpha,M_z\cos\alpha)$

→ ***Echo formation***

Signal $S(\tau,t) \propto M_{\perp}(t) = \int\int m(x,y,t)dxdy$

$= \int\int m(x,y,0)\ e^{\gamma(n\Delta G y y)\tau} e^{\gamma G x x(\tau+t)} dxdy$

$S(k_x,k_y) \propto \int\int m_{\perp}(x,y)\ e^{-k_x x} e^{-k_y y} dxdy$

MRI measures the ***2D Fourier transformation*** of the object

(measuring the 2nd dimension requires time!)