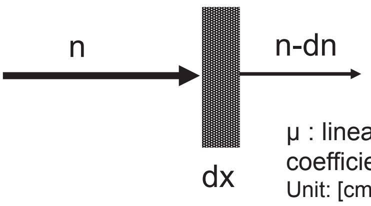
4: From x-ray to image – Computed Tomography

1. What factors influence contrast in x-ray imaging ?
   - Beam hardening
   - Sensitivity and resolution considerations
2. What influences CNR of x-ray imaging ?
3. What is the fundamental basis for image reconstruction using x-ray absorption ?
   - Radon Transform
4. How can x-ray images be reconstructed?
   - Sinogram
   - Backprojection vs. filtered backprojection
   - Central Slice Theorem
5. Examples & Summary

After this course you
1. Understand the consequences of the Bremsstrahlung continuum on image contrast
2. Understand how Compton scattering reduces image contrast and how its influence can be reduced
3. Are familiar with the Radon transform
4. Understand the principle of matrix reconstruction and backprojection
5. Understand the major mechanisms leading to CT contrast

4-1. What does absorption in the real world imply ?

Linear attenuation coefficient μ



μ : linear attenuation coefficient
Unit: [cm^{-1}]

If μ is constant in x

$$n(x) = N_0 e^{-\mu x}$$

The measurement that is **wanted**:

$\mu(x,y)$

What is **measured**: n(x)

$$\ln\left(\frac{n(x)}{N_0}\right) = -\mu x$$

(μ for a homogeneous object of thickness x)

Contrast is "well-defined" for monochromatic x-rays

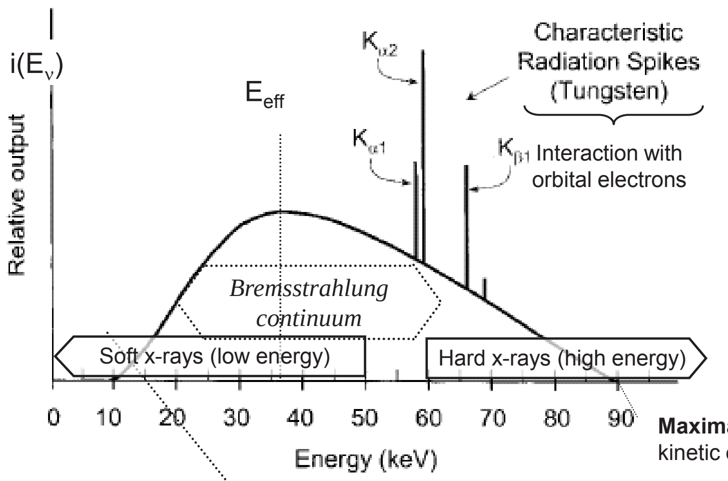
But, $\mu = f(E_\nu, Z, \rho)$

Two consequences:

Beam hardening

Depth dependent contrast

What does the Energy Spectrum of an x-ray tube really look like ?
filtered Bremsstrahlung and characteristic emission



$$I_0 = \int_0^\infty i(E_\nu)dE_\nu$$

$i(E_\nu)$: complex function

Define effective photon energy E_{eff}

Maximal x-ray energy is = kinetic energy of e^- ($eV_{cathode}$)

Minimal energy is zero, but:
Soft x-rays (low energy) are filtered by instrument

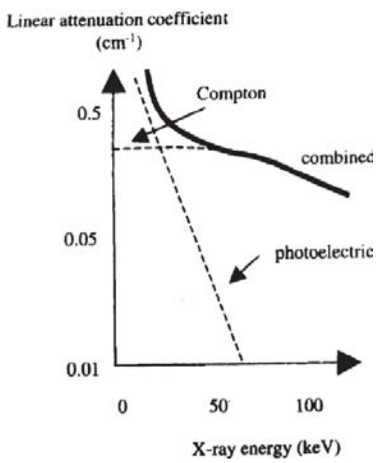
E_{eff} is increased by instrument (filtering of soft x-rays)

What is the consequence of energy-dependent absorption ?
Beam Hardening - Effective energy depends on depth

A similar consequence arises in tissue:

Ideal:
Monochromatic x-rays
($E_\nu(\lambda) = \delta(\lambda_0)$)

Reality:
Polychromatic, multienergetic $i(E_\nu)$

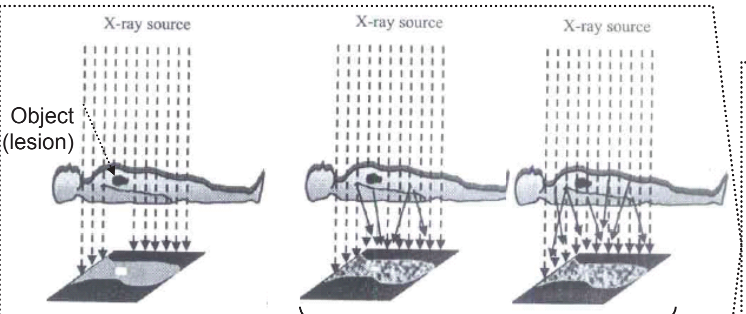


Absorption is not uniform with E_ν
- Contrast changes with large objects and depth
- Excessive radiation dose to superficial tissue

"Solution": Reduce $i(E_\nu)$ for soft x-rays

(e.g. 3mm Al eliminates 90% of 20keV photons)

4-2. How does x-ray scattering impact CNR ?

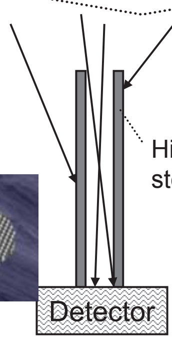


Photoelectric effect: complete absorption of photons → Object (lesion) easily detected

With E_ν ↑: Compton scattering → increased masking of object.

Scattering increases with FOV (field-of-view) of irradiation
⇒ Further reduction of image contrast

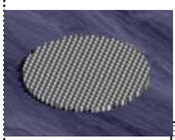
Solution: Anti-scatter grid (collimator)



High Z, ρ: stop x-rays

Slice selection

Collimation principle: establishes directionality of x-rays



How is CNR quantified ?

Signal:

∝ I(d) (no. photons detected)

∝ I_0 (no. photons irradiated)

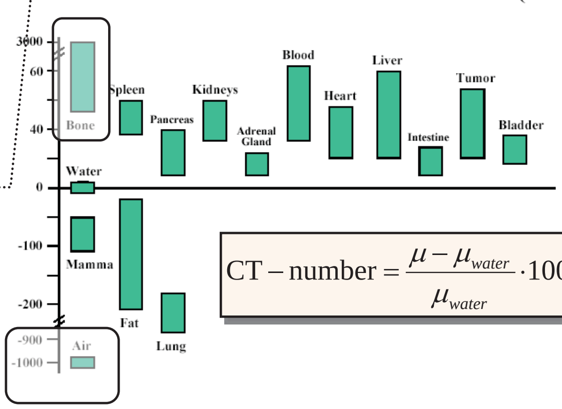
Contrast: ΔI(d) due to μ(d) differences

- $i(E_\nu)$, μ_C produces reduced contrast
- Compton scattering: Antiscatter grid

CT intensity can be measured in absolute terms (CT-number)

Soft tissue: Typically has weak contrast (small **Hounsfield units**)

CT-numbers of tissue in Hounsfield units (HU)

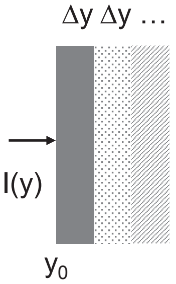


$$\text{CT-number} = \frac{\mu - \mu_{water}}{\mu_{water}} \cdot 1000$$

HU : attenuation normalized to water (≡0)
range from -1000 (air) to +3000 (bone and contrast agents)
soft tissues: -300 to +100

4-3. What is the basis of image reconstruction ?

The Radon transform



Given a certain beam intensity (no. of photons) I at a given position y, $I(y_0)$, the beam intensity at y+Δy is

$$I(y_0+\Delta y)=I(y_0)e^{-\mu(y_0)\Delta y}$$

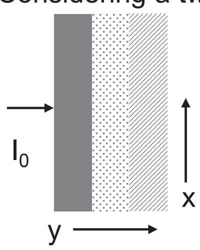
Recursive application to derive $I(y_0+2\Delta y)$

$$I(y_0+2\Delta y)=I(y_0)e^{-(\mu(y_0+\Delta y)\Delta y+\mu(y_0)\Delta y)}$$

lim Δy→0

$$I_{detected}=I(y_0)e^{-\int_{source}^{detector}\mu(y')dy'}$$

Considering a two-dimensional object:



$$I_{detected}(x)=I_0e^{-\int_{source}^{detector}\mu(x,y)dy}$$

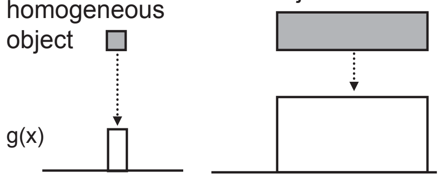
I_0: intensity of incident x-ray beam

Radon transform g(x)

Definition $$g(x)=\int_{\infty}^{-\infty}\mu(x,y')dy'$$

Radon transform of a point-like homogeneous object

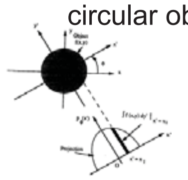
rectangular object



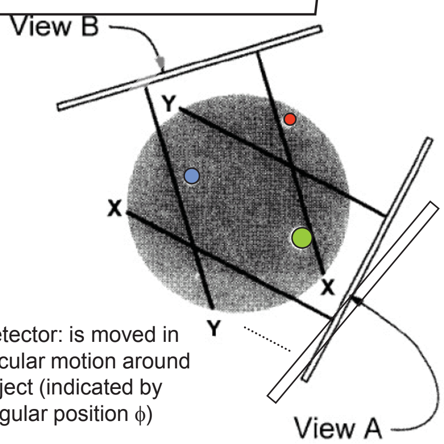
Does each pixel have a unique trajectory ?

Sinogram

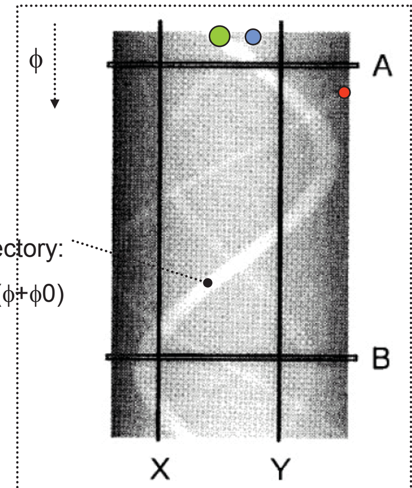
Radon transform of a circular object



Radon transform = projection of object



Detector: is moved in circular motion around object (indicated by angular position ϕ)

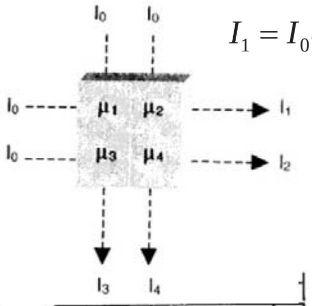


Each point in space is uniquely represented by Amplitude R and phase ϕ_0 of sinusoidal trajectory in **Sinogram** (sic!): (x,y) → (R,ϕ_0)

Can a CT image be constructed by Matrix inversion ?

Decomposing an object into a 2x2 matrix requires a minimum of 4 measurements:

$$I_1 = I_0 e^{-(\mu_1 \Delta x + \mu_2 \Delta x)}$$



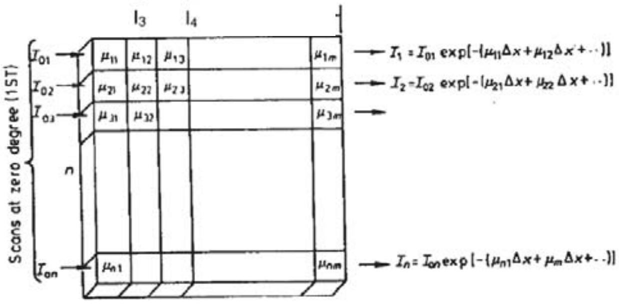
$\ln(I_1/I_0) = -\mu_1\Delta x - \mu_2 \Delta x$

$\ln(I_2/I_0) = -\mu_3\Delta x - \mu_4 \Delta x$

$\ln(I_3/I_0) = -\mu_1\Delta y - \mu_3 \Delta y$

$\ln(I_4/I_0) = -\mu_2\Delta y - \mu_4 \Delta y$

Setting $\Delta x = \Delta y$ yields a linear 2x2 inversion problem linking μ_k to I_k



In principle such an n^2 matrix can be inverted.

⇒ Too complex, computationally intensive and unstable

CT was introduced in 1970 ⇒ simple reconstruction algorithm!

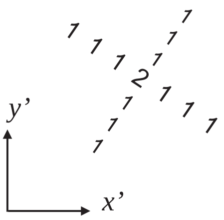
4-4. What algorithm is adapted to 1970's computing power ?

Backprojection reconstruction

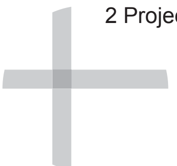
Basic reconstruction principle: Along the measured projection direction fill in each pixel constant numbers corresponding to the Radon transform (projection intensity).

Repeat for next orientation of the projection, sum the values in overlapping pixels.

Illustration with gray shades (point-like object):



2 Projections

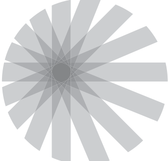


4 Projections

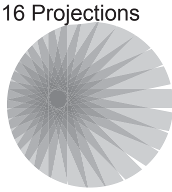
$$R_\theta(x') = \int_{-\infty}^{+\infty} f(x'\cos\theta - y'\sin\theta, x'\sin\theta + y'\cos\theta)\,dy'$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

8 Projections

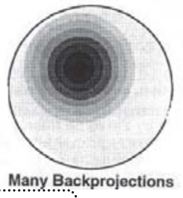


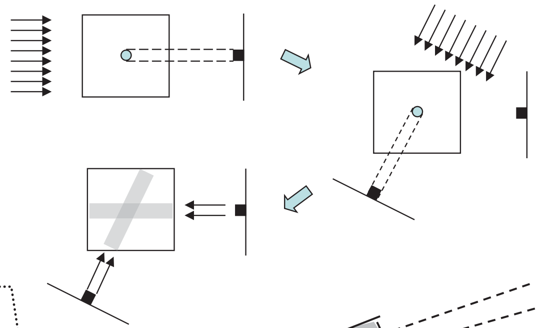
16 Projections



Why does simple Backprojection have poor spatial resolution ?

Backprojection has poor spatial resolution:





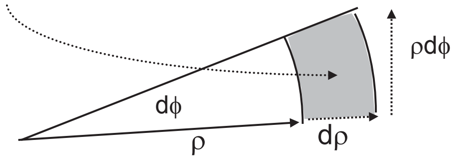
Reconstruction of a point-object falls of with 1/r

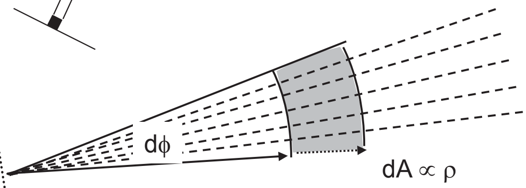
WHY?

The reconstruction falls off with 1/r

(in analogy to the decrease of light intensity in 2D)

$dxdy = dA = \rho \sin(d\phi) d\rho \cong \rho d\phi d\rho$





Number of rays (projections): constant with $d\phi$

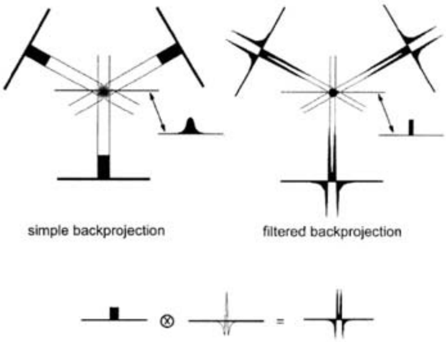
But:

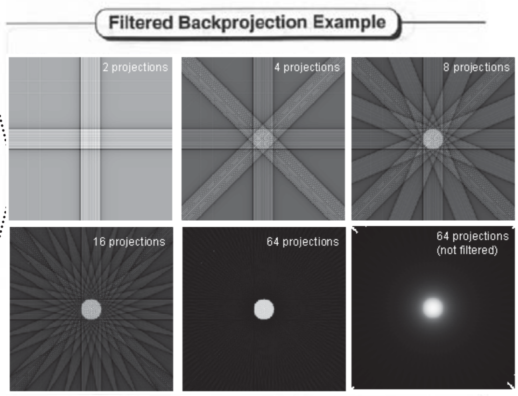
pixel size = $dxdy \propto dA$ = const

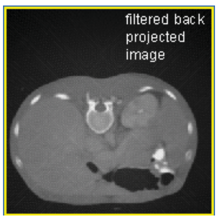
$\Rightarrow$ No. of rays $\propto 1/\rho$

How can good image resolution be maintained ?

Filtered Backprojection

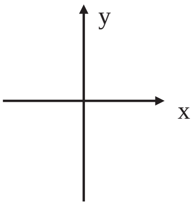






How is Backprojection linked to Fourier transform ?
Central Slice Theorem

Image space



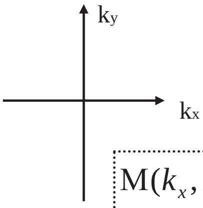
$\mu(x,y)$

FT

IFT

Final Image
(CT: acquired data)

k-space (Fourier space)



$$\mathrm{M}(k_x,k_y)=\iint \mu(x,y)e^{-ik_x x}e^{-ik_y y}dxdy$$

MRI: Acquired Data

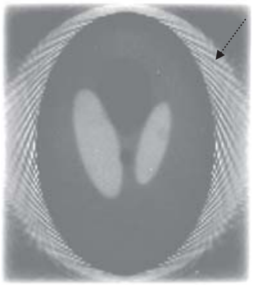
$$M(k_x,k_y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mu(x,y)e^{-2\pi i(xk_x+yk_y)}dxdy$$

k_x=0

$$M(0,k_y)=\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty}\mu(x,y)dx\right]e^{-2\pi iyk_y}dy$$

FT of projection → 1 line in k-space

⇒ Reconstruction using FT :



artifacts

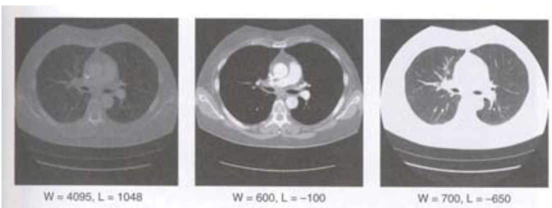
See also: Signals and Systems (SV)

4-5. X-ray CT : Examples (Human)

Bone (calcification) :
bright (high absorption)

Different densities of tissue give inter-mediate results

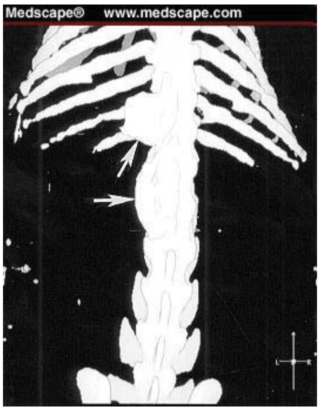
Air is dark

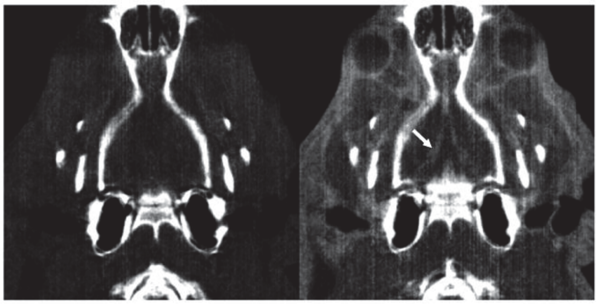


Imaging of mummified bodies

Dislodged arrow head

CT: Examples (mouse)



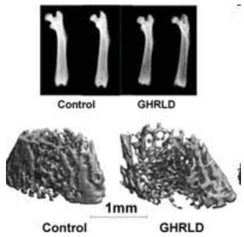


Pre-

Post-contrast

3D CT scan of rodent spine
- treated with human mesenchymal stem cells
  (transduced with the human BMP-9 gene via an adenoviral vector)
- significant bone formation at the treatment sites (arrows)

13µm micro CT of mouse placenta vasculature



Micro-CT of mouse femor bone

CT: Summary

Main **contrast** is bone vs. soft tissue (or air) (calcium content i.e. e^- density ρ)

Contrast agents (increase Z_{eff}) allow depiction of vessel architecture and lesions

SNR and CNR:

1. Intensity can be increased by cathode current
2. High spatial resolution possible
   (limited only by radiation dose in humans)

How have CT scanners evolved ?
Generations of CT scanners

First Generation
- Parallel beam design
- One/two detectors
- Translation/rotation

2nd Generation
- Small fan beam
- Translation/rotation
- Larger no. of detectors

3rd Generation
- Multiple detectors
- Large fan beam

4th Generation
- Detector ring
- Large fan beam